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Characterizing General-Relativistic Transonic Astrophysical Accretion in Kerr Black Holes

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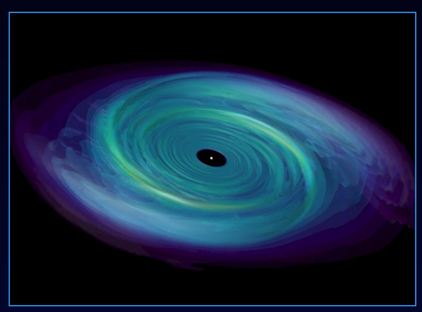
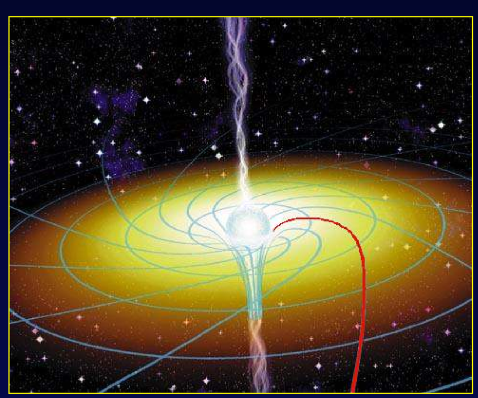
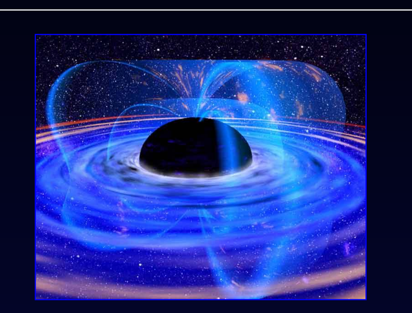


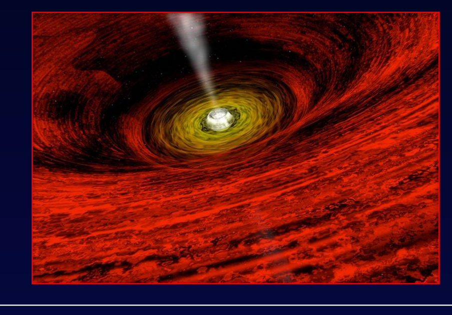


Abstract

We are investigating various aspects of transonic astrophysical accretion around Kerr Black Holes (BHs) to examine the dependence of accretion properties on BH spin. Here we consider general relativistic, multi-transonic, hydrodynamic advective accretion in the Kerr metric. Following standard accretion disk theory, we use the vertically integrated and averaged model to describe the thin flow of the BH accretion disk. We performed analytical and numerical calculations describing the behavior of some dynamic and thermodynamic properties of low angular momentum accreting matter very close to the Kerr BH event horizon, and analyzed how these properties depend on the BH spin. We could self-consistently discriminate between prograde and retrograde relativistic accretion by their different trends of terminal properties. This provides a potential theoretical framework for determining the spin of a BH, if one can observe some of the properties of the accreting matter, e.g. variation of temperature, pressure and Mach number with distance.

We examined the energy -- angular momentum -- adiabatic index -- black hole spin parameter space much more extensively, to classify the parameter space according to the occurrence of sets of critical points and their nature (single- or multi-sonic points, inner or outer, accretion or wind). Using these results we are studying the dependence of Analog Hawking radiation properties on the spin of the BH.





Introduction

- Mach number $M(r) = \frac{u(r)}{a_s(r)}$
- Accretion flow
  - Subsonic: $M < 1$
  - Supersonic: $M > 1$
  - Transonic: crosses $M = 1$
- Transition
  - Smooth -- Sonic point
  - Discontinuous – Shock
- Multi-transonic flow ≡ 3 sonic points

Motivation

- Transonic Flow is relevant in many astrophysical situations:
  - BH or Neutron Star accretion
  - Collapse / Explosion of stars
  - Solar / Stellar winds
  - Formation of protostars & galaxies
  - Interaction of supersonic galactic (or extragalactic) jets with ambient medium
- BH inner boundary condition : Supersonic flow at Event Horizon (EH)
- Far away from EH – subsonic
- Hence BH accretion is essentially transonic
  - Except cases where already supersonic initially
- Multi-transonic flow → Shock Formation

Goal

- Study variation of dynamic and thermodynamic properties of accretion flow very close to EH & their dependence on spin of BH
- Found: Higher possibility of shock formation in retrograde accretion flow (i.e. BH spin is opposite to rotation of accreting matter) as compared to prograde flow

What do we do?

- Formulate & solve conservation equations governing general relativistic, multi-transonic, advective accretion flow in Kerr metric
- Calculate behavior of accretion flow properties very close (≈ 0.01 r_g) to EH as function of BH spin
- Can be done at any length scale

Notations

$$r_g = \frac{GM_{BH}}{c^2}$$

- Gravitational Radius = r_g
- Black Hole Spin / Kerr Parameter = a

Describing Our System

- No self-gravity, no magnetic field
- Units: $G = c = M_{BH} = 1$
- Boyer-Lindquist coordinates (– + + +)
- Observer frame corotates with accreting fluid
- λ = Specific angular momentum of flow -- aligned with a
- Stationary & axisymmetric flow
- Euler & Continuity eqns $\nabla^{\mu}\mathfrak{I}^{\mu\nu} = 0 \quad , \quad (\rho v^{\mu})_{;\mu} = 0$
- Polytropic equation of state $p = K\rho^{\gamma}$
- K ~ specific entropy density
- γ = adiabatic index, n = polytropic index $n = \frac{1}{\gamma - 1}$
- Specific proper flow enthalpy, h
- Polytropic sound speed, a_s

$$a_s = \left(\frac{\partial p}{\partial \varepsilon}\right)_S^{1/2} = \Psi_1(p,r,\gamma) = \Psi_2(p,\rho,\gamma) \; ; \; a_s^2(r) = \frac{\gamma K_B T(r)}{\mu m_H}$$

- Frame dragging neglected
- Weak viscosity limit
- Very large radial velocity close to BH → Timescale (viscous >> infall)
- Effect of Viscosity → λ ↓ → Flow behavior as function of λ provides information on viscous transonic flow

Metric

- Kerr metric in equatorial plane of BH (Novikov & Thorne 1973)

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = -\frac{r^2\Delta}{A}dt^2 + \frac{A}{r^2}(d\phi - \omega dt)^2 + \frac{r^2}{\Delta}dr^2 + dz^2$$

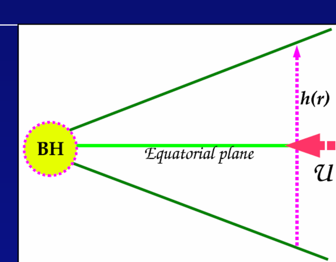
$$\Delta = r^2 - 2r + a^2 \qquad A = r^4 + r^2a^2 + 2ra^2 \qquad \omega = \frac{2ar}{A}$$

$$g_{tt} = \frac{A\omega^2}{r^2} - \frac{r^2\Delta}{A} \qquad g_{t\phi} = -\frac{A\omega}{r^2} \qquad g_{\phi\phi} = \frac{A}{r^2}$$

- Angular velocity: $\Omega = \frac{u^{\phi}}{u^t} = -\frac{(g_{t\phi} + \lambda g_{tt})}{(g_{\phi\phi} + \lambda g_{t\phi})}$
- Normalization: $v_{\mu}v^{\mu} = -1$
- 4th Component of velocity:

$$v_t = \left[\frac{\Delta}{(1-u^2)(1-\Omega\lambda)(g_{\phi\phi} + \lambda g_{t\phi})}\right]^{1/2}$$

Accretion Disk



- Vertically integrated model (Matsumoto et al. 1984)
  - Flow in hydrostatic equilibrium in transverse direction
  - Equations of motion apply to equatorial plane of BH
- Thermodynamic quantities vertically averaged over disk height
  - Consider quantities on equatorial plane
  - 1-dimensional quantities, vertically avg.

$$\bar{\rho} = \frac{\int_{-H/2}^{H/2}\rho(h)dh}{\int_{-H/2}^{H/2}dh}$$

- Disk height, *H(r)* (Abramowicz et al. 1997)

$$h_{disk}(r) = H(r) = r^2\left[\frac{2na_s^2}{(n+1)(1-na_s^2)\{\lambda^2 v_t^2 - a^2(v_t - 1)\}}\right]^{1/2}$$

Conserved Quantities

- Specific energy (including rest mass)

$$\varepsilon = hv_t = \left[\frac{\gamma - 1}{\gamma - (1 + \theta^2 T)}\right]v_t$$

- Mass accretion rate

$$\dot{M}(r) = 4\pi\Delta^{1/2}H(r)\rho\frac{u}{\sqrt{1-u^2}}$$

- Entropy accretion rate

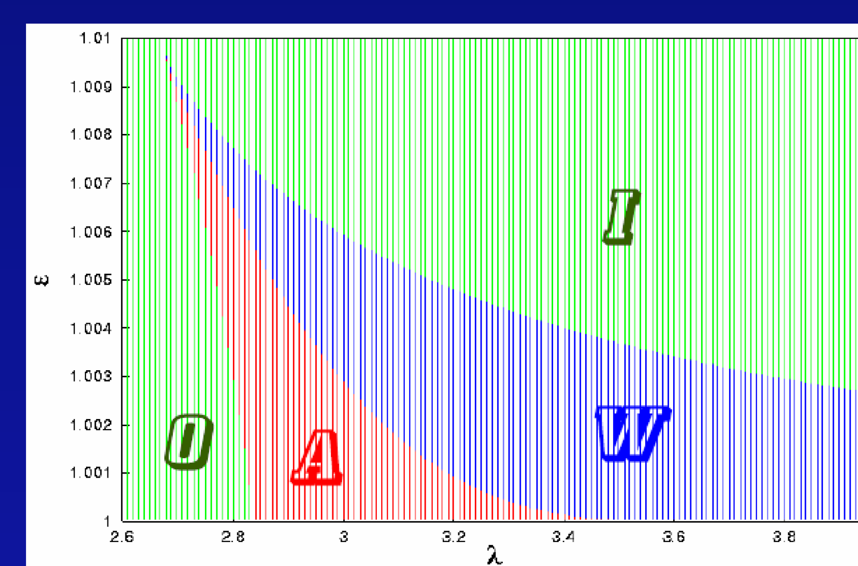
$$\dot{\Xi} = K^{1/(\gamma-1)}\dot{M}_{in} = 4\pi\left(\frac{n}{n+1}\right)^n \Delta^{1/2}\left(\frac{a_s^2}{1-na_s^2}\right)^n H(r)\frac{u}{\sqrt{1-u^2}}$$

Sonic Point Quantities

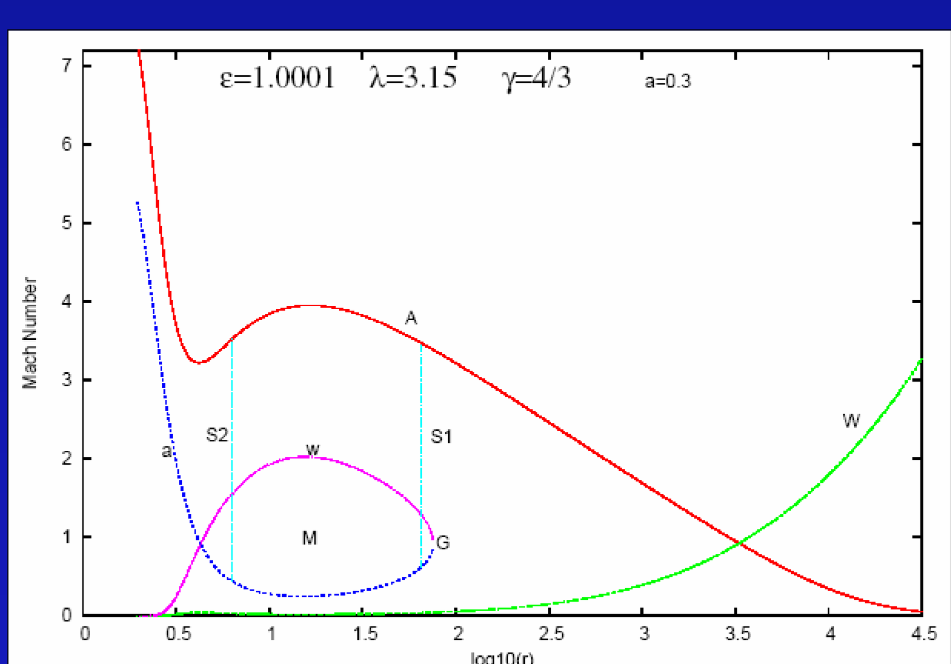
$$\chi = \frac{1}{\Delta}\frac{d\Delta}{dr} + \frac{\lambda}{(1-\Omega\lambda)}\frac{d\Omega}{dr} - \frac{\left(\frac{dg_{\phi\phi}}{dr} + \lambda\frac{dg_{t\phi}}{dr}\right)}{(g_{\phi\phi} + \lambda g_{t\phi})}$$

$$\psi = \lambda^2 v_t^2 - a^2(v_t - 1) \qquad \sigma = 2\lambda^2 v_t - a^2$$

- Radial fluid velocity $u_c = \left[\frac{\chi\Delta r}{2r(r-1) + 4\Delta}\right]_c^{1/2}$
- Sound speed $a_s|_c = \left[\frac{u^2(\gamma+1)\psi}{2\psi - u^2 v_t\sigma}\right]_c^{1/2}$
- Quadratic eqn. for *(du/dr)*$_c$ $\alpha\left(\frac{du}{dr}\right)_c^2 + \beta\left(\frac{du}{dr}\right)_c + \zeta = 0$
- α, β, ζ = Complicated functions (Barai, Das & Wiita 2004)



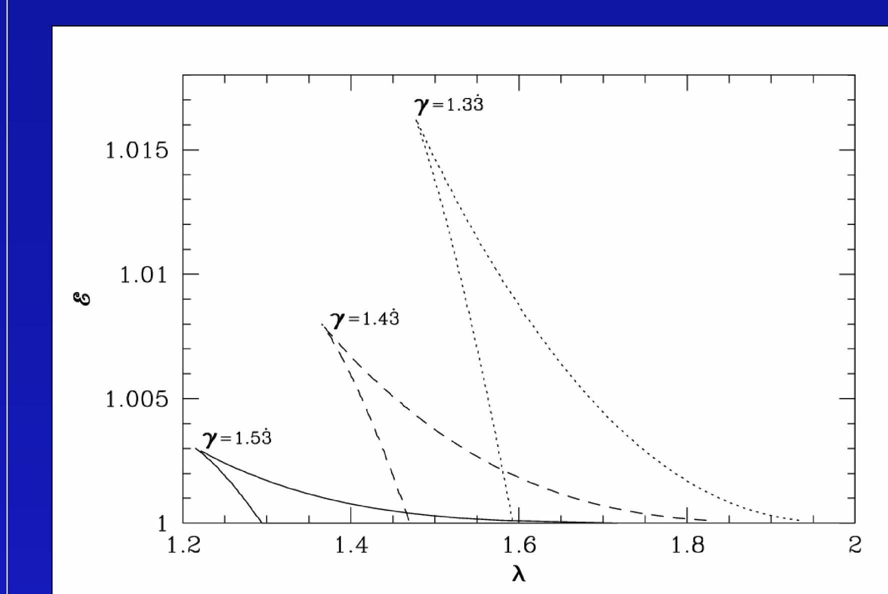
[ε-λ] parameter space (γ=4/3, a=0.3) for mono- and multi-transonic accretion & wind



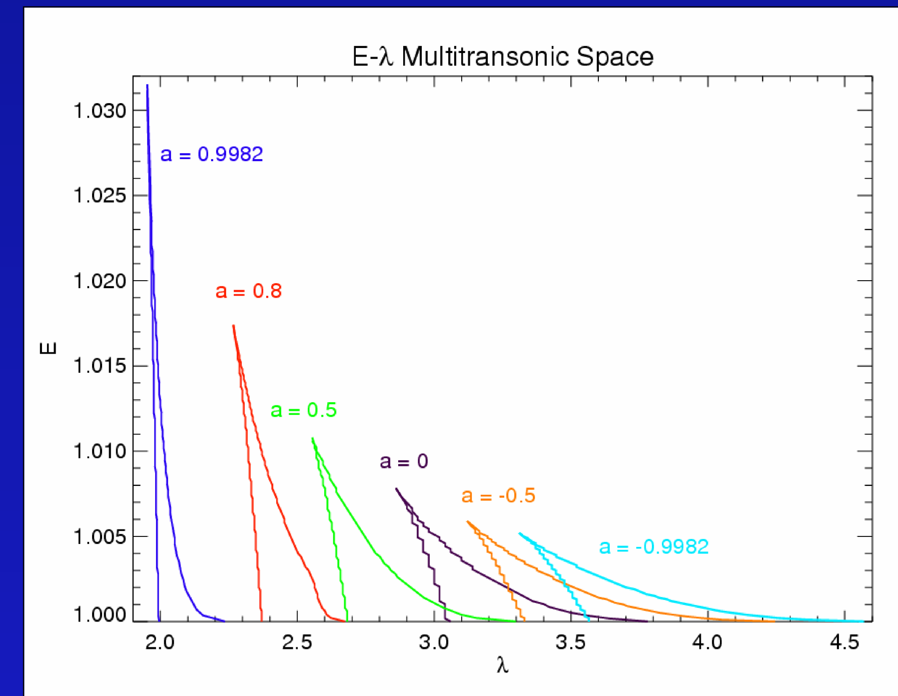
Solution topology for multi-transonic accretion in Kerr geometry

Results

- For some [ε-λ-γ-a] get 3 sonic points on solving eq.
  - $r_{out} > r_{mid} > r_{in}$
- r_{out}, r_{in} : X-type sonic points
- r_{mid} : O-type sonic pt (unphysical – no steady transonic soln. passes thru. it)
- Multi-transonic Accretion: $\Xi(r_{in}) > \Xi(r_{out})$
- Multi-transonic Wind : $\Xi(r_{in}) < \Xi(r_{out})$
- General astrophysical accretion → Flow thru. rout
- Flow thru. r_{in} possible only in case of a shock
  - If supersonic flow thru. r_{out} is perturbed to produce entropy = [$\Xi(r_{in}) - \Xi(r_{out})$], it joins subsonic flow thru r_{in} forming a standing shock
  - Shock details from GR Rankine-Hugoniot conditions



[ε-λ] parameter space for multi-transonic BH accretion in Schwarzschild geometry (a=0)

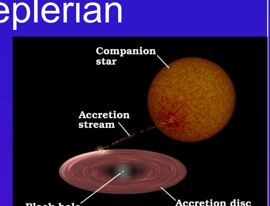


[ε-λ] parameter space for multi-transonic BH accretion for different values of Kerr parameter

Pro- vs. Retro-Grade Flows

Weakly rotating flow is found in several astrophysical situations:

- Detached binary systems fed by accretion from OB stellar winds
- Semidetached low-mass non-magnetic binaries
- Supermassive BHs fed by accretion from slowly rotating central stellar clusters
- Turbulence in standard Keplerian accretion disk



We found:

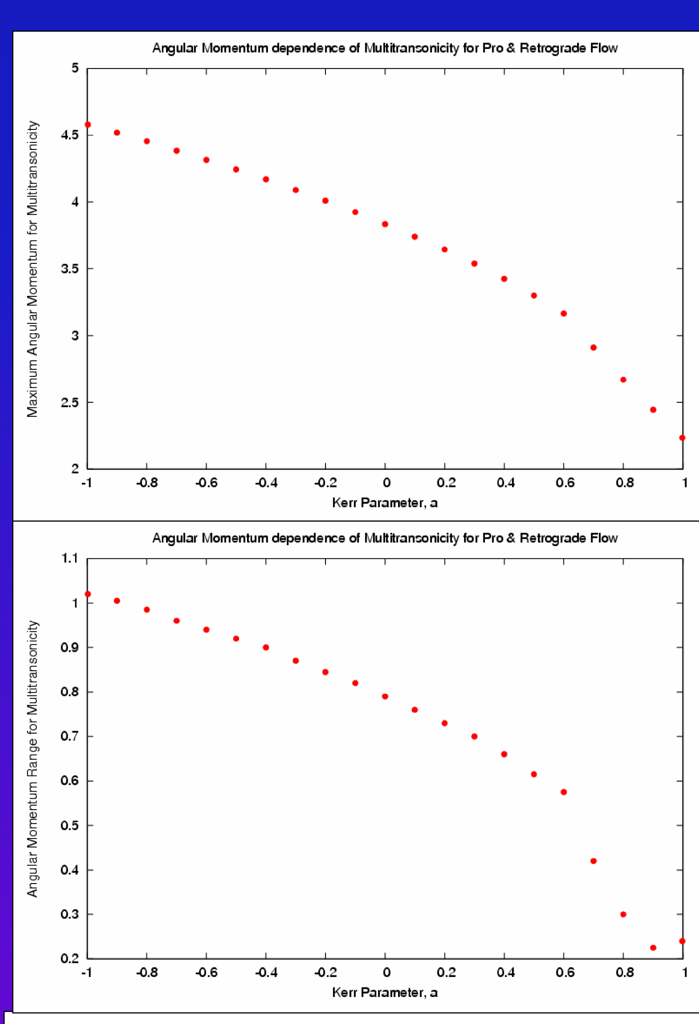
At higher values of angular momentum multi-transonicity is more common for retrograde flow

Prograde Accretion Flow:

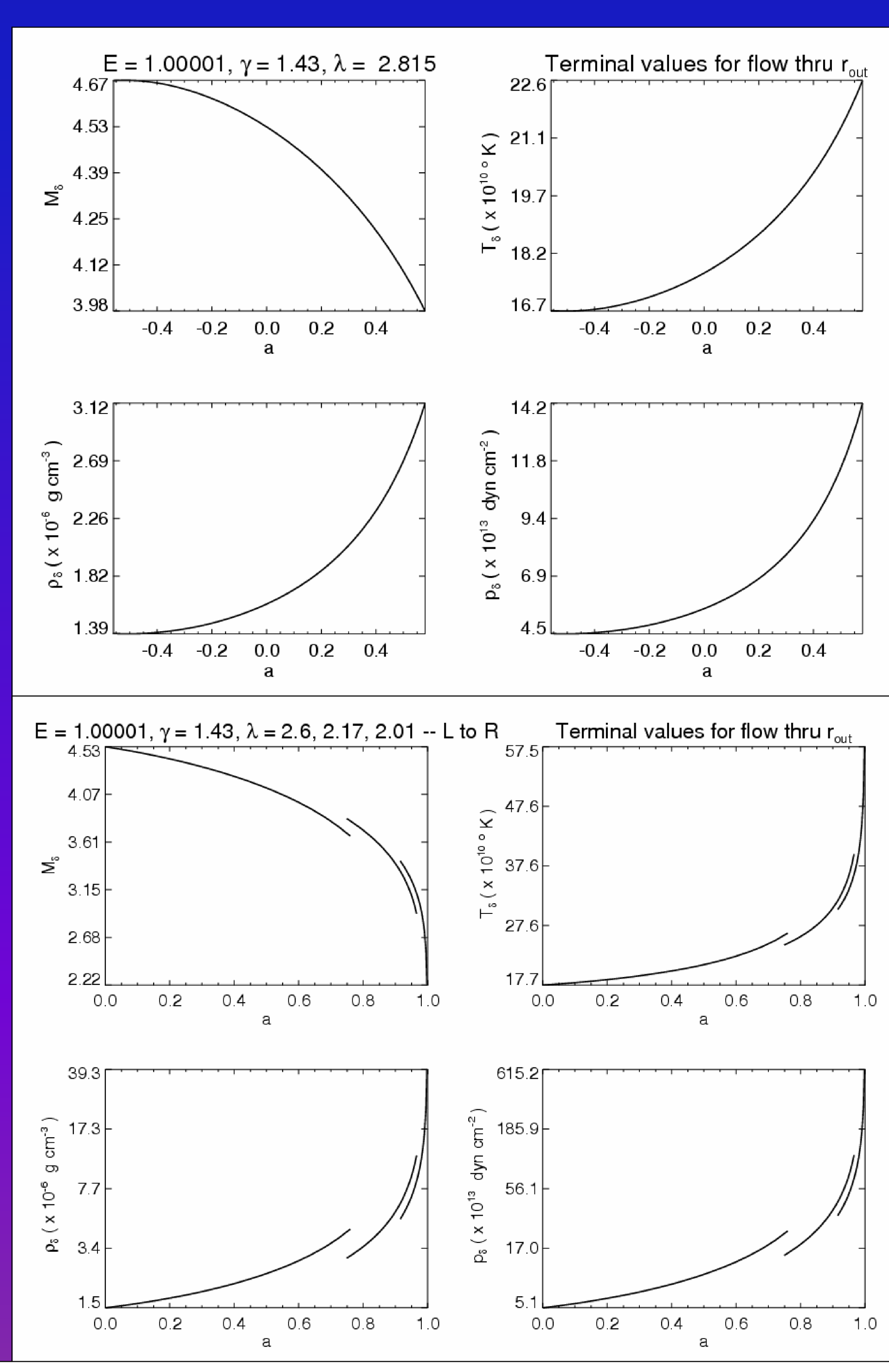
- Multi-transonic regions at lower λ

Retrograde Accretion Flow:

- Multi-transonicity much more common
- Covers higher value of λ



Angular momentum trends differentiating accretion properties of co- & counter-rotating BH



Terminal values of accretion variables (Mach number, temperature, density & pressure) as a function of BH spin

Terminal Behavior of Accretion Variables

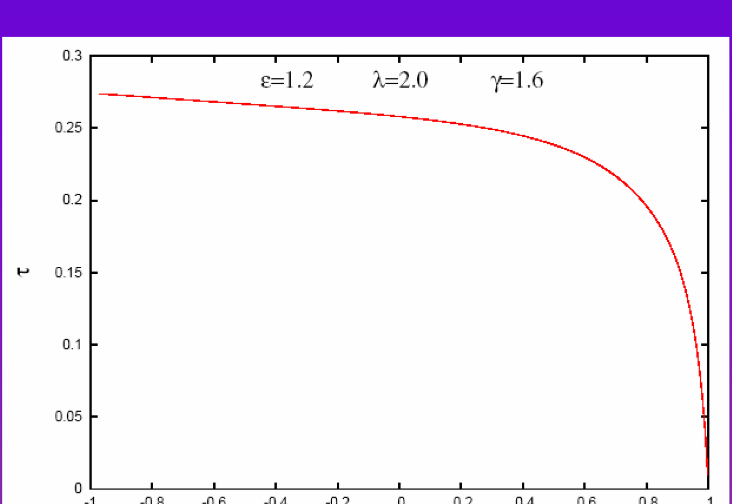
Terminal value:

$A_\delta = A$ (at $r_\delta = r_{EH} + \delta$)

- Integrate flow from r_c down to r_δ → get A_δ
- Studied variation of A_δ with a → BH spin dependence of accretion variables very close to event horizon
- Can be done for any r_δ → dependence of flow behavior on BH spin at any radial distance from singularity

Astrophysical Implications

- Preliminary step toward understanding how BH spin affects astrophysical accretion and related phenomena
  - Shock waves in BH accretion disks must form through multi-transonic flows
- Study of the post shock flow helpful in explaining:
  - Spectral properties of BH candidates
  - Formation & dynamics of cosmic (galactic & extragalactic) jets powered by accretion
  - Origin of Quasi Periodic Oscillations in galactic sources



Variation of the ratio of analog to actual Hawking temperature with black hole spin

Analog Hawking Radiation

Das, Bilic & Dasgupta, 2006

- Hawking Temperature $T_H = \frac{\hbar c^3}{8\pi K_B G M_{BH}}$
- Analog Hawking Temperature $T_{AH} = \frac{\hbar}{4\pi K_B}\left[\frac{1}{c_s}\frac{du_{\perp}^2}{d\eta}\right]_{\text{Acoustic Horizon}}$
- Ratio $\tau = \frac{T_{AH}}{T_H}$

Conclusions

1. Study dependence of multi-transonic accretion properties on BH spin at any radial distance / accretion length scale
   - Very close to event horizon
   - Possible shock location
2. Found non-trivial difference in the accretion behavior for co- & counter-rotating BH
   - Prograde flow (co-rotating BH) – low λ
   - Retrograde flow (counter-rotating BH) – high λ & greater possibility of shock formation
3. Retrograde flow enhances analog gravity effect

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