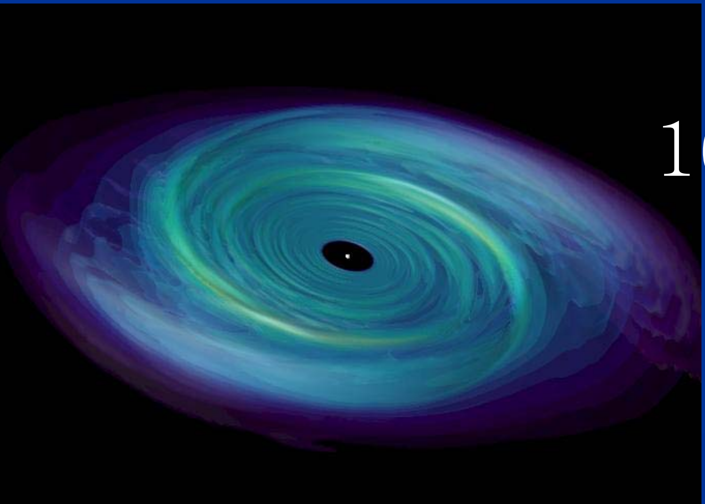
Investigating General-Relativistic Transonic Accretion in Kerr Black Holes

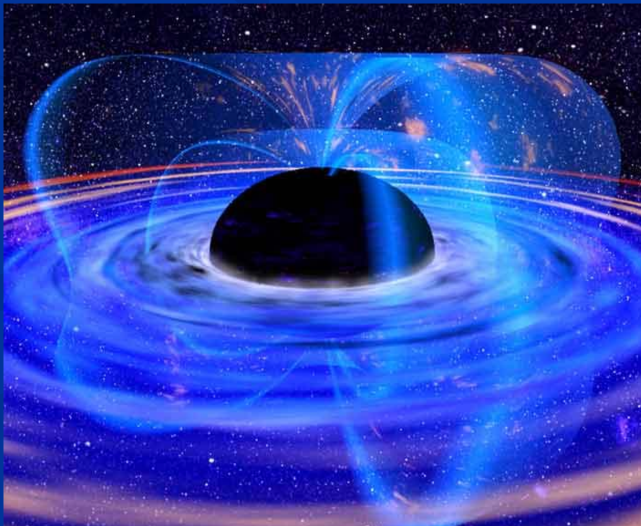
Paramita Barai -- Université Laval

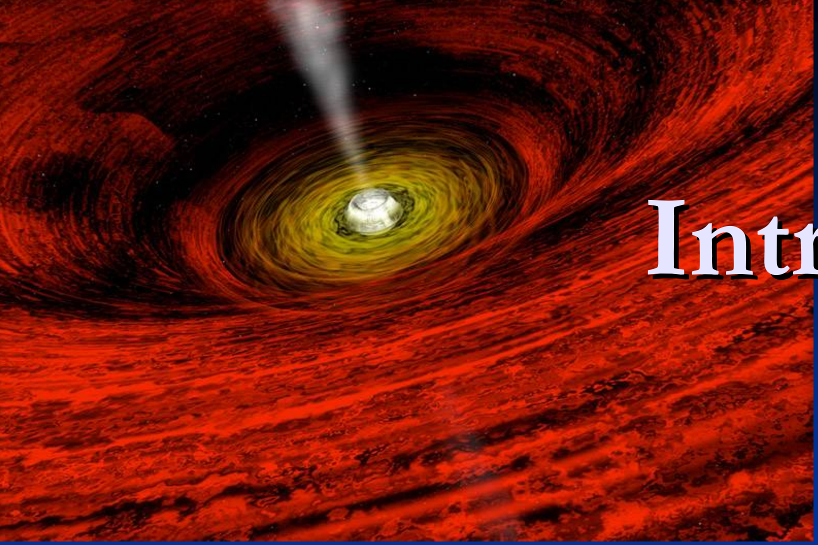
Collaborator: Tapas K. Das,

Harish Chandra Research Institute, India.

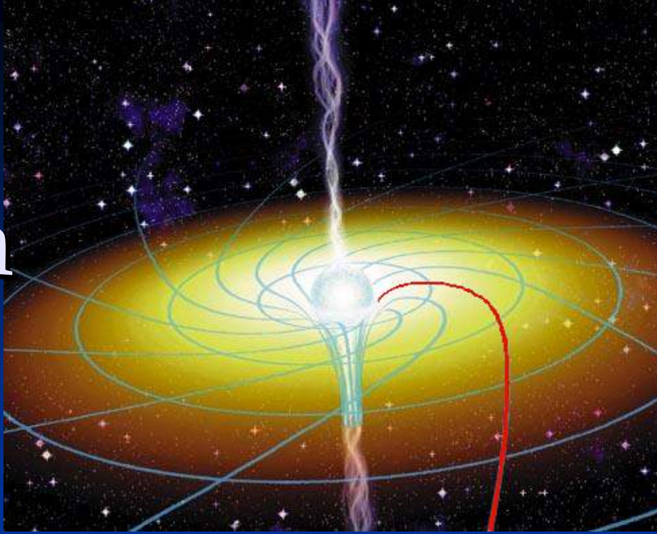


16th March, 2007





Introduction



- Radial fluid velocity, u
- Sound speed, a_s
- Mach number $\boxed{M(r) = \frac{u(r)}{a_s(r)}}$
- Accretion flow
  - Subsonic: $M < 1$
  - Supersonic: $M > 1$
  - Transonic: crosses $M = 1$

- Transition
  - Smooth -- Sonic point
  - Discontinuous – Shock
- Multi-transonic flow ≡ 3 sonic points

Motivation

- Black Hole (BH) inner boundary condition : Supersonic flow at Event Horizon (EH)
- Far away from EH – subsonic
- Hence BH accretion is essentially transonic
  - Except cases where already supersonic initially
- Shock Formation → Multi-transonic flow

Accretion in Kerr (rotating) Black Holes:
- Transonic
- Consider General-Relativistic effects

Notations

- Gravitational Radius

$$r_g = \frac{GM_{BH}}{c^2}$$

- Black Hole Spin, Kerr Parameter = a
- $|a| \in (0, 0.9982)$
- λ = Specific angular momentum of flow, is aligned with a
- BH
    - Co-rotating : (+ve) a
    - Counter-rotating : (-ve) a

Goal

- Study behavior of dynamic and thermodynamic properties of accretion flow very close ($\approx 0.01\ r_g$) to EH & their dependence on spin of BH

What do we do?

- Formulate & solve conservation equations governing general relativistic, multi-transonic, advective accretion flow in Kerr metric

Metric

- Kerr metric in equatorial plane of BH
  (Novikov, I.D. & Thorne, K.S. 1973, in Black Holes, 343)

$$\boxed{ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -\frac{r^2\Delta}{A} dt^2 + \frac{A}{r^2}(d\phi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2}$$

$$\boxed{\Delta = r^2 - 2r + a^2}$$

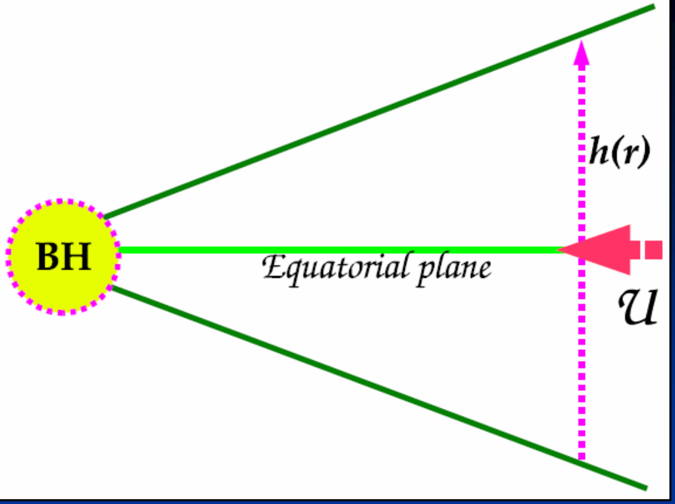
$$\boxed{A = r^4 + r^2 a^2 + 2ra^2}$$

$$\boxed{\omega = \frac{2ar}{A}}$$

$$\boxed{g_{tt} = \frac{A\omega^2}{r^2} - \frac{r^2\Delta}{A}}$$

$$\boxed{g_{t\phi} = -\frac{A\omega}{r^2}}$$

$$\boxed{g_{\phi\phi} = \frac{A}{r^2}}$$



Accretion Disk

$$\bar{\rho} = \frac{\int_{-H/2}^{H/2} \rho(h)\,dh}{\int_{-H/2}^{H/2} dh}$$

- Vertically integrated model (Matsumoto, R. et al. 1984, PASJ, 36, 71)
  - Flow in hydrostatic equilibrium in transverse direction
  - Thermodynamic quantities vertically averaged over disk height
  - Calculate quantities on equatorial plane of BH
- Disk height (Abramowicz, M.A. et al. 1997, ApJ, 479, 179)

$$h_{disk}(r) = H(r) = r^2 \left[\frac{2na_s{}^2}{(n+1)(1-na_s{}^2)\{\lambda^2 v_t{}^2 - a^2(v_t - 1)\}} \right]^{1/2}$$

Conserved Quantities

- Specific energy (including rest mass)

$$\varepsilon = hv_t = \left[\frac{\gamma - 1}{\gamma - \left(1 + \theta^2 T\right)}\right] v_t$$

- Mass accretion rate

$$\dot{M}(r) = 4\pi \Delta^{1/2} H(r) \rho \frac{u}{\sqrt{1 - u^2}}$$

- Entropy accretion rate

$$\dot{\Xi} = K^{1/(\gamma - 1)} \dot{M}_{in} = 4\pi \left(\frac{n}{n+1}\right)^n \Delta^{1/2} \left(\frac{a_s{}^2}{1 - n a_s{}^2}\right)^n H(r) \frac{u}{\sqrt{1 - u^2}}$$

Methodology

- Solve conservation eqns. to get *du*/*dr*

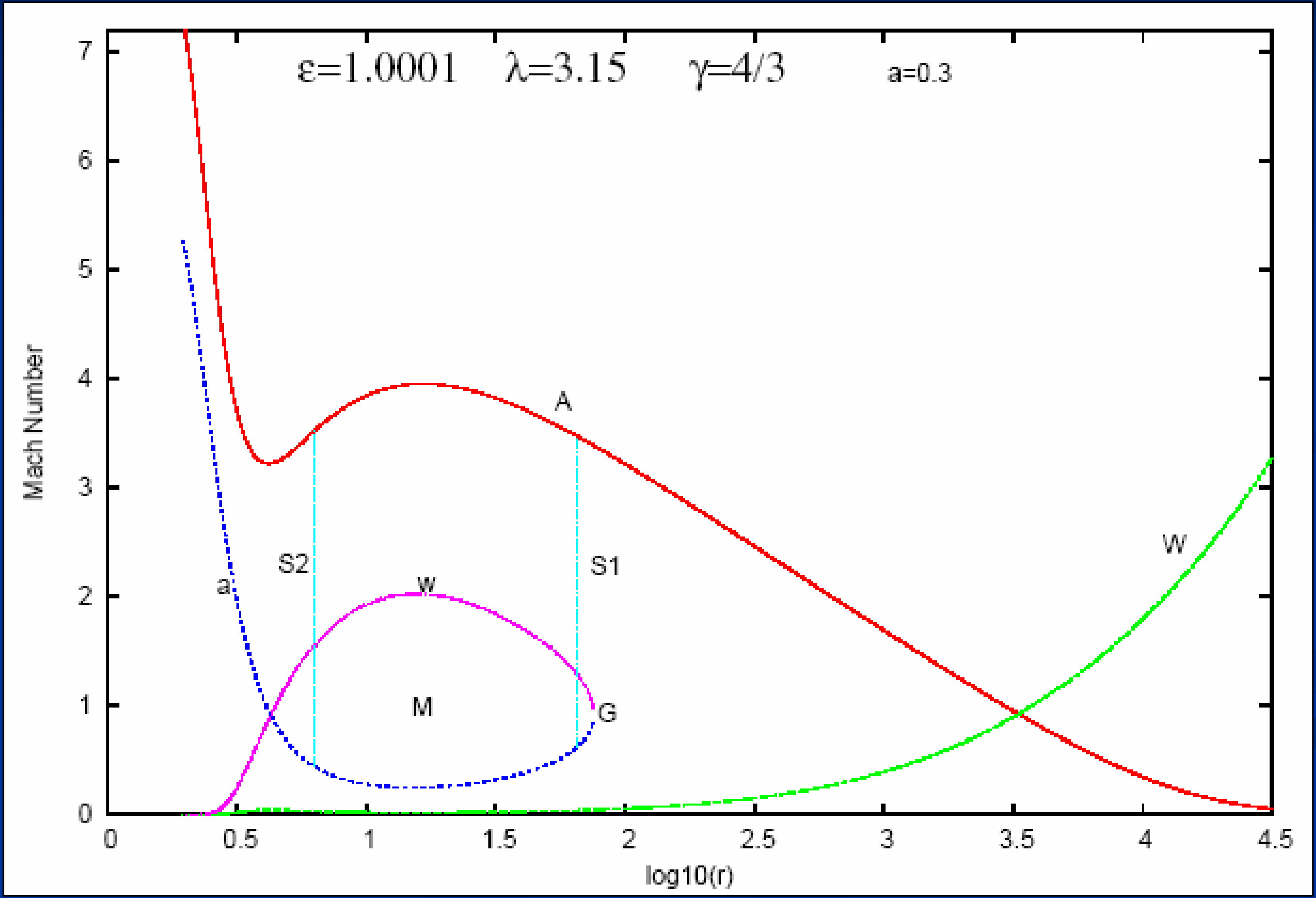
$$\frac{d\varepsilon}{dr} = 0, \frac{d\dot{\Xi}}{dr} = 0.$$

$$\frac{du}{dr} = \frac{N}{D} = fn(r, u, a_s, \varepsilon, \lambda, \gamma, a)$$

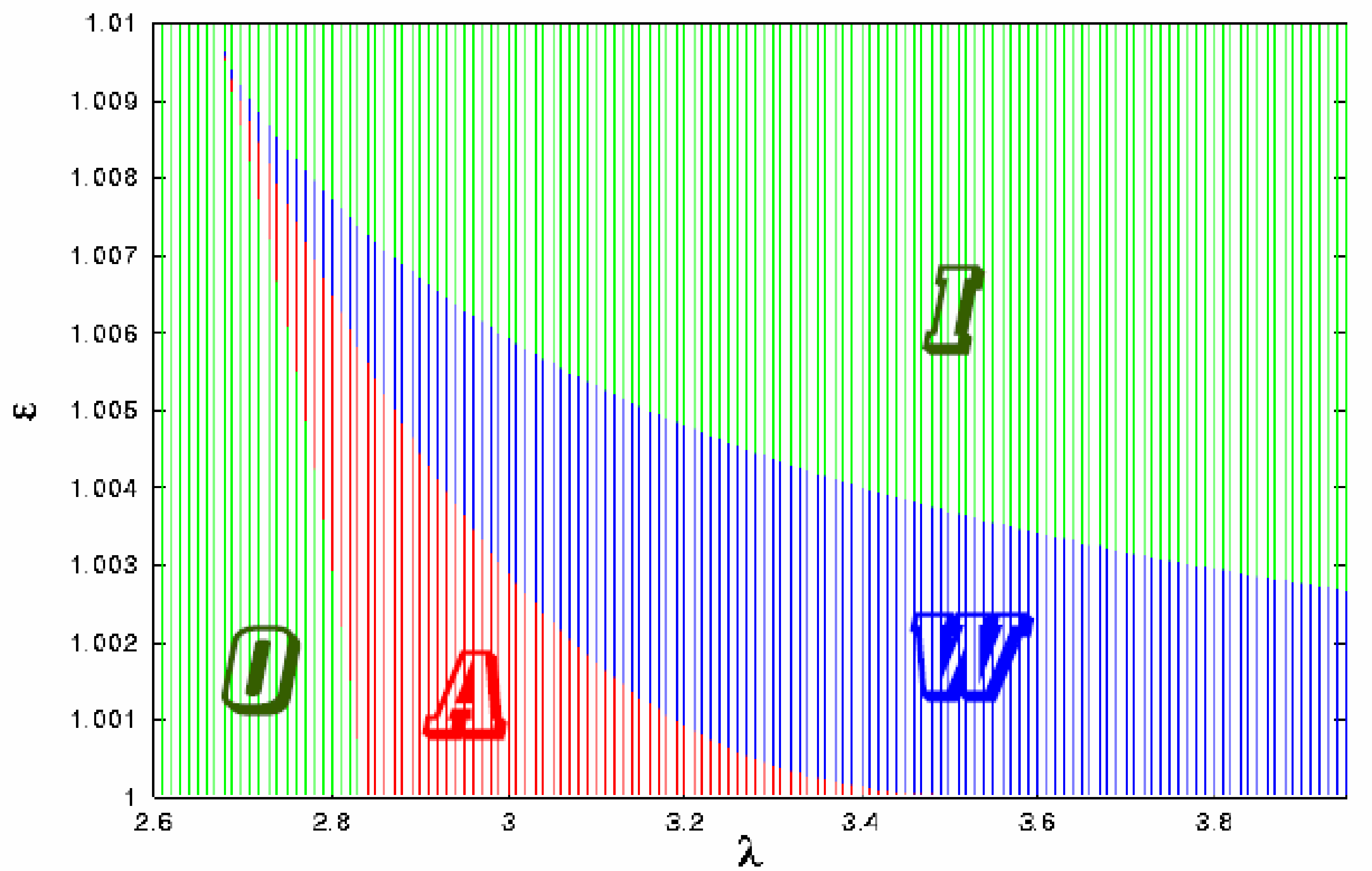
- Start from sonic point and integrate to get flow properties at any *r*
- For a range of [ε-λ-γ-a] → get 3 sonic points on solving eqns.
  - $r_{out} > r_{mid} > r_{in}$

Result plots for black hole of mass, $M_{BH}=10\,M_{sun}$

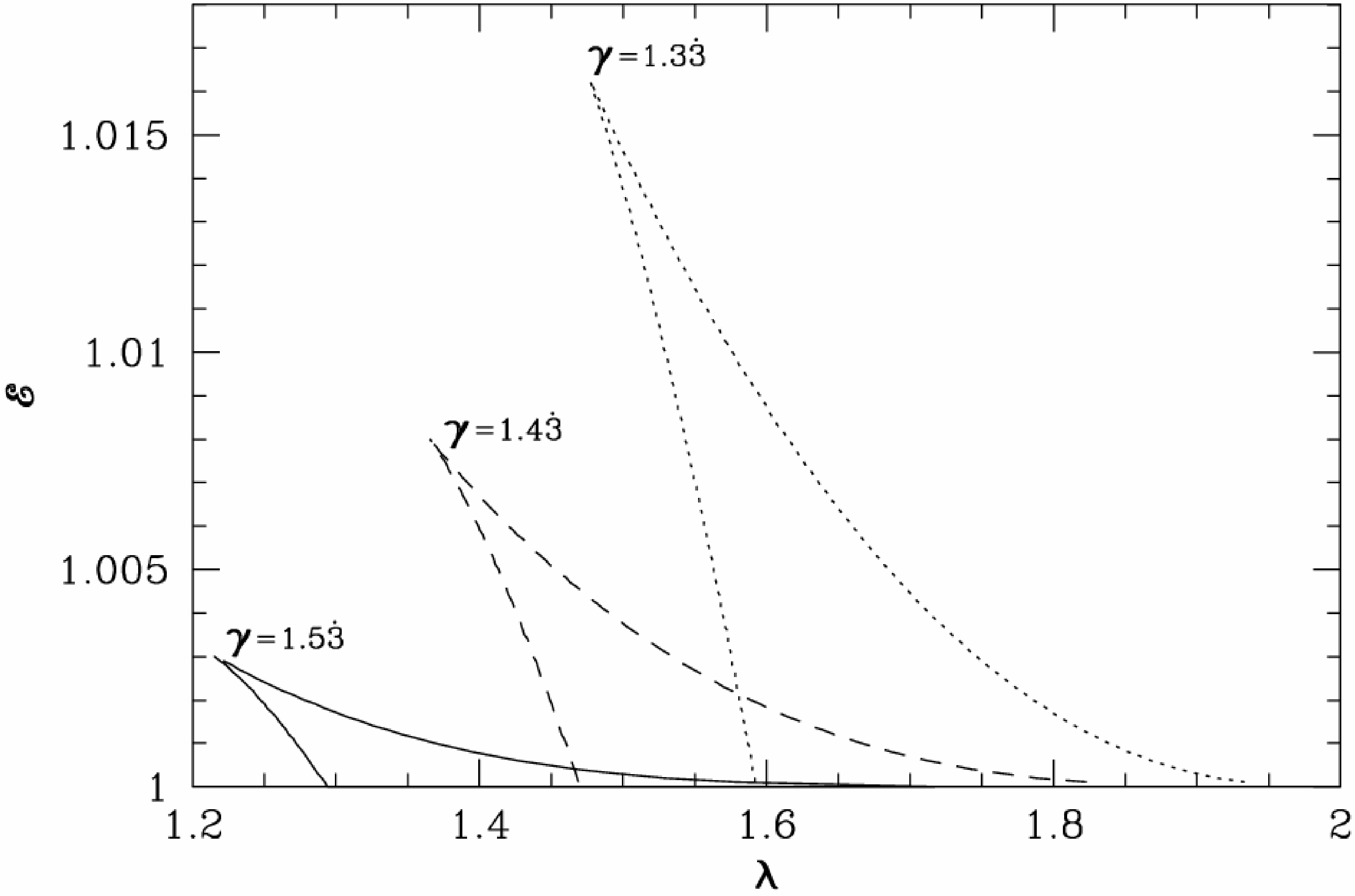
Solution topology for multi-transonic accretion in Kerr geometry



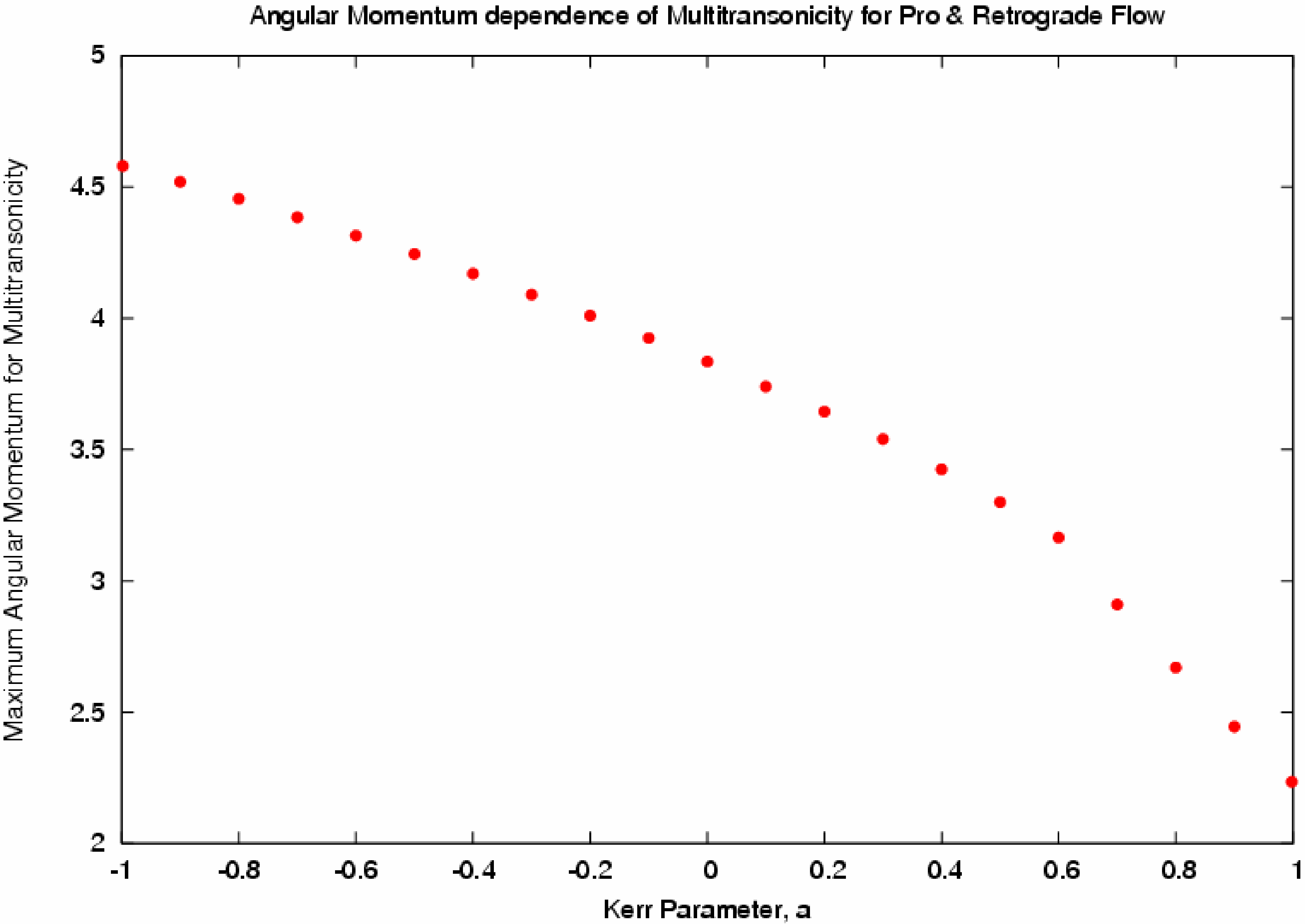
[ε-λ] parameter space (γ=4/3, a=0.3) for mono- and multi-transonic accretion & wind



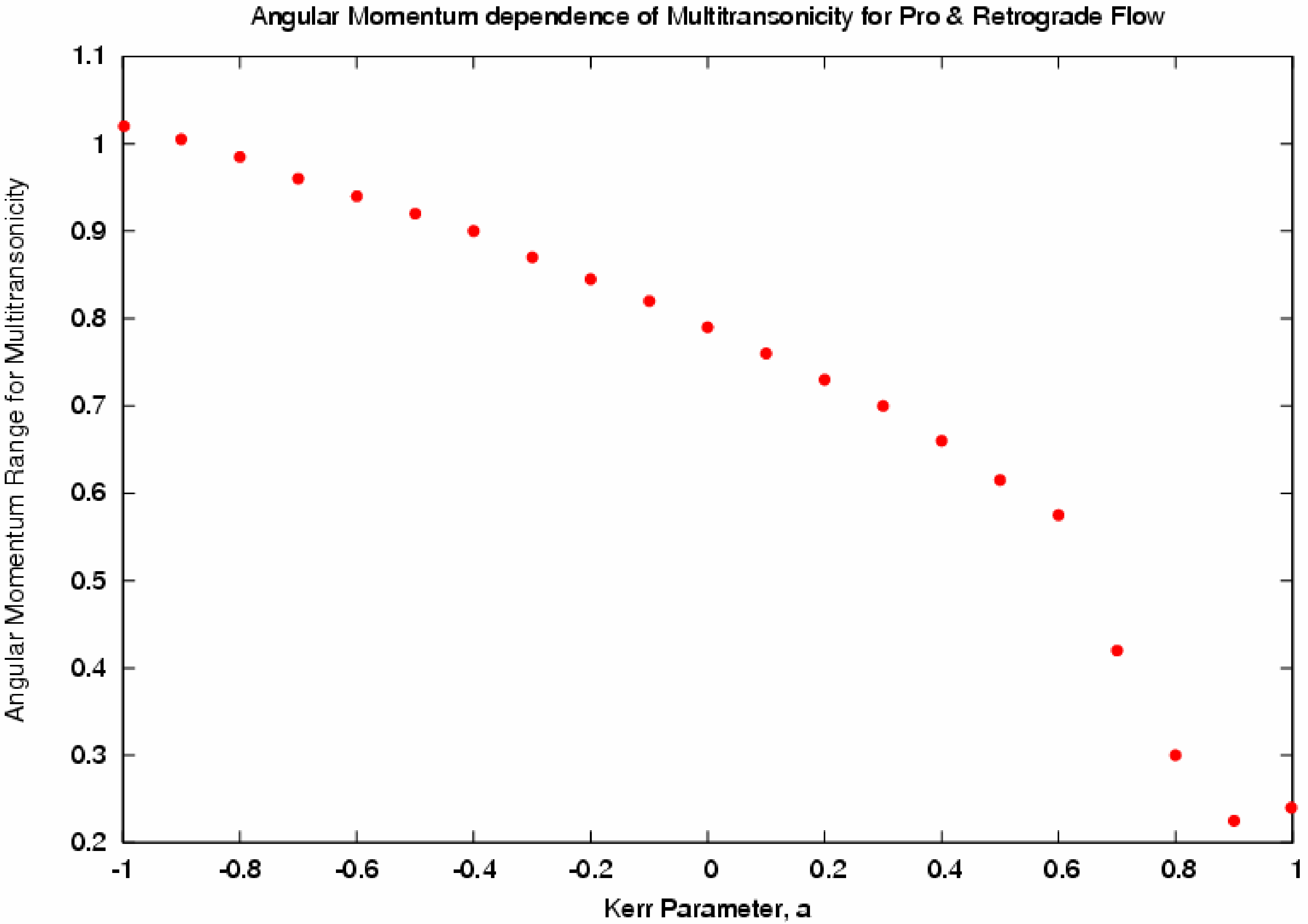
ε-λ multi-transonic space for different γ



Differentiating Accretion Properties of Co- & Counter-Rotation of Central Black Hole



Angular Momentum dependence of Multitransonicity for Pro & Retrograde Flow


Angular Momentum dependence of Multitransonicity for Pro & Retrograde Flow
Angular Momentum Range for Multitransonicity
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Kerr Parameter, a

Pro- vs. Retro-Grade Flows

Prograde Accretion Flow (co-rotating)

- Multi-transonic regions at lower value of λ

Retrograde Accretion Flow (counter-rotating)

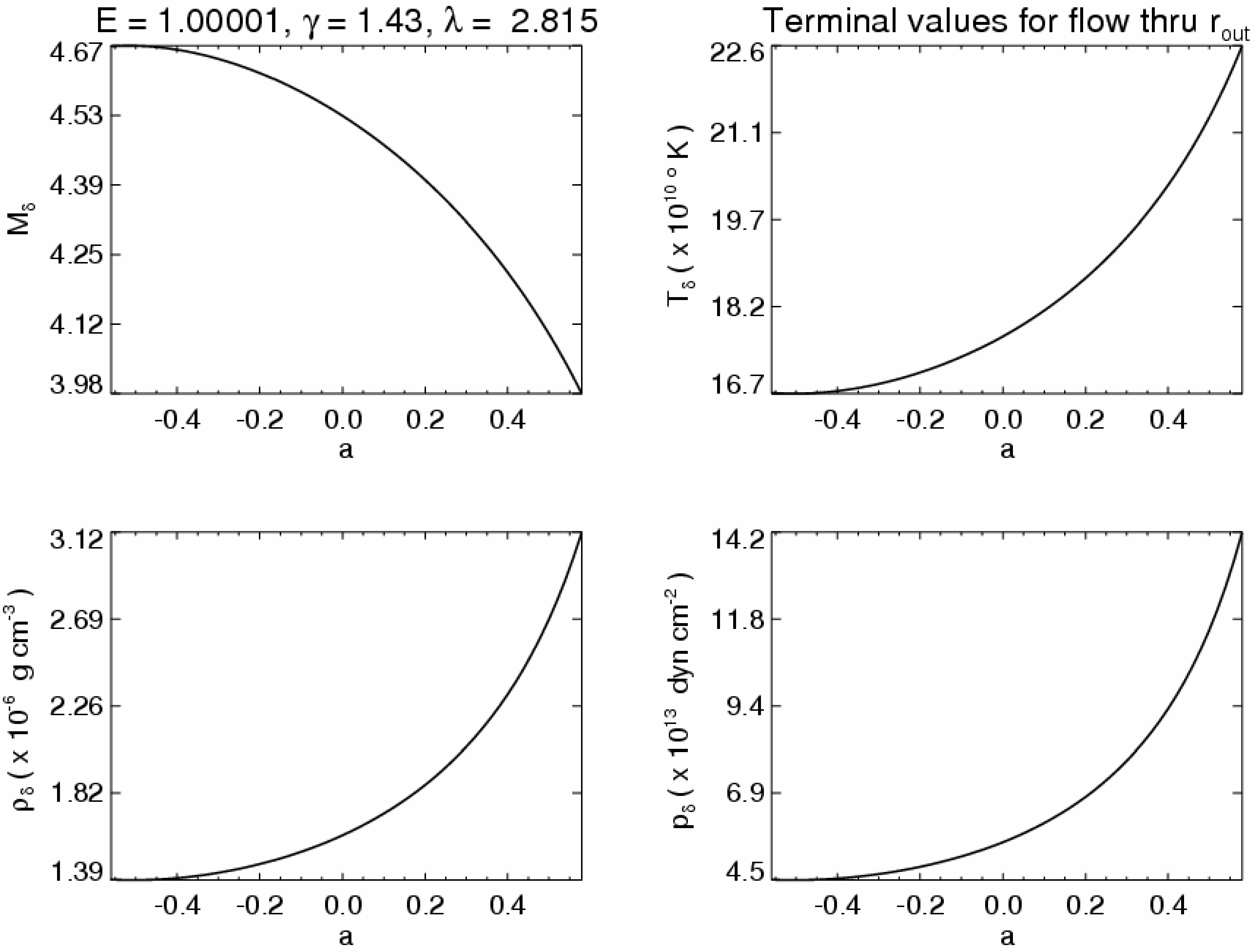
- Multi-transonicity much more common
- Covers higher value of angular momentum (λ)

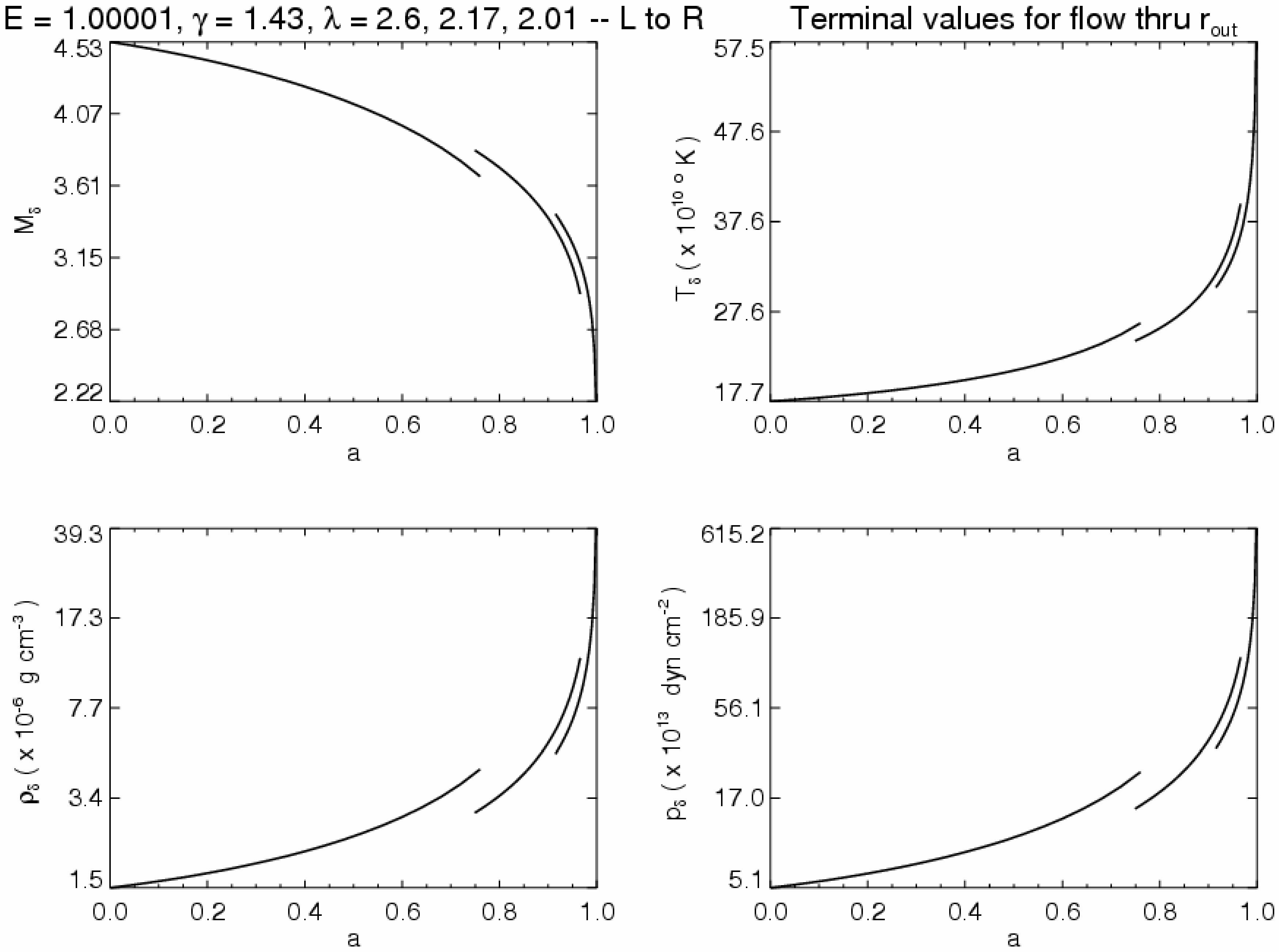
Terminal Behavior of Accretion Variables

- Terminal value of accretion variable A :

  $A_\delta = A$ (at $r_\delta = r_e + \delta$)

- Integrate flow from r_c down to r_δ → get A_δ
- Studied variation of A_δ with a → BH spin dependence of accretion variables very close to event horizon
- For M_{BH}=10M_{sun}, and δ=0.01 → plots next …


E = 1.00001, γ = 1.43, λ = 2.815
Terminal values for flow thru r_out
M_δ
4.67
4.53
4.39
4.25
4.12
3.98
T_δ (x 10^10 ° K)
22.6
21.1
19.7
18.2
16.7
ρ_δ (x 10^-6 g cm^-3)
3.12
2.69
2.26
1.82
1.39
p_δ (x 10^13 dyn cm^-2)
14.2
11.8
9.4
6.9
4.5
-0.4
-0.2
0.0
0.2
0.4
a


E = 1.00001, γ = 1.43, λ = 2.6, 2.17, 2.01 -- L to R
Terminal values for flow thru r_out
M_δ
4.53
4.07
3.61
3.15
2.68
2.22
T_δ (x 10^10 ° K)
57.5
47.6
37.6
27.6
17.7
ρ_δ (x 10^-6 g cm^-3)
39.3
17.3
7.7
3.4
1.5
p_δ (x 10^13 dyn cm^-2)
615.2
185.9
56.1
17.0
5.1
a
0.0
0.2
0.4
0.6
0.8
1.0

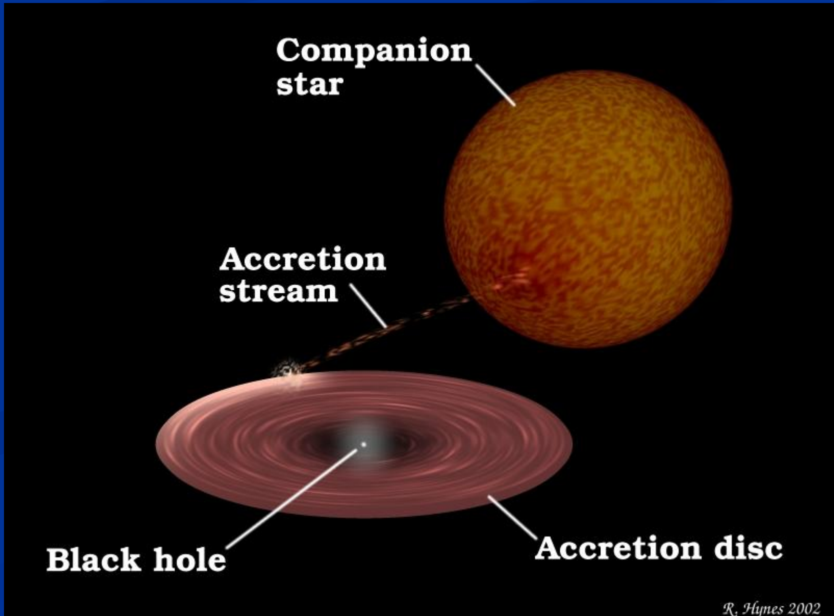
Astrophysical Implications

- Preliminary step towards understanding how BH spin affects astrophysical accretion
  - Shock waves in BH accretion disks must form through multi-transonic flows
- Study of the post shock flow helpful in explaining:
  - Spectral properties of BH candidates
  - Cosmic (galactic & extragalactic) jets powered by accretion (formation & dynamics)
  - Origin of Quasi Periodic Oscillations in galactic sources

Summary

1. Study dependence of multi-transonic accretion properties on BH spin at any radial distance
   - Very close to event horizon
2. Found non-trivial difference in,
   - ❑ Prograde flow (co-rotating BH) : low-λ
   - ❑ Retrograde flow (counter-rotating BH) : multi-transonicity at high-λ & greater possibility of shock formation
3. Future work:
   - ❑ Shock location
   - ❑ Analog gravity

     Analog Hawking radiation



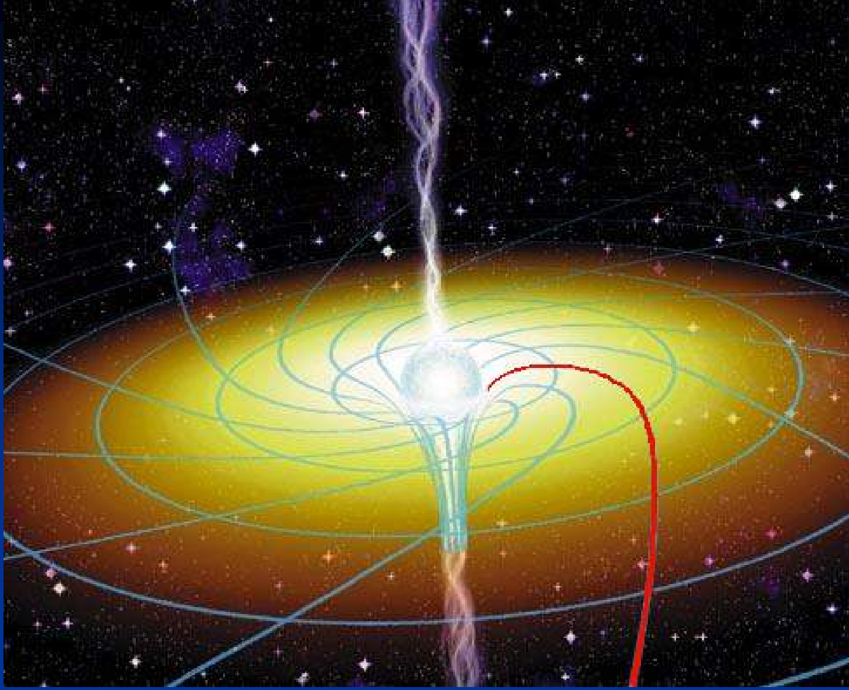
References

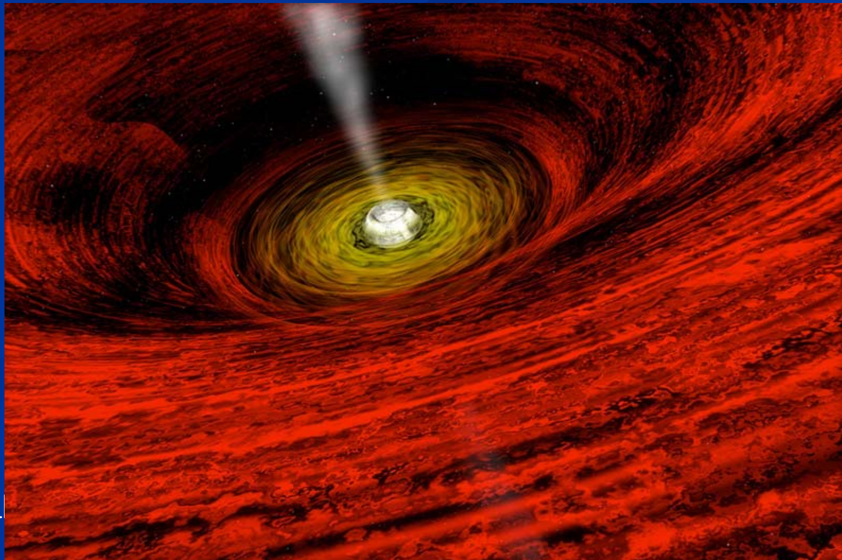
- Abramowicz, M.A. et al. 1997, ApJ, 479, 179
- Barai, P., Das, T.K. & Wiita, P.J. 2004, ApJ, 613, L49
- Das, T.K., Bilic, N. & Dasgupta, S. 2006, astro-ph/0604477
- Matsumoto, R. et al. 1984, PASJ, 36, 71
- Novikov, I.D. & Thorne, K.S. 1973, in Black Holes, 343

Thank You All

Contents

- Introduction & Motivation
  - What are we doing?
- Our system formulations
- Results
- Conclusions





Motivation

- Transonic Flow is relevant in various astrophysical situations:
  - Black Hole (BH) or Neutron Star accretion
  - Collapse / Explosion of stars
  - Solar / Stellar winds
  - Formation of protostars & galaxies
  - Interaction of supersonic galactic (or extragalactic) jets with ambient medium
- BH inner boundary condition : Supersonic flow at Event Horizon (EH)
- Far away from EH – subsonic
- Hence BH accretion is essentially transonic
  - Except cases where already supersonic initially
- Multi-transonic flow → Shock Formation

Describing Our System

- No self-gravity, no magnetic field
- Units: $G = c = M_{BH} = 1$
- Boyer-Lindquist coordinates (– + + +)
- Observer frame corotating with accreting fluid
- λ = Specific angular momentum of flow -- aligned with a
- Stationary & axisymmetric flow
- Euler & Continuity eqns :

$$\boxed{\nabla^{\mu} \Im^{\mu\nu} = 0 \quad , \quad \left(\rho v^{\mu}\right)_{;\mu} = 0}$$

- Polytropic equation of state
  - K ~ specific entropy density
  - γ = adiabatic index, n = polytropic index

$$\boxed{p = K\rho^{\gamma}}$$

$$\boxed{n = \frac{1}{\gamma - 1}}$$

Our System …

- Specific proper flow enthalpy, h
- Polytropic sound speed, a_s

$$a_s = \left(\frac{\partial p}{\partial \varepsilon}\right)_S^{1/2} = \Psi_1(p, r, \gamma) = \Psi_2(p, \rho, \gamma) \quad ; \quad a_s^{\,2}(r) = \frac{\gamma K_B T(r)}{\mu m_H}$$

- Frame dragging neglected
- Weak viscosity limit
  - Very large radial velocity close to BH → Timescale (viscous >> infall)
  - Effect of Viscosity → λ ↓ → Flow behavior as function of λ provides information on viscous transonic flow

Metric & Others

- Kerr metric in equatorial plane of BH (Novikov & Thorne 1973)

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\frac{r^2\Delta}{A}dt^2 + \frac{A}{r^2}\left(d\phi - \omega dt\right)^2 + \frac{r^2}{\Delta}dr^2 + dz^2$$

$$\Delta = r^2 - 2r + a^2$$

$$A = r^4 + r^2a^2 + 2ra^2$$

$$\omega = \frac{2ar}{A}$$

$$g_{tt} = \frac{A\omega^2}{r^2} - \frac{r^2\Delta}{A}$$

$$g_{t\phi} = -\frac{A\omega}{r^2}$$

$$g_{\phi\phi} = \frac{A}{r^2}$$

- Angular velocity, Ω

$$\Omega = \frac{u^\phi}{u^t} = -\frac{\left(g_{t\phi} + \lambda g_{tt}\right)}{\left(g_{\phi\phi} + \lambda g_{t\phi}\right)}$$

- 4th Component of velocity:

$$v_t = \left[\frac{\Delta}{\left(1-u^2\right)\left(1-\Omega\lambda\right)\left(g_{\phi\phi} + \lambda g_{t\phi}\right)}\right]^{1/2}$$

Methodology

- Solve conservation eqns. to get du/dr

$$\frac{d\varepsilon}{dr} = 0, \frac{d\dot{\Xi}}{dr} = 0.$$

$$\frac{du}{dr} = \frac{N}{D} = fn(r, u, a_s, \varepsilon, \lambda, \gamma, a)$$

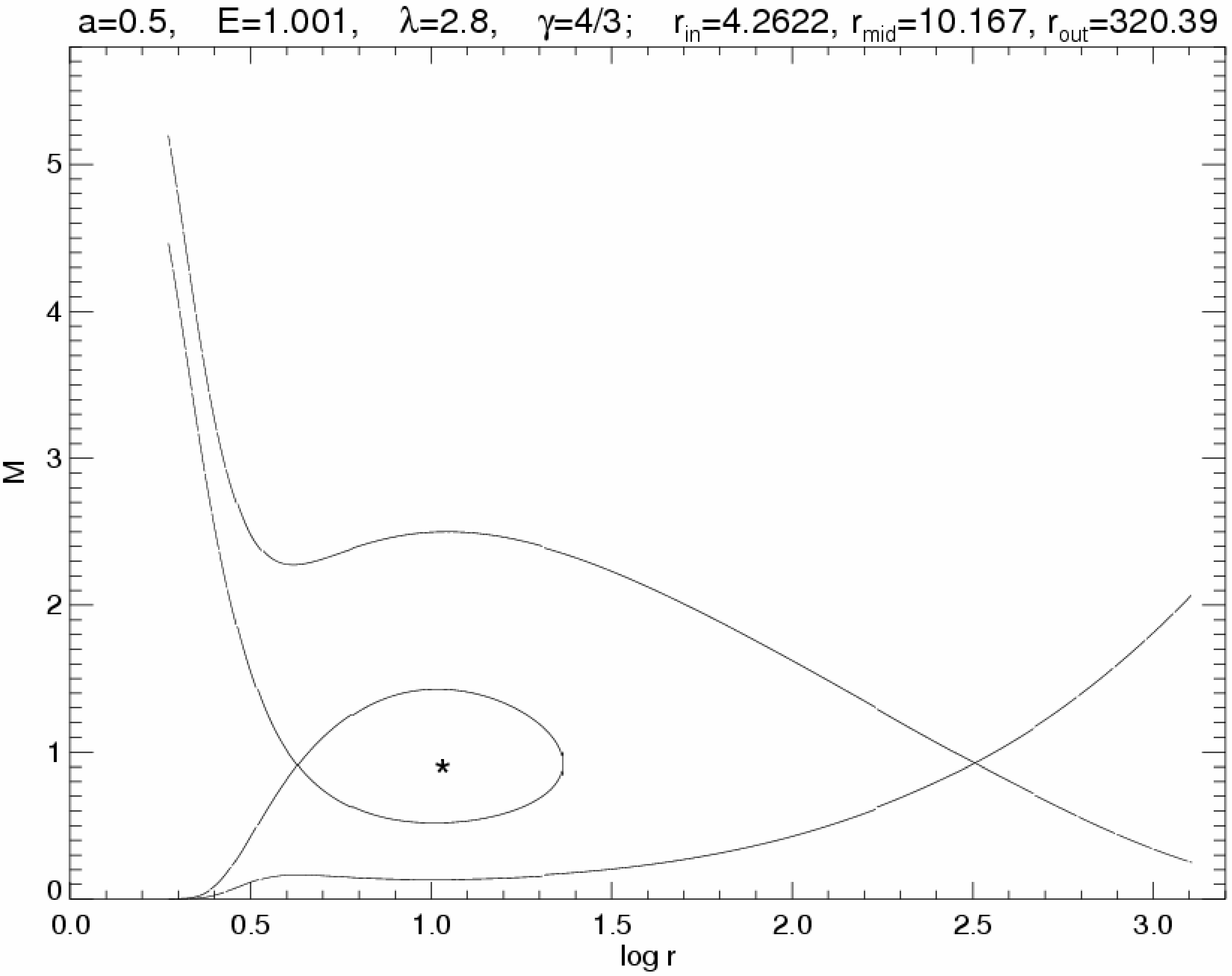
- Start from sonic point and integrate to get flow properties at any r
- By setting N=0, D=0 get sonic point quantities:
  - u_c, $a_{s|c}$
  - Quadratic eqn. for $(du/dr)_c$
- For some [ε-λ-γ-a] → get 3 sonic points on solving eqns.
  - $r_{out} > r_{mid} > r_{in}$

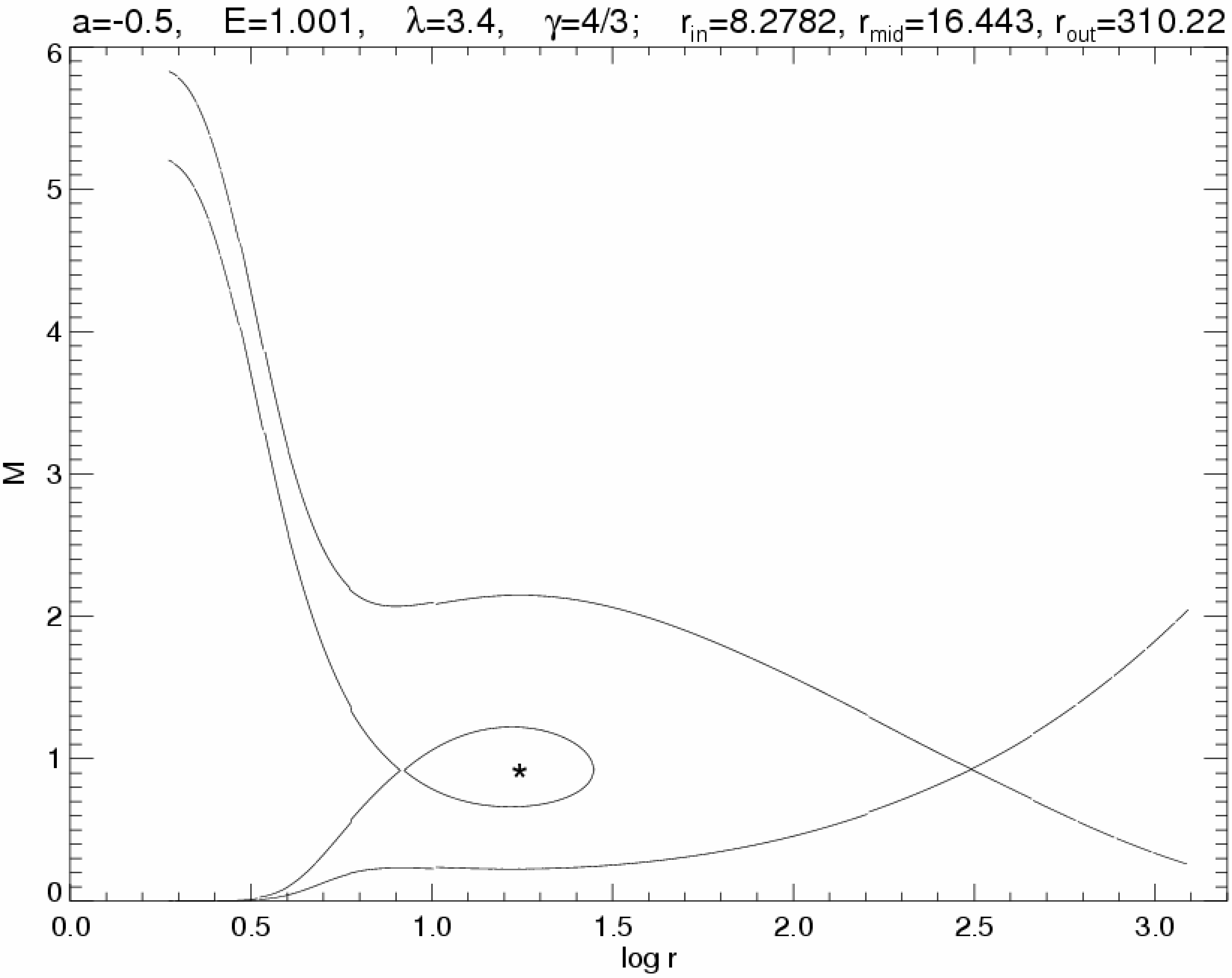
Observational Consequences

- Debate: Determining BH spin from observations
- Most popular approach: study of skew shaped fluorescent iron lines
- Our approach presents the potential to deal with this problem
  - We predict behavior of flow properties
    - For hot, low-λ prograde accretion flow & high-λ retrograde flow

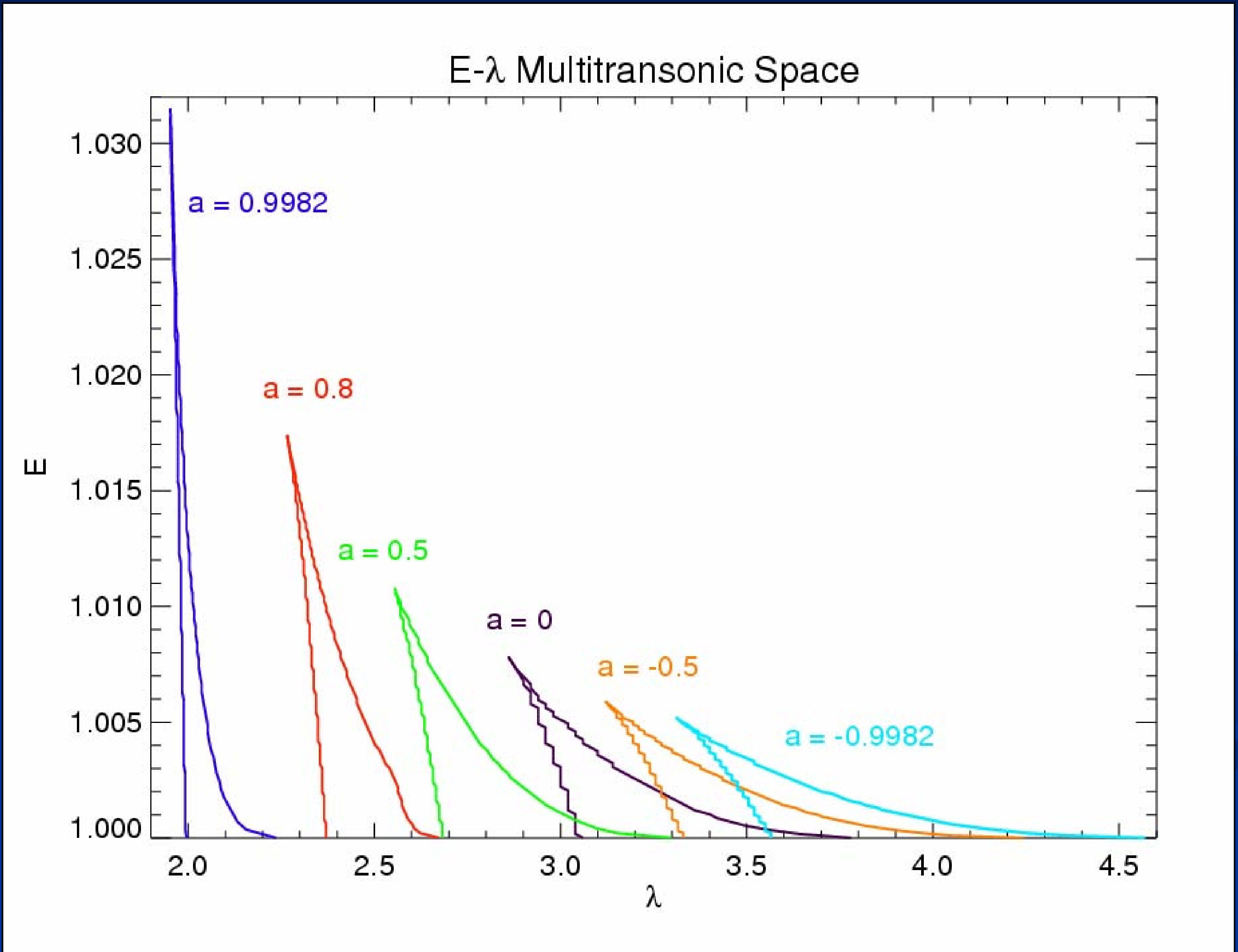
Results

- For some [ε-λ-γ-a] -- get 3 sonic points on solving eqn
  - $r_{out} > r_{mid} > r_{in}$
- r_{out}, r_{in} : X-type sonic points
- r_{mid} : O-type sonic pt (unphysical – no steady transonic soln passes thru it)
- Multitransonic Accretion: $\Xi(r_{in}) > \Xi(r_{out})$
- Multitransonic Wind : $\Xi(r_{in}) < \Xi(r_{out})$
- General astrophysical accretion → Flow thru r_{out}
- Flow thru r_{in} possible only in case of a shock
  - If supersonic flow thru r_{out} is perturbed to produce entropy = $[\Xi(r_{in}) - \Xi(r_{out})]$, it joins subsonic flow thru r_{in} forming a standing shock
  - Shock details from GR Rankine-Hugoniot conditions


a=0.5, E=1.001, λ=2.8, γ=4/3; r_in=4.2622, r_mid=10.167, r_out=320.39
M
0
1
2
3
4
5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
log r
*


a=-0.5, E=1.001, λ=3.4, γ=4/3; r_in=8.2782, r_mid=16.443, r_out=310.22
M
0
1
2
3
4
5
6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
log r
*

E-λ Multi-Transonic Space for different *a*

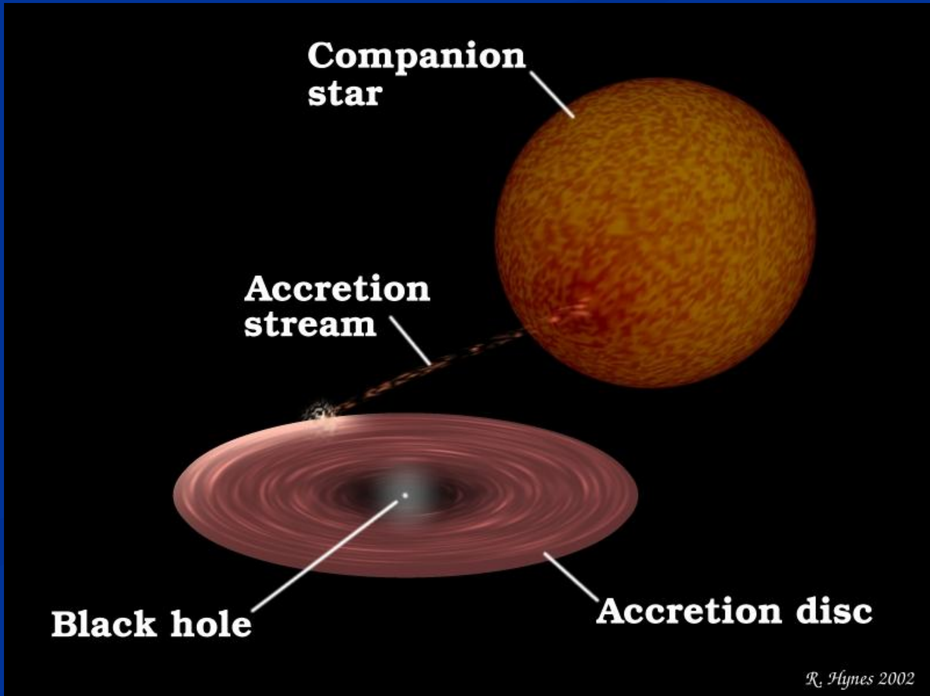


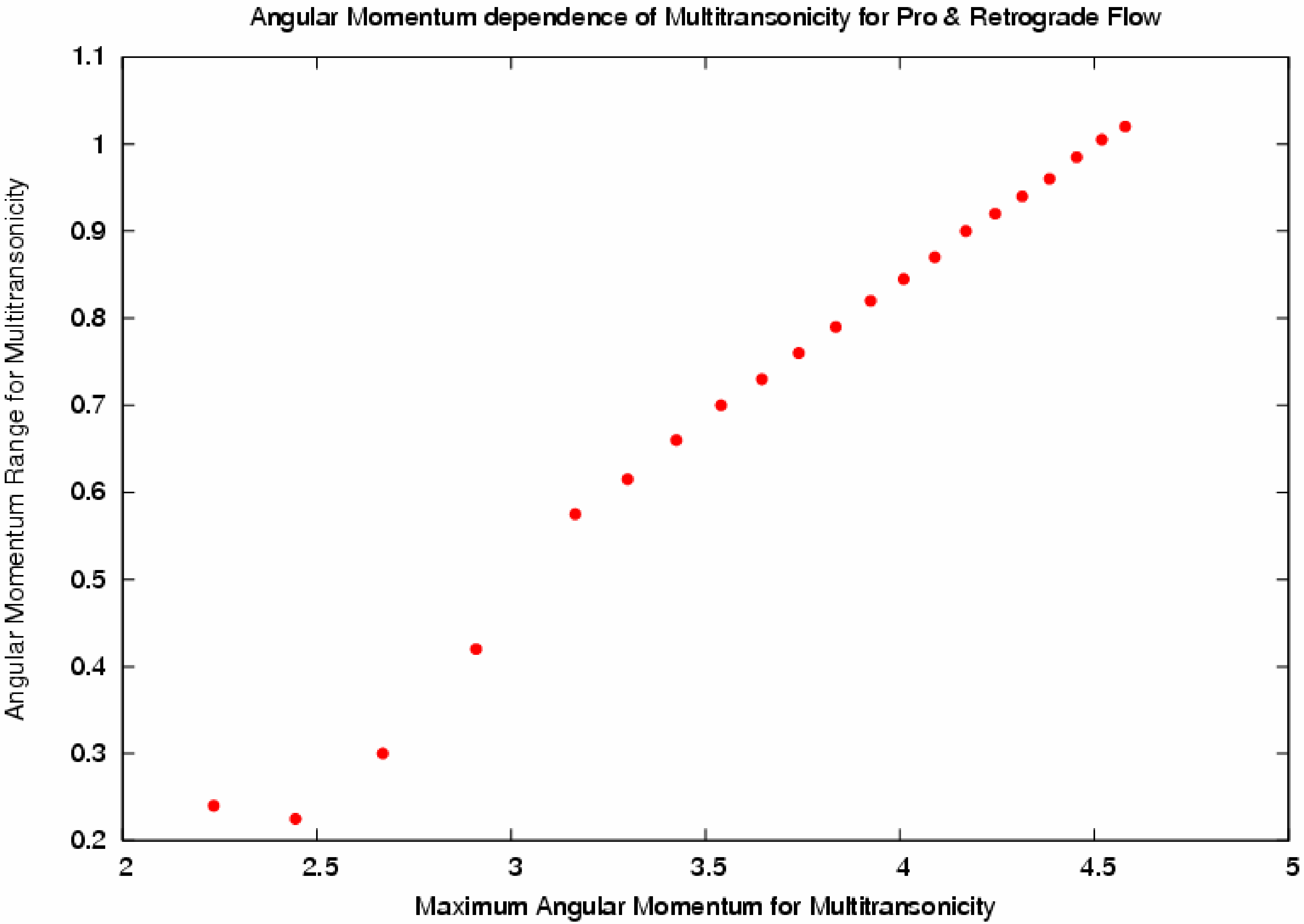
Angular Momentum

- Weakly rotating flows, found in several physical situations:
  - Detached binary systems fed by accretion from OB stellar winds
  - Semidetached low-mass non-magnetic binaries
  - Supermassive BHs fed by accretion from slowly rotating central stellar clusters
  - Turbulence in standard Keplerian accretion disk

We found:

- At higher values of angular momentum multi-transonicity is more common for retrograde flow





Angular Momentum dependence of Multitransonicity for Pro & Retrograde Flow

Analog Hawking Radiation

(Das, Bilic & Dasgupta, 2006)

- Hawking Temperature

$$\boxed{T_H = \frac{\hbar c^3}{8\pi K_B G M_{BH}}}$$

- Analog Hawking Temperature

$$\boxed{T_{AH} = \frac{\hbar}{4\pi K_B}\left[\frac{1}{c_s}\frac{du_\perp^2}{d\eta}\right]_{\text{Acoustic Horizon}}}$$

- Ratio

$$\boxed{\tau = \frac{T_{AH}}{T_H}}$$

Variation of the ratio of analog to actual Hawking temperature with black hole spin