Investigating General-Relativistic Transonic Accretion in Kerr Black Holes

Paramita Barai -- Université Laval
Collaborator: Tapas K. Das,
Harish Chandra Research Institute, India.

16th March, 2007
Radial fluid velocity, $u$

Sound speed, $a_s$

Mach number $M(r) = \frac{u(r)}{a_s(r)}$

Accretion flow
- Subsonic: $M < 1$
- Supersonic: $M > 1$
- Transonic: crosses $M = 1$

Transition
- Smooth -- Sonic point
- Discontinuous -- Shock

Multi-transonic flow $\equiv 3$ sonic points
Motivation

- Black Hole (BH) inner boundary condition: Supersonic flow at Event Horizon (EH)
- Far away from EH – subsonic
- Hence BH accretion is essentially transonic
  - Except cases where already supersonic initially
- Shock Formation $\Rightarrow$ Multi-transonic flow

Accretion in Kerr (rotating) Black Holes:
- Transonic
- Consider General-Relativistic effects
Notations

- Gravitational Radius

$$r_g = \frac{GM_{BH}}{c^2}$$

- Black Hole Spin, Kerr Parameter = $a$

- $|a| \in (0, 0.9982)$

- $\lambda$ = Specific angular momentum of flow, is aligned with $a$

- BH

  - Co-rotating : (+ve) $a$
  - Counter-rotating : (-ve) $a$

Goal

- Study behavior of dynamic and thermodynamic properties of accretion flow very close ($\approx 0.01\, r_g$) to EH & their dependence on spin of BH

What do we do?

- Formulate & solve conservation equations governing general relativistic, multi-transonic, advective accretion flow in Kerr metric
Metric

- Kerr metric in equatorial plane of BH
  (Novikov, I.D. & Thorne, K.S. 1973, in Black Holes, 343)

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\frac{r^2\Delta}{A} dt^2 + \frac{A}{r^2} (d\phi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2
\]

\[
\Delta = r^2 - 2r + a^2
\]

\[
A = r^4 + r^2 a^2 + 2ra^2
\]

\[
\omega = \frac{2ar}{A}
\]

\[
g_{tt} = \frac{A\omega^2}{r^2} - \frac{r^2\Delta}{A}
\]

\[
g_{t\phi} = -\frac{A\omega}{r^2}
\]

\[
g_{\phi\phi} = \frac{A}{r^2}
\]
Accretion Disk

- **Vertically integrated model** (Matsumoto, R. et al. 1984, PASJ, 36, 71)
  - Flow in hydrostatic equilibrium in transverse direction
  - Thermodynamic quantities vertically averaged over disk height
  - Calculate quantities on equatorial plane of BH


\[ h_{\text{disk}}(r) = H(r) = r^2 \left[ \frac{2n\alpha_s^2}{(n+1)(1-n\alpha_s^2)} \left( \lambda^2 v_t^2 - a^2 (v_t - 1) \right) \right]^{1/2} \]
Conserved Quantities

- **Specific energy (including rest mass)**
  \[ \varepsilon = h\nu_t = \left[ \frac{\gamma - 1}{\gamma - (1 + \theta^2 T)} \right] \nu_t \]

- **Mass accretion rate**
  \[ \dot{M}(r) = 4\pi \Delta^{1/2} H(r) \rho \frac{u}{\sqrt{1-u^2}} \]

- **Entropy accretion rate**
  \[ \dot{\Xi} = K^{1/(\gamma-1)} \dot{M}_{in} = 4\pi \left( \frac{n}{n+1} \right)^n \Delta^{1/2} \left( \frac{a_s^2}{1-na_s^2} \right)^n H(r) \frac{u}{\sqrt{1-u^2}} \]
Methodology

- Solve conservation eqns. to get $du/dr$

\[
\frac{d\varepsilon}{dr} = 0, \quad \frac{d\dot{\varepsilon}}{dr} = 0.
\]

\[
\frac{du}{dr} = \frac{N}{D} = fn(r, u, a_s, \varepsilon, \lambda, \gamma, a)
\]

- Start from sonic point and integrate to get flow properties at any $r$

- For a range of $[\varepsilon-\lambda-\gamma-a] \rightarrow$ get 3 sonic points on solving eqns.
  - $r_{out} > r_{mid} > r_{in}$
Result plots for black hole of mass, \( M_{BH} = 10 M_{\text{sun}} \)
Solution topology for multi-transonic accretion in Kerr geometry
[\varepsilon-\lambda] parameter space (\gamma=4/3, \ a=0.3) for mono- and multi-transonic accretion & wind
ε-λ multi-transonic space for different γ
Differentiating Accretion Properties of Co- & Counter-Rotation of Central Black Hole
Angular Momentum dependence of Multitransonicity for Pro & Retrograde Flow

Maximum Angular Momentum for Multitransonicity

Kerr Parameter, $a$

-1  -0.8  -0.6  -0.4  -0.2  0   0.2   0.4   0.6   0.8   1

2.0  2.5  3.0  3.5  4.0  4.5  5.0
Angular Momentum dependence of Multitransonicity for Pro & Retrograde Flow

Angular Momentum Range for Multitransonicity

Kerr Parameter, a
Pro- vs. Retro-Grade Flows

Prograde Accretion Flow (co-rotating)

- Multi-transonic regions at lower value of $\lambda$

Retrograde Accretion Flow (counter-rotating)

- Multi-transonicity much more common
- Covers higher value of angular momentum ($\lambda$)
Terminal Behavior of Accretion Variables

Terminal value of accretion variable $A$:

$A_\delta = A$ (at $r_\delta = r_e + \delta$)

Integrate flow from $r_e$ down to $r_\delta \rightarrow$ get $A_\delta$

Studied variation of $A_\delta$ with $a \rightarrow$ BH spin dependence of accretion variables very close to event horizon

For $M_{BH} = 10M_{sun}$, and $\delta = 0.01 \rightarrow$ plots next …
E = 1.00001, $\gamma = 1.43$, $\lambda = 2.6, 2.17, 2.01$ -- L to R

Terminal values for flow thru $r_{out}$
Astrophysical Implications

- Preliminary step towards understanding how BH spin affects astrophysical accretion
  - Shock waves in BH accretion disks must form through multi-transonic flows

- Study of the post shock flow helpful in explaining:
  - Spectral properties of BH candidates
  - Cosmic (galactic & extragalactic) jets powered by accretion (formation & dynamics)
  - Origin of Quasi Periodic Oscillations in galactic sources
Summary

1. Study dependence of multi-transonic accretion properties on BH spin at any radial distance
   - Very close to event horizon

2. Found non-trivial difference in,
   - Prograde flow (co-rotating BH) : low-$\lambda$
   - Retrograde flow (counter-rotating BH) : multi-transonicity at high-$\lambda$ & greater possibility of shock formation

3. Future work:
   - Shock location
   - Analog gravity
     Analog Hawking radiation
References

- Matsumoto, R. et al. 1984, PASJ, 36, 71
- Novikov, I.D. & Thorne, K.S. 1973, in Black Holes, 343
Thank You All
Contents

- Introduction & Motivation
  - What are we doing?
- Our system formulations
- Results
- Conclusions
Motivation

- Transonic Flow is relevant in various astrophysical situations:
  - Black Hole (BH) or Neutron Star accretion
  - Collapse / Explosion of stars
  - Solar / Stellar winds
  - Formation of protostars & galaxies
  - Interaction of supersonic galactic (or extragalactic) jets with ambient medium

- BH inner boundary condition: Supersonic flow at Event Horizon (EH)

- Far away from EH – subsonic

- Hence BH accretion is essentially transonic
  - Except cases where already supersonic initially

- Multi-transonic flow → Shock Formation

Transonic Flow is relevant in various astrophysical situations:

- Black Hole (BH) or Neutron Star accretion
- Collapse / Explosion of stars
- Solar / Stellar winds
- Formation of protostars & galaxies
- Interaction of supersonic galactic (or extragalactic) jets with ambient medium

BH inner boundary condition: Supersonic flow at Event Horizon (EH)

Far away from EH – subsonic

Hence BH accretion is essentially transonic
  -Except cases where already supersonic initially

Multi-transonic flow → Shock Formation
Describing Our System

- No self-gravity, no magnetic field
- Units: \( G = c = M_{BH} = 1 \)
- Boyer-Lindquist coordinates \((- + + +)\)
- Observer frame corotating with accreting fluid
- \( \lambda \) = Specific angular momentum of flow -- aligned with \( a \)
- Stationary & axisymmetric flow
- Euler & Continuity eqns:

\[
\nabla^\mu \mathcal{S}^{\mu\nu} = 0 \quad , \quad \left( \rho v^\mu \right)_{;\mu} = 0
\]

- Polytropic equation of state
  - \( K \sim \) specific entropy density
  - \( \gamma = \) adiabatic index, \( n = \) polytropic index

\[
p = K \rho^\gamma
\]

\[
n = \frac{1}{\gamma - 1}
\]
Our System ...

- Specific proper flow enthalpy, $h$
- Polytropic sound speed, $a_s$

\[
a_s = \left( \frac{\partial p}{\partial \varepsilon} \right)^{1/2}_s = \Psi_1(p, r, \gamma) = \Psi_2(p, \rho, \gamma) ; \quad a_s^2(r) = \frac{\gamma K_B T(r)}{\mu m_H}
\]

- Frame dragging neglected
- Weak viscosity limit
  - Very large radial velocity close to BH $\rightarrow$ Timescale (viscous $\gg$ infall)
  - Effect of Viscosity $\rightarrow \lambda \downarrow$ $\rightarrow$ Flow behavior as function of $\lambda$ provides information on viscous transonic flow
\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\frac{r^2 \Delta}{A} dt^2 + \frac{A}{r^2} (d\phi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2 \]

\[ \Delta = r^2 - 2r + a^2 \]

\[ A = r^4 + r^2 a^2 + 2ra^2 \]

\[ \omega = \frac{2ar}{A} \]

\[ g_{tt} = \frac{A \omega^2}{r^2} - \frac{r^2 \Delta}{A} \]

\[ g_{t\phi} = -\frac{A \omega}{r^2} \]

\[ g_{\phi\phi} = \frac{A}{r^2} \]

\[ \Omega = \frac{u^\phi}{u^t} = -\frac{\left( g_{t\phi} + \lambda g_{tt} \right)}{\left( g_{\phi\phi} + \lambda g_{t\phi} \right)} \]

\[ \nu_t = \left[ \frac{\Delta}{\left( 1 - u^2 \right) \left( 1 - \Omega \lambda \right) \left( g_{\phi\phi} + \lambda g_{t\phi} \right)} \right]^{1/2} \]
Methodology

- Solve conservation eqns. to get $du/dr$

$$\frac{d\varepsilon}{dr} = 0, \quad \frac{d\Xi}{dr} = 0.$$ \hspace{2cm} \frac{du}{dr} = \frac{N}{D} = fn(r, u, a_s, \varepsilon, \lambda, \gamma, a)

- Start from sonic point and integrate to get flow properties at any $r$

- By setting $N=0$, $D=0$ get sonic point quantities:
  - $u_0$, $a_s|_c$
  - Quadratic eqn. for $(du/dr)_c$

- For some $[\varepsilon-\lambda-\gamma-a]$ → get 3 sonic points on solving eqns.
  - $r_{out} > r_{mid} > r_{in}$
Observational Consequences

- Debate: Determining BH spin from observations

- Most popular approach: study of skew shaped fluorescent iron lines

- Our approach presents the potential to deal with this problem
  - We predict behavior of flow properties
    - For hot, low-$\lambda$ prograde accretion flow & high-$\lambda$ retrograde flow
Results

- For some $[\varepsilon-\lambda-\gamma-a]$ -- get 3 sonic points on solving eqn
  - $r_{\text{out}} > r_{\text{mid}} > r_{\text{in}}$
- $r_{\text{out}}, r_{\text{in}}$: X-type sonic points
- $r_{\text{mid}}$: O-type sonic pt (unphysical – no steady transonic soln passes thru it)
- Multitransonic Accretion: $\Xi(r_{\text{in}}) > \Xi(r_{\text{out}})$
- Multitransonic Wind: $\Xi(r_{\text{in}}) < \Xi(r_{\text{out}})$
- General astrophysical accretion $\rightarrow$ Flow thru $r_{\text{out}}$
- Flow thru $r_{\text{in}}$ possible only in case of a shock
  - If supersonic flow thru $r_{\text{out}}$ is perturbed to produce entropy $= [\Xi(r_{\text{in}}) - \Xi(r_{\text{out}})]$, it joins subsonic flow thru $r_{\text{in}}$ forming a standing shock
  - Shock details from GR Rankine-Hugoniot conditions
$a = -0.5, \ E = 1.001, \ \lambda = 3.4, \ \gamma = 4/3; \ \ r_{in} = 8.2782, \ r_{mid} = 16.443, \ r_{out} = 310.22$
E-λ Multi-Transonic Space for different $a$
Angular Momentum

- Weakly rotating flows, found in several physical situations:
  - Detached binary systems fed by accretion from OB stellar winds
  - Semidetached low-mass non-magnetic binaries
  - Supermassive BHs fed by accretion from slowly rotating central stellar clusters
  - Turbulence in standard Keplerian accretion disk

We found:
- At higher values of angular momentum multi-transonicity is more common for retrograde flow
Analog Hawking Radiation

(Das, Bilic & Dasgupta, 2006)

- **Hawking Temperature**
  \[ T_H = \frac{\hbar c^3}{8\pi K_B GM_{BH}} \]

- **Analog Hawking Temperature**
  \[ T_{AH} = \frac{\hbar}{4\pi K_B} \left[ \frac{1}{c_s} \frac{du^2}{d\eta} \right] \]
  (Acoustic Horizon)

- **Ratio**
  \[ \tau = \frac{T_{AH}}{T_H} \]
Variation of the ratio of analog to actual Hawking temperature with black hole spin