# The Universe in a Box: Simulations of Large Scale Structure in Cosmology

Seminar

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# *Outline*

- Introduction
- Current Cosmological Scenario
- Simulations: Procedures and Results
- The Millennium Simulation
- Summary

# *Introduction*

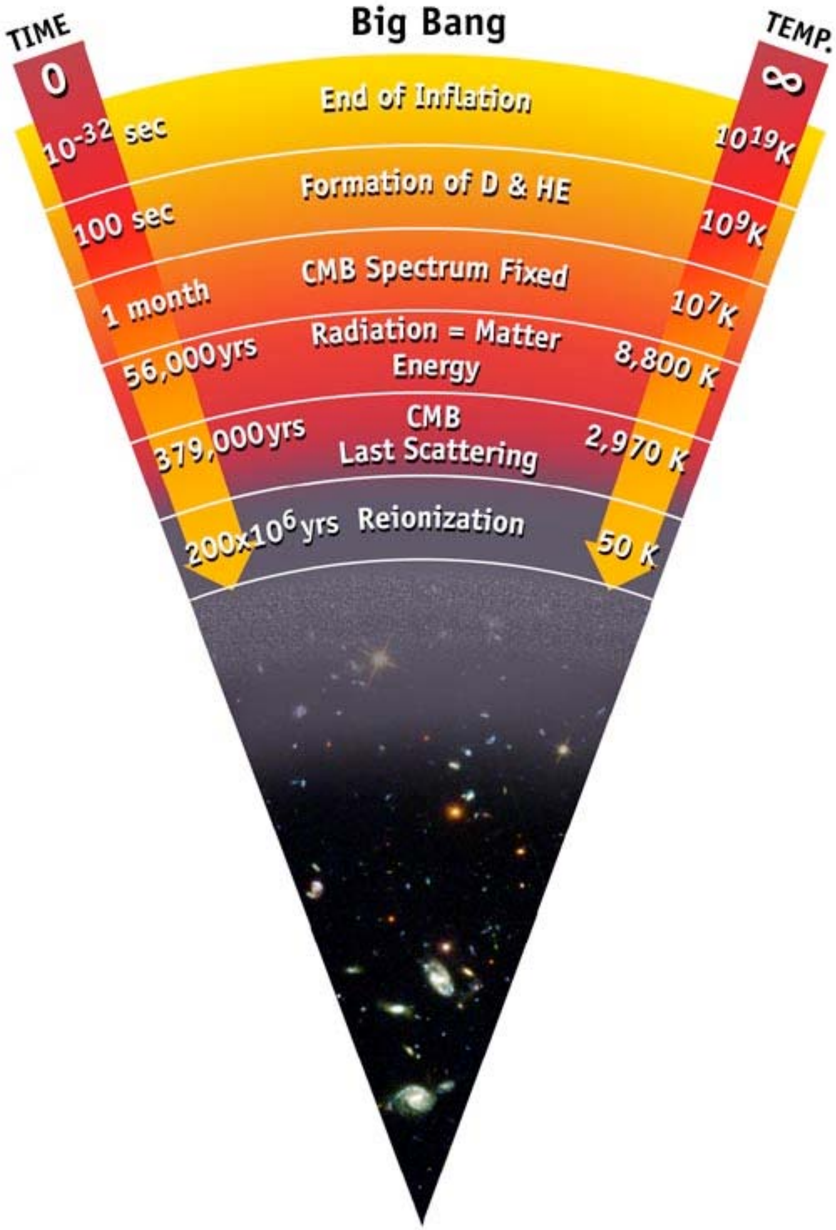
- Cosmological principle
  - Universe is Homogeneous and Isotropic on large scales (> 100 Mpc)
  - Laws of science are universal
- Structure formation in small scales (< 10-100 Mpc) by gravitational self organization
- Our Universe
  - Started from a very dense, hot initial state **(Big Bang)**
  - Had an early period of **Inflation**
  - Cold dark matter dominated
  - Having accelerated expansion now due to dark energy
- Such a model has been conclusively verified in the last few years → the era of **Precision Cosmology**

# *Units*

- 1 pc (parsec)
  = 3.26 light-years
  = $3.09 \times 10^{18}$ cm
  = $2.06 \times 10^{5}$ a.u.

- Mass of Sun = $M_{\odot} = 1.99 \times 10^{33}$ g

- Luminosity of Sun = $L_{\odot} = 3.83 \times 10^{33}$ erg/s



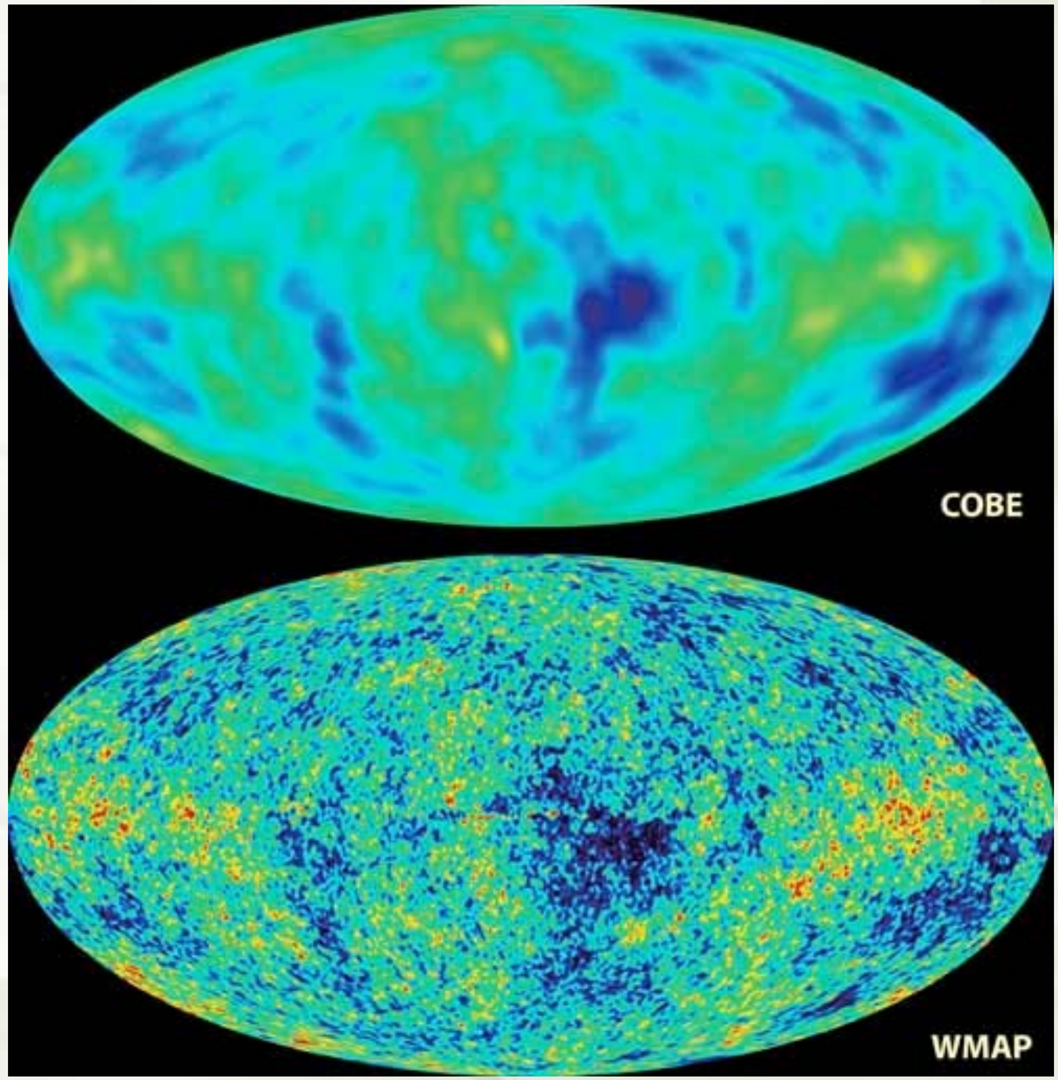

Age of our Universe, from the Big Bang

= 13.7 Gyrs

= $13.7 \times 10^9$ yrs

# Cosmic Microwave Background Radiation (CMBR)

- Radiation left over from an early very hot and dense epoch
  - 300,000 years after the Big Bang
- Blackbody: 2.7 °K
- Anisotropies in the CMB give us the nature of initial density fluctuations



# *Simulate the Universe on Computers*

- Why?
  - Best way to study origin and evolution of galaxies
- Bridge gap between observations of early epochs
  - Oldest stars: $10^9$ yrs after Big Bang (HST)
  - CMBR: radiation from $3 \times 10^5$ yrs after
- The purpose of a cosmological simulation is to model the growth of structures in the Universe
- Simulations are the only experiments to verify theories of the origin and evolution of the Universe
- Can run many experiments over cosmic ages in practical times

# *Hubble Expansion*

- Galaxies moving away from us, further galaxies move away faster

→ The Universe (space itself) is expanding

- Hubble Law
  - $H_0$ = Hubble's Constant

$$v = H_0 r$$

- This recession affects the light emitted by the distant galaxies, stretching the wavelengths of emitted photons due to the Doppler shift effect

# Cosmological Redshift

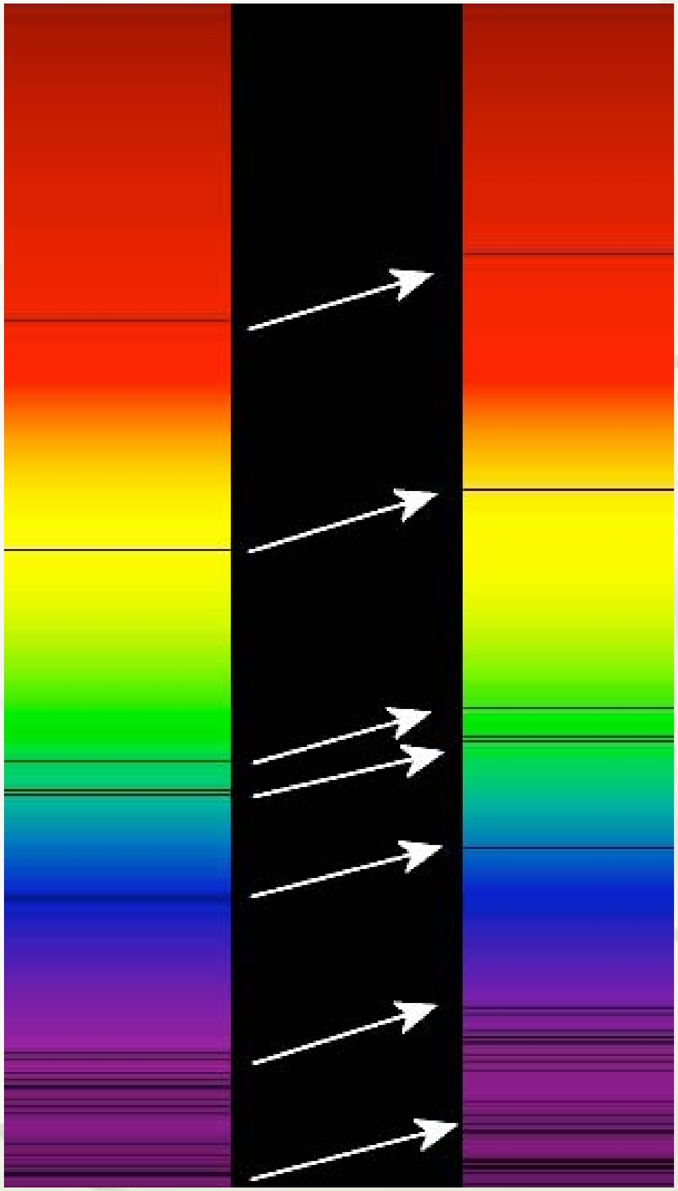
$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}}$$

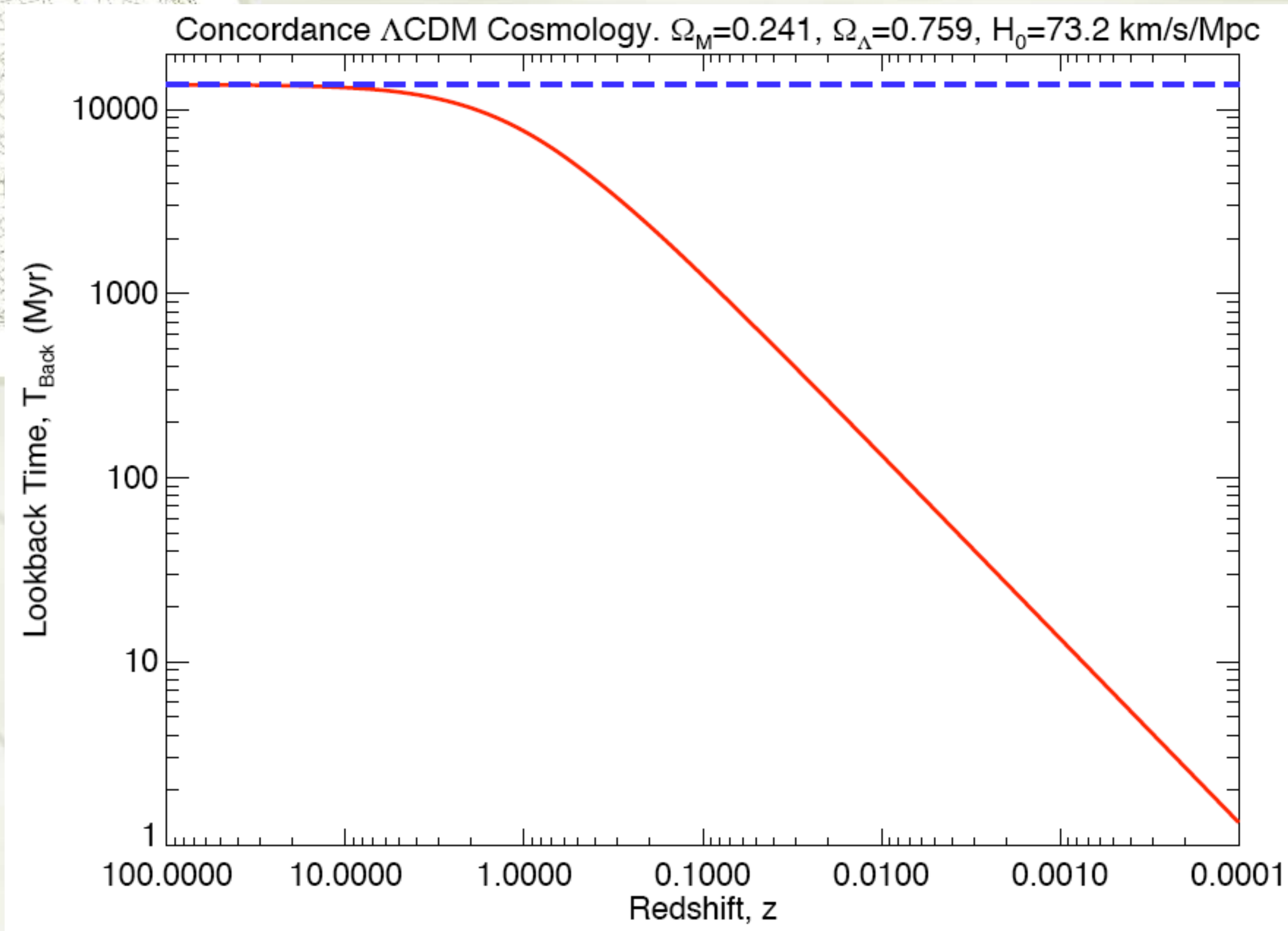
$$1 + z = \frac{a(z=0)}{a(z)}$$

- Stretching of light waves traveling through space
- Caused by expansion of the Universe
- Determined by observing an object's spectrum

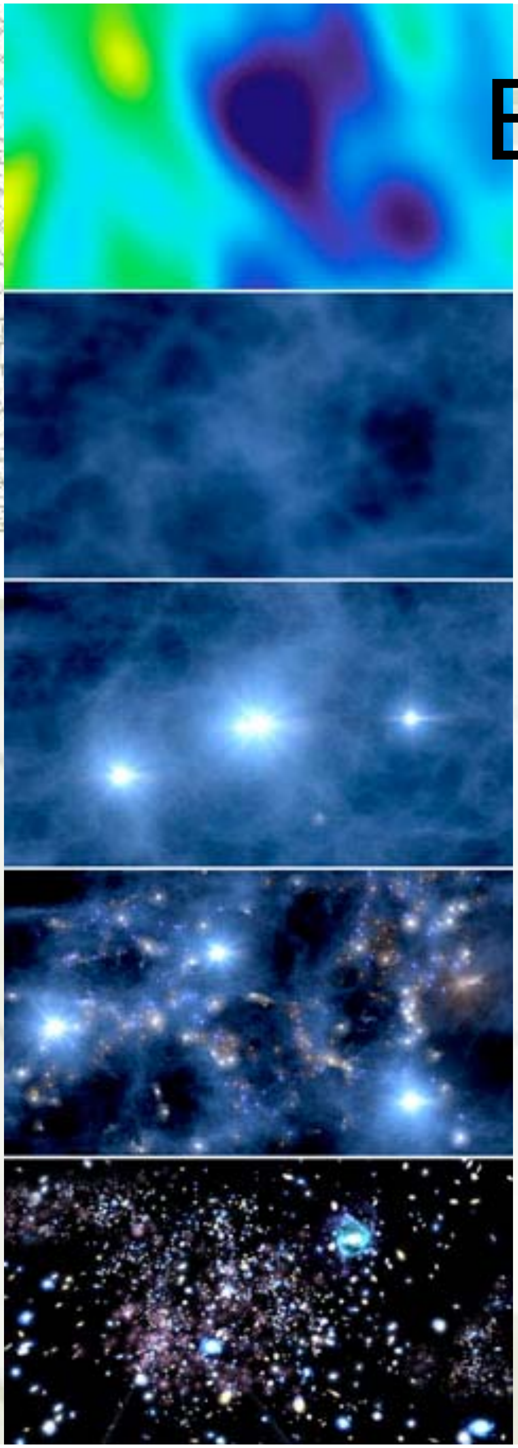
- Comoving (line-of-sight) distance:

$$D_{comov}(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_M (1+z')^3 + \Omega_\Lambda}}$$





Concordance ΛCDM Cosmology. $\Omega_M$=0.241, $\Omega_\Lambda$=0.759, $H_0$=73.2 km/s/Mpc
10000
1000
100
10
1
Lookback Time, $T_{Back}$ (Myr)
100.0000
10.0000
1.0000
0.1000
0.0100
0.0010
0.0001
Redshift, z



# Evolution of our Universe

- Our (observable) Universe expanded from a very dense, hot initial state (*Big Bang*)
- Structures formed in small scales by self-gravity
- Evolution — from the early stage of density fluctuations into what we observe today: galaxies, clusters, and **We** – the people on planet Earth

# *Large Scale Structure Formation*

$\Lambda$CDM Universe ⇒ CDM (cold dark matter) + Cosmological constant / Dark energy

- Quantum fluctuations shortly after the Big Bang give rise to primordial density perturbations in the early radiation and matter density fields
- Inflation expands the perturbations, which form the seeds for later growth
- Structures grow from gravitational clumping of matter from these initial density fluctuations
- Main forces driving evolution
  - Gravity : affects dark matter and baryons
  - Gas dynamics : only baryons

# *How do we Simulate? -- Basics*

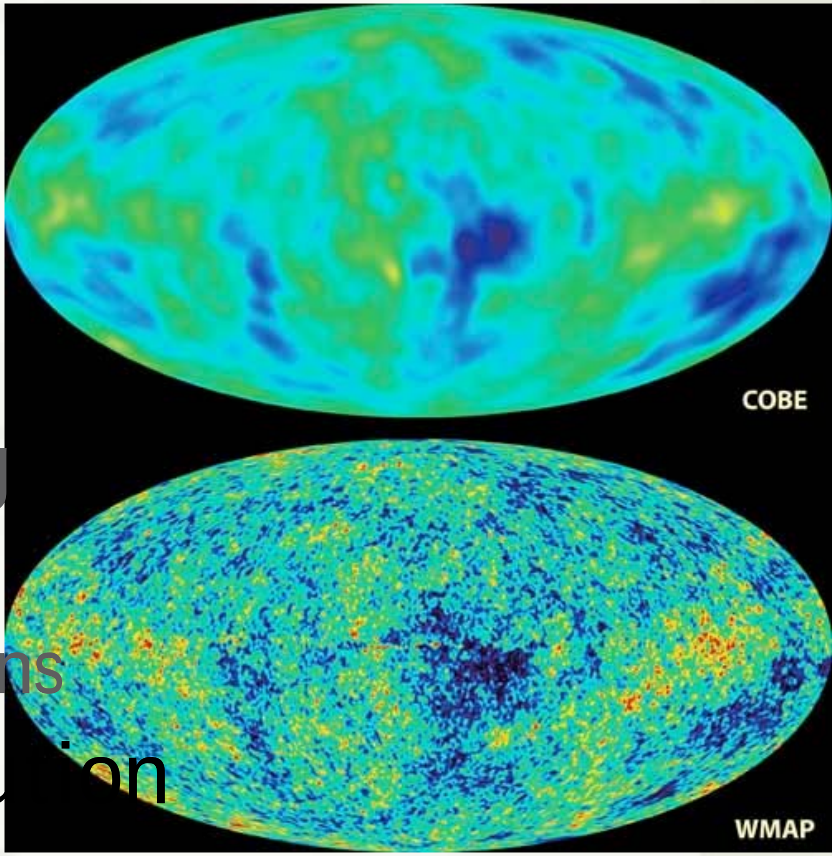
- A computational box ⇔ the Universe
- Particles in the box ⇔ matter in the Universe
- Assume: can model Universe as many massive particles each representing about $10^{10}$–$10^{12}$ $M_\odot$
  - Treat particles as average density over a finite volume (not point particles)
- Physical laws governing the nature determines the dynamics of the particles in the box
- Cosmological simulation → tool to investigate the evolution of millions of particles
- To run these simulations we need supercomputers
  - High speed and high processing power
  - Weeks to months of time

# *How do we Simulate? -- Procedures*

- Must follow the non-linear evolution of the early density fields using numerical methods
- A twofold process:
  - Generate the initial conditions for the density fluctuations, which are then used as an input to the evolution code
  - Evolve the system by following the trajectories of particles under their mutual gravity
    - Should get the final particle distribution consistent with observations of the Universe

# *Simulation -- 1st step*

- Assume a cosmological model
  - ΛCDM parameters constrained by various observations:
    - CMBR (WMAP)
    - SN
    - Galaxy clusters
    - Gravitational lensing
- Set initial conditions using
  - Cosmological parameters
  - Gaussian density fluctuations
- Underlying matter distribution



# *Current Cosmological Parameters (from WMAP)*

- $H_0$ = 71 (+ 4 – 3) km/s/Mpc

- Density parameter, $\Omega_{TOT} = \rho/\rho_C = 1.00 \pm 0.02$
- $\Omega_\Lambda = 0.73 \pm 0.04 = \Lambda \,/\, 3H_0^2$
- $\Omega_M = 0.27 \pm 0.04 = \rho_M/\rho_C$
- $\Omega_{Baryon} = 0.044 \pm 0.004$
  - $n_b$ (baryon density) = $(2.5 \pm 0.1) \times 10^{-7}$ cm$^{-3}$

$$\rho_C = \frac{3H_0^2}{8\pi G}$$

- $t_{Universe} = 13.7 \pm 0.2$ Gyr
- $T_{decoupling} = (379 \pm 8)$ kyr
- $T_{reionization}$ = 180 (+220 – 80) Myr

# *Initial Conditions*

- Overdensity, δ
- When δ << 1 : linear regime
  - Use Zeldovich approximation
- In galaxies, δ >> 1 : linear approx. breaks down

$$\delta \equiv \frac{\rho}{\bar{\rho}} - 1$$

- Inflation ⇒ near homogeneous initial particle distribution
  - Small perturbations
- Linear regime
  - Λ dominated
  - EdS/Matter dominated

$$\delta_k(t) = A + Be^{-2Ht}$$

$$\delta_k(t) = At^{\frac{2}{3}} + Bt^{-1}$$

- Need to be in growing mode (for eventual collapse)
  - From Virial theorem: linearized overdensity > 1.7

# *Methods for Generating IC's*

- Begin with particles on a lattice
- Generate an initial gravitational potential to ensure growing mode
  - Place particles on a cubic grid
  - They are randomly perturbed
- Calculate particle velocities to put system in growing mode (from initial potential)
- Displace the particles from uniform positions by amount corresponding to assigned velocity

# *Cosmological Simulation*

- Dark matter (collision-less)
  - Gravity-only
  - Particle *N*-body method
- Baryon / Gas evolution
  - Gravity + Hydrodynamics
- Add source & sink terms
  - Cooling and heating of gas
  - Star formation and stellar feedback
- Numerically integrate dynamical equations
- Evolve box from $z$=100 up to $z$=0
  - From after Big Bang ($t \rightarrow 0$) to present ($t$=13.7 Gyr)

# *Simulation Volume*

- The computational box has Hubble expansion just like the real Universe

$$L_{box}(z)_{before} = \frac{L_{box}(z=0)_{now}}{1+z}$$

  - Always encompasses the same mass
  - Mean density decreases

- The expansion is taken out from computations, s.t. the box appears static
- Coordinate system that expands (or co-moves) with the Universe (the comoving coordinates) is used

# *Comoving Coordinate, x(t)*

- Coordinate moving with the Hubble flow
- Distance between 2 points measured now, at $z = 0$
- Remains constant for objects in Hubble expansion

$$x(t) = \frac{r(t)}{a(t)}$$

$$r = \int_{t_{emit}}^{t_{now}} \frac{c\,dt'}{a(t')}$$

- $r(t)$ = Proper Distance
- $a(t)$ = Scale factor of the Universe

# *N-body (collision-less)*

- *N* number of dark matter particles
  - No thermal velocities, no other interactions
- Gravitational interactions only
- Equations of particle motion

$$\frac{d\vec{x}}{dt} = \vec{u} \qquad \frac{d\vec{u}}{dt} = -\nabla\Phi$$

- Poisson's equation

$$\nabla^2\Phi = 4\pi G\rho$$

- During late evolution the relevant particle velocities are non-relativistic $\Rightarrow$ Newtonian limit

# *N-body*

- Each evolution timestep has 2 parts:
  - Gravity-solver : solve Poisson's equation & calculate particle accelerations
  - Time integrator : calculate particle positions & velocities
- Different codes, run in parallel processors
  - PP : Particle-Particle
  - PM : Particle-Mesh
  - P3M : PM+PP
  - AP3M : Adaptive P3M
  - ART : Adaptive Refinement Tree
  - TreePM

# *Hydrodynamics (baryons only)*

- Collisional particles with ideal gas properties
- Continuity

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

- Euler

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right)\vec{u} = -\vec{\nabla}\Phi - \frac{\vec{\nabla} P}{\rho}$$

- Energy

$$\frac{\partial E}{\partial t} + \vec{\nabla} \cdot \left[(E + P)\vec{u}\right] = -\rho \vec{u} \cdot \vec{\nabla}\Phi$$

- Equation of state, $\varepsilon = f(\rho, P)$
  - Ideal gas

$$\varepsilon = \frac{1}{(\gamma - 1)} \frac{P}{\rho}$$

  - Polytropic

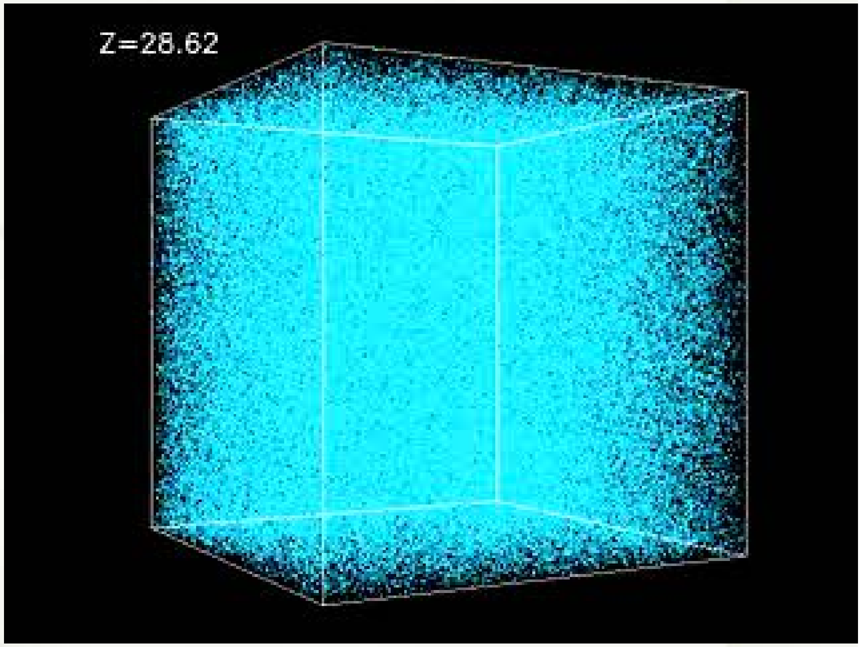
$$P = K\rho^{\gamma}$$

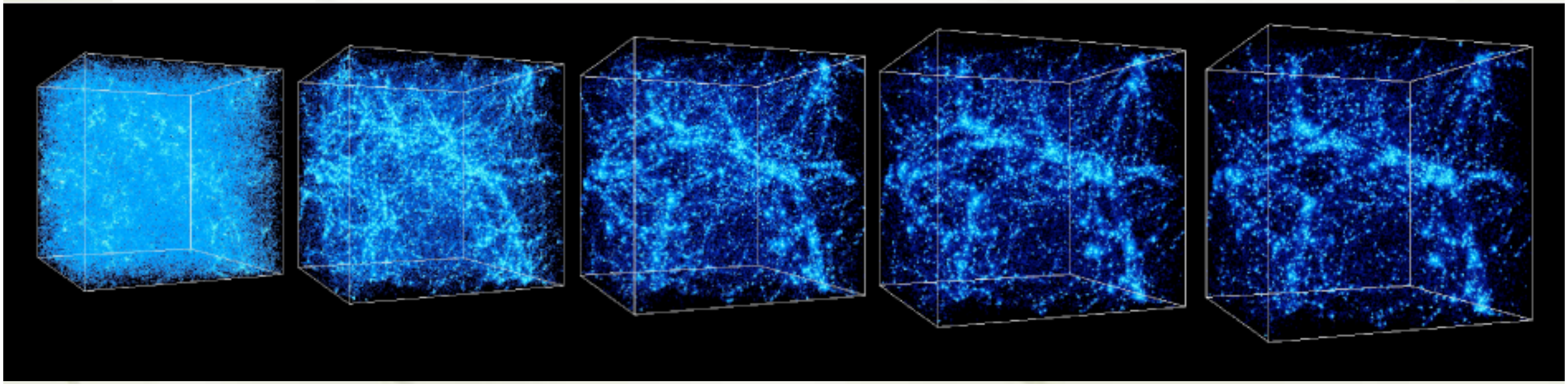
# *Successes of Cosmological Simulations*

- Can reproduce the distribution and structure of galaxies in very large scales as seen in observations
- On small scales
  - Stellar evolution, black holes, star clusters
- Distribution of matter in the Universe
  - How the matter collapses via gravitational forces into filaments. Galaxies form in these filaments
- Galaxy clustering at all z (z~0, z~1, z~4-5) observed in large scale surveys is well reproduced

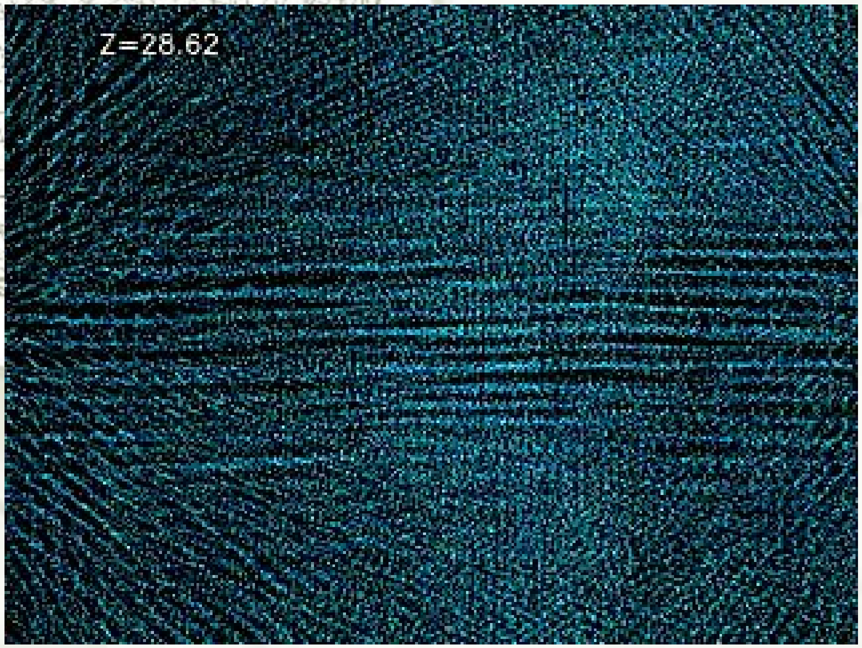
# *Large-scale filaments*

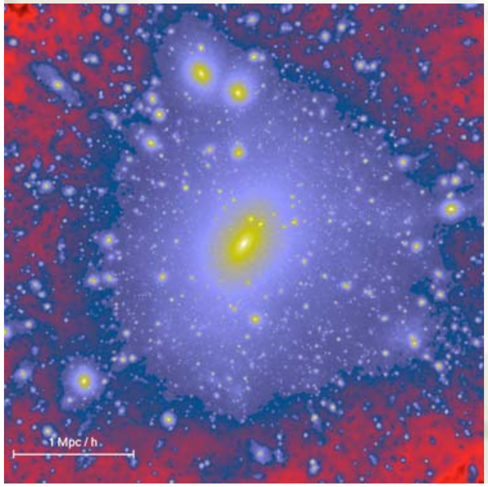
- A 43 Mpc box
- From z=30 to z=0
- The frames below show formation of large scale structure from z=10 to the present

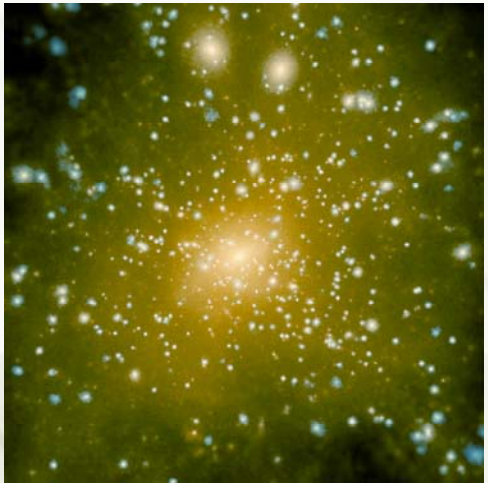


# *Formation of a Galaxy Cluster*



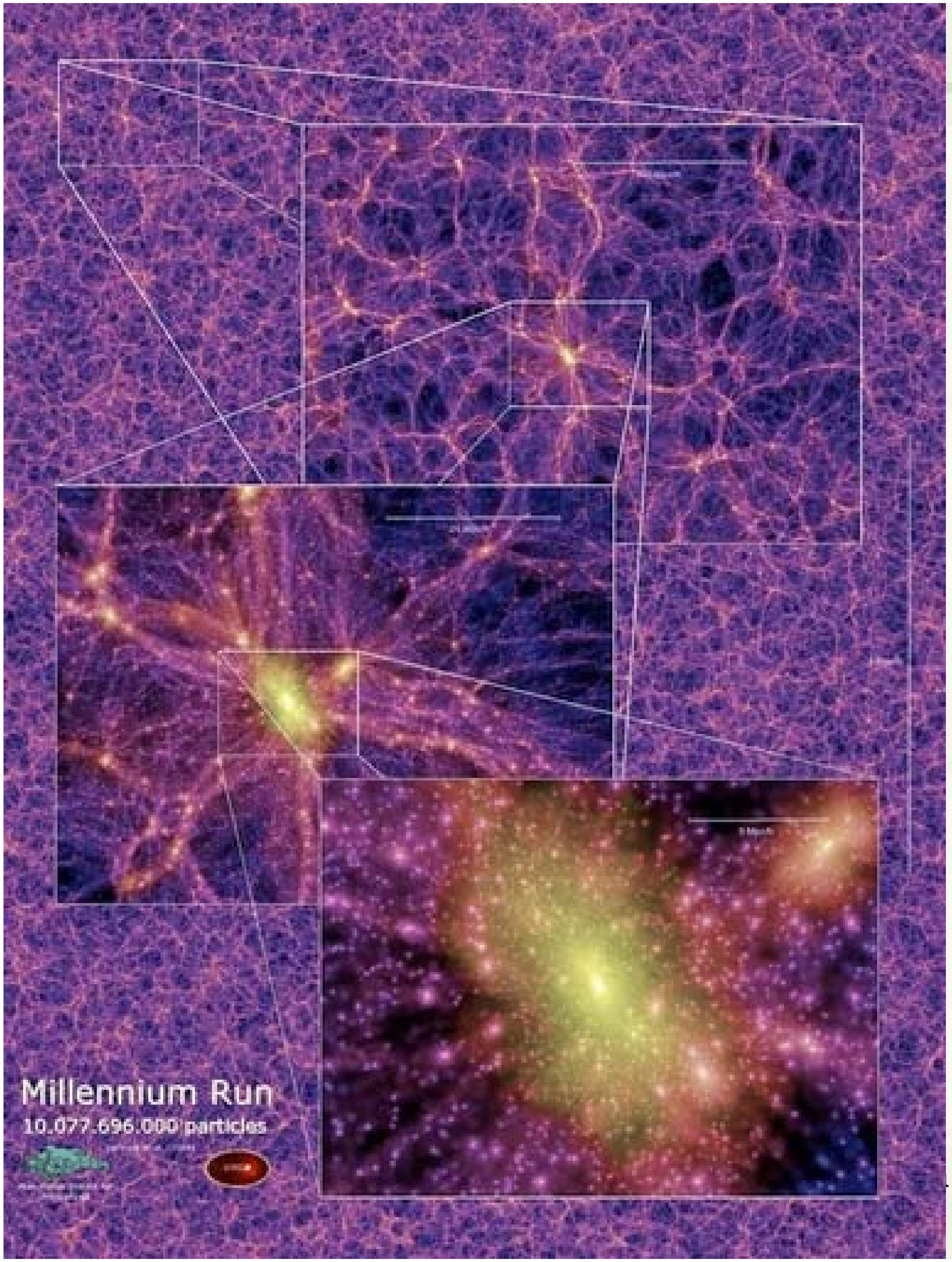



- 4.3 Mpc region of box
- The formation of a cluster proceeds hierarchically
  - Small-mass objects form first at z>5
  - Quickly grow in size and violently merge with each other, creating increasingly larger and larger system
- This galactic "cannibalism" continues up to z=0

# *The Millennium Simulation*

- One of the largest cosmological N-body run
  - Dark matter only
  - Gravity only, no hydrodynamics
- Done by the Virgo Consortium
  - Springel et al. 2005, Nature, 435, 629
- Box comoving side = 500 $h^{-1}$ Mpc
- $2160^3 \sim 10^{10}$ particles
- Particle mass = 8.6 x $10^8$ $h^{-1}$ $M_\odot$
  - A galaxy = few particles
- Comoving softening length, $\varepsilon$ = 5 $h^{-1}$ kpc
- From $z$=127 to $z$=0
- Took 28 days on 512 CPUs

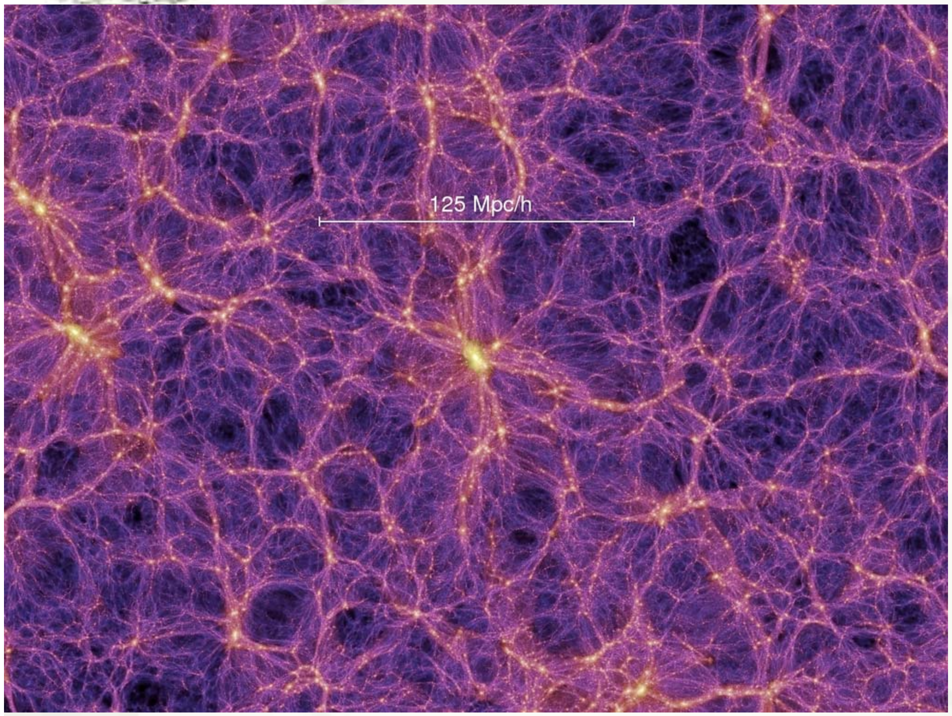



- A projected density field for a 15 Mpc/h thick slice of the $z$=0 output

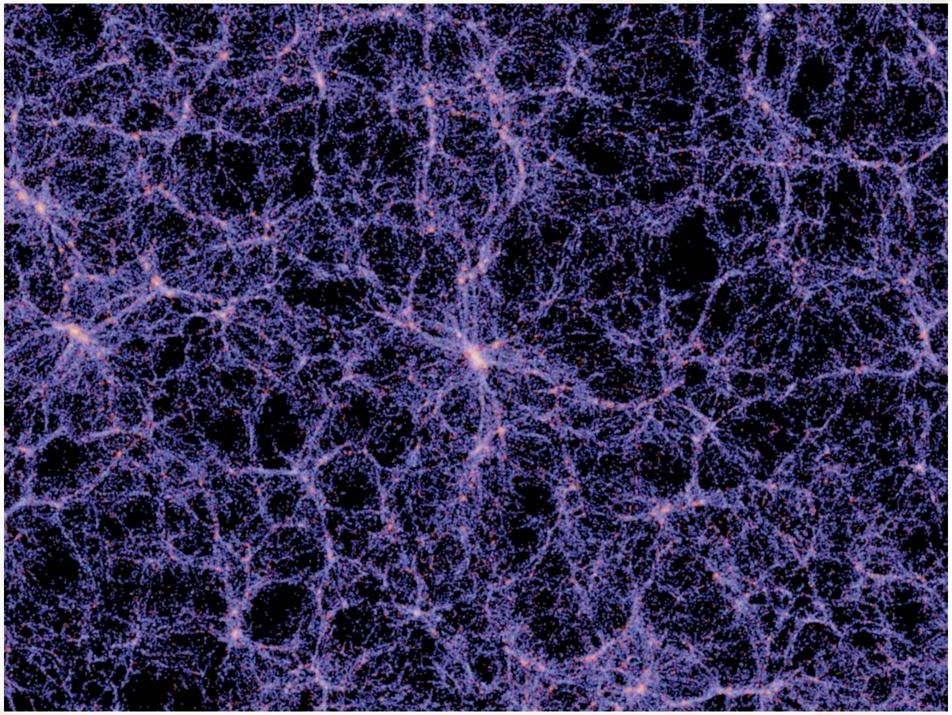
- The overlaid panels zoom in by factors of 4 in each case, enlarging the regions indicated by the white squares

# *The Large-scale Matter Distribution in the Millennium Simulation*

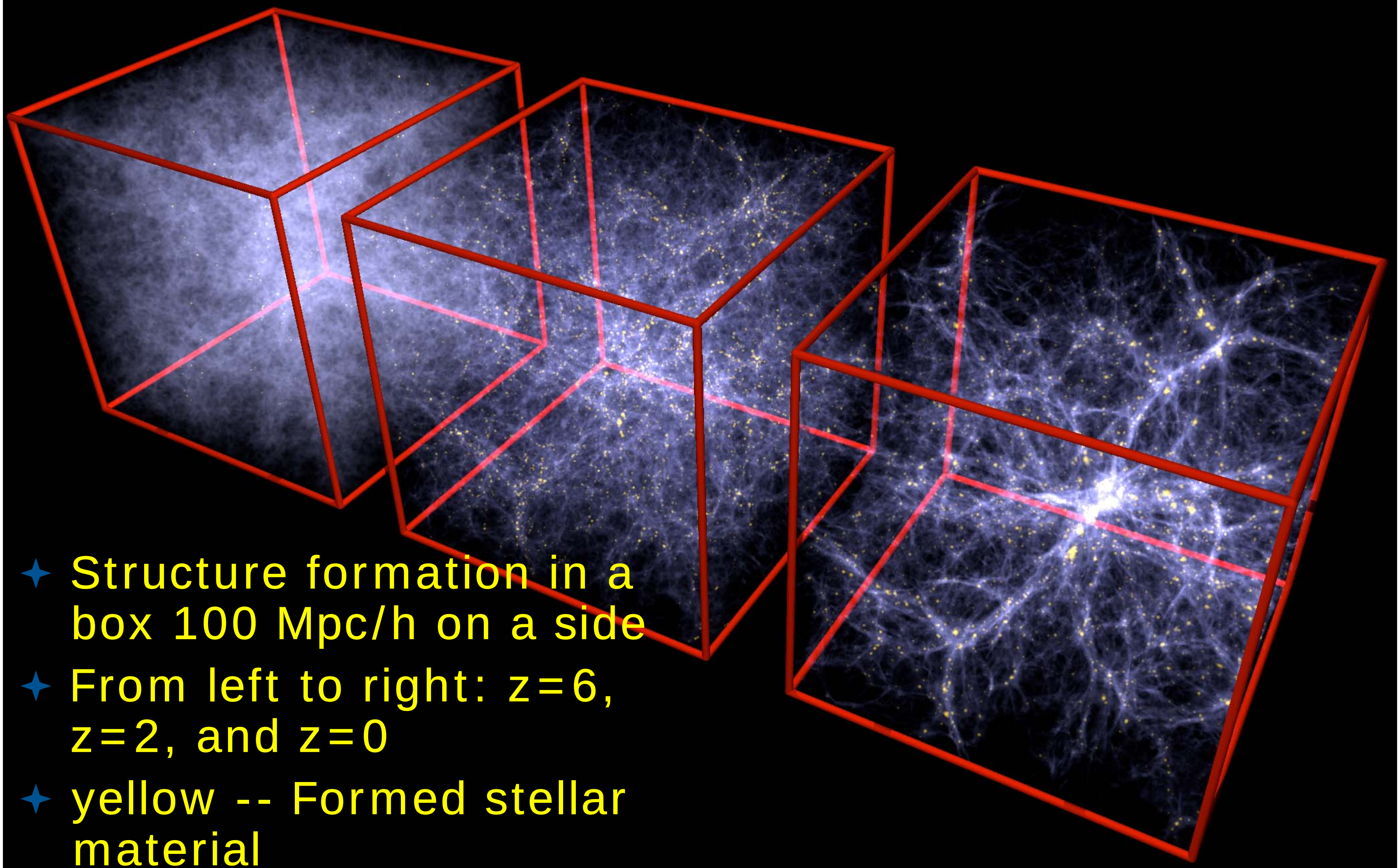
Dark Matter




Galaxies

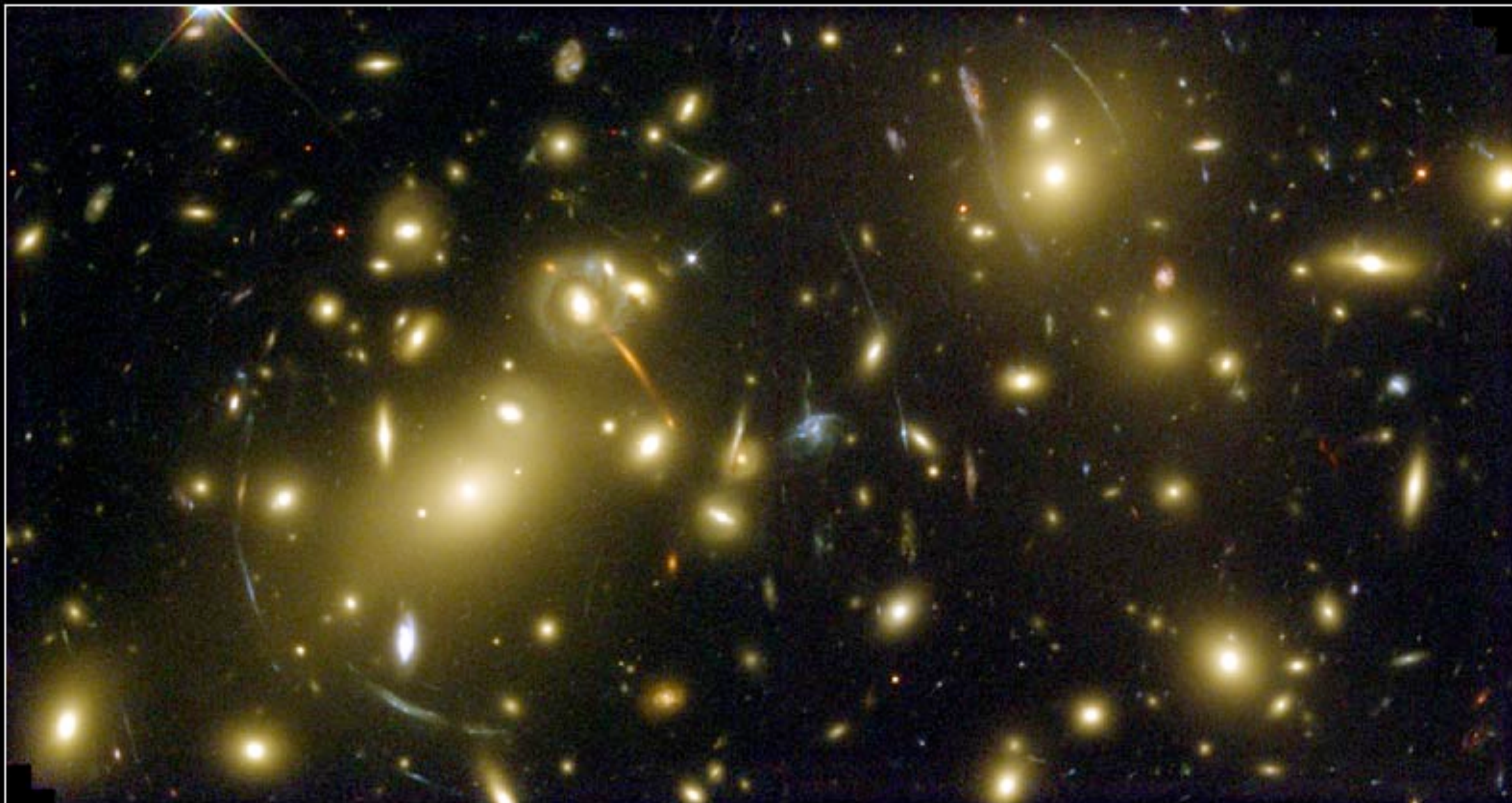


# *Cosmic structure in 3D*



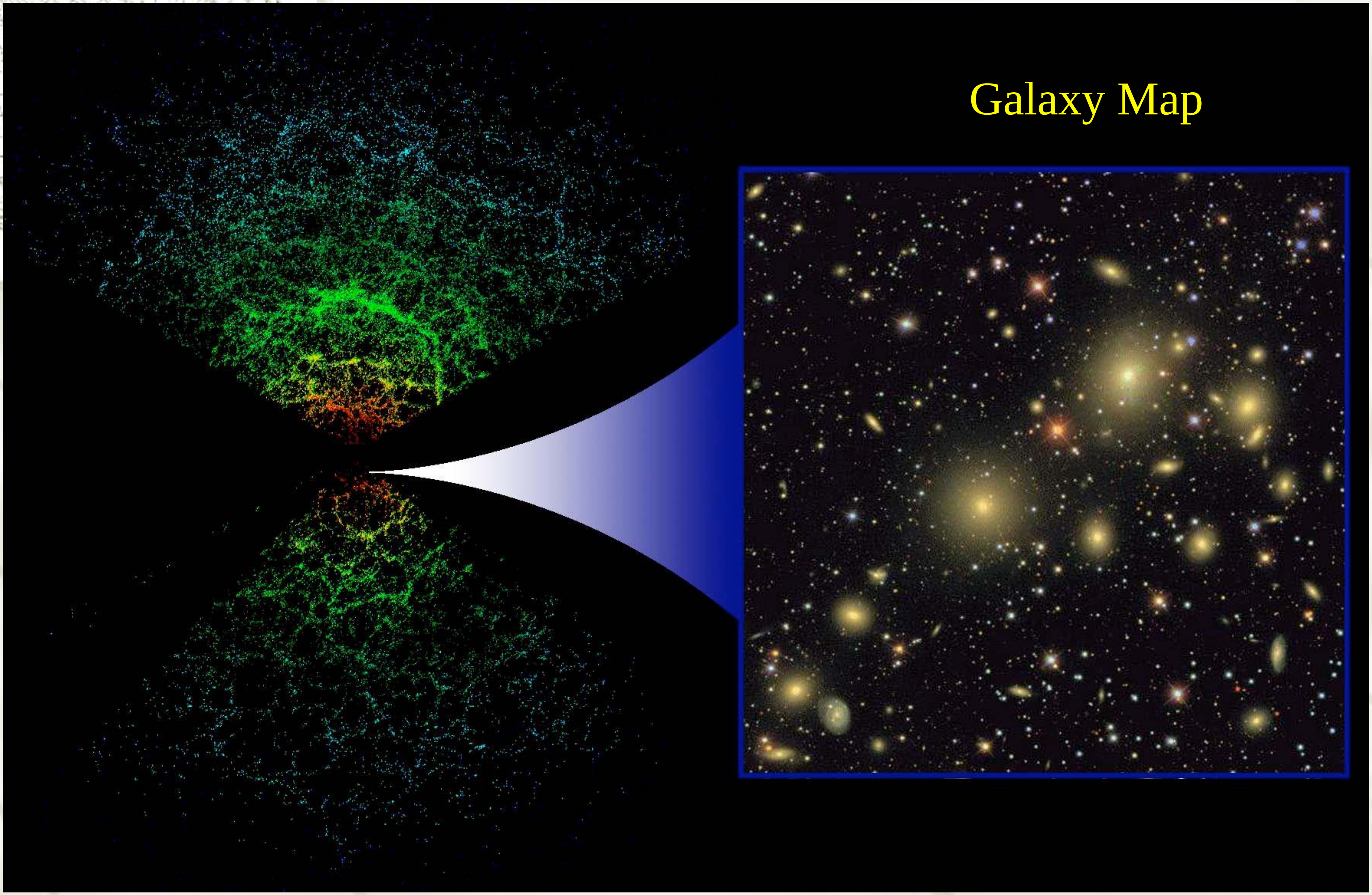
- Structure formation in a box 100 Mpc/h on a side
- From left to right: z=6, z=2, and z=0
- yellow -- Formed stellar material

# *Resolving galaxy sub-structures at the same time is tricky ...*

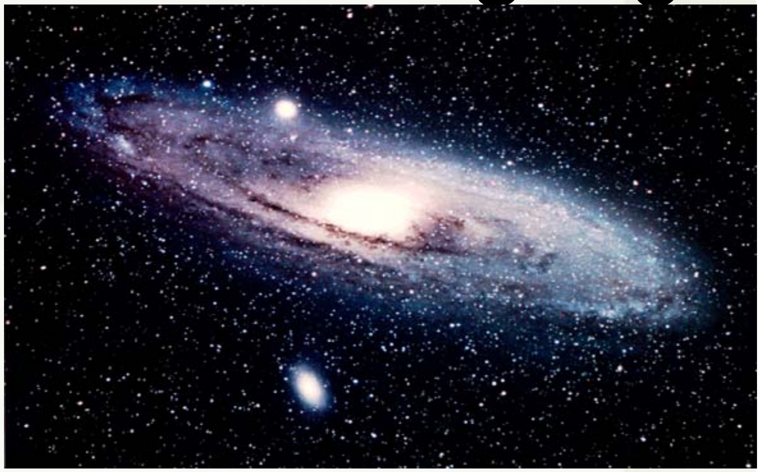


**Galaxy Cluster Abell 2218** **HST** • WFPC2
NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

# Sloan Digital Sky Survey (SDSS)



# *Predicting right internal structure in galaxies is challenging!*



- Galaxy interior
  - Star & gas distribution
  - Star formation & feedback
- Why?
  - Need very large resolution and dynamic range
- Star formation needs resolution of 10 pc ($10^6$ resolution in a 10 Mpc box)
- Scale height of star forming gas disk in Milky Way is 100 pc

# *Research in India*

- HRI - Jasjeet Bagla
- IUCAA - T. Padmanabhan
- RRI
- IIA

# Summary

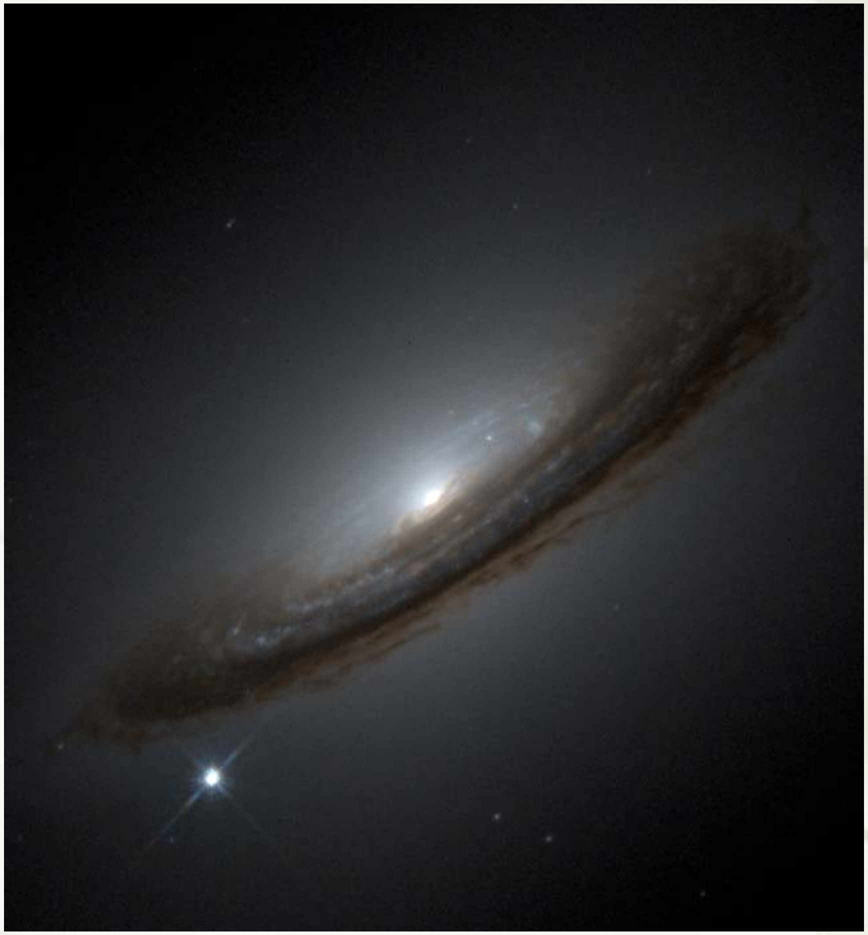
- Observations have given a well constrained cosmological picture
  - Flat, dark-energy & dark-matter dominated
  - Primordial density fluctuations
- Evolve a computational box using non-relativistic equations for particle dynamics
  - Reproduce the observed large-scale structures in the Universe
- Several challenges and open questions remain for future research

# *Extra Slides*

General Funda or More Explanation

# SN Type Ia

Giant star accreting onto white dwarf
Standard candles
Compare observed luminosity with predicted
Too faint
From z → expansion rate of distant SN < closer SN



# *Initial Conditions*

## Cubic grid

- Particles are placed on a (uniform) cubic grid
- Particle mass is varied to match potential
- Assign velocities to particles to put system in growing mode

## Random Distribution

- Start with a uniform distribution of identical particles
- Calculate particle velocities to put system in growing mode (from initial potential)
- Displace the particles from uniform positions by amount corresponding to assigned velocity
- Check displacements << inter-particle separation
  - Otherwise will violate initial potential

# *Dark Matter Candidates*

- Hot
  - Remain relativistic until recombination
  - Dampen initial fluctuation
  - Could be a neutrino with 10 eV < $m$ < 100 eV
- Warm
  - $m \approx 1$ eV
  - Interact more weakly than neutrinos!
  - Less abundant than neutrinos
  - There are few candidate particles with these properties
- Cold
  - Any particles which became nonrelativistic early on
  - Possible to calculate the appropriate initial density perturbation
  - Thus, the evolution of a universe dominated by CDM can be treated as an initial value problem. In particular,