Galaxy dynamics as a probe of cluster mass

andrea biviano
INAF/Oss.Astr.Trieste

Galaxy dynamics as a probe of cluster mass

i.e.:

determine a cluster mass by using the l.o.s. velocities (and projected positions) of galaxies that are estimated to be cluster members

➔ must define a cluster center
in projected phase-space
(e.g. position and velocity of BCG,
or position of BCG and mean velocity)

This is generally not a
problematic issue for
cluster mass determination

➔ must determine which galaxies are members of the cluster (e.g. use galaxy locations in projected phase-space, and intrinsic galaxy properties)

This IS a problematic issue for cluster mass determination:

contamination

Contamination

Two kinds of contamination:

- 'average' field contamination
- 'catastrophic' contamination by big external structures

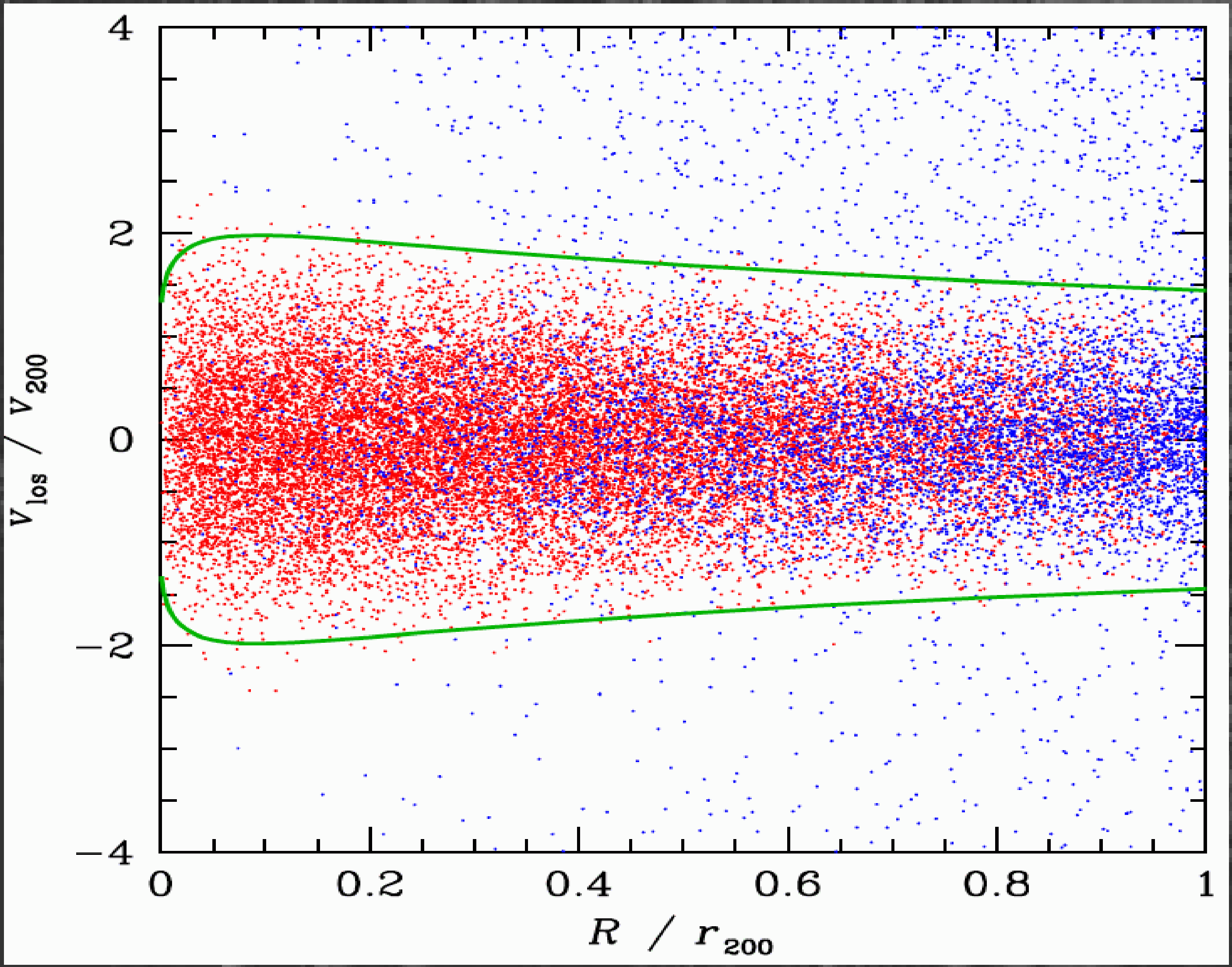
Can characterize statistically the former
but most troubles come from the latter

Contamination

Particles $r \leq r_{200}$

Particles $r > r_{200}$

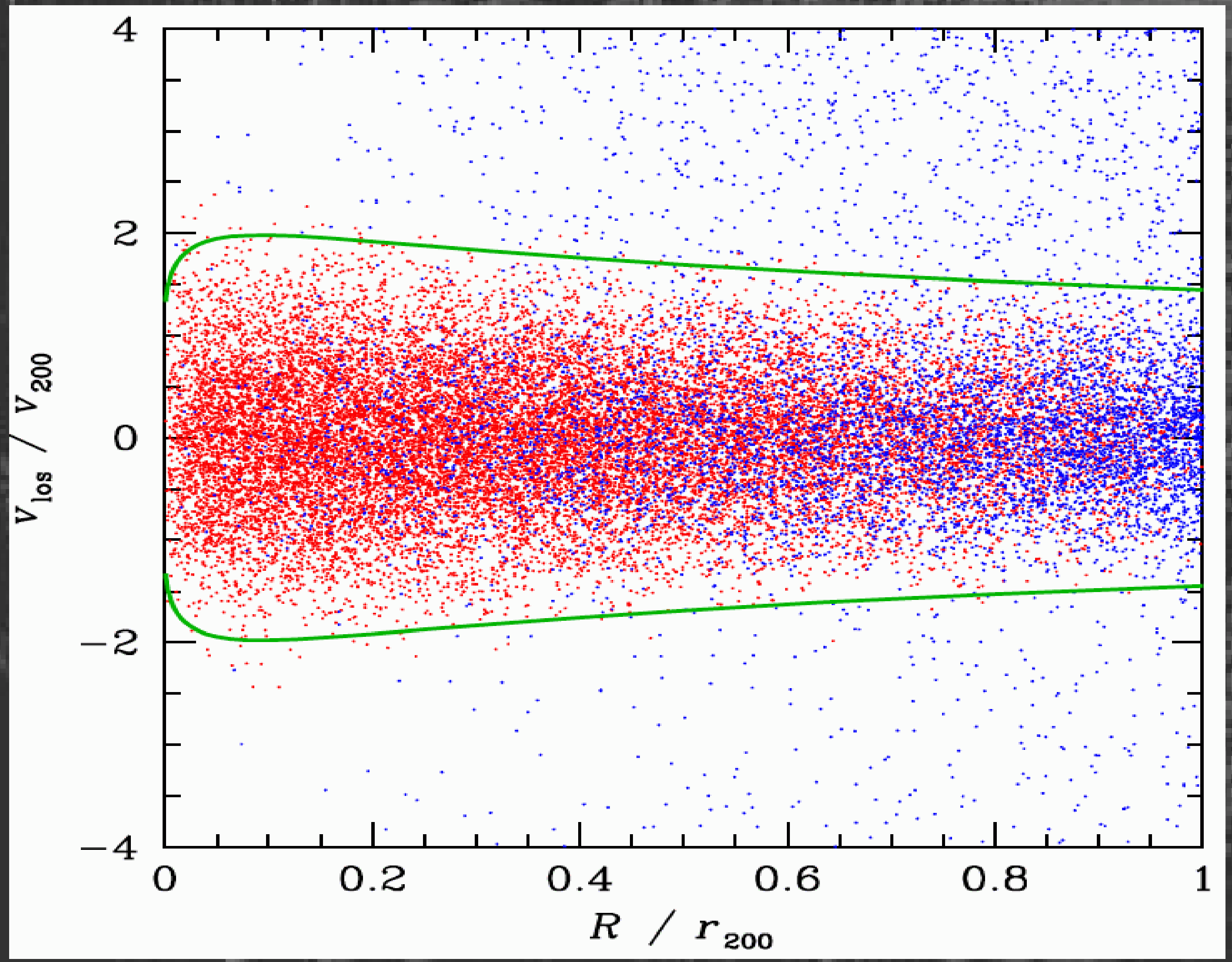
Selection of cluster members (Mamon, ab, Murante+10)



Stack of cluster-sized halos from cosmological simulation (Borgani+04)

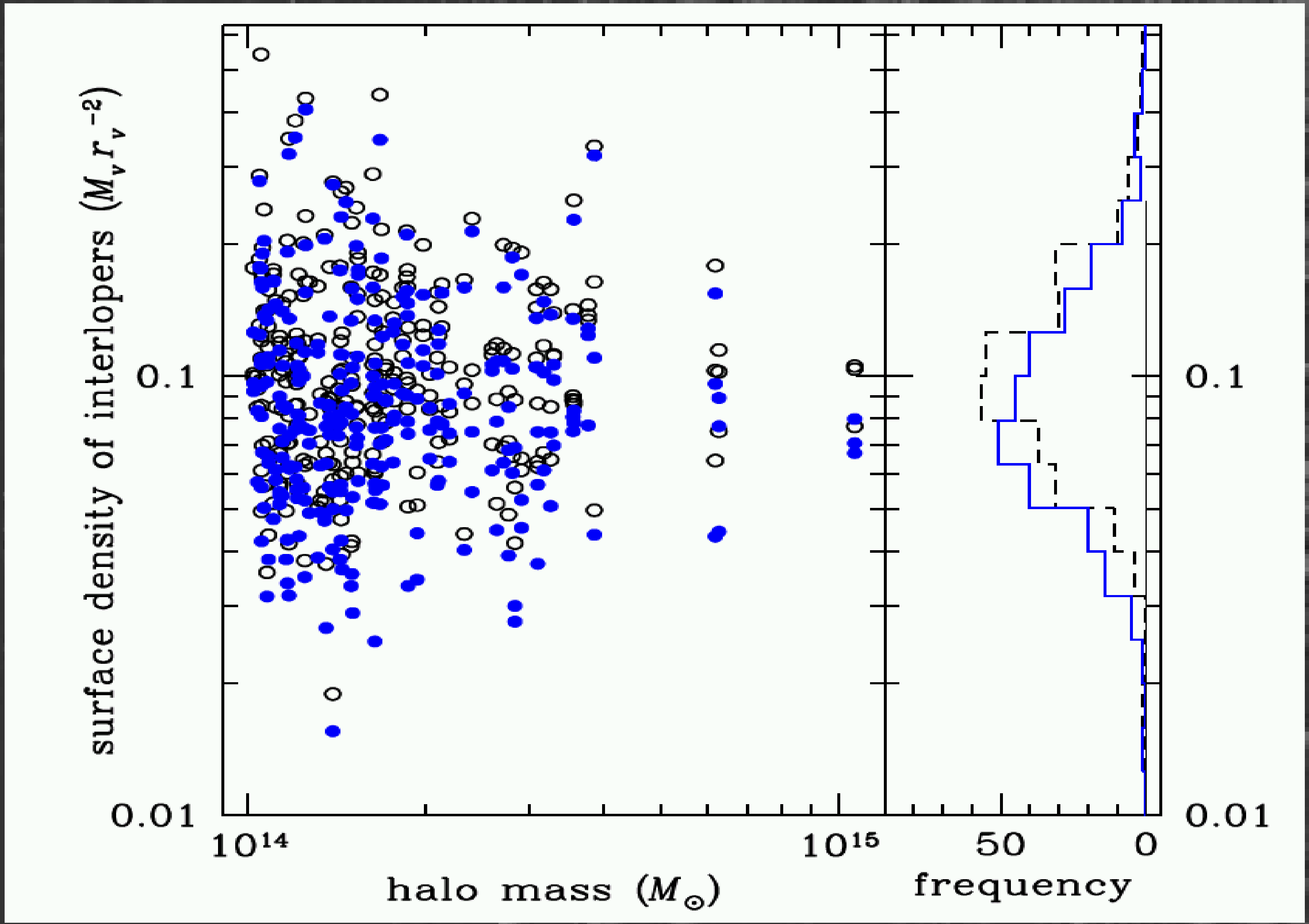
Contamination

~20% selected members are interlopers (ab+06; Wojtak+07; Mamon+10) which are impossible to remove by velocity cut



(Cen 97), regardless of the method (Wojtak+07)

Contamination

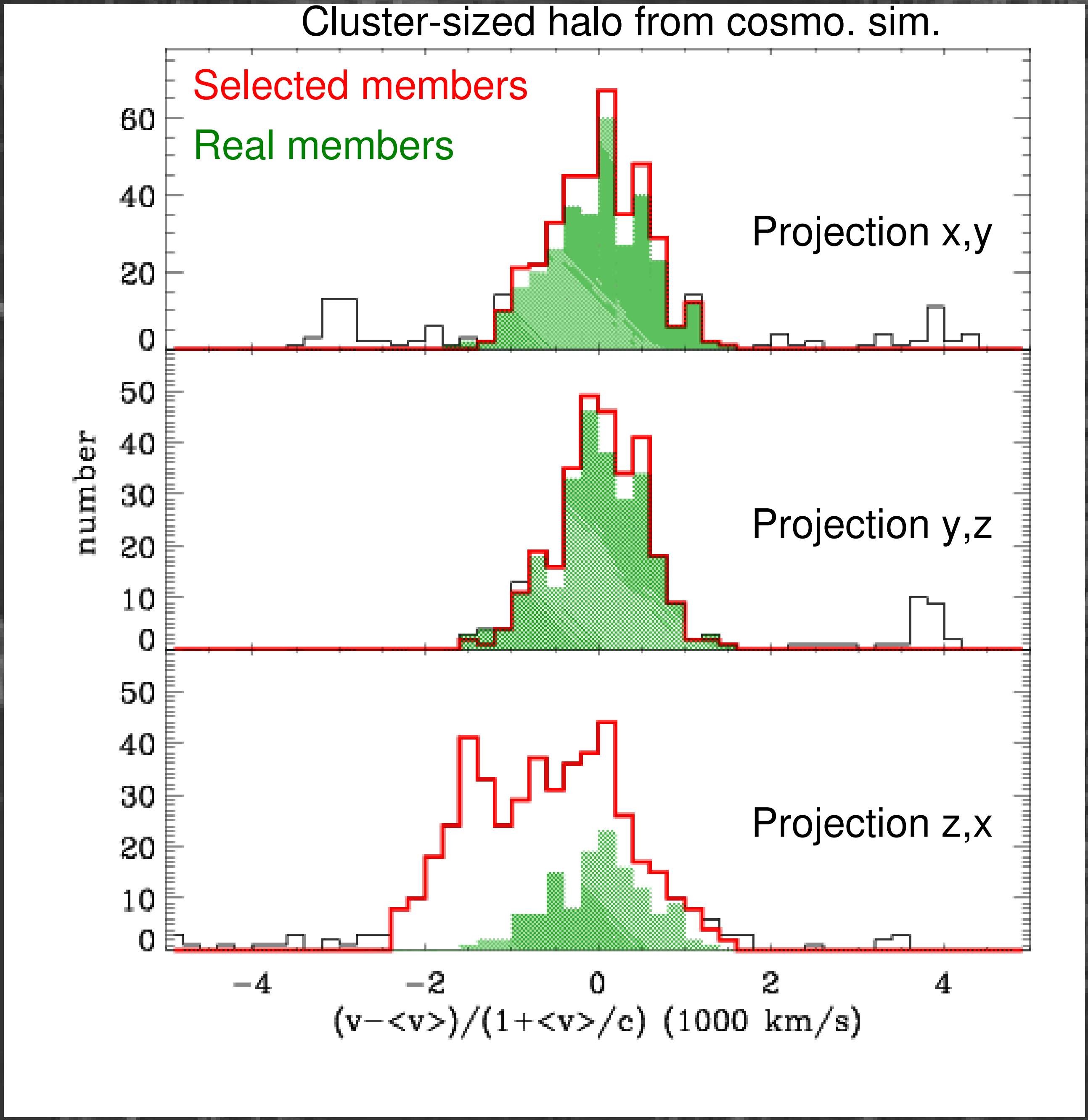


...but with a large cosmic variance! (Mamon+10)

Contamination

can be catastrophic!

...but can be found with subclustering analysis (ab+06;



Katgert & ab, work in progress)

METHODS:

- Virial theorem
- M from $\sigma_{\mathrm{v,los}}$
- Jeans equation
- Dispersion + Kurtosis
- Distribution functions E,L
- MAMPOSSt
- Caustics

Virial theorem

(Zwicky 33, 37; Limber & Mathews 60; The & White 86)

$$M = 3\pi \quad \sigma_p^2 R_h / G$$

$$R_h = \frac{1}{2} N(N-1) \sum_{i>j} R_{ij}^{-1}$$

Virial theorem

(Zwicky 33, 37; Limber & Mathews 60; The & White 86)

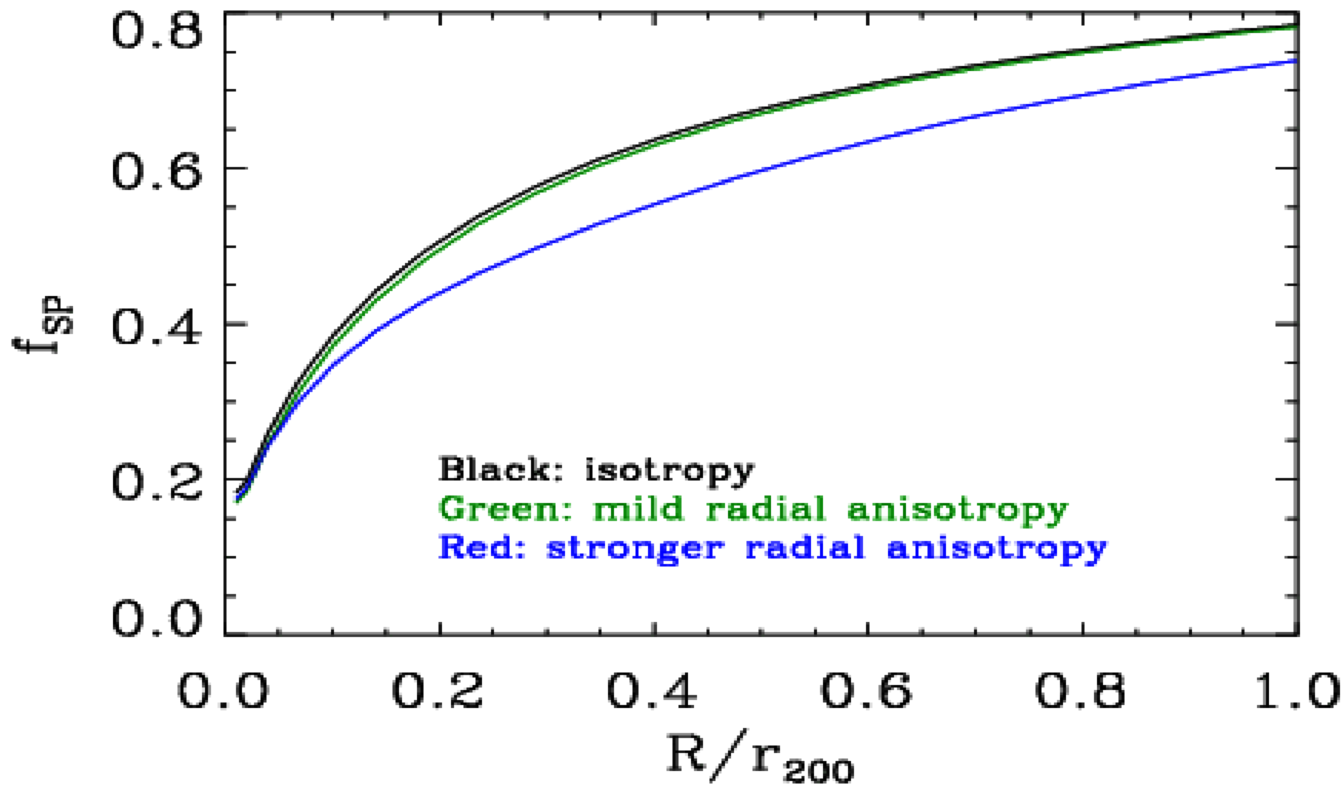
$$M = 3\pi\, f_{sp}\, \sigma_p^2\, R_h / G$$

$$R_h = \frac{1}{2} N(N-1) \sum_{i>j} R_{ij}^{-1}$$

$$f_{sp} = 1 - 4\pi r_l^3 \frac{\rho(r_l)}{\int_0^{r_l} 4\pi x^2 \rho dx} \frac{\sigma_r^2(r_l)}{\sigma^2(< r_l)}$$

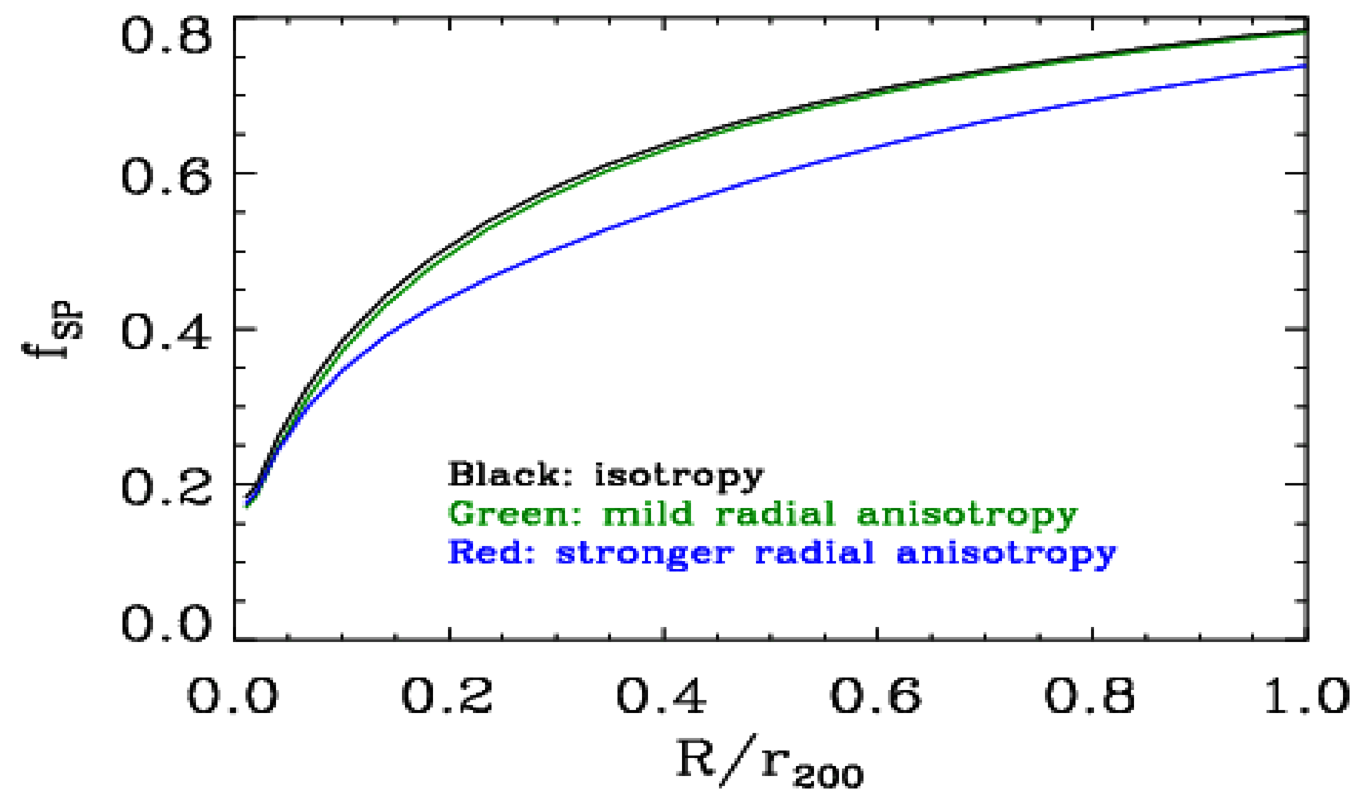
$f_{sp} \approx 0.8\text{-}0.9$ at $r_l \approx r_{200}$

Virial theorem



$$f_{sp} = 1 - 4\pi r_l^3 \frac{\rho(r_l)}{\int_0^{r_l} 4\pi x^2 \rho dx} \frac{\sigma_r^2(r_l)}{\sigma^2(< r_l)}$$

Virial theorem



Example of mild

Mass-anisotropy (M-β) degeneracy

Virial theorem

$$f_{sp} = 1 - 4\pi r_l^3 \frac{\rho(r_l)}{\int_0^{r_l} 4\pi x^2 \rho dx} \frac{\sigma_r^2(r_l)}{\sigma^2(< r_l)}$$

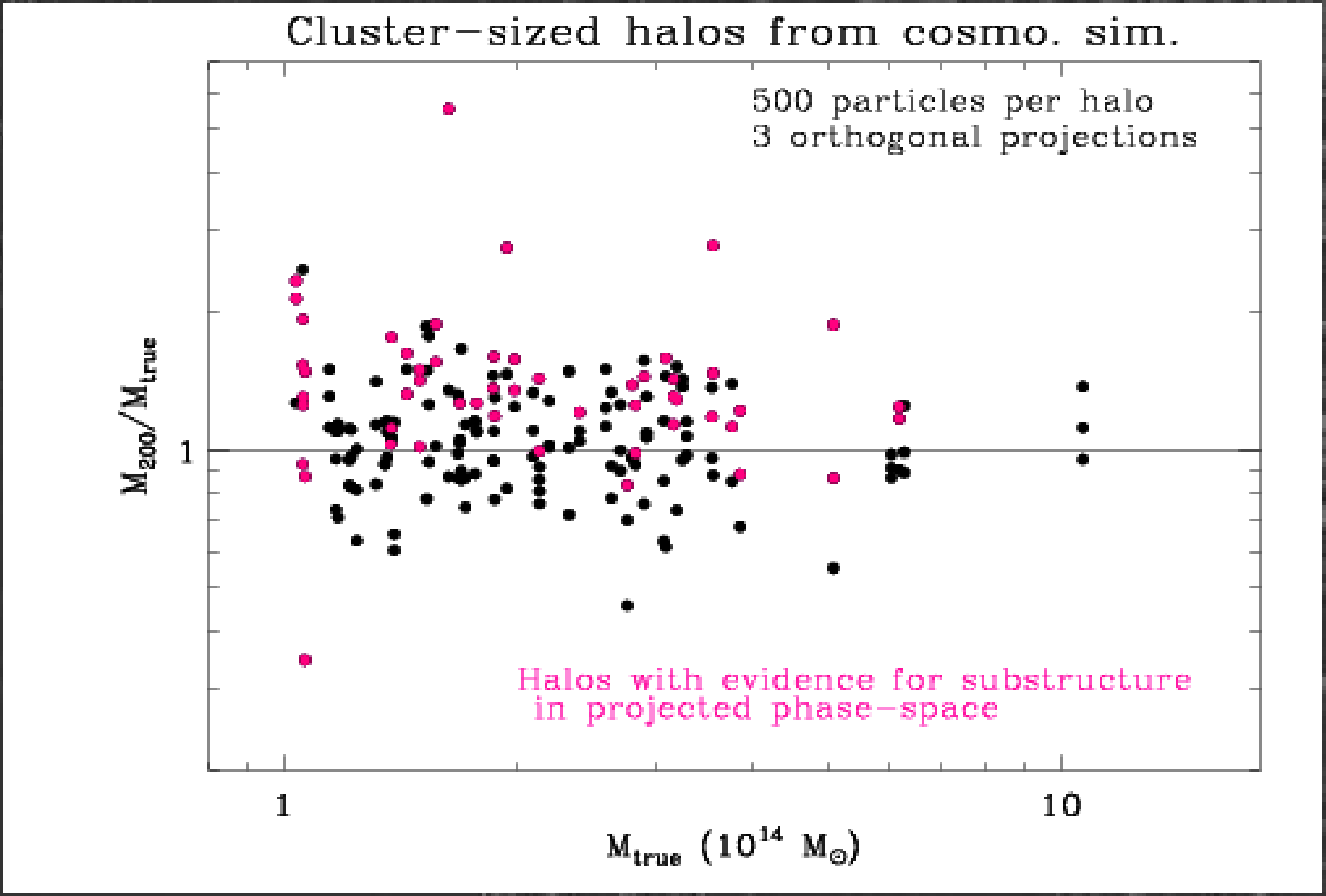
Dependence on unknown mass-density profile:
must be assumed, e.g. from theoretical predictions.
Example of mild dependence
from the cosmological framework

Virial theorem

$$R_h = \frac{1}{2} N(N-1) \sum_{i>j} R_{ij}^{-1}$$

Harmonic mean radius estimate
affected by incomplete spatial sampling.
Can be corrected for,
using photometric samples.

Virial theorem



Virial theorem

2 T + W = 0 → $$M = 3\pi\, f_{sp}\, \sigma_p^2\, R_h / G$$

(Limber & Mathews 60)

if galaxies are the mass carriers
(if they are distributed like the mass);
if not, mass estimate is biased
(The & White 86, Merritt 87)

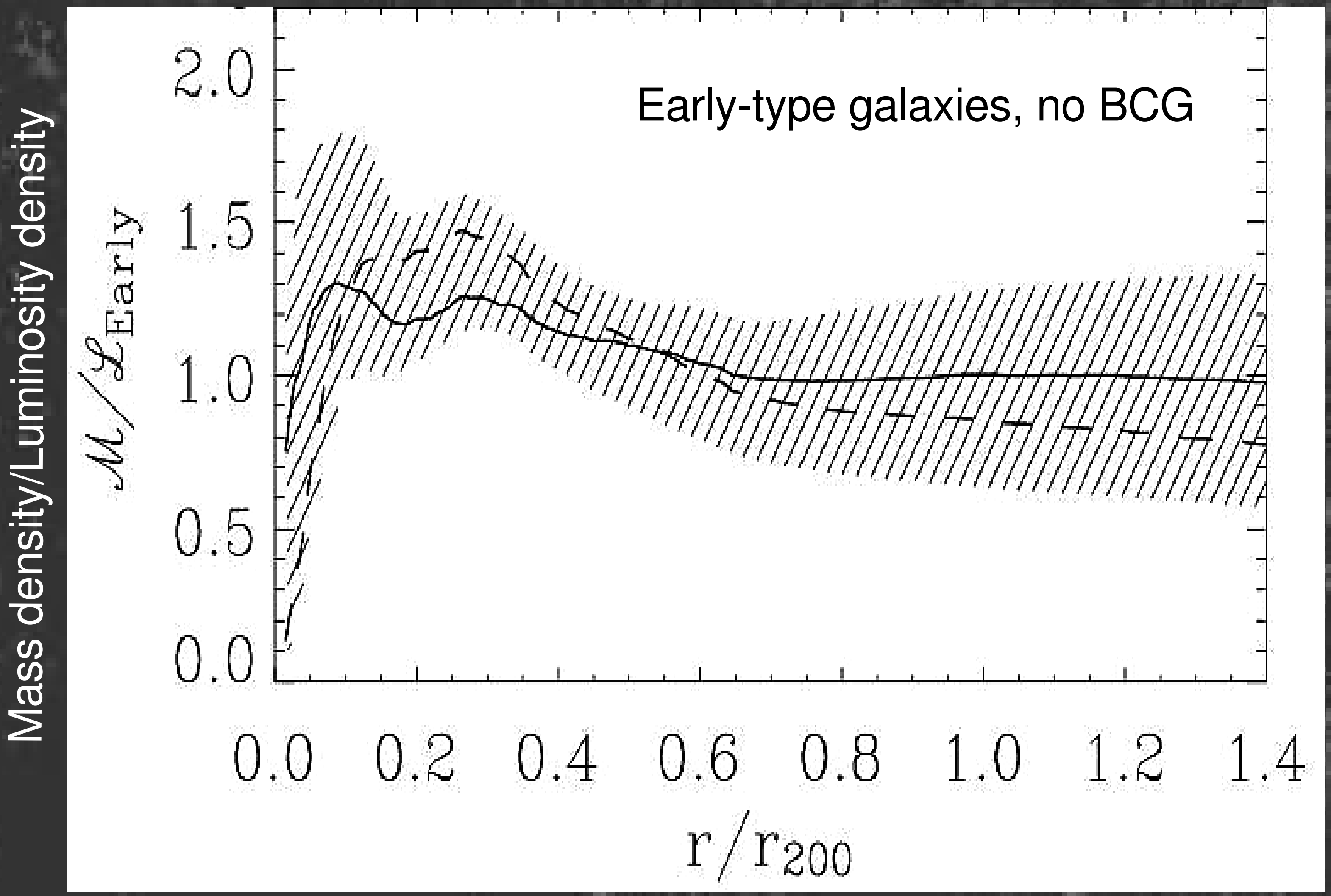
Biased distribution of the tracers?

Similar phase-space distributions for 'galaxies' and DM particles in cluster-size simulated halos (ab+06)...

...Mass/Number density ratio changes little with clustercentric distance in real clusters...

...but different cluster galaxy populations have different distributions!

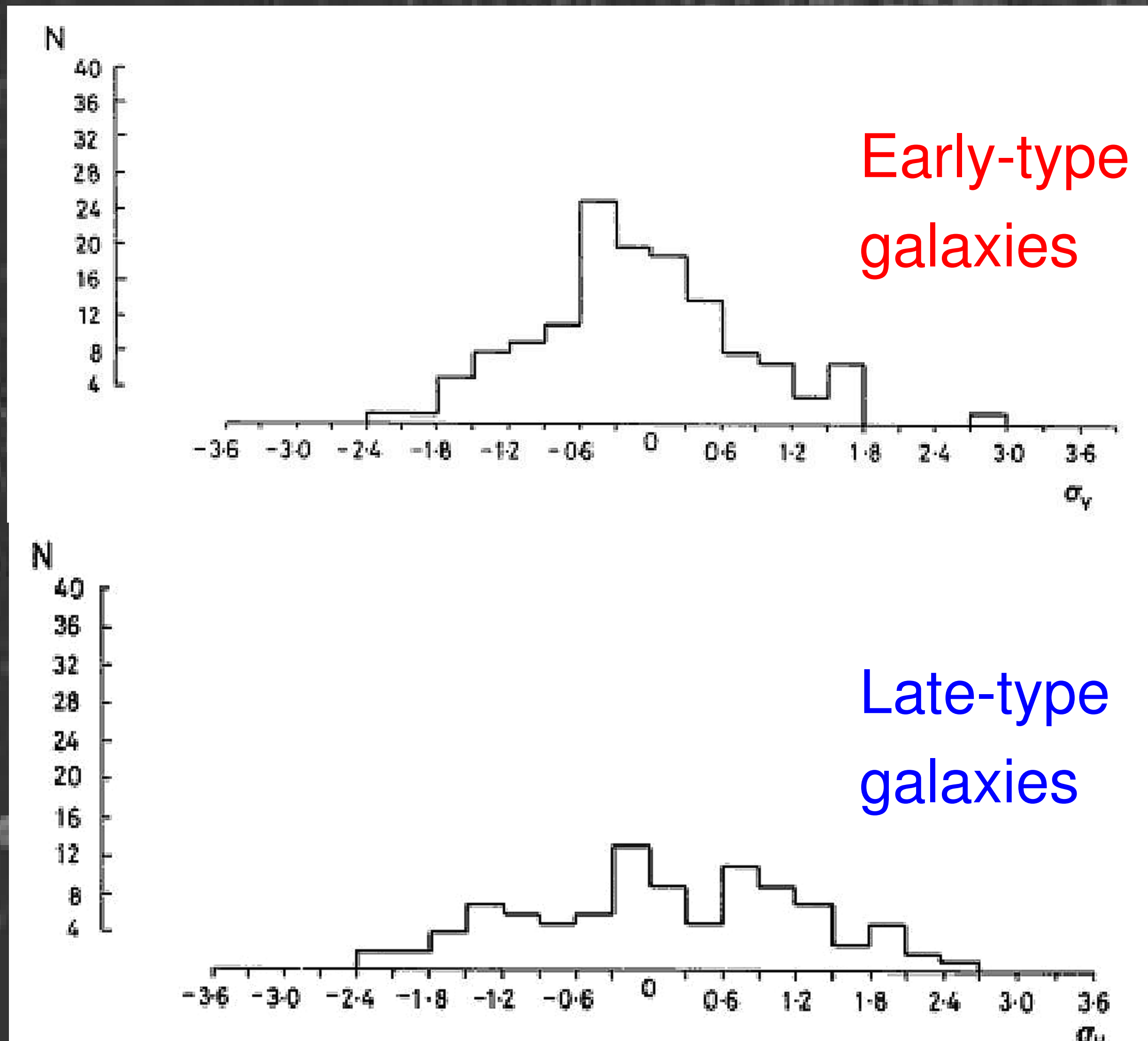
Biased distribution of the tracers?



(stack of 59 ENACS clusters; Katgert+04)

Biased distribution of the tracers?

(Moss & Dickens 77)

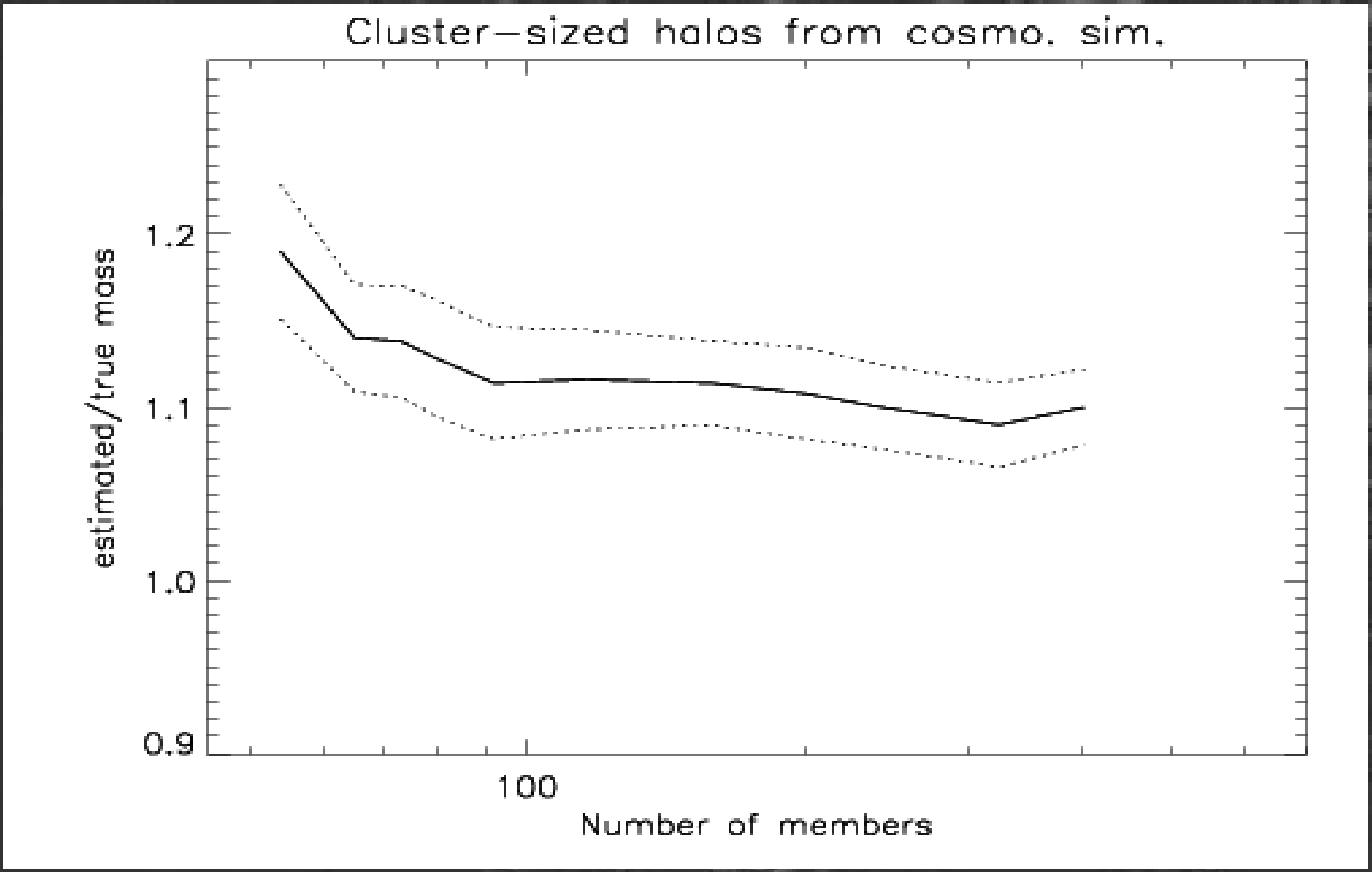


Velocity wrt cluster mean

Virial theorem

500 members: bias ~ +10%, scatter ~ 30%

100 members: bias ~ +10%, scatter ~ 46%

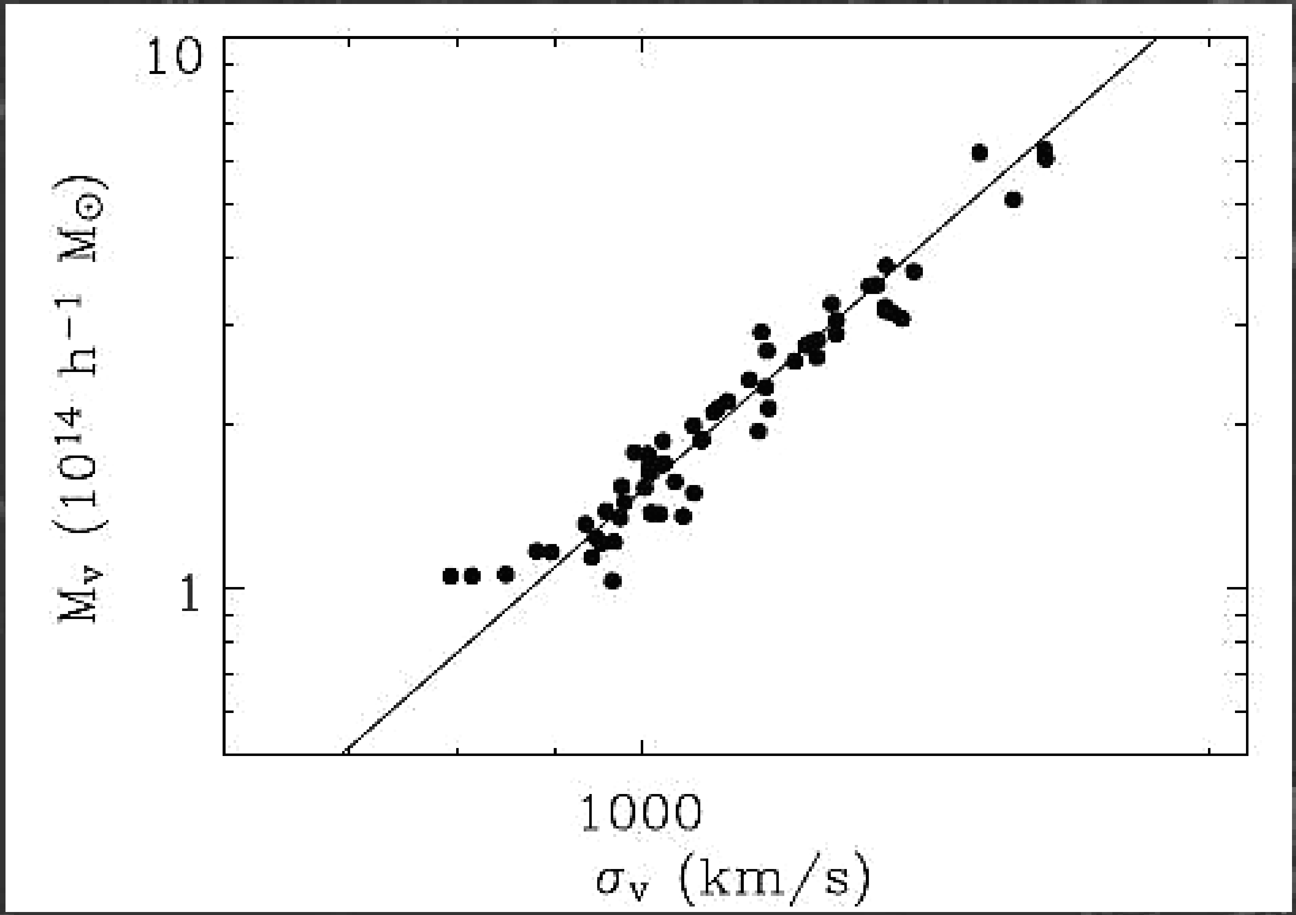


(ab+06)

Methods vs. problems:

	Virial	Mσ	Jeans	D+K	DF	MAMPOSSt	Caustic
M-β degeneracy	🟡	⚫	⚫	⚫	⚫	⚫	⚫
Cosmo. framework	🟡	⚫	⚫	⚫	⚫	⚫	⚫
Incompleteness	🔴	⚫	⚫	⚫	⚫	⚫	⚫
Contamination	🔴	⚫	⚫	⚫	⚫	⚫	⚫
No equilibrium	🔴	⚫	⚫	⚫	⚫	⚫	⚫
Biased tracer	🔴	⚫	⚫	⚫	⚫	⚫	⚫
Poor statistics	🟡	⚫	⚫	⚫	⚫	⚫	⚫

$M_\sigma \equiv$ M from $\sigma_{v,los}$



True
mass – vel. disp.
relation for
cluster-sized
halos from
cosmo. sim.
(ab+06)

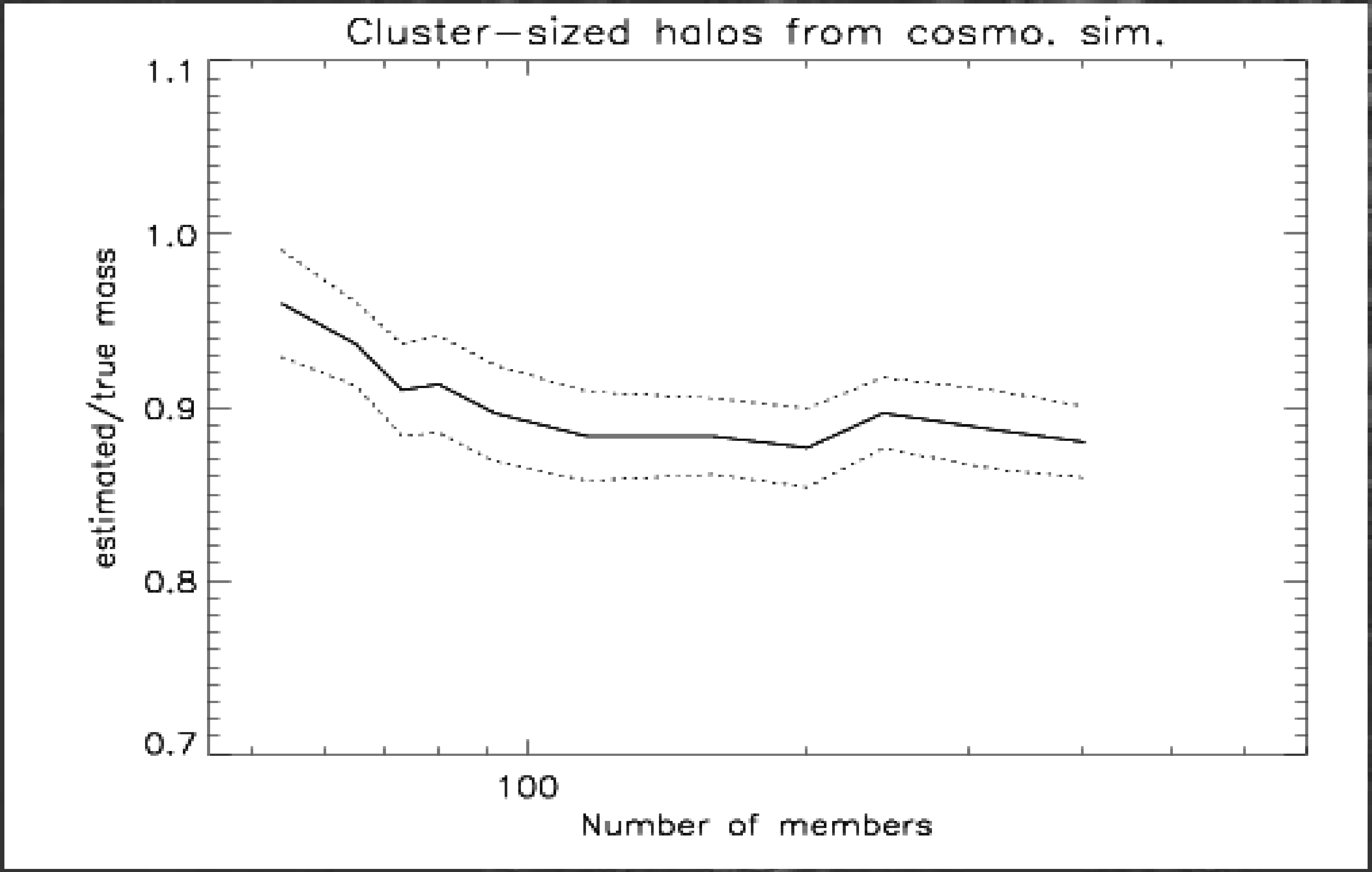
Advantage: get rid of spatial distribution (R_h)

Disadvantage: based on numerical simulations

$M_\sigma \equiv$ M from $\sigma_{v,los}$

500 members: bias ~ -12%, scatter ~ 28%

100 members: bias ~ -12%, scatter ~ 38%



(ab+06)

Methods vs. problems:

	Virial	Mσ	Jeans	D+K	DF	MAMPOSSt	Caustic
M-β degeneracy	● (yellow)	● (yellow)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)
Cosmo. framework	● (yellow)	● (red)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)
Incompleteness	● (red)	● (green)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)
Contamination	● (red)	● (yellow)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)
No equilibrium	● (red)	● (red)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)
Biased tracer	● (red)	● (yellow)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)
Poor statistics	● (yellow)	● (yellow)	● (grey)	● (grey)	● (grey)	● (grey)	● (grey)

Jeans equation [Binney & Tremaine 87]

$$M(<r) = -\frac{r\sigma_r^2}{G}\left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta\right)$$

velocity anisotropy

$$\beta(r) \equiv 1 - \sigma_t^2/\sigma_r^2$$

Jeans equation

Observables
N(R), σ_p(R)

+β(r)

M(r)

Mamon & Boué 08

Observables
N(R), σ_p(R)

+M(r)

β(r)

Binney & Mamon 82

Observables
N(R), σ_p(R)

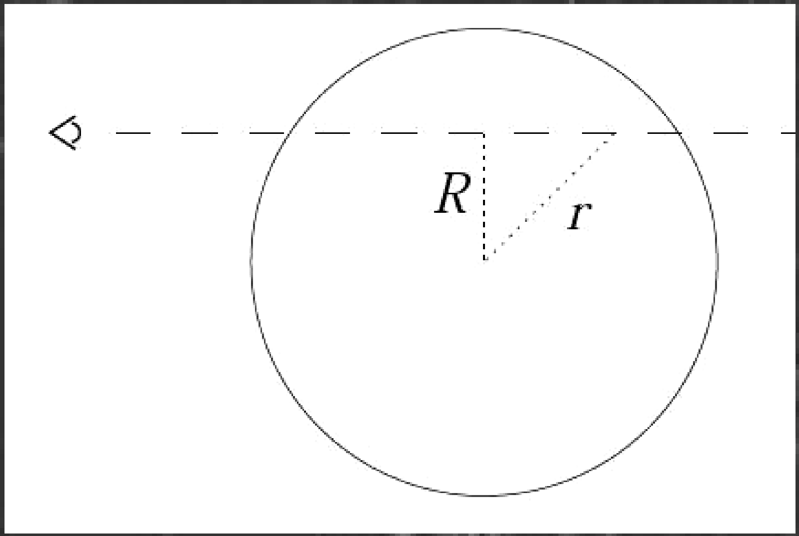
M(r)+β(r)

Bacon+83

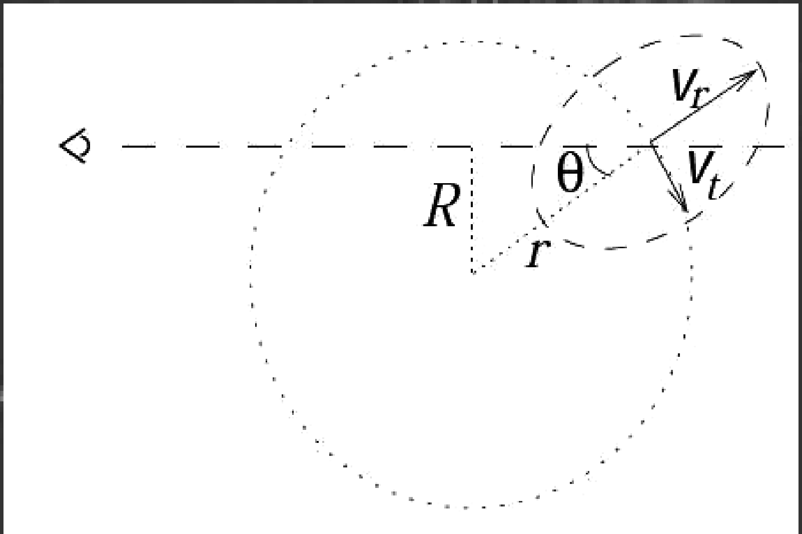
$$\nu\,\sigma_r^2 = -G\int_r^\infty \nu(\xi)\frac{M(<\xi)}{\xi^2}\exp\left[2\int_r^\xi \frac{\beta dx}{x}\right]d\xi$$

van der Marel 94

Jeans equation



Direct Abel deprojection of projected number density profile is possible:
N(R) → ν(r)



Deprojecting the l.o.s. velocity-dispersion profile requires knowledge of the velocity-anisotropy profile

➔ M(r) - β(r) degeneracy

Jeans equation

(Partially) breaking the M-β degeneracy:

- Use several tracers of same grav. potential
  (biased tracers are not a problem)
  [Carlberg+97; Battaglia+ 08; ab & Katgert 04; ab & Poggianti 09]

- Compare full galaxy velocities distribution with predictions from models
  [Carlberg+97; van der Marel+00; Katgert+04]

Methods vs. problems:

	Virial	Mσ	Jeans	D+K	DF	MAMPOSSt	Caustic
M-β degeneracy	yellow	yellow	red	grey	grey	grey	grey
Cosmo. framework	yellow	red	green	grey	grey	grey	grey
Incompleteness	red	green	red	grey	grey	grey	grey
Contamination	red	yellow	red	grey	grey	grey	grey
No equilibrium	red	red	red	grey	grey	grey	grey
Biased tracer	red	yellow	green	grey	grey	grey	grey
Poor statistics	yellow	yellow	red	grey	grey	grey	grey

Dispersion + Kurtosis

[Łokas & Mamon 03; Sanchis+04; Łokas+06]

$$\overline{v_{\rm los}^4}(R) = \frac{6G^2}{I(R)} \int_R^\infty \frac{r^{-2\beta+1}}{\sqrt{r^2-R^2}} g(r,R,\beta)\,\mathrm{d}r \times \int_r^\infty \frac{\nu(q)M(q)}{q^{2-2\beta}}\,\mathrm{d}q \int_r^q \frac{M(p)}{p^2}\,\mathrm{d}p$$

$$g(r,R,\beta) = 1 - 2\beta\frac{R^2}{r^2} + \frac{\beta(1+\beta)}{2}\frac{R^4}{r^4}$$

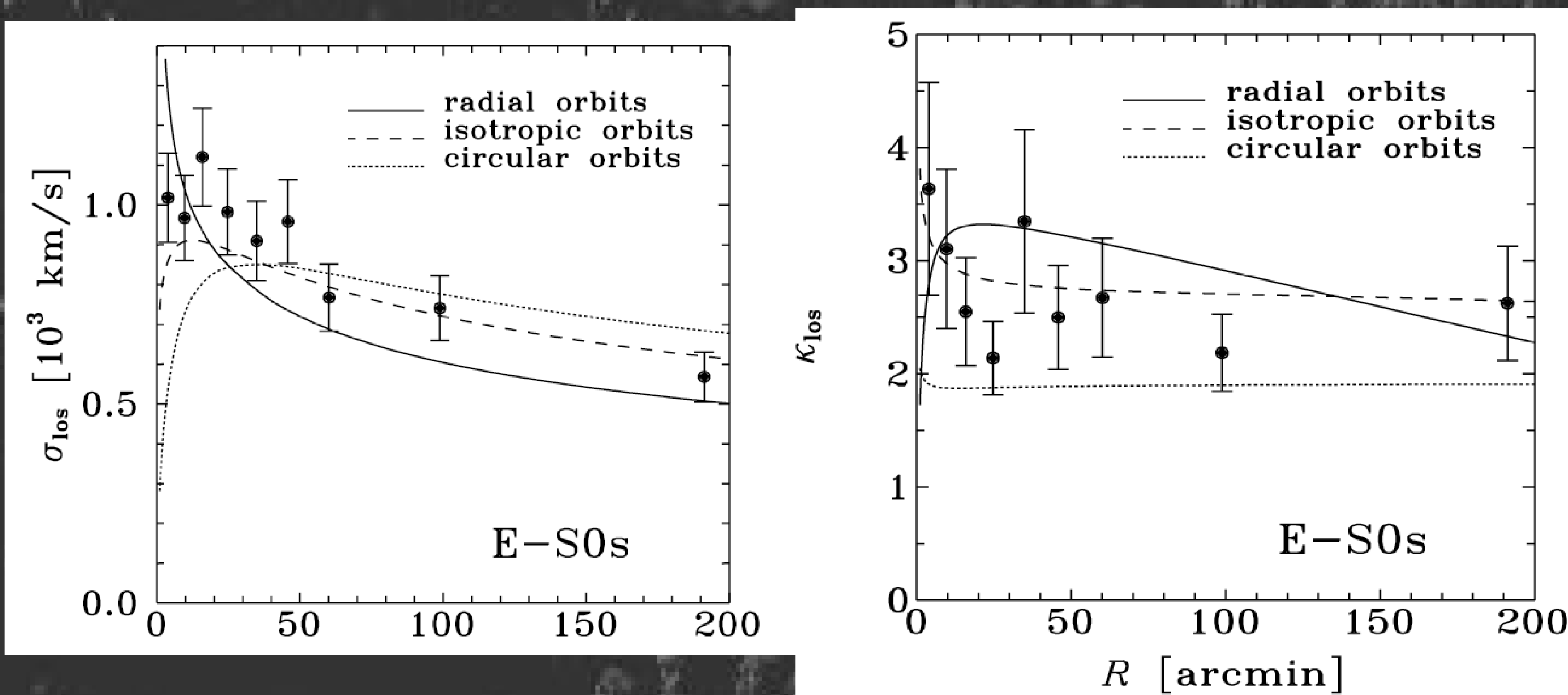
$$\kappa_{\rm los}(R) = \frac{\overline{v_{\rm los}^4}(R)}{\sigma_{\rm los}^4(R)}$$

Breaks the M-β degeneracy by adding the 4th moment eq. to the Jeans eqs.

assuming constant β

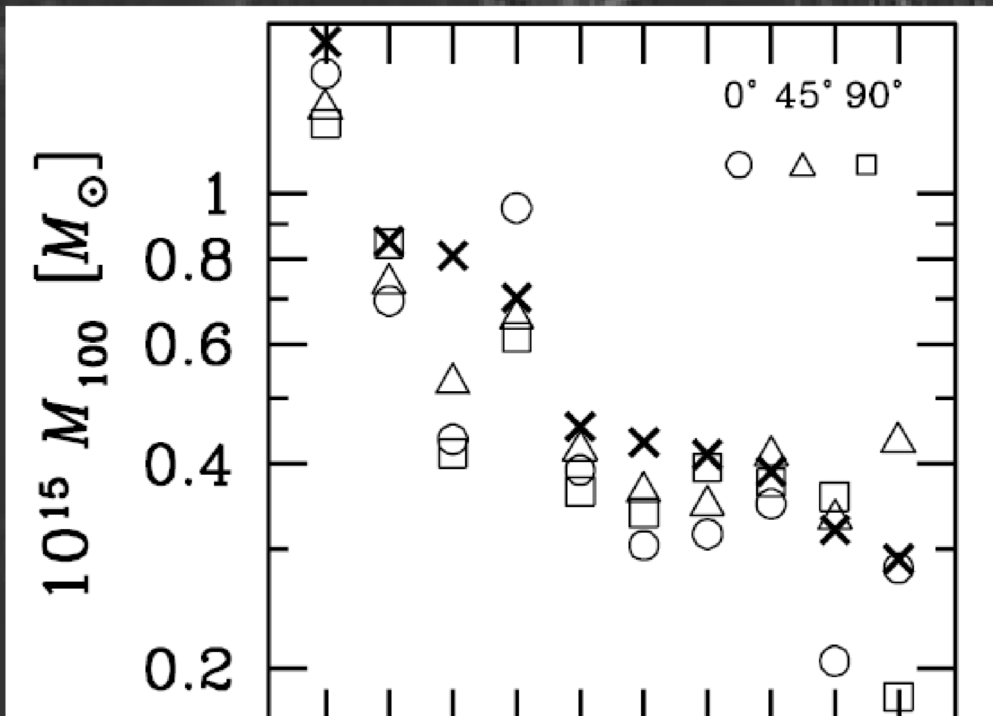
Dispersion + Kurtosis

Find best M(r)+β model by simultaneous fitting to vel. dispersion and kurtosis profiles

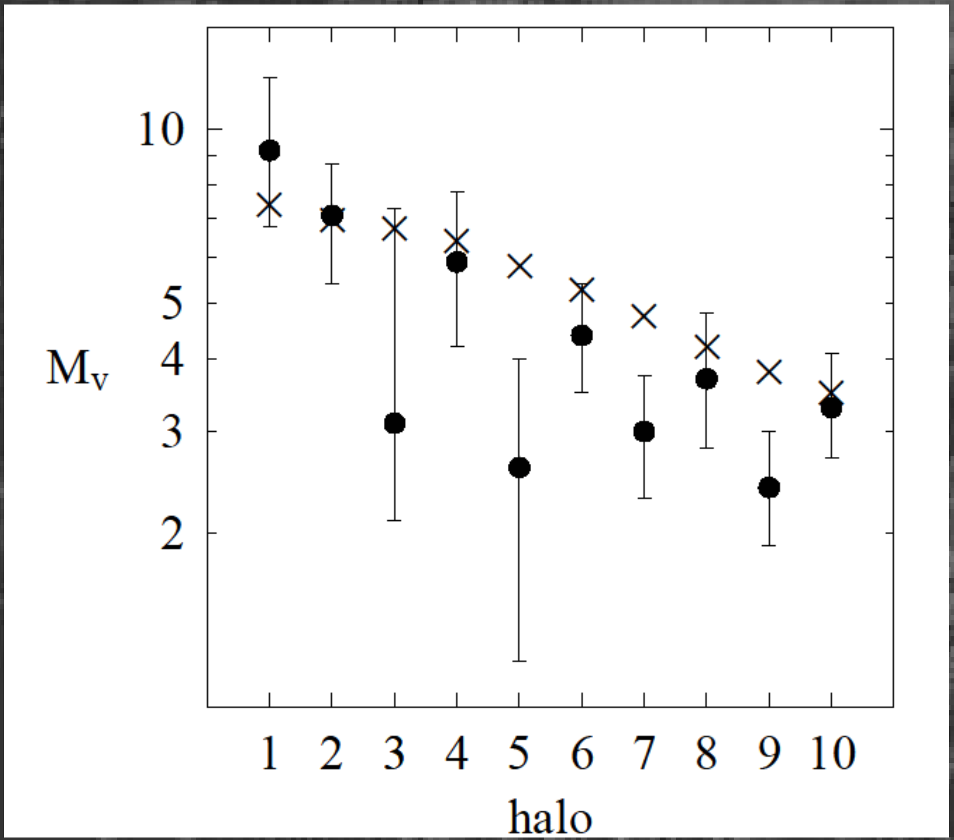


[Coma cluster data; Łokas & Mamon 03]

Dispersion + Kurtosis



[Cluster-size halos from cosmo. sim.; Sanchis + 04; Łokas + 06]



300-400 members: bias ~ -20%, scatter ~ 25%

Methods vs. problems:

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Cosmo. framework							
Incompleteness							
Contamination							
No equilibrium							
Biased tracer							
Poor statistics							

Distribution function methods

[Dejonghe & Merritt 92; Merritt & Saha 93; Mahdavi & Geller 04; van der Marel + 00; Wojtak+09]

Spherical system ⇔ its distribution function (DF)

DF depends on phase-space coords through E, L.

Generally assume: $$f(E, L) = f_{\mathrm{E}}(E) f_{\mathrm{L}}(L)$$

$$f_{\mathrm{los}}(R, v_{\mathrm{los}}) = 2\pi R \int_{-z_{\mathrm{max}}}^{z_{\mathrm{max}}} \mathrm{d}z \iint_{E>0} \mathrm{d}v_R \mathrm{d}v_\phi \, f_{\mathrm{E}}(E) f_{\mathrm{L}}(L)$$

$$2 \int_0^{R_{\mathrm{max}}} \mathrm{d}R \int_0^{\sqrt{2\Psi(R)}} f_{\mathrm{los}}(R, v_{\mathrm{los}}) \mathrm{d}v_{\mathrm{los}} = 1$$

Distribution function methods

[Wojtak+08; Wojtak+09]

DF model that fits cluster-sized halos from cosmological simulations:

$$f_{\mathrm{L}}(L) = \left(1 + \frac{L^2}{2L_0^2}\right)^{-\beta_\infty + \beta_0} L^{-2\beta_0}$$

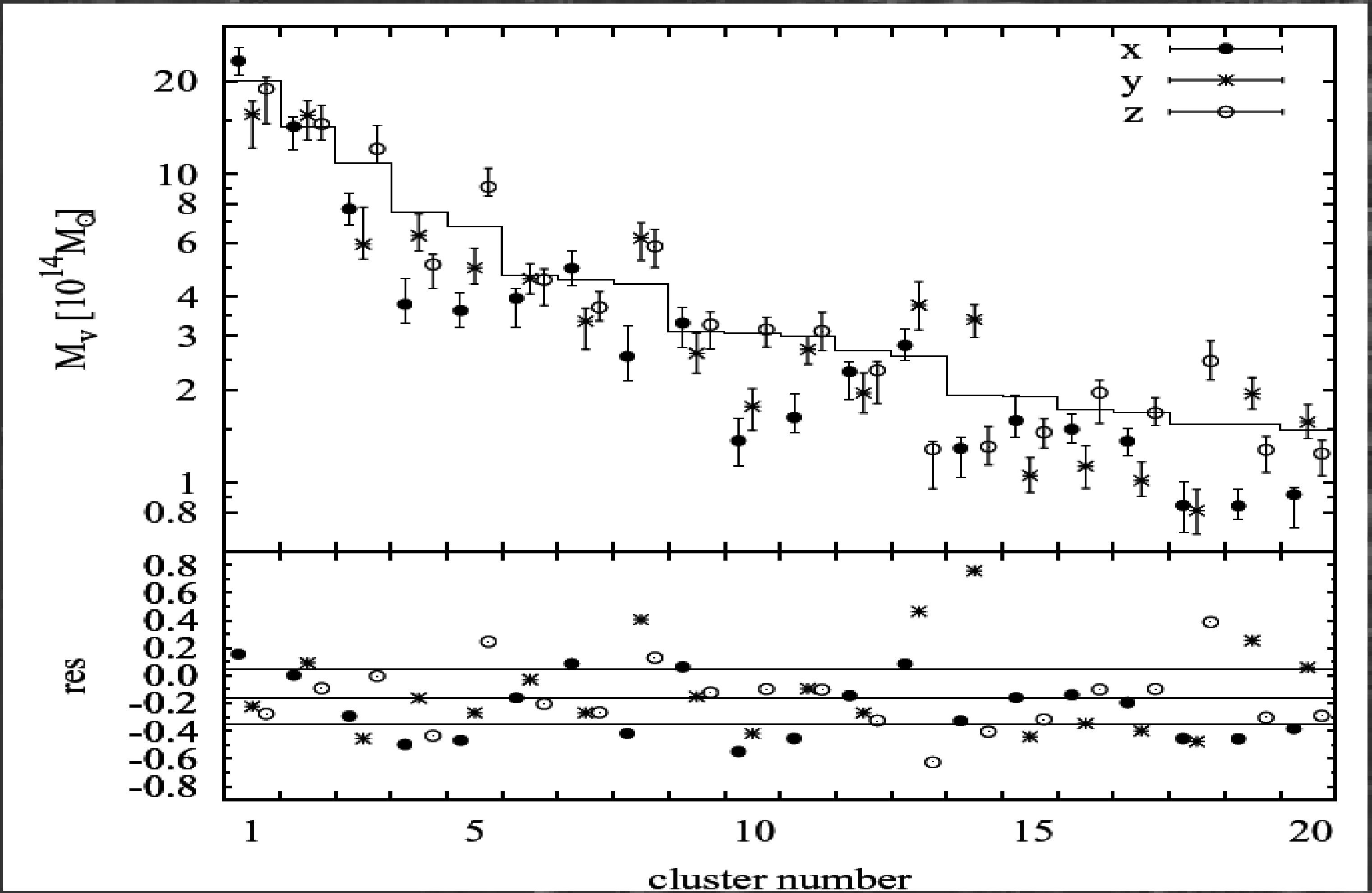
$$\beta(r) = 1 - \frac{\sigma_\theta^2(r)}{\sigma_{\mathrm{r}}^2(r)}$$

$f_E(E)$ found by solving:

$$\rho(r) = \iiint f_{\mathrm{E}}(E)\left(1 + \frac{L^2}{2L_0^2}\right)^{-\beta_\infty + \beta_0} L^{-2\beta_0}\,\mathrm{d}^3 v$$

Distribution function methods

[Wojtak+08; Wojtak+09]



300 members: bias ~ -10%, scatter ~30%

Methods vs. problems:

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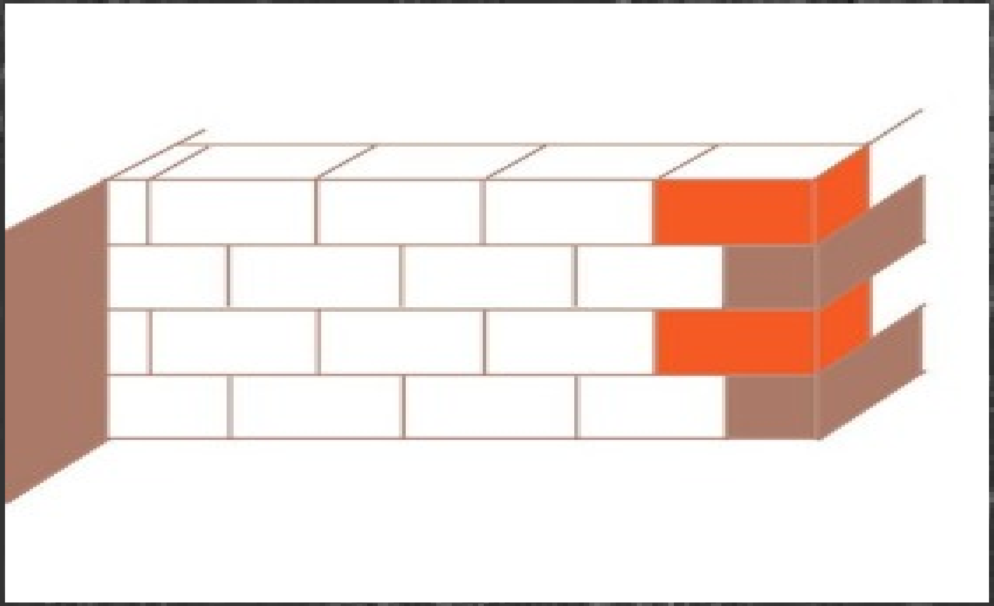
Biased tracer	red	yellow	green	green	green	grey	grey
Poor statistics	yellow	yellow	red	red	red	grey	grey

MAMPOSSt:

*M*odelling *A*nisotropy and *M*ass *P*rofiles of *O*bserved *S*pherical *S*ys*t*ems

[Mamon, ab, Boué 2010]



Like a *lampost*... ...or like *mampostería* (masonry)

MAMPOSSt:

[Mamon, ab, Boué 2010]

$$\left(\frac{dN}{dv_z}\right)_{r,R} = \int_{-\infty}^{+\infty} dv_\perp \int_{-\infty}^{+\infty} f_v(v_z, v_\perp, v_\phi)\, dv_\phi$$

distrib. of los velocities

$$\begin{aligned} g(R, v_z) &= \Sigma(R) \left\langle \frac{dN}{dv_z} \right\rangle_{\rm l.o.s.} \\ &= 2 \int_R^\infty \frac{r\,\nu(r)}{\sqrt{r^2 - R^2}} \left(\frac{dN}{dv_z}\right)_{r,R} dr \\ &= 2 \int_R^\infty \frac{r\,dr}{\sqrt{r^2 - R^2}} \int_{-\infty}^{+\infty} dv_\perp \int_{-\infty}^{+\infty} f(r, v_z, v_\perp, v_\phi)\, dv_\phi \end{aligned}$$

surface density of objects in projected phase space

MAMPOSSt:

Rather than replace velocities by E, L
use known 3D velocity distributions, e.g. Gaussian:

$$f_v(v_r, v_\theta, v_\phi) = \frac{1}{(2\pi)^{3/2}\sigma_r\sigma_\theta^2} \exp\left[-\frac{v_r^2}{2\sigma_r^2} - \frac{v_\theta^2 + v_\phi^2}{2\sigma_\theta^2}\right]$$

MAMPOSSt:

so that the surface density of objects becomes:

$$g(R,v_z) = \sqrt{\frac{2}{\pi}} \int_R^\infty \frac{r\,\nu}{\sqrt{r^2-R^2}} \frac{\left(1-\beta R^2/r^2\right)^{-1/2}}{\sigma_r} \times \exp\left[-\frac{v_z^2}{2\,\left(1-\beta R^2/r^2\right)\,\sigma_r^2}\right] dr$$

and we maximize the probability of finding the observed los velocities at the observed projected radii:

$$p(v_z|R) = \frac{g(R,v_z)}{\Sigma(R)} = \frac{1}{\sqrt{2\pi}} \frac{\int_0^\infty (\nu/\sigma_z)\,\exp\left[-v_z^2/\left(2\,\sigma_z^2\right)\right]\,dz}{\int_0^\infty \nu\,dz}$$

$$\sigma_z^2(R,r) = \left[1-\beta(r)\,\left(\frac{R}{r}\right)^2\right]\,\sigma_r^2(r)$$

where ν and σ_r are obtained from the Abel and Jeans eqs.

MAMPOSSt:

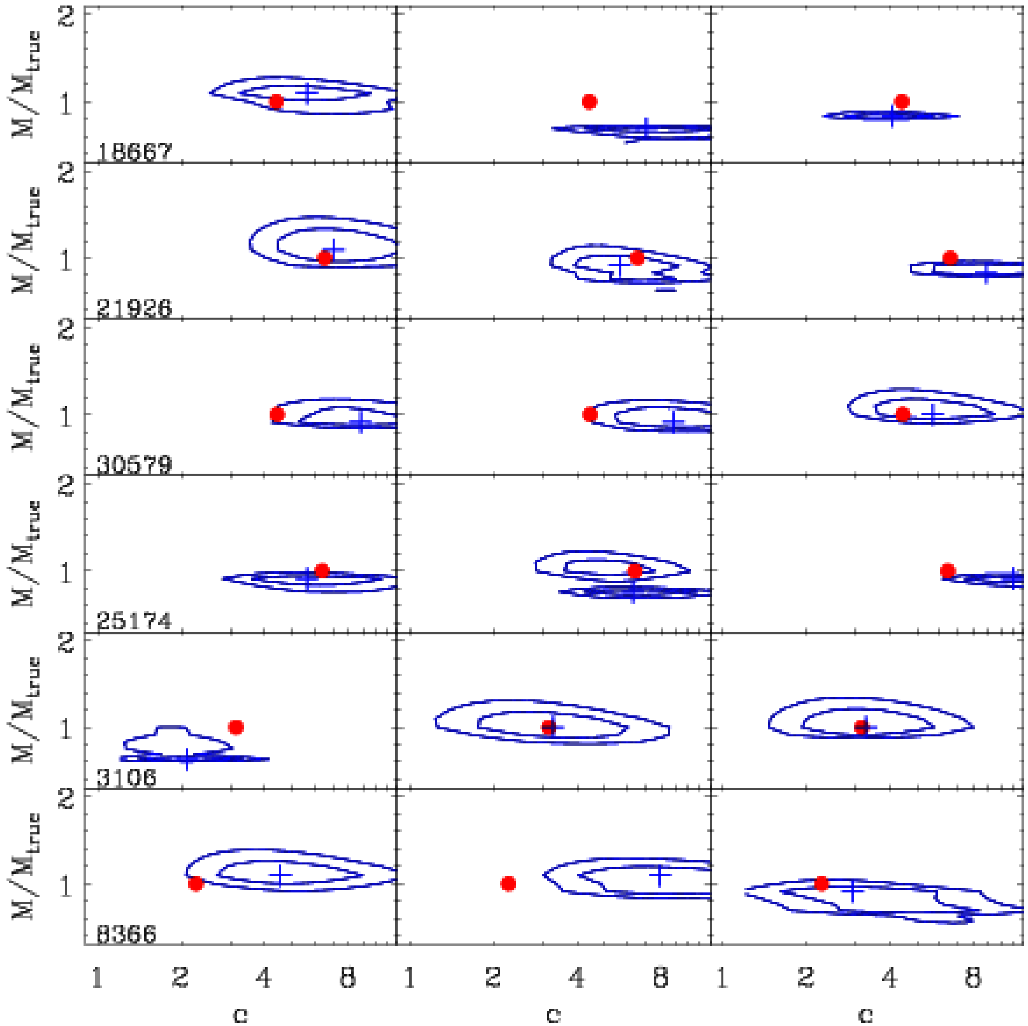
Red dot:

true values

Blue contours

68% and 95% confidence levels

~400 particles per halo



concentration

Mass

Test on 11x3 cluster-sized halos from cosmo. sim.

MAMPOSSt:

Red dot:

true values

Blue contours

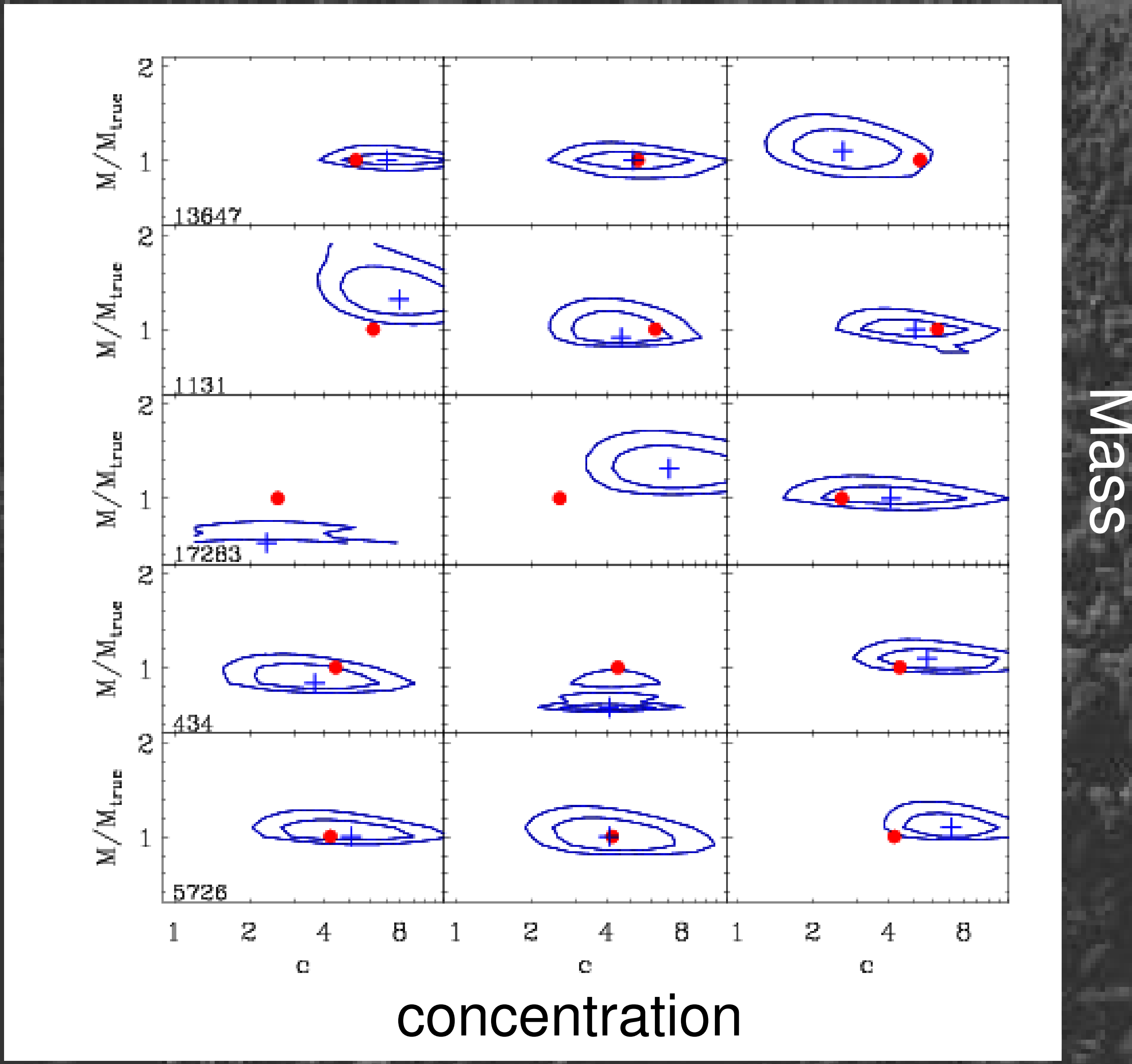
68% and 95%

confidence

levels

~400 particles

per halo

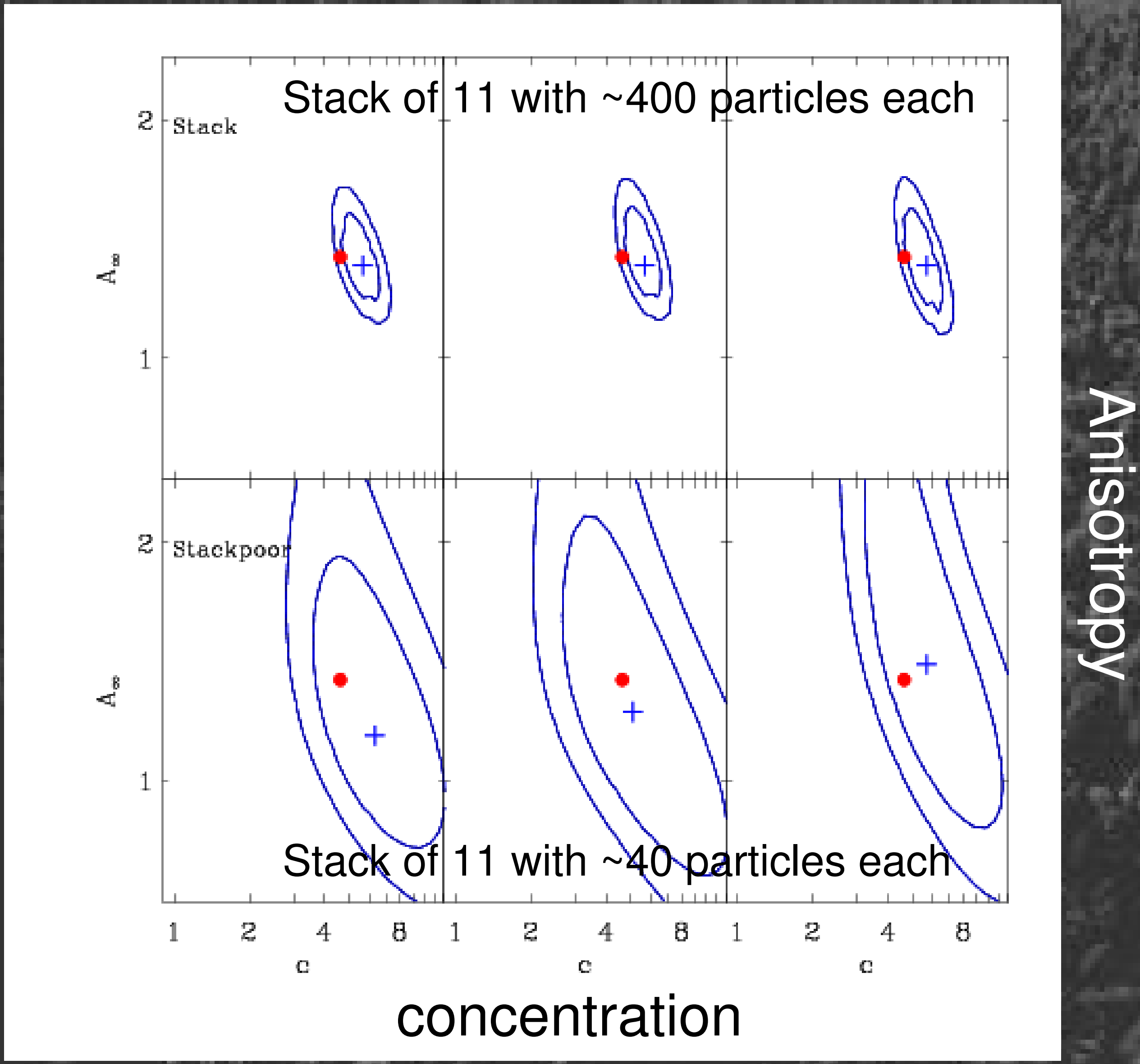


Test on 11x3 cluster-sized halos from cosmo. sim.

MAMPOSSt:

Red dot:
true values

Blue contours
68% and 95%
confidence
levels

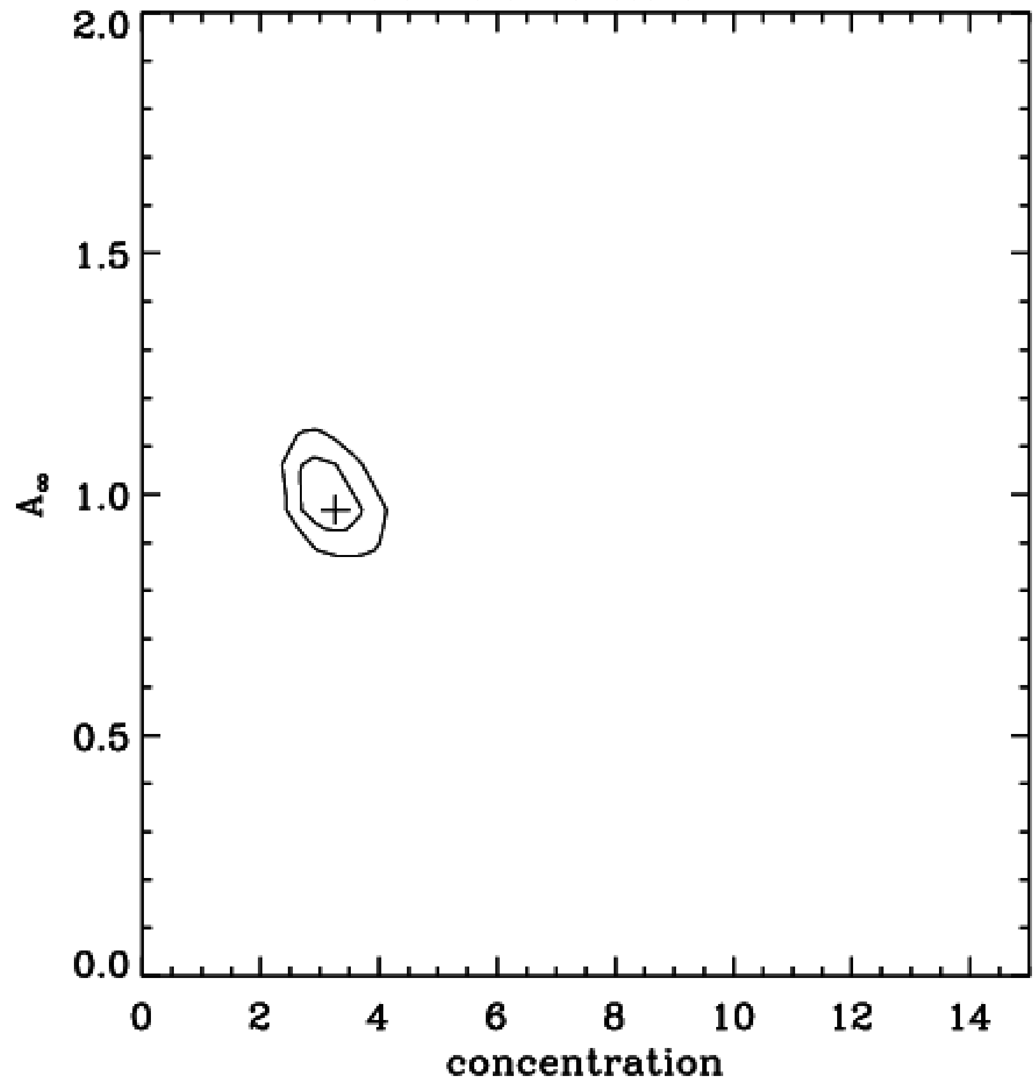


Test on 11x3 cluster-sized halos from cosmo. sim.

MAMPOSSt:

concentration

Stack of 55 CIRS clusters (using MAMPOSSt estimates for individual cluster M_{200}): 3800 galaxies with $R<r_{200}$



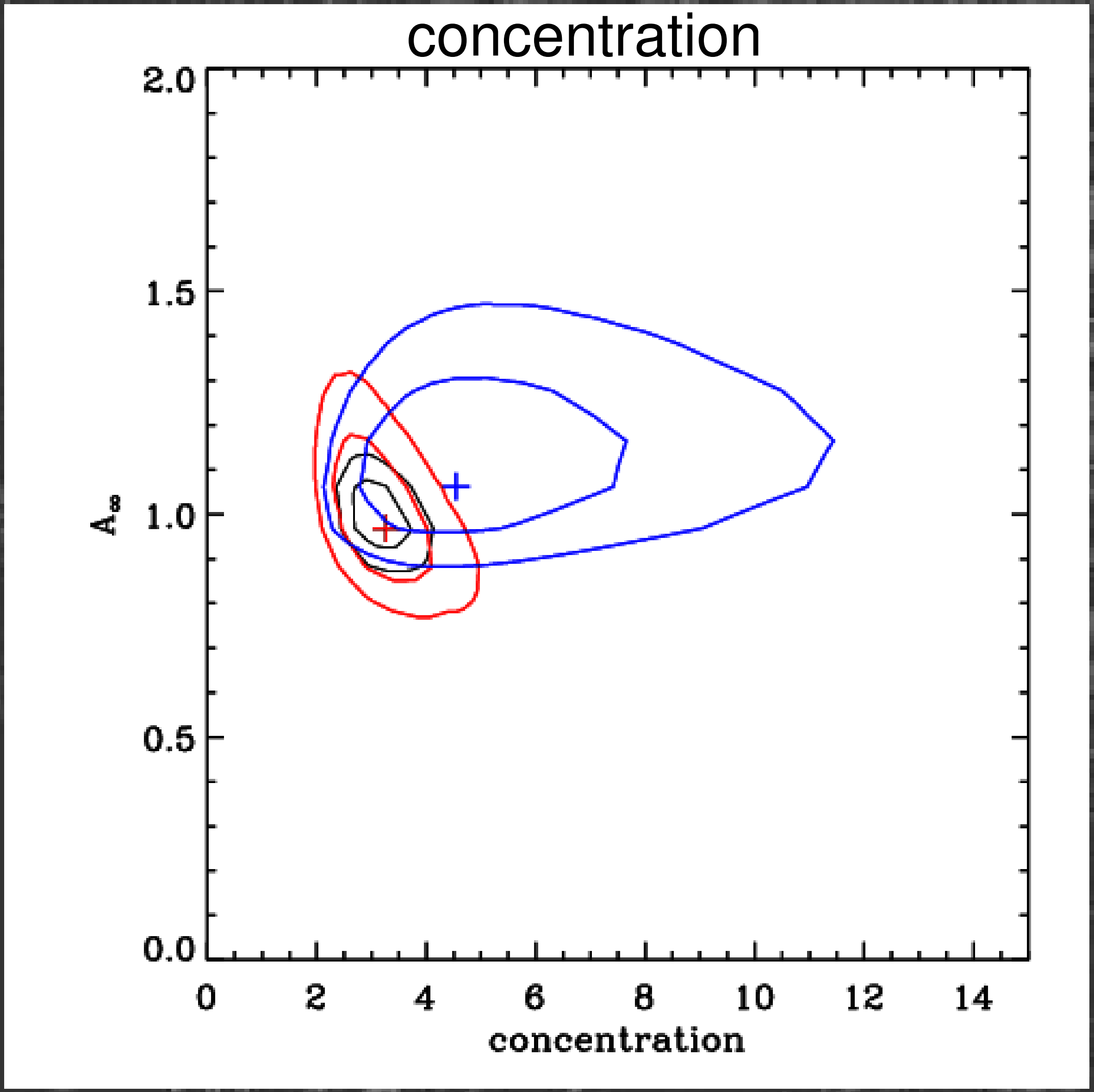
Anisotropy

[ab, Diaferio, Mamon & Rines, in prep.]

MAMPOSSt:

Stack of 55
CIRS clusters

3800 galaxies
with $R<r_{200}$
of which:
1400 red seq.
700 blue cloud



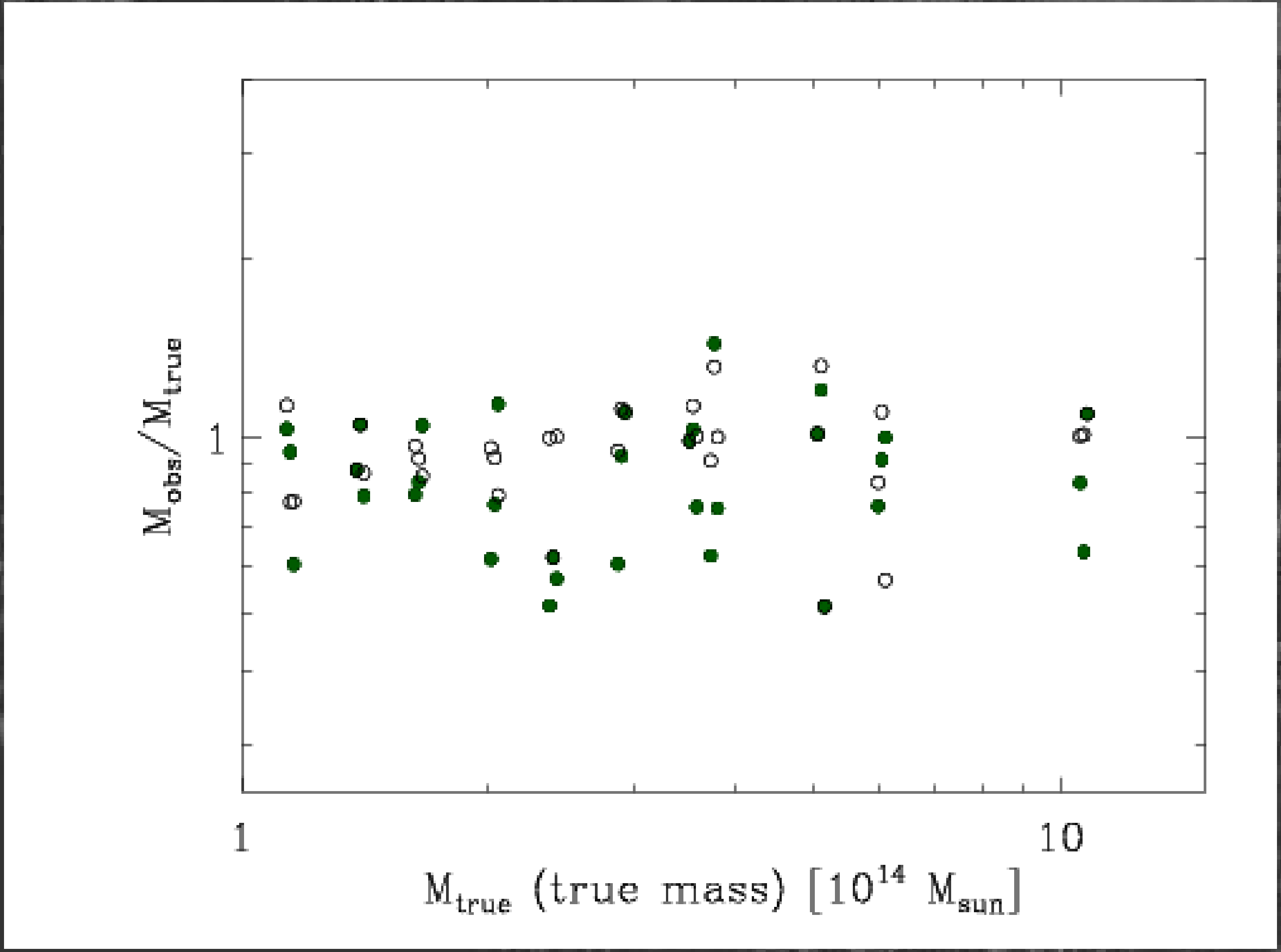
[ab, Diaferio, Mamon & Rines, in prep.]

MAMPOSSt:

11x3 halos from cosmo. sim.

~400 particles per halo

~100 particles per halo



~400 members: bias -5%, scatter 18%

~100 members: bias -14%, scatter 22%

Methods vs. problems:

	Virial	Mσ	Jeans	D+K	DF	MAMPOSSt	Caustic
M-β degeneracy	yellow	yellow	red	yellow	green	green	dark
Cosmo. framework	yellow	red	green	green	green	green	dark
Incompleteness	red	green	red	red	red	red	dark
Contamination	red	yellow	red	red	red	red	dark
No equilibrium	red	red	red	red	red	red	dark
Biased tracer	red	yellow	green	green	green	green	dark
Poor statistics	yellow	yellow	red	red	red	yellow	dark

Caustic technique [Diaferio & Geller 97; Diaferio 99]

$A(r)$

↓

$\phi(r)$

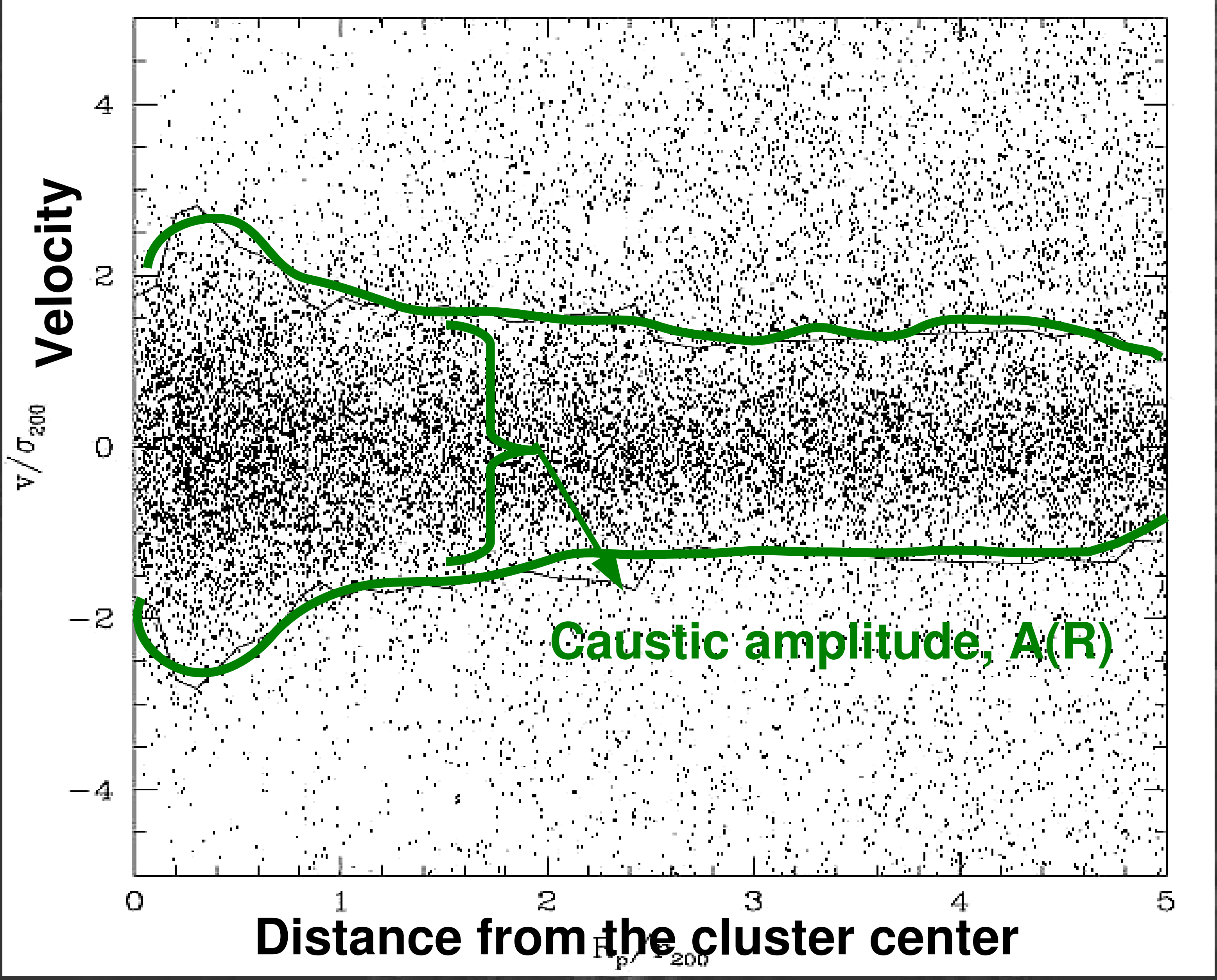
through

$F(r;\beta,\phi)$

$\approx$ const

outside

the center



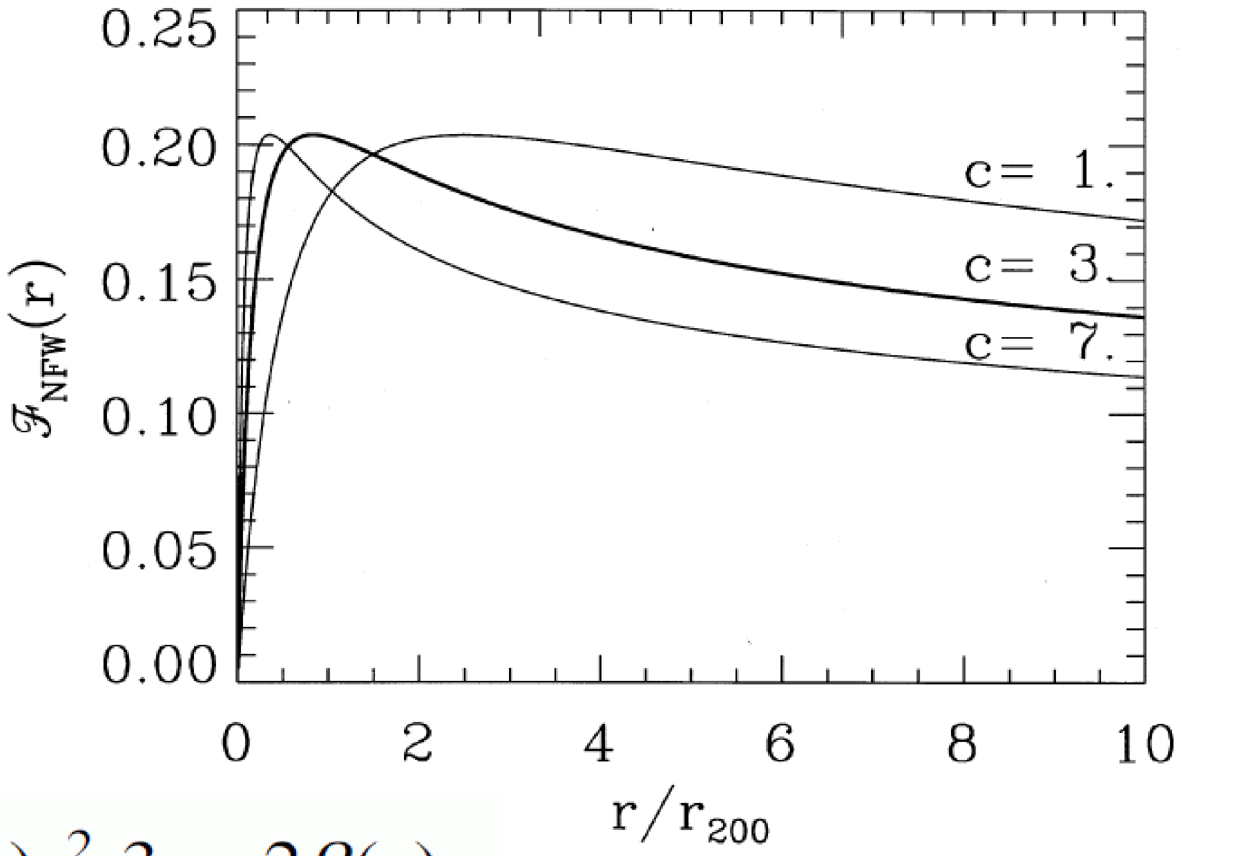
(Rines & Diaferio 06)

Caustic problems

$$GM(<r) - GM(<r_0) = \int_{r_0}^{r} \mathcal{A}^2(x)\mathcal{F}_\beta(x)\,\mathrm{d}x$$

Assume a constant $\mathcal{F}(r)$?

[cf. ab+Girardi 03]

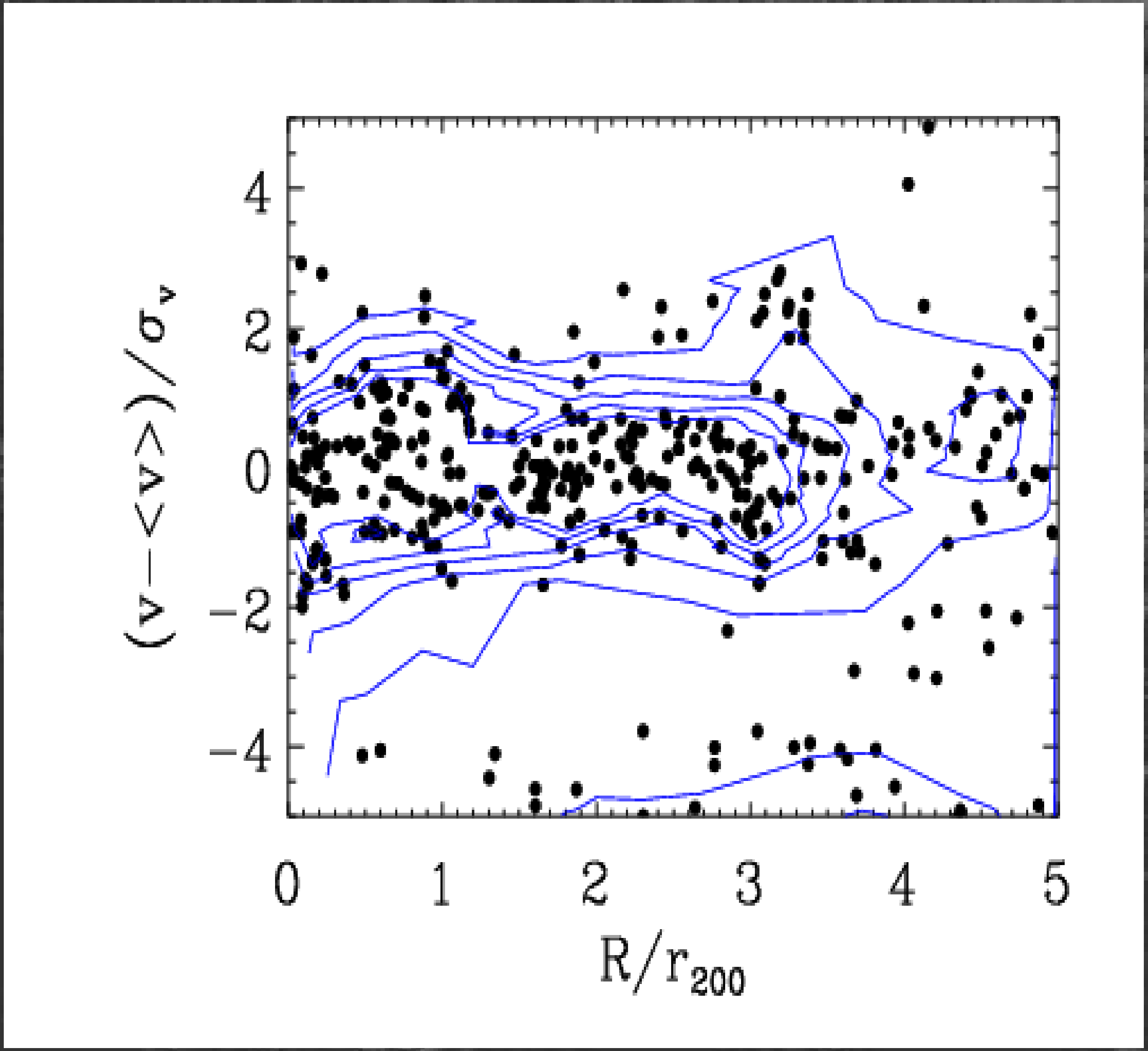


$$\mathcal{F}_\beta(r) = -2\pi G \frac{\rho(r) r^2}{\phi(r)} \frac{3 - 2\beta(r)}{1 - \beta(r)}$$

Caustic problems

Which is *the* caustic?

Ansatz requires identification of cluster members near the center and symmetrization of amplitude wrt v=0 axis



Caustic advantages

- Completeness not required

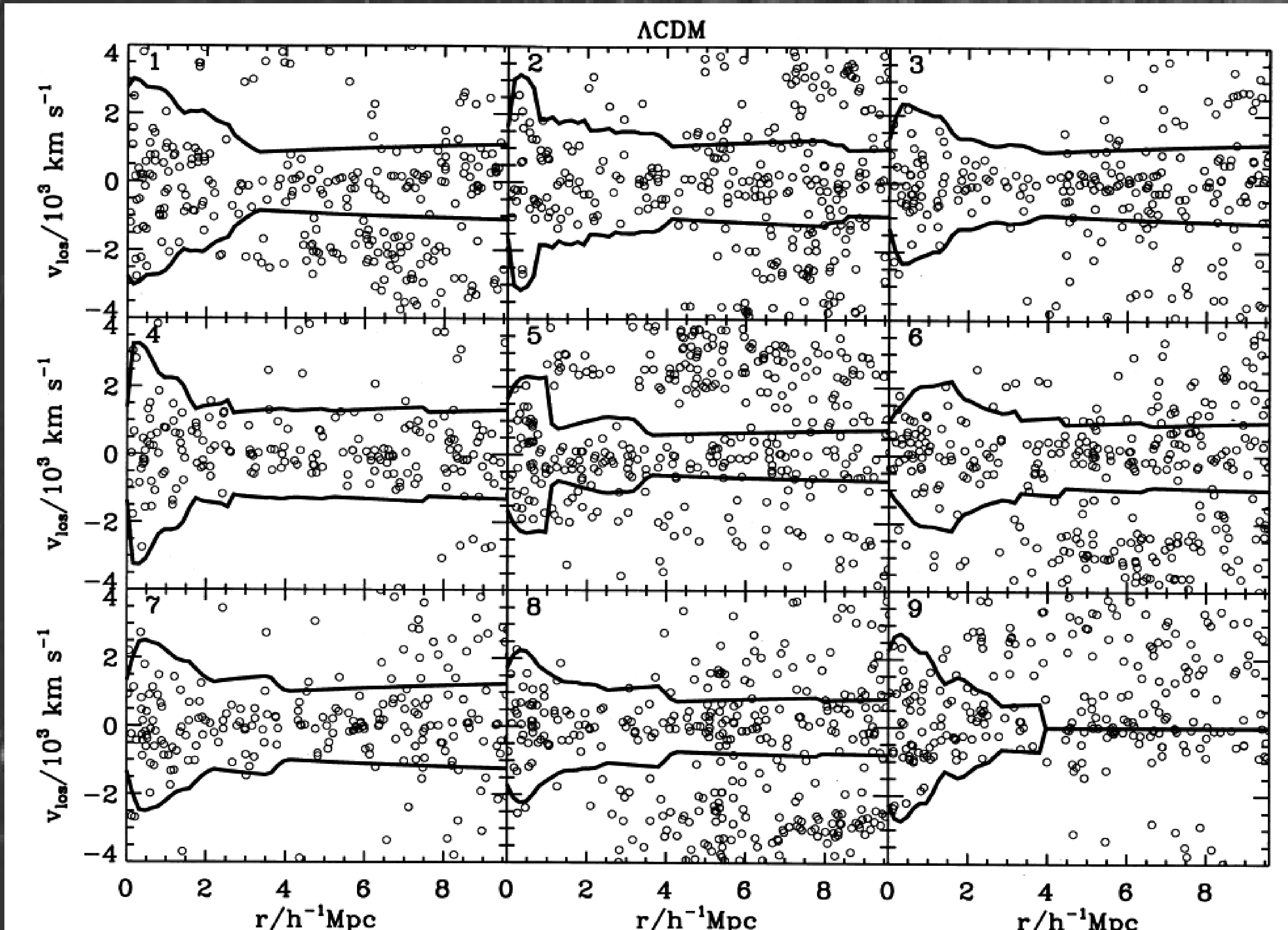
- Dynamical equilibrium
- Definition of membership

} required only in the inner virial region

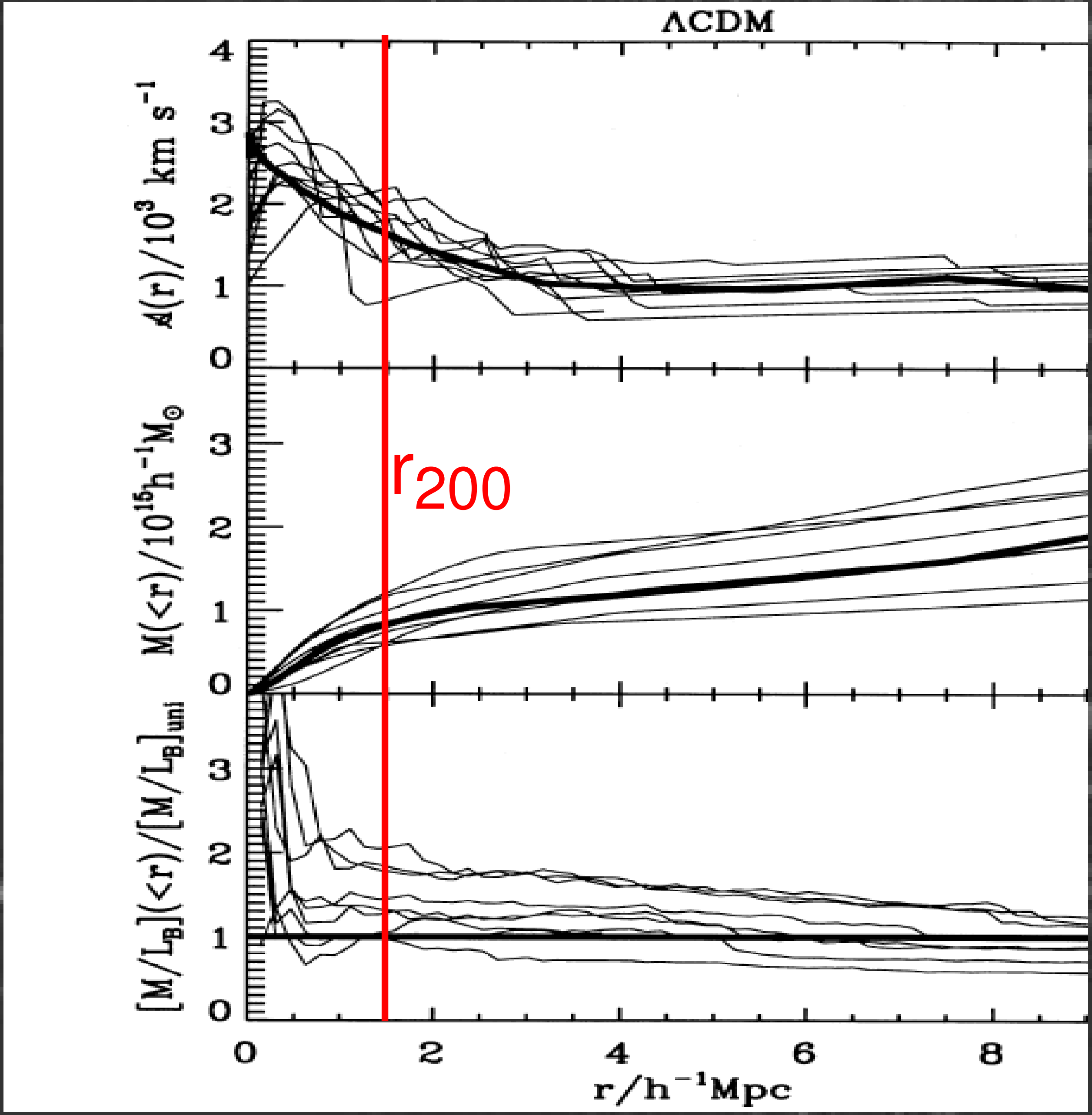
- Precise knowledge of $\beta(r)$ not required

- Mass determined at $r \gg r_{200}$

Caustic: Simulated halo, 9 los [Diaferio 99]



Caustic: 2 simulated halos, 9 los [Diaferio 99]



~300 particles: bias ~0%, scatter ~25%

Methods vs. problems:

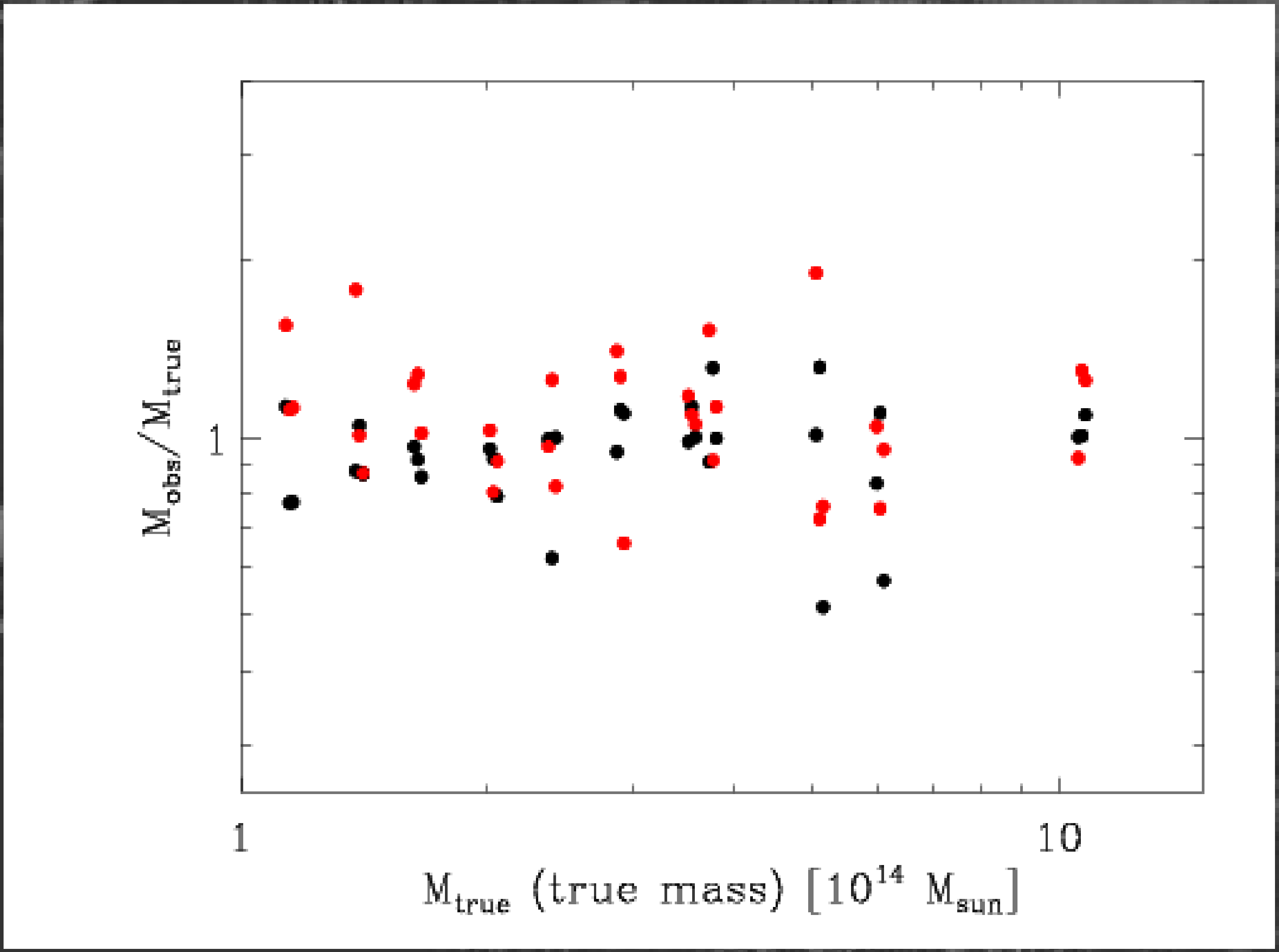
	Virial	Mσ	Jeans	D+K	DF	MAMPOSSt	Caustic
M-β degeneracy	yellow	yellow	red	yellow	green	green	yellow
Cosmo. framework	yellow	red	green	green	green	green	red
Incompleteness	red	green	red	red	red	red	green
Contamination	red	yellow	red	red	red	red	yellow
No equilibrium	red	red	red	red	red	red	yellow
Biased tracer	red	yellow	green	green	green	green	green
Poor statistics	yellow	yellow	red	red	red	yellow	yellow

COMPARING THE DIFFERENT METHODS

MAMPOSSt vs. Virial

11x3 halos from cosmo. sim.

~400 particles per halo



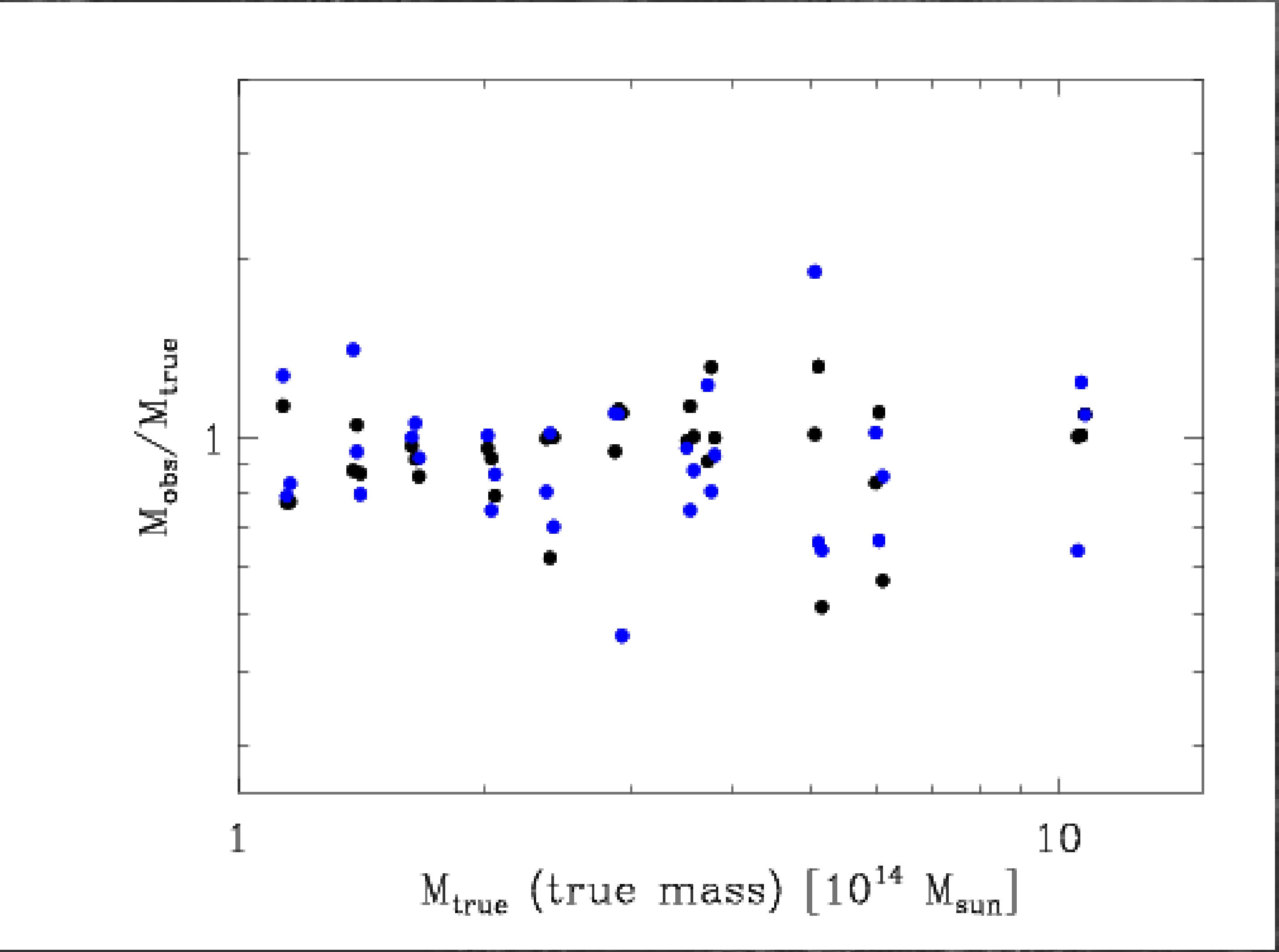
MAMPOSSt: bias -5%, scatter 18%

Virial: bias +10%, scatter 28%

MAMPOSSt vs. M_σ

11x3 halos from cosmo. sim.

~400 particles per halo



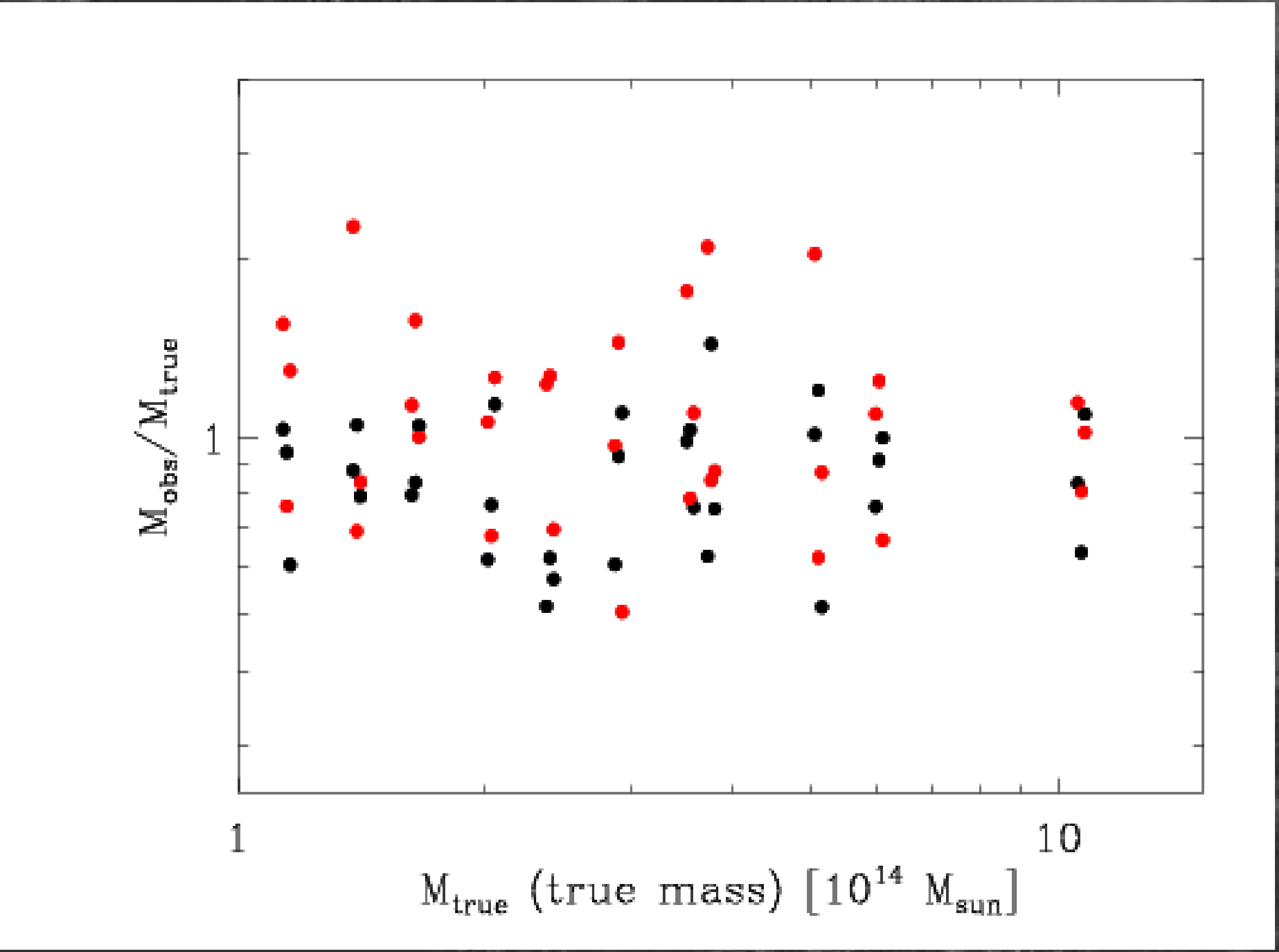
MAMPOSSt: bias -5%, scatter 18%

$M\sigma$: bias -6%, scatter 26%

MAMPOSSt vs. Virial

11x3 halos from cosmo. sim.

~100 particles per halo



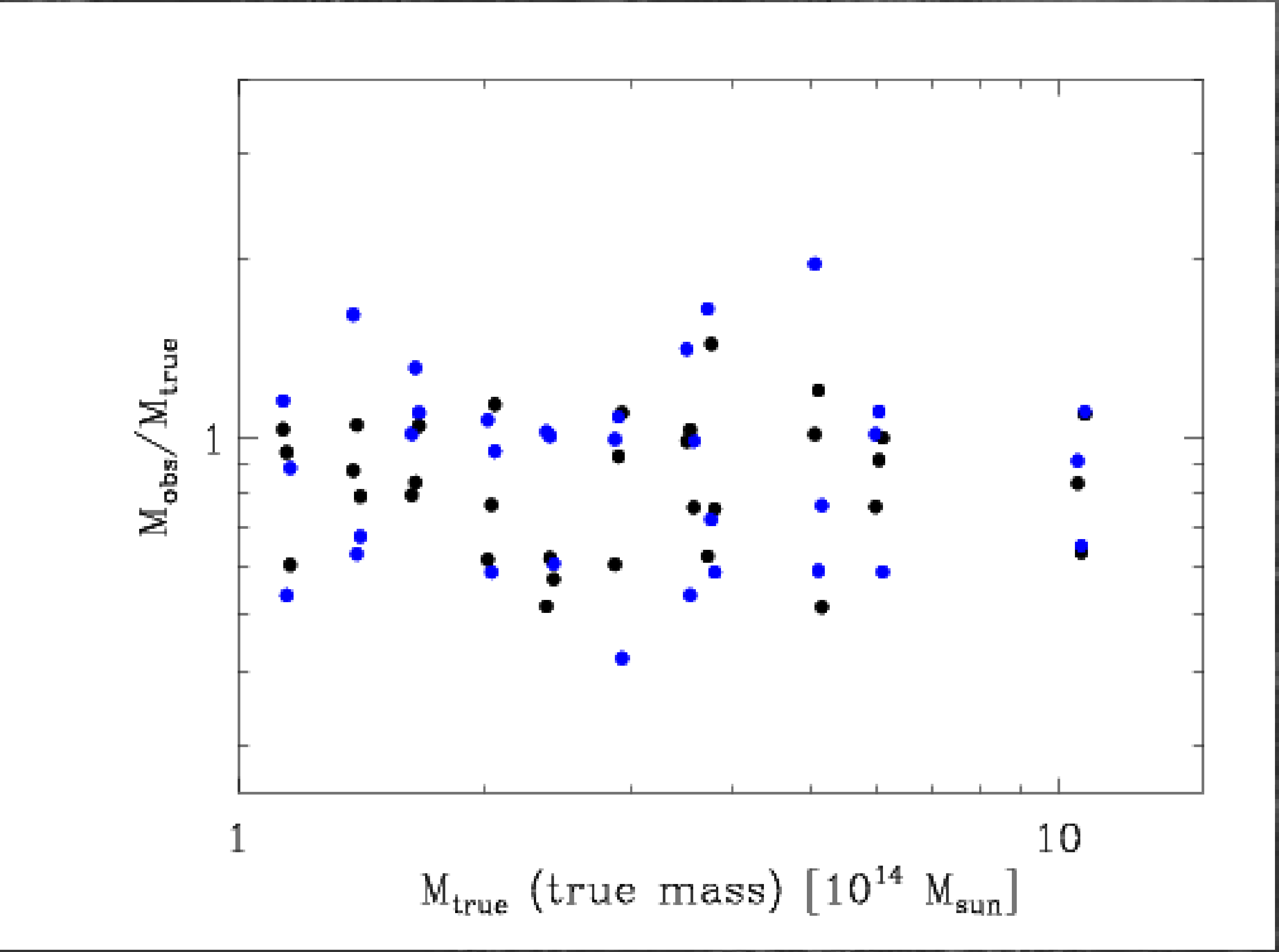
MAMPOSSt: bias -14%, scatter 22%

Virial: bias +13%, scatter 43%

MAMPOSSt vs. M_σ

11x3 halos from cosmo. sim.

~100 particles per halo



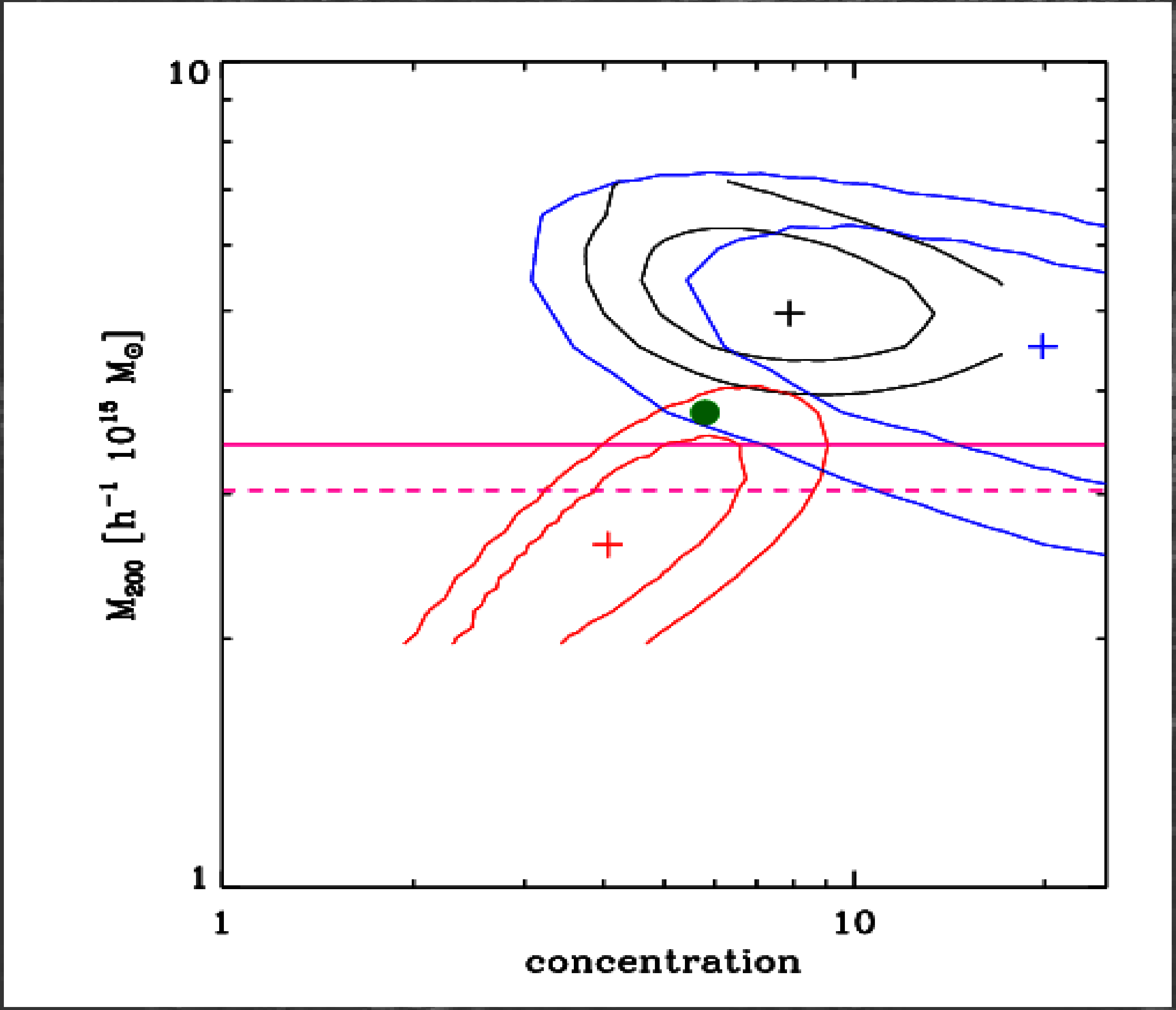
MAMPOSSt: bias -14%, scatter 22%

$M\sigma$: bias -6%, scatter 35%

MAMPOSSt, D+K, Caustic, Virial, M_σ, true

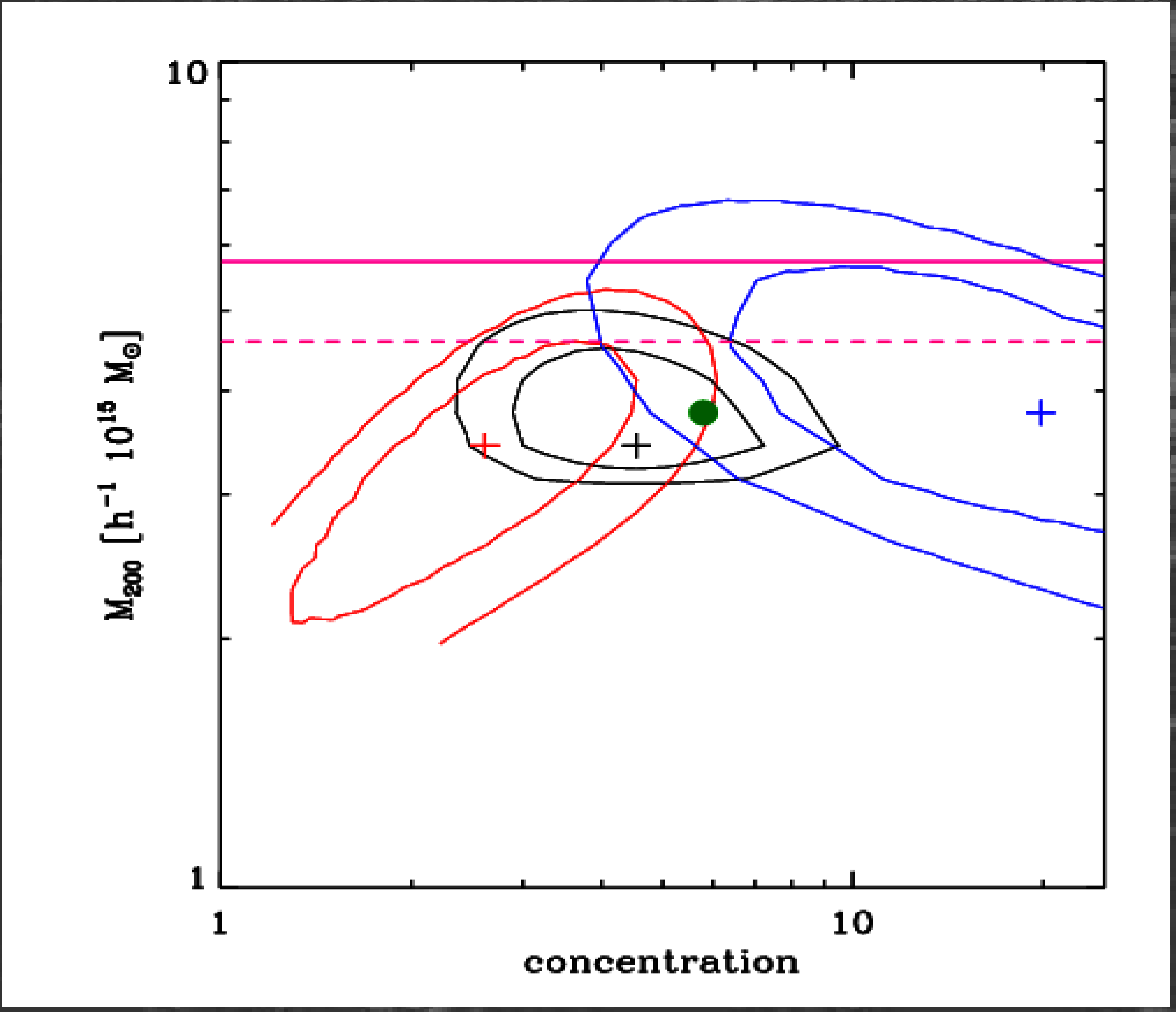
1 halo from cosmo. sim., 1st projection axis

~500 particles randomly selected with $R \leqslant r_{200}$ (except for Caustic method, $R \leqslant 5\ r_{200}$)



MAMPOSSt, D+K, Caustic, Virial, M_σ, true

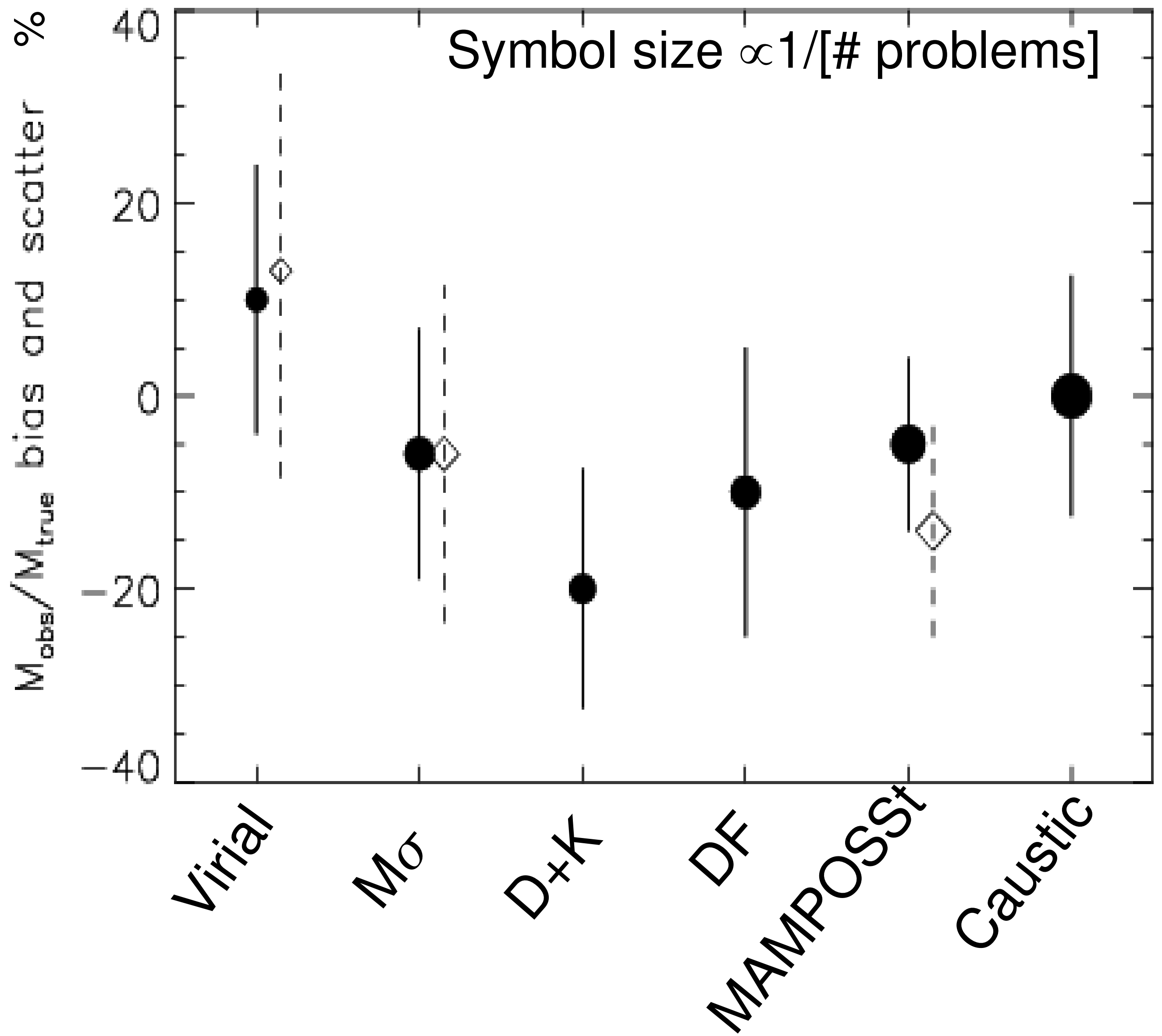
1 halo from cosmo. sim.,
2nd projection axis
~500 particles randomly selected with $R \leqslant r_{200}$
(except for Caustic method, $R \leqslant 5\ r_{200}$)



Methods vs. problems and performances:

Solid lines:
~400 particles
per halo

Dashed lines:
~100 particles
per halo



Conclusions:

- Reduce bias ($\rightarrow$0%) by combining different methods
- Reduce scatter ($\rightarrow$15%) by removing out-of-equilibrium halos
- Use different methods in different observational situations
- Problematic with sample sizes $\ll$100 velocities
- Need to (re)calibrate methods using (simulated) galaxies