The Universe in a Box: Simulations of Large Scale Structure in Cosmology

Seminar

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Outline

- Introduction
- Current Cosmological Scenario
- Simulations: Procedures and Results
- The Millennium Simulation
- Summary

Introduction

- Cosmological principle
  - Universe is Homogeneous and Isotropic on large scales (> 100 Mpc)
  - Laws of science are universal
- Structure formation in small scales (< 10-100 Mpc) by gravitational self organization
- Our Universe
  - Started from a very dense, hot initial state **(Big Bang)**
  - Had an early period of **Inflation**
  - Cold dark matter dominated
  - Having accelerated expansion now due to dark energy
- Such a model has been conclusively verified in the last few years → the era of **Precision Cosmology**

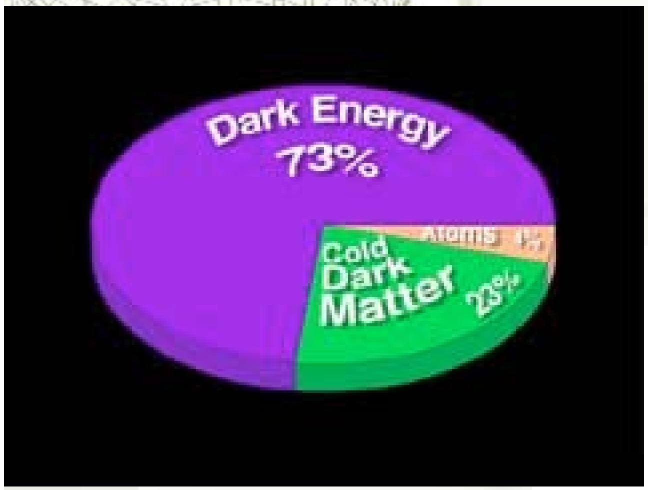
Units

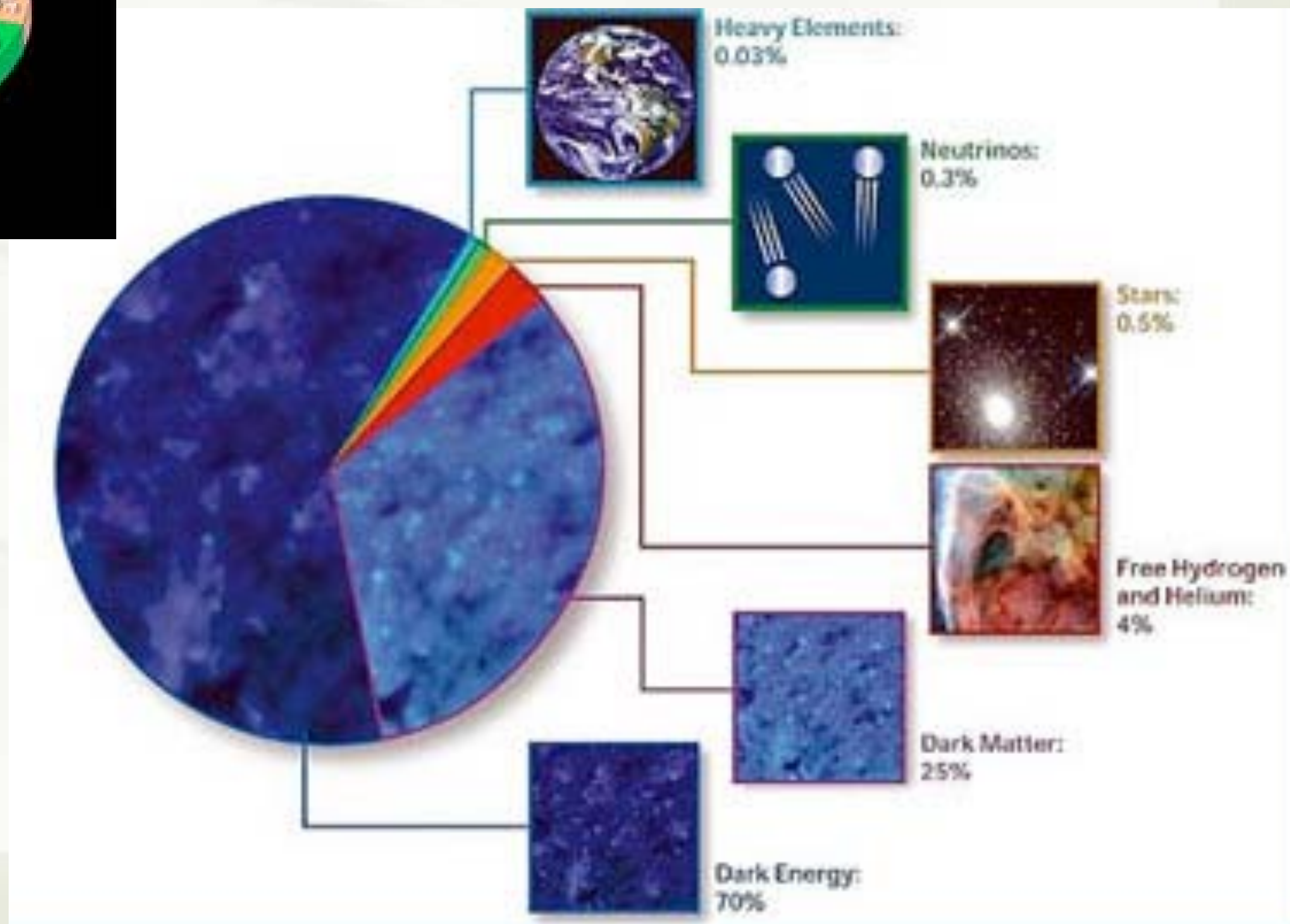
- 1 pc (parsec)
  = 3.26 light-years
  = 3.09×10^{18} cm
  = 2.06×10^{5} a.u.

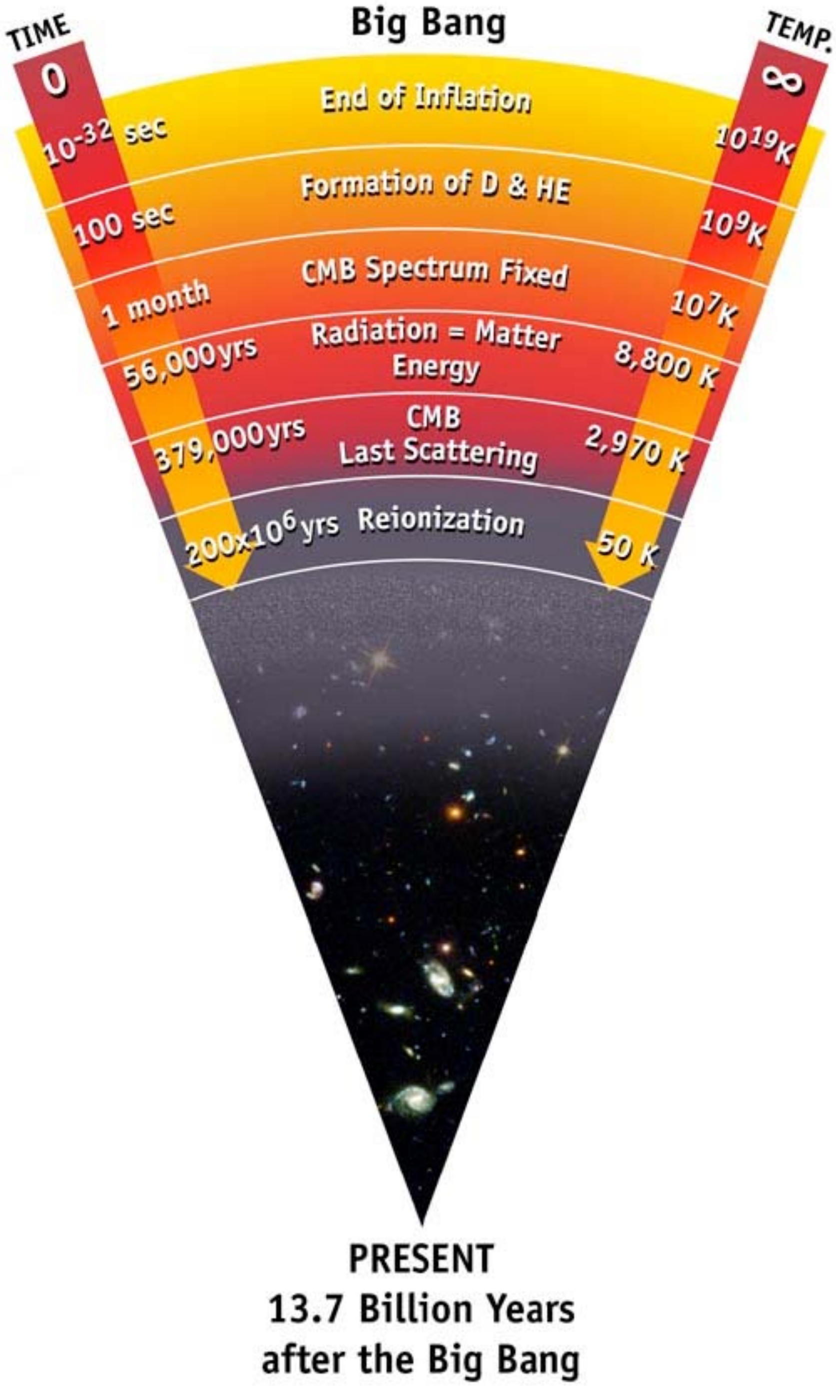
- Mass of Sun = $M_{\odot} = 1.99 \times 10^{33}$ g

- Luminosity of Sun = $L_{\odot} = 3.83 \times 10^{33}$ erg/s

Cosmological Mass/Energy Budget







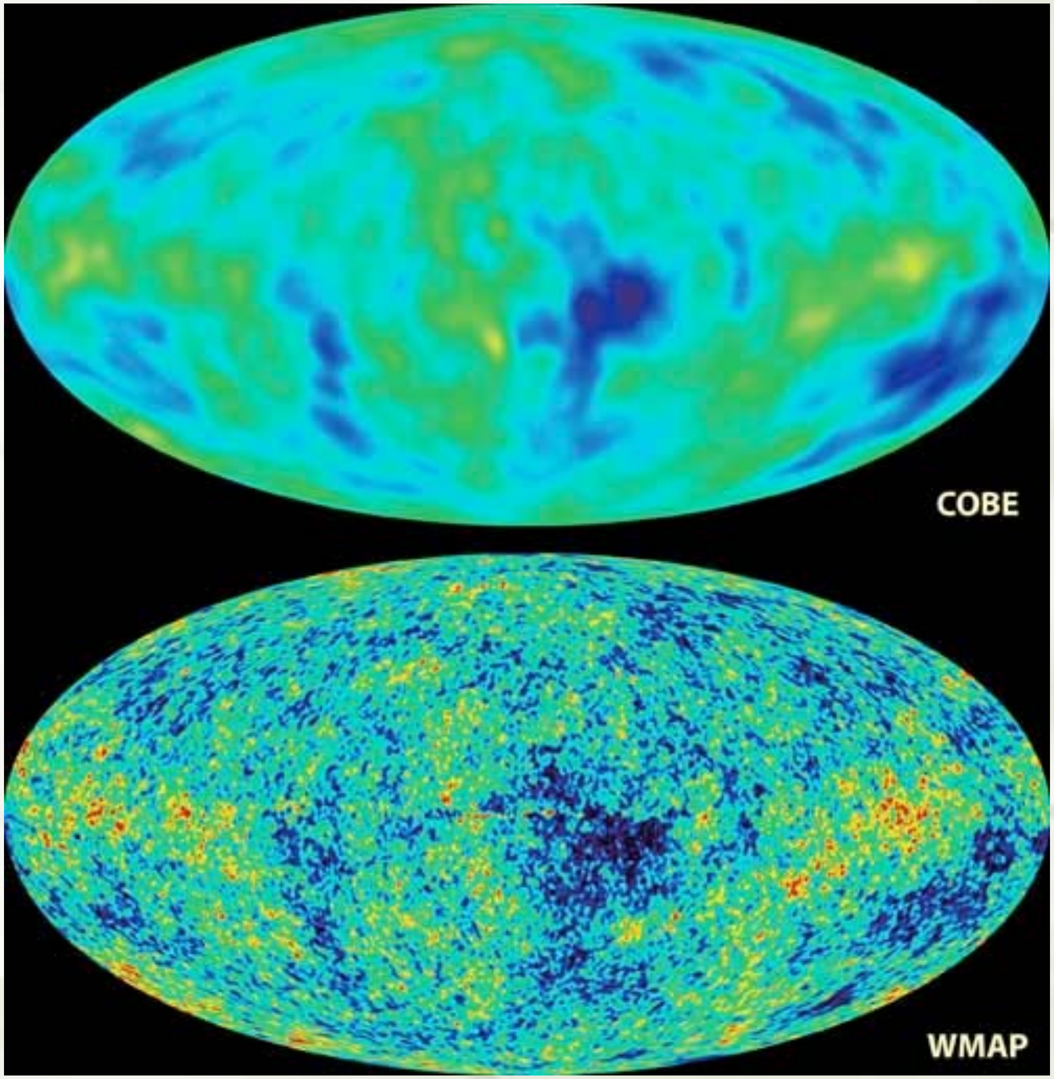
Age of our Universe, from the Big Bang

$= 13.7$ Gyrs

$= 13.7 \times 10^9$ yrs

Cosmic Microwave Background Radiation (CMBR)

- Uniform faint background of microwaves throughout space
- Radiation left over from an early very hot and dense epoch
- Baby picture of the Universe
  - 300,000 years after the Big Bang
- Blackbody: 2.7 °K



Simulate the Universe on Computers

- Why?
  - Best way to study origin and evolution of galaxies
- Bridge gap between observations of early epochs
  - Oldest stars: 10^9 after Big Bang
  - CMBR: radiation from 3×10^5 yrs after
- The purpose of a cosmological simulation is to model the growth of structures in the Universe
- Simulations are the only experiments to verify theories of the origin and evolution of the Universe
- Can run many experiments over cosmic ages in practical times

Hubble Expansion

- Galaxies moving away from us, further galaxies move away faster
- Distance between distant galaxies increases with time

→ The Universe (space itself) is expanding

- Hubble Law
  - H_0 = Hubble's Constant

$$v = H_0 r$$

- This recession affects the light emitted by the distant galaxies, stretching the wavelengths of emitted photons due to the Doppler shift effect

Cosmological Redshift

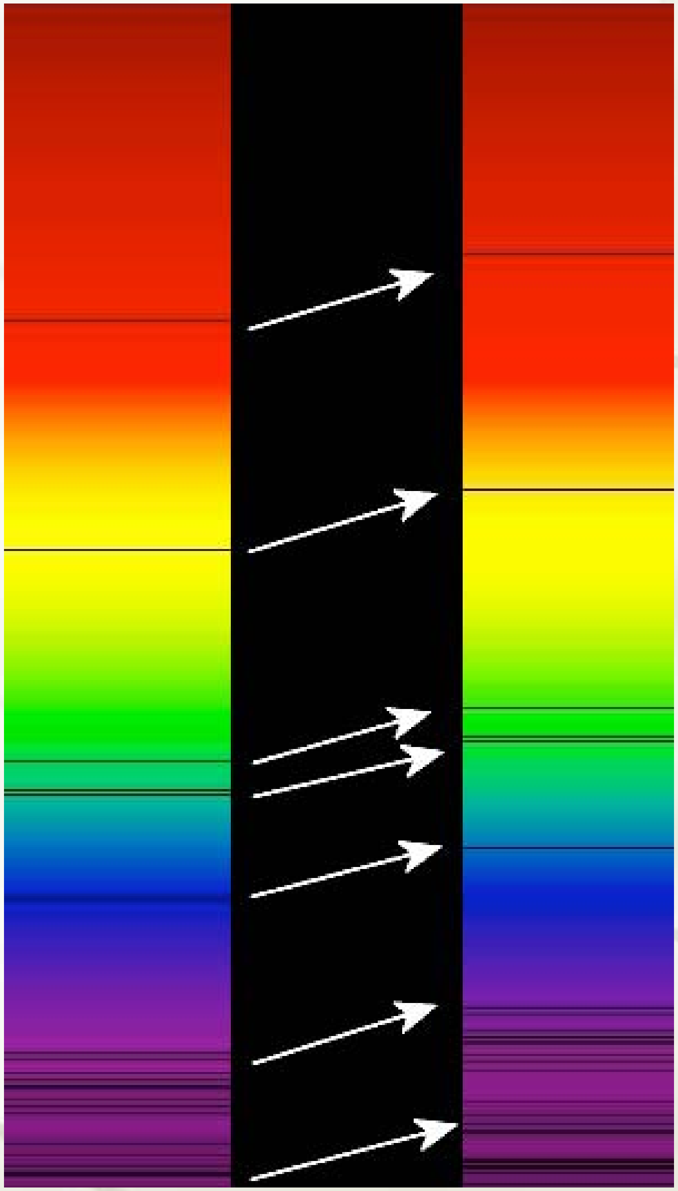
$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

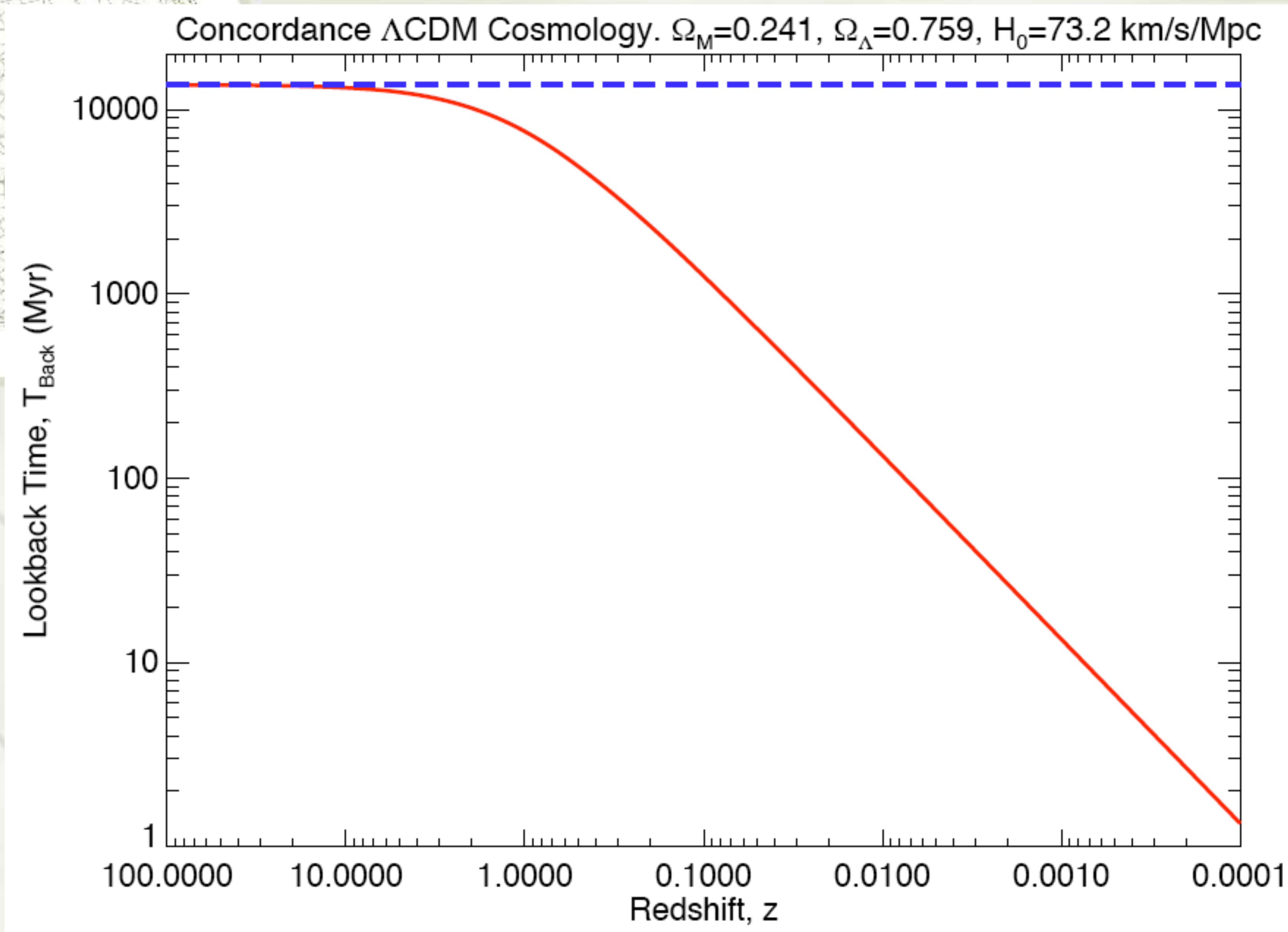
$$1 + z = \frac{a(z=0)}{a(z)}$$

- Stretching of light waves traveling through space
- Caused by expansion of the Universe
- Determined by observing an object's spectrum

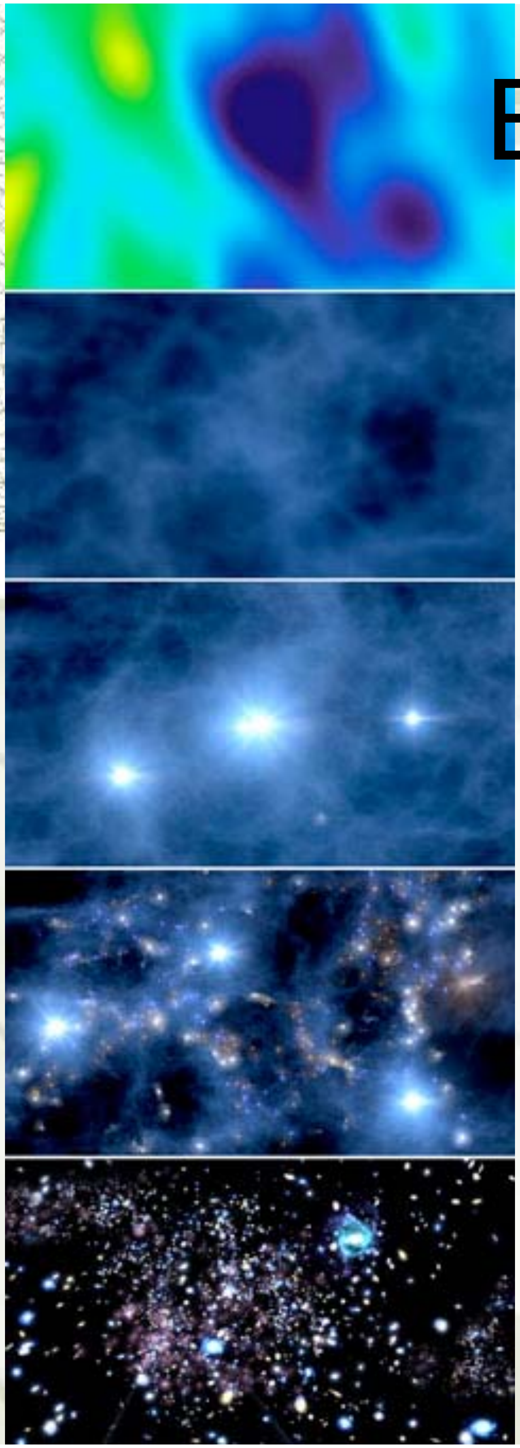
- Comoving (line-of-sight) distance:

$$D_{comov}(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}}$$





Concordance ΛCDM Cosmology. Ω_M=0.241, Ω_Λ=0.759, H_0=73.2 km/s/Mpc
Lookback Time, T_Back (Myr)
10000
1000
100
10
1
100.0000
10.0000
1.0000
0.1000
0.0100
0.0010
0.0001
Redshift, z



Evolution of our Universe

• Our (observable) Universe expanded from a very dense, hot initial state (*Big Bang*)

• Structures formed in small scales by self-gravity

• Evolution — from the early stage of density fluctuations into what we observe today: galaxies, clusters, and **We** – the people on planet Earth

Large Scale Structure Formation

ΛCDM Universe ⇒ CDM (cold dark matter) + Cosmological constant / Dark energy

- Quantum fluctuations shortly after the Big Bang give rise to primordial density perturbations in the early radiation and matter density fields
- Inflation expands the perturbations, which form the seeds for later growth
- Structures grow from gravitational clumping of matter from these initial density fluctuations
- Main forces driving evolution
  - Gravity : affects dark matter and baryons
  - Gas dynamics : only baryons

How do we Simulate? -- Basics

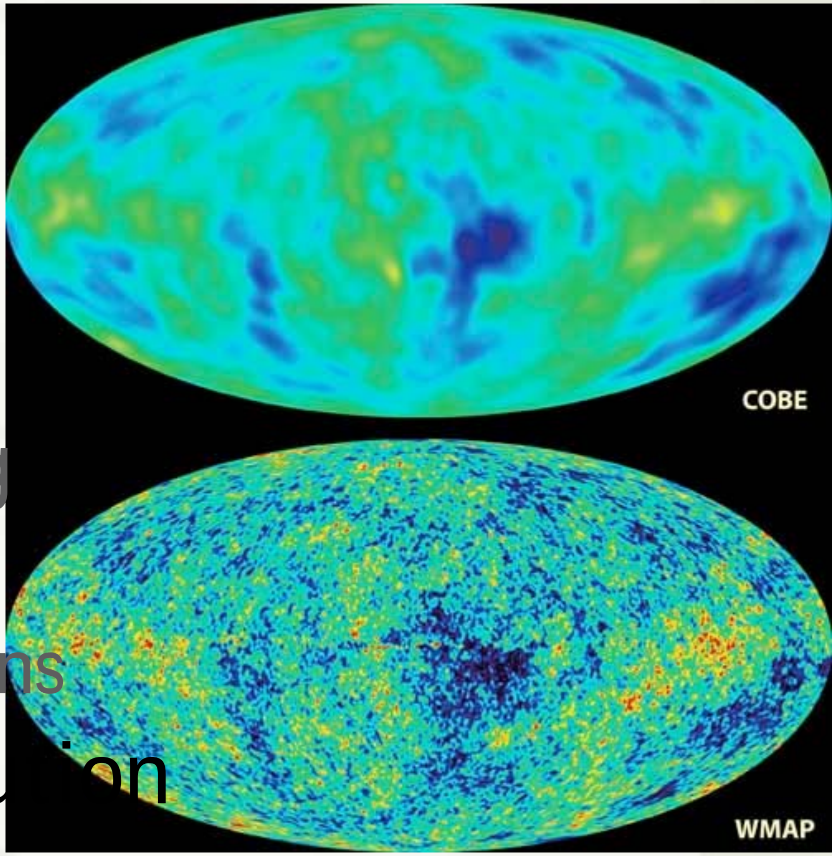
- A computational box ⇔ the Universe
- Particles in the box ⇔ matter in the Universe
- Assume: can model Universe as few massive particles each representing about 10^{12} - 10^{15} $M_\odot$
  - Treat particles as average density over a finite volume (not point particles)
- Physical laws governing the nature determines the dynamics of the particles in the box
- Cosmological simulation → tool to investigate the evolution of millions of particles
- To run these simulations we need supercomputers
  - High speed and high processing power
  - Weeks to months of time

How do we Simulate? -- Procedures

- Must follow the non-linear evolution of the early density fields using numerical methods
- A twofold process:
  - Generate the initial conditions for the density fluctuations, which are then used as an input to the evolution code
  - Evolve the system by following the trajectories of particles under their mutual gravity
    - Should get the final particle distribution consistent with observations of the Universe

Simulation -- 1st step

- Assume a cosmological model
  - ΛCDM parameters constrained by various observations:
    - CMBR (WMAP)
    - SN
    - Galaxy clusters
    - Gravitational lensing
- Set initial conditions using
  - Cosmological parameters
  - Gaussian density fluctuations
- Underlying matter distribution



Current Cosmological Parameters (from WMAP)

- $H_0 = 71\ (+4 - 3)$ km/s/Mpc

- Density parameter, $\Omega_{TOT} = \rho/\rho_C = 1.00 \pm 0.02$
- $\Omega_\Lambda = 0.73 \pm 0.04 = \Lambda\ /\ 3H_0^2$
- $\Omega_M = 0.27 \pm 0.04 = \rho_M/\rho_C$
- $\Omega_{Baryon} = 0.044 \pm 0.004$
  - n_b (baryon density) $= (2.5 \pm 0.1) \times 10^{-7}\ cm^{-3}$

$$\rho_C = \frac{3H_0^2}{8\pi G}$$

- $t_{Universe} = 13.7 \pm 0.2$ Gyr
- $T_{decoupling} = (379 \pm 8)$ kyr
- $T_{reionization} = 180\ (+220 - 80)$ Myr

Cosmological Simulation

- Dark matter (dissipation-less, collision-less)
  - Gravity-only
  - Particle *N*-body method
- Baryon / Gas evolution
  - Gravity + Hydrodynamics
- Add source & sink terms
  - Cooling and heating of gas
  - Star formation and stellar feedback
- Numerically integrate dynamical equations
- Evolve box from z=100 up to z=0

Simulation Volume

- The computational box has Hubble expansion just like the real Universe
  - Always encompasses the same mass

- The expansion is taken out from computations, s.t. the box appears static
- Coordinate system that expands (or co-moves) with the Universe (the comoving coordinates) is used

Comoving Coordinate, x(t)

- Coordinate moving with the Hubble flow
- Distance between 2 points measured now, at *z = 0*
- Remains constant for objects in Hubble expansion

$$x(t) = \frac{r(t)}{a(t)}$$

$$r = \int_{t_{emit}}^{t_{now}} \frac{c\,dt'}{a(t')}$$

- *r(t)* = Proper Distance
- *a(t)* = Scale factor of the Universe

N-body (collision-less)

- N number of particles
  - Dark matter (+ Baryons)
- Gravitational interactions only
- Equations of particle motion

$$\frac{d\vec{x}}{dt} = \vec{u}$$

$$\frac{d\vec{u}}{dt} = -\nabla\Phi$$

- Poisson's equation

$$\nabla^2\Phi = 4\pi G\rho$$

- During late evolution the relevant velocities are non-relativistic

N-body

- Each evolution timestep has 2 parts:
  - Gravity-solver : solve Poisson's equation & calculate particle accelerations
  - Time integrator : calculate particle positions & velocities
- Different codes, run in parallel processors
  - PP : Particle-Particle
  - PM : Particle-Mesh
  - P3M : PM+PP
  - AP3M : Adaptive P3M
  - ART : Adaptive Refinement Tree
  - TreePM

Hydrodynamics (baryons only)

- Collisional particles with ideal gas properties
- Continuity

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

- Euler

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right)\vec{u} = -\vec{\nabla}\Phi - \frac{\vec{\nabla} P}{\rho}$$

- Energy

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \left[(E + P)\vec{u}\right] = -\rho \vec{u} \cdot \vec{\nabla}\Phi$$

- Equation of state, $\varepsilon = f(\rho, P)$
  - Ideal gas

$$\varepsilon = \frac{1}{(\gamma - 1)} \frac{P}{\rho}$$

  - Polytropic

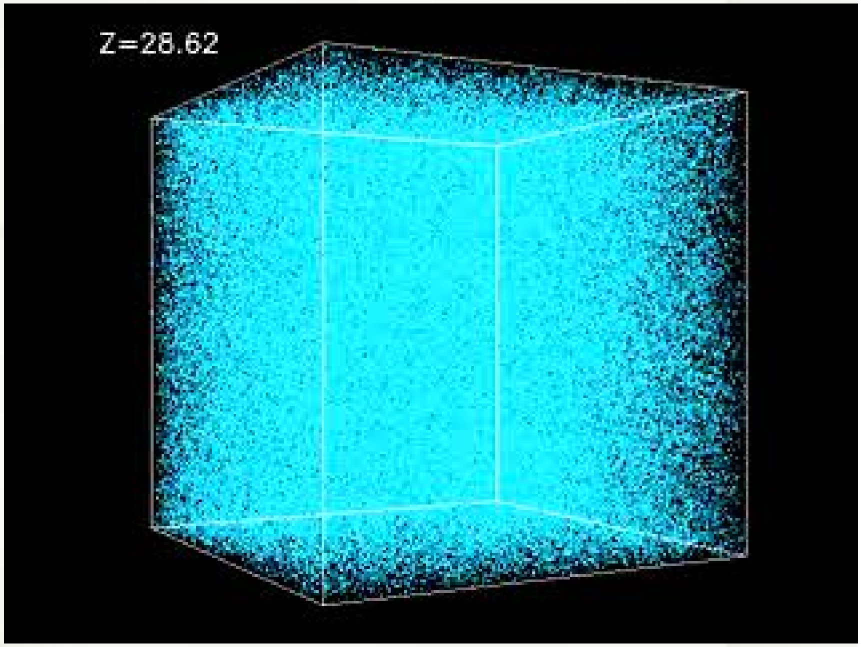
$$P = K\rho^{\gamma}$$

Successes of Cosmological Simulations

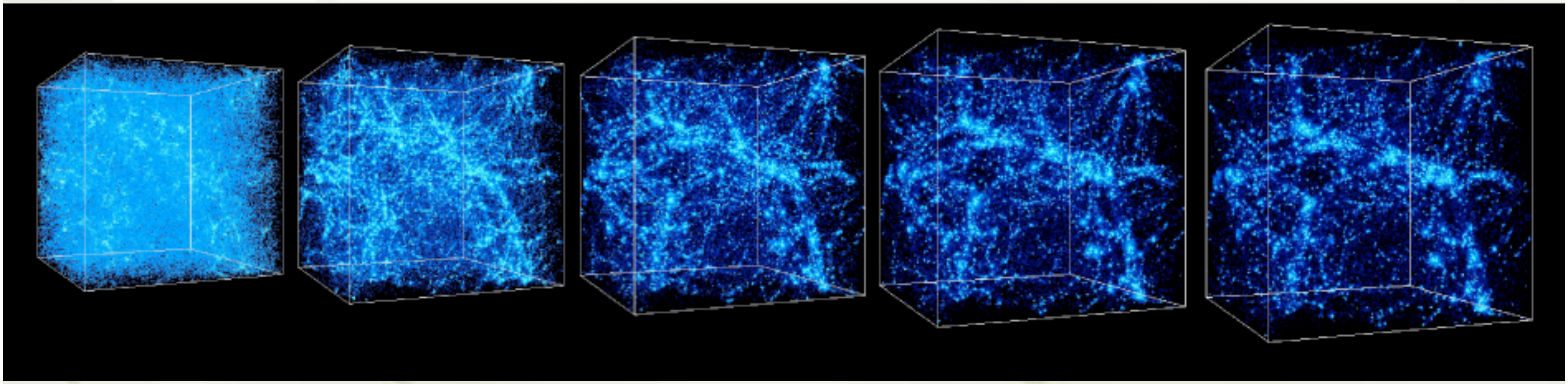
- Can reproduce the distribution and structure of galaxies in very large scales as seen in observations
- On small scales
  - Stellar evolution, black holes, star clusters
- Distribution of matter in the Universe
  - How the matter collapses via gravitational forces into filaments. Galaxies form in these filaments
- Galaxy clustering at all z (z~0, z~1, z~4-5) observed in large scale surveys is well reproduced

Large-scale filaments

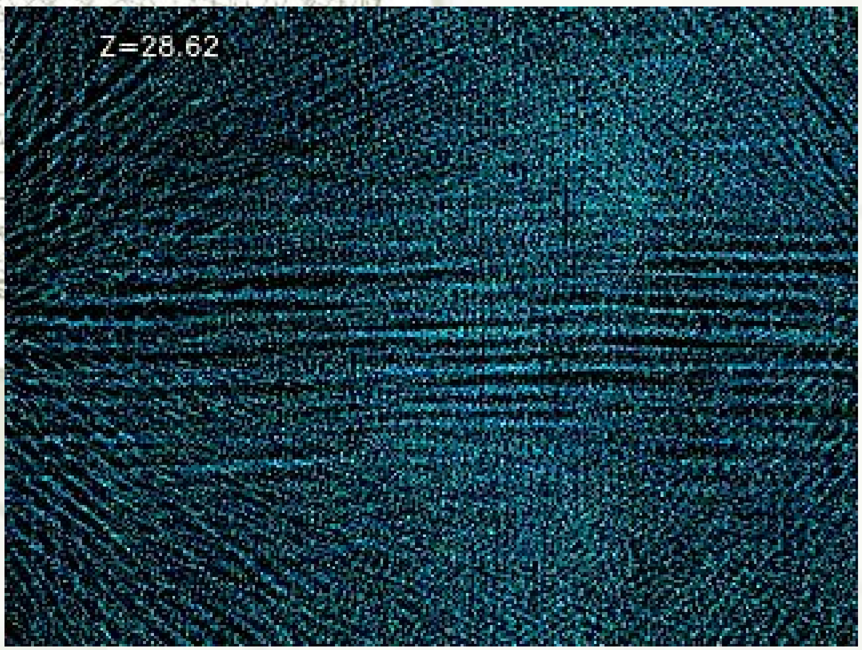
- A 43 Mpc box
- From z=30 to z=0

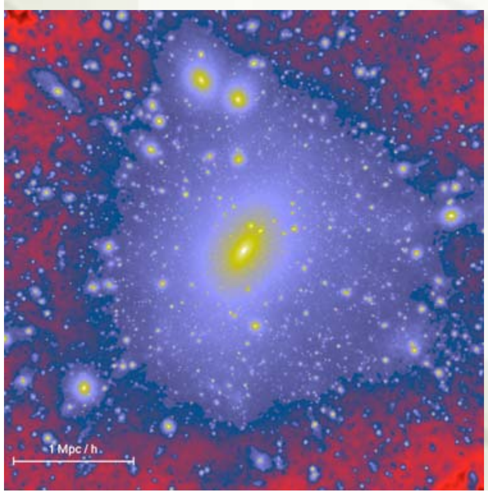


- The frames below show formation of large scale structure from z=10 to the present



Formation of a Galaxy Cluster

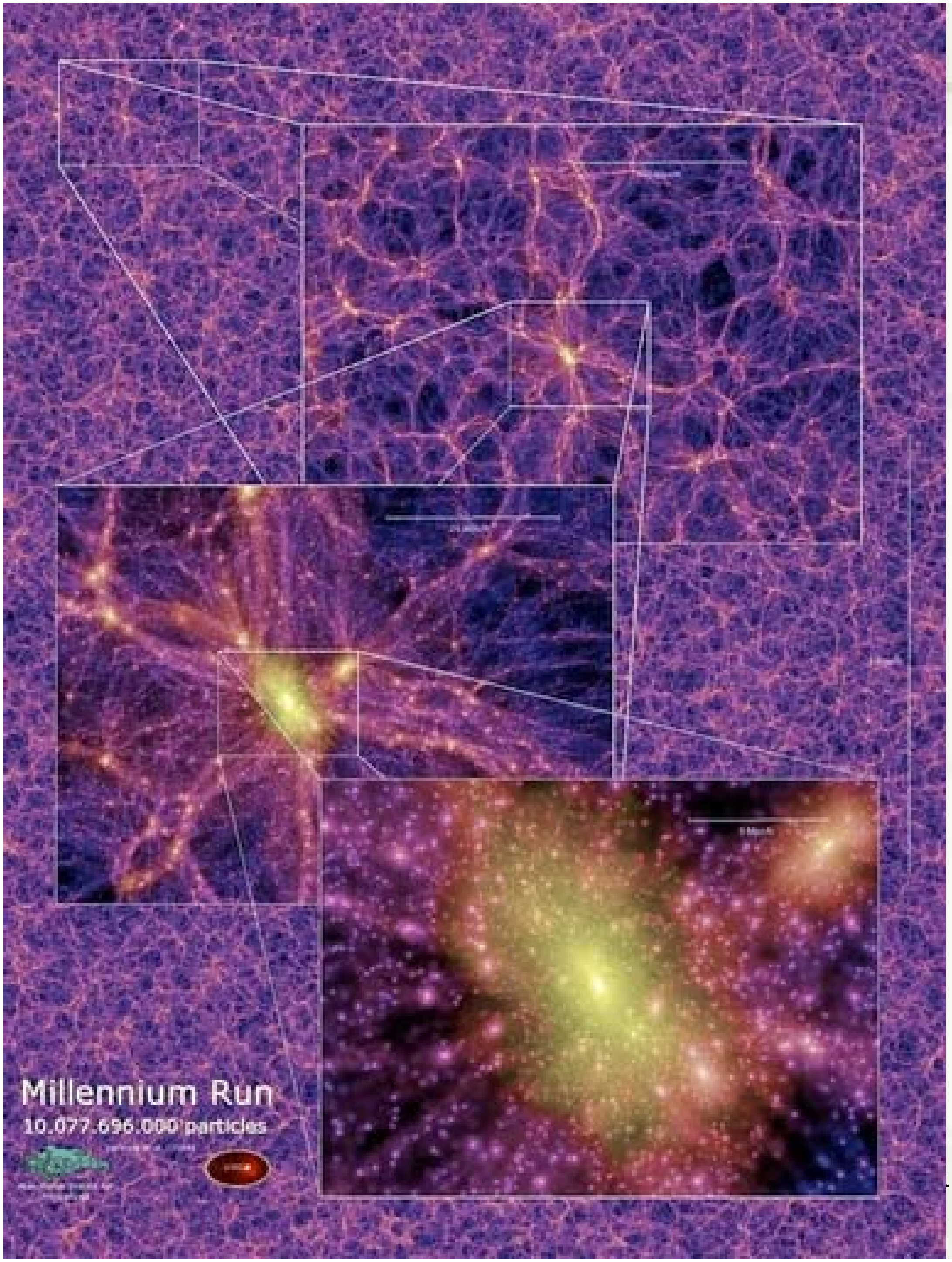




- 4.3 Mpc region of box
- The formation of a cluster proceeds hierarchically
  - Small-mass objects form first at z>5
  - Quickly grow in size and violently merge with each other, creating increasingly larger and larger system
- This galactic "cannibalism" continues up to z=0

The Millennium Simulation

- One of the largest cosmological N-body run
  - Dark matter only
  - Gravity only, no hydrodynamics
- Done by the Virgo Consortium
  - Springel et al. 2005, Nature, 435, 629
- Box comoving side = 500 h^{-1} Mpc
- $2160^3 \sim 10^{10}$ particles
- Particle mass = 8.6 x 10^8 h^{-1} $M_\odot$
- Comoving softening length, ε = 5 h^{-1} kpc
- From z=127 to z=0
- Took 28 days on 512 CPUs

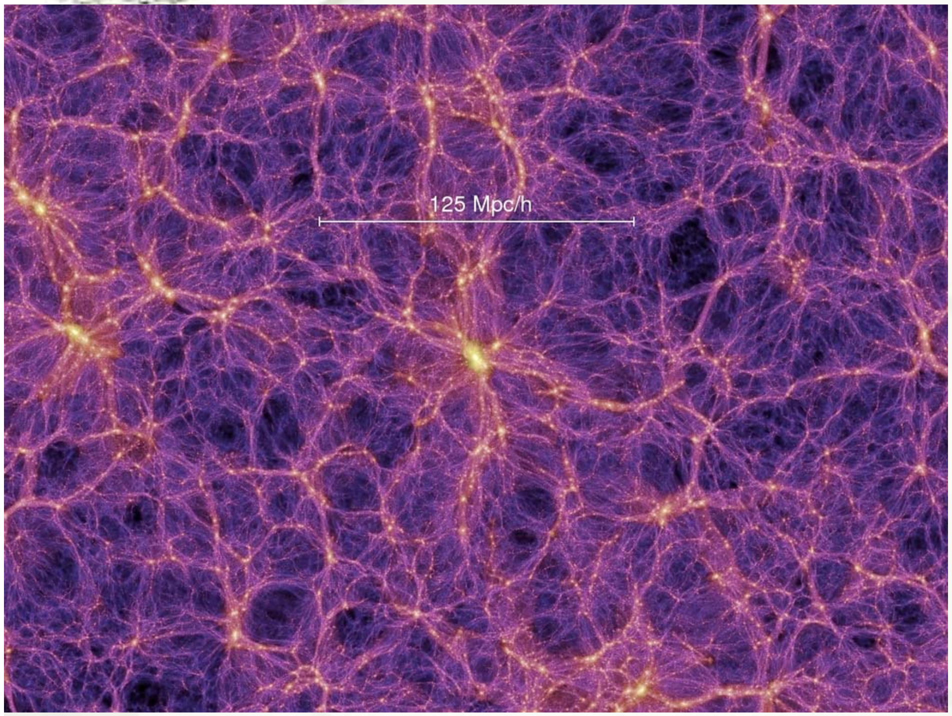


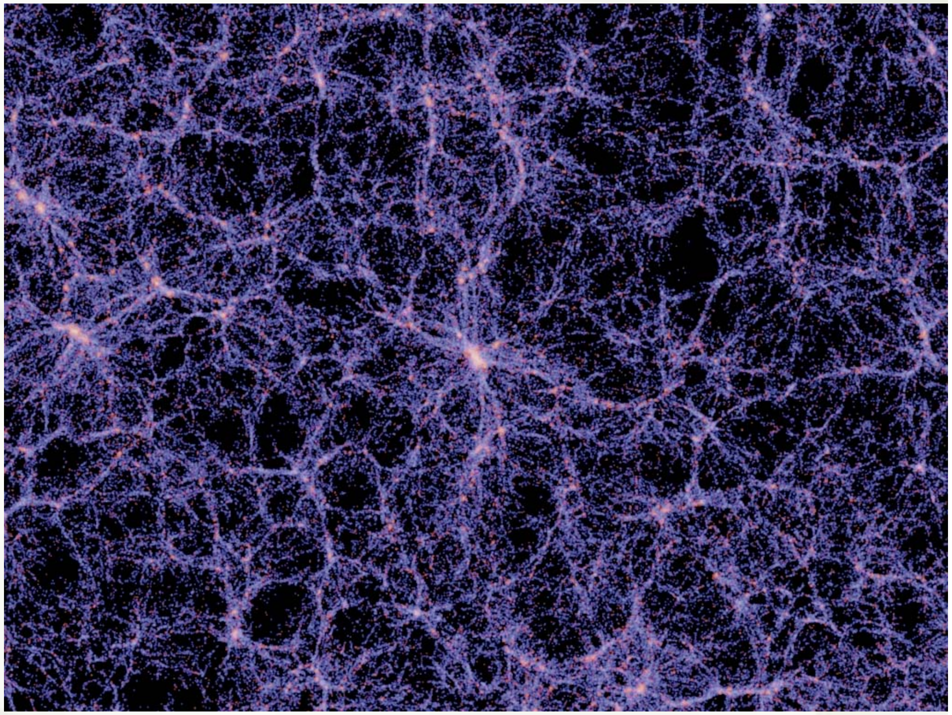
- A projected density field for a 15 Mpc/h thick slice of the z=0 output
- The overlaid panels zoom in by factors of 4 in each case, enlarging the regions indicated by the white squares

The Large-scale Matter Distribution in the Millennium Simulation

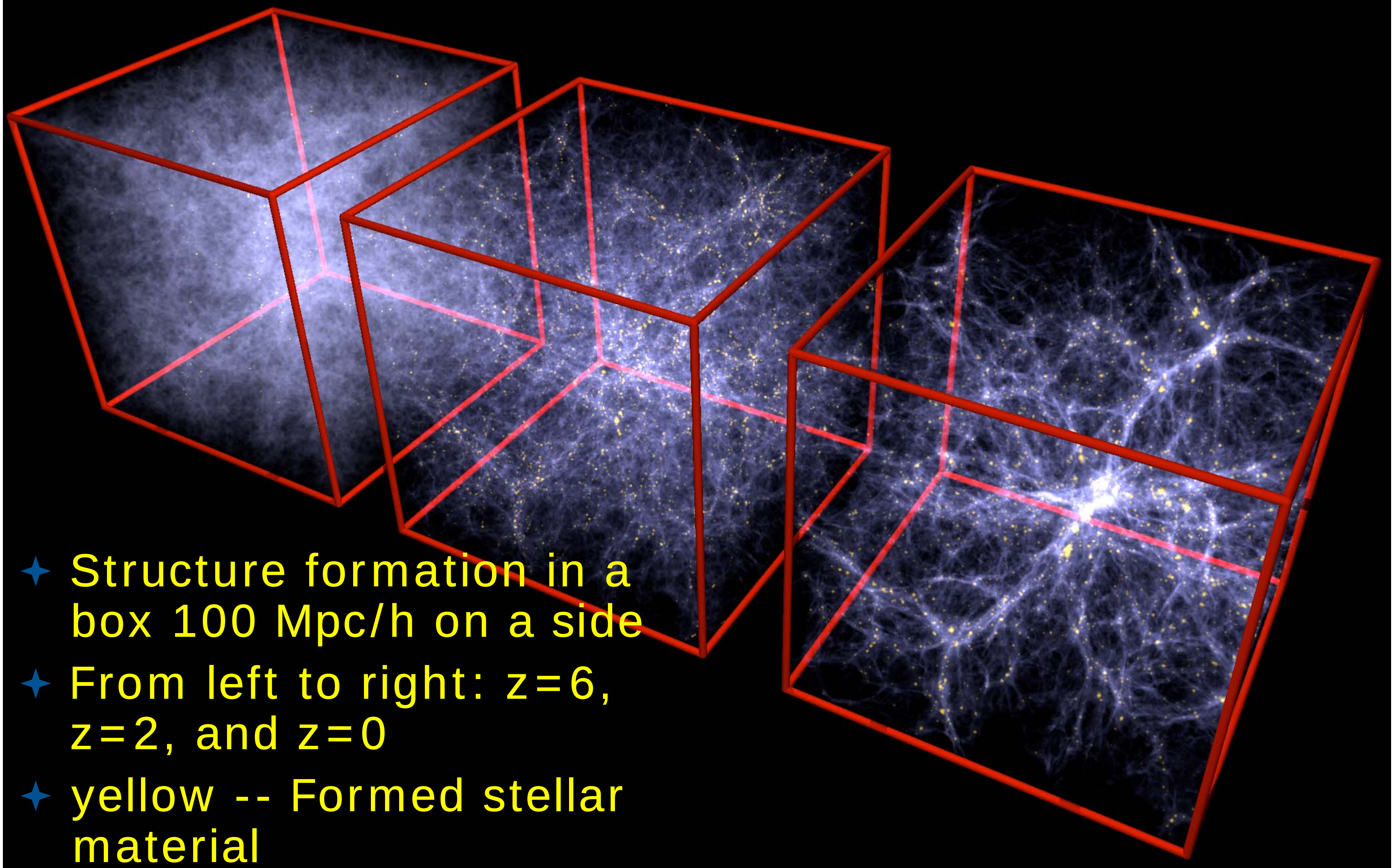
Dark Matter

Galaxies



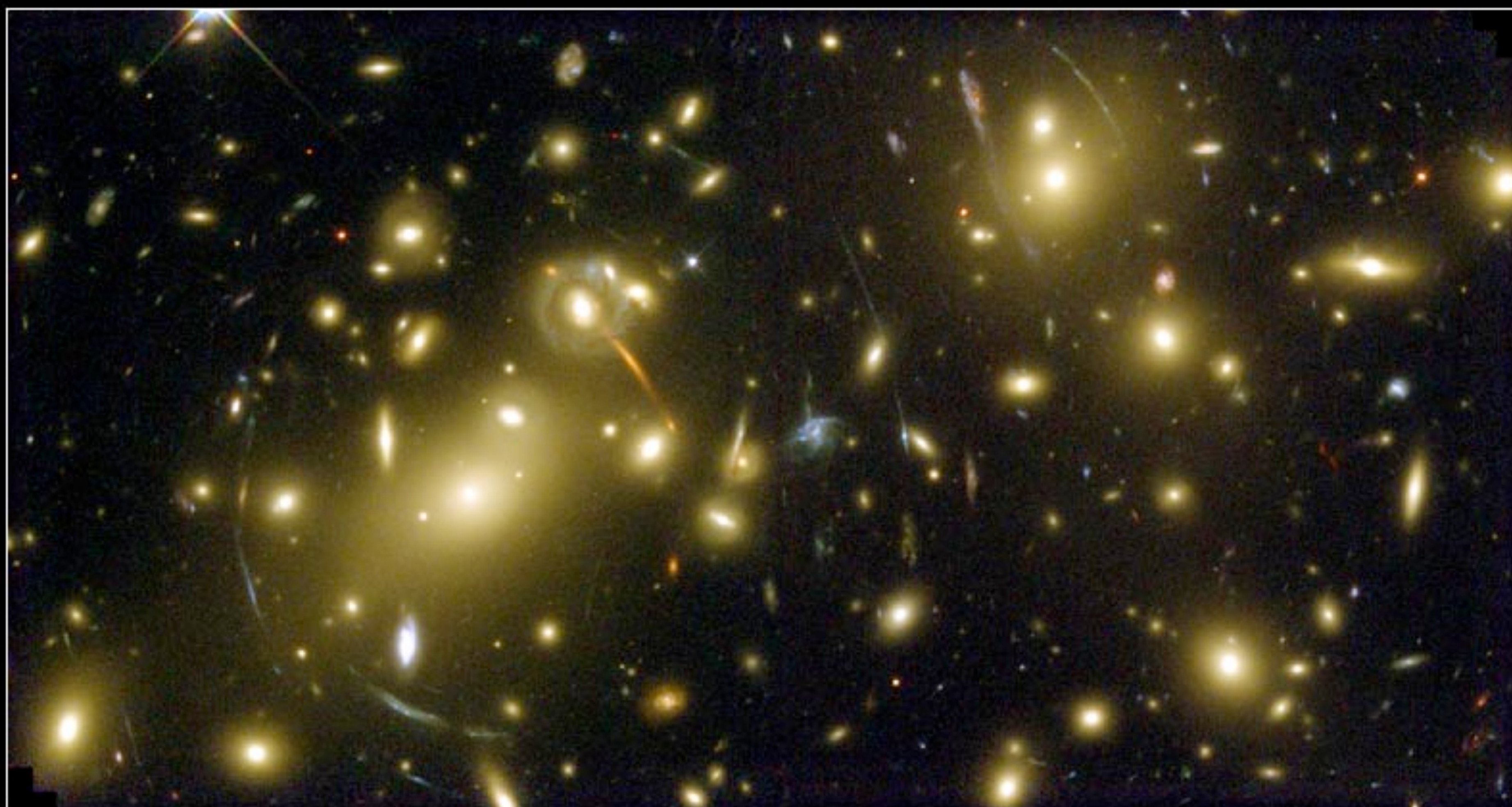


Cosmic structure in 3D



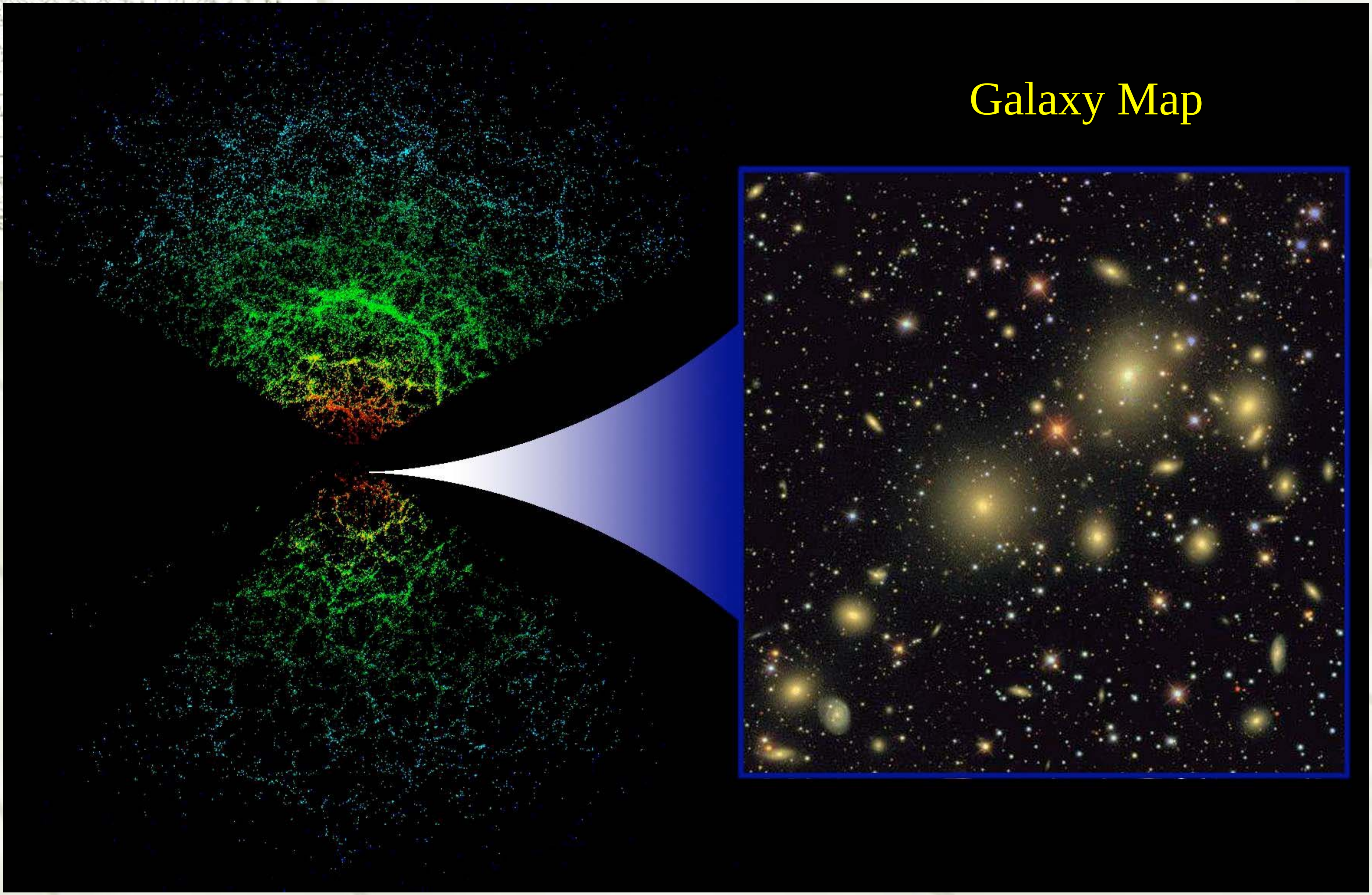
- Structure formation in a box 100 Mpc/h on a side
- From left to right: z=6, z=2, and z=0
- yellow -- Formed stellar material

Resolving galaxy sub-structures at the same time is tricky ...

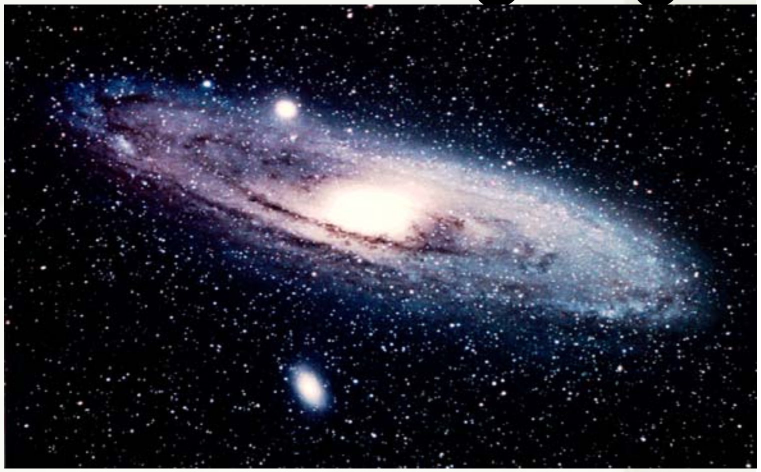


Galaxy Cluster Abell 2218 **HST** • WFPC2
NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Sloan Digital Sky Survey (SDSS)



Predicting right internal structure in galaxies is challenging!



- Galaxy interior
  - Star & gas distribution
  - Star formation & feedback
- Why?
  - Need very large resolution and dynamic range
- Star formation needs resolution of 10 pc (10^6 resolution in a 10 Mpc box)
- Scale height of star forming gas disk in Milky Way is 100 pc

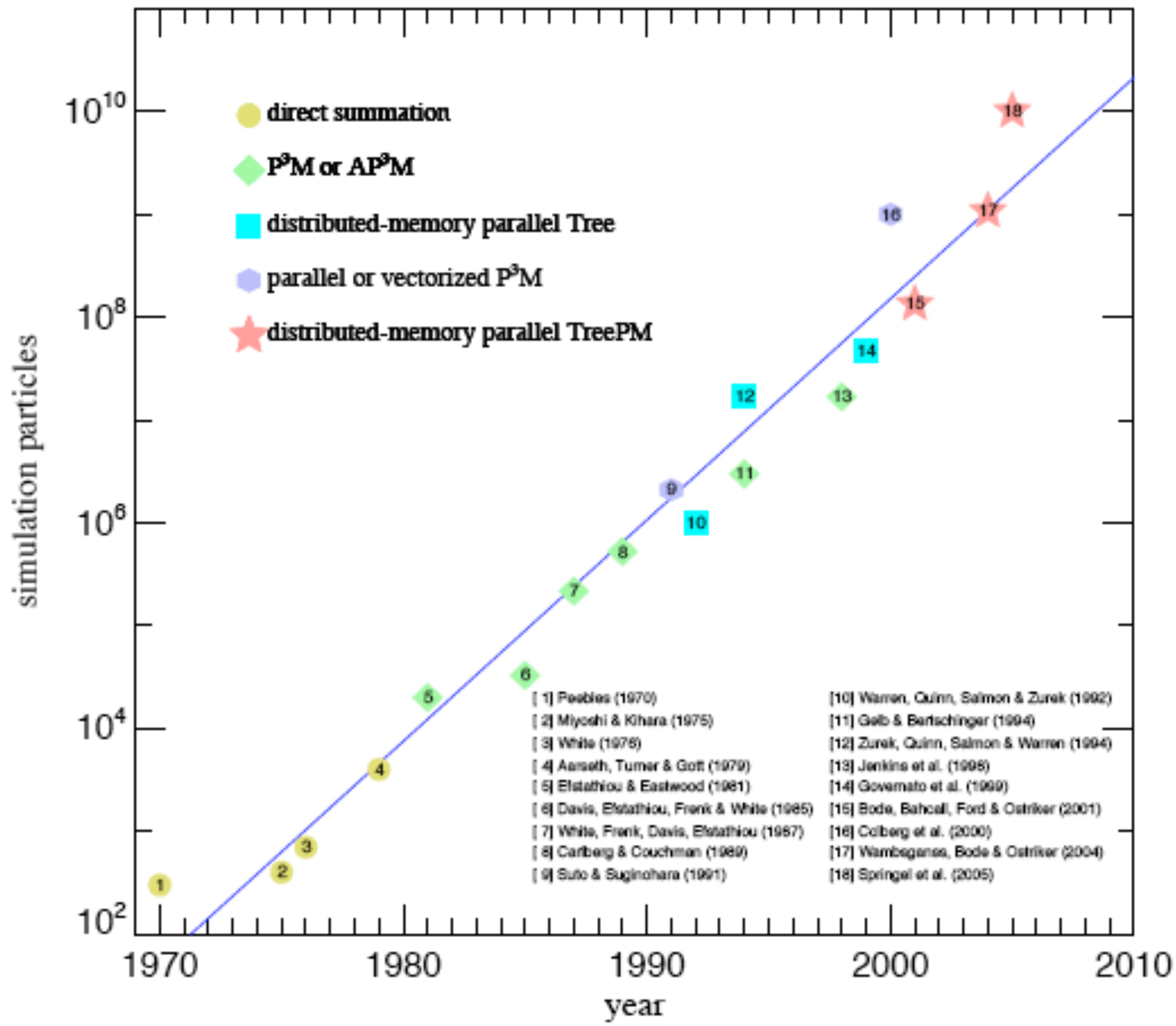


Fig. 7 Moore's empirical law shows that the computing power typically doubles every 18 months. This figure shows the size of N-body simulations as a function of their running date. Clearly, specially recently, the improvement in the algorithms allowed the simulation to grow faster than the improvement of the underlying CPU power. Kindly provided by Volker Springel.

N-body simulations have grown rapidly in size over last decades

Research in India

- HRI - Jasjeet Bagla
- IUCAA - T. Padmanabhan
- RRI
- IIA

Summary

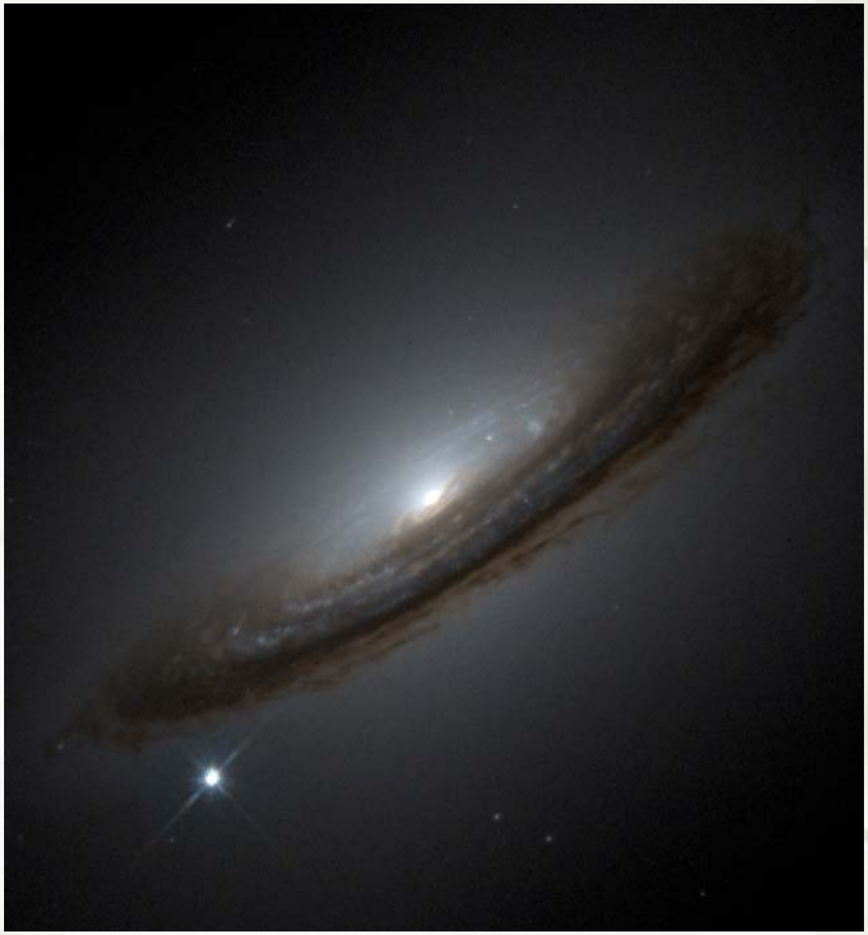
- Observations have given a well constrained cosmological picture
  - Flat, dark-energy & dark-matter dominated
  - Primordial density fluctuations
- Evolve a computational box using non-relativistic equations for particle dynamics
  - Reproduce the observed large-scale structures in the Universe
- Several challenges and open questions remain for future research

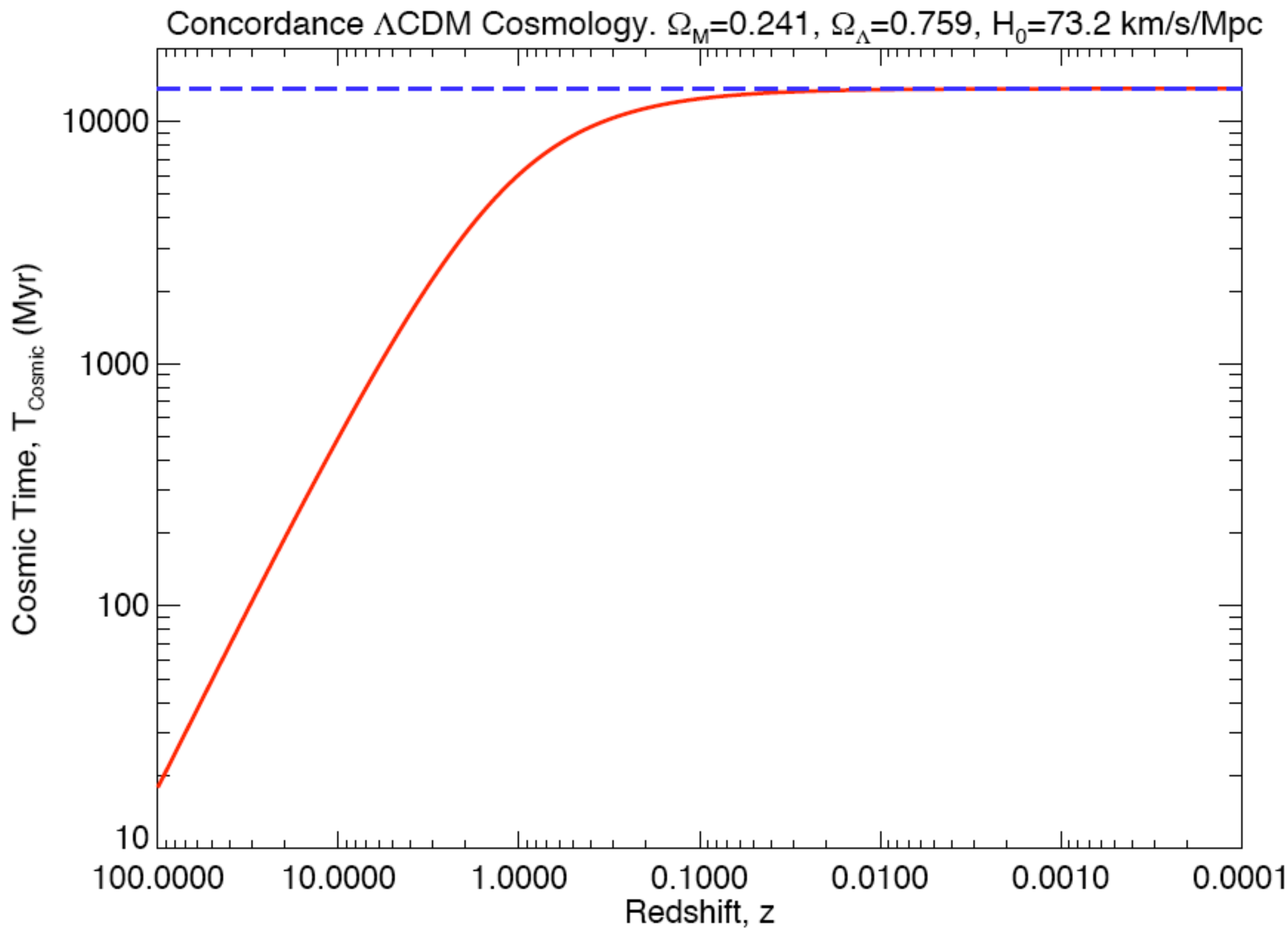
Extra Slides

General Funda or More Explanation

SN Type Ia

- Giant star accreting onto white dwarf
- Standard candles
- Compare observed luminosity with predicted
- Too faint
- From z → expansion rate of distant SN < closer SN





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Cosmic Time, T_Cosmic (Myr)
10000
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0.0010
0.0001
Redshift, z