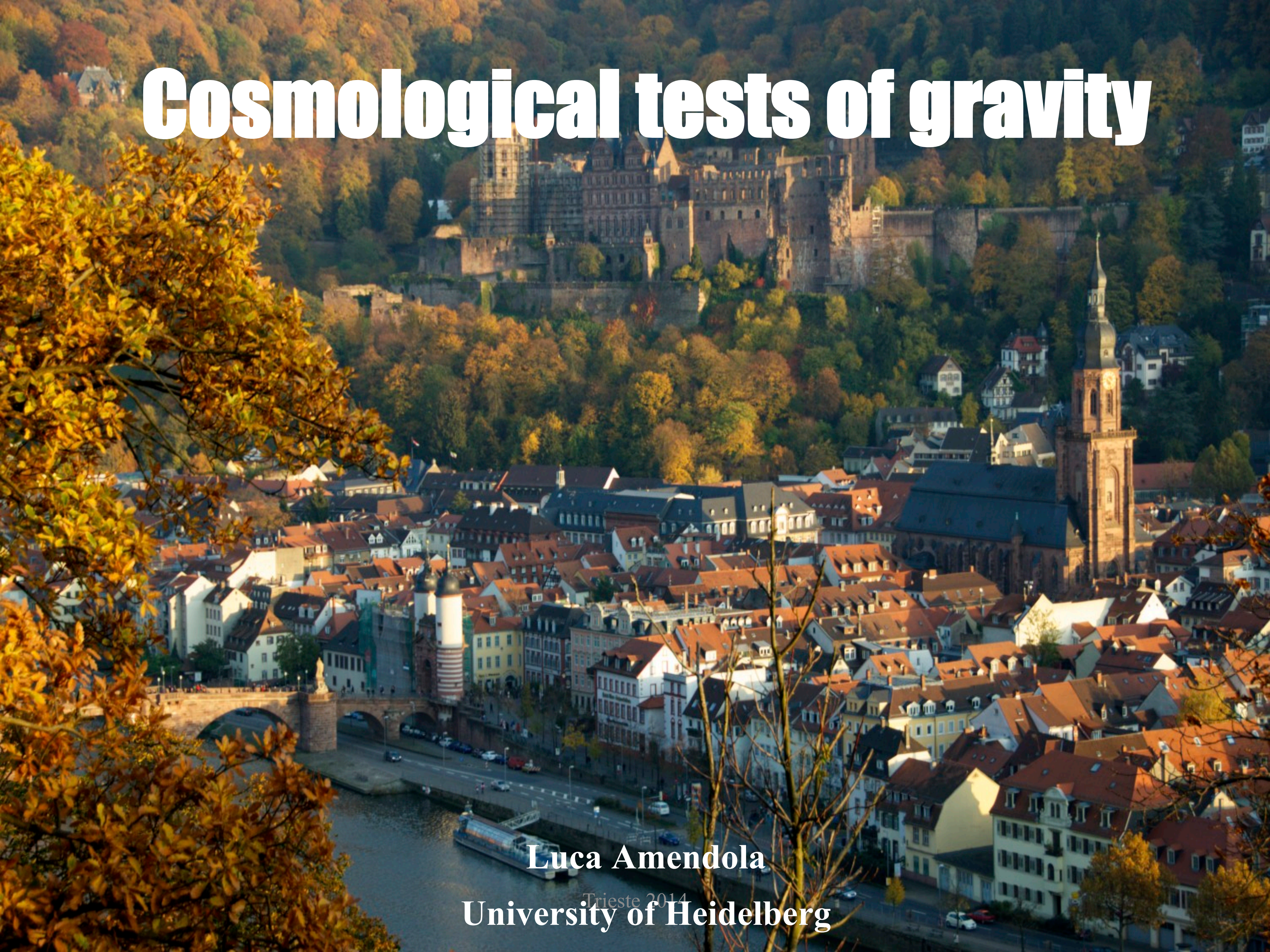


Cosmological tests of gravity

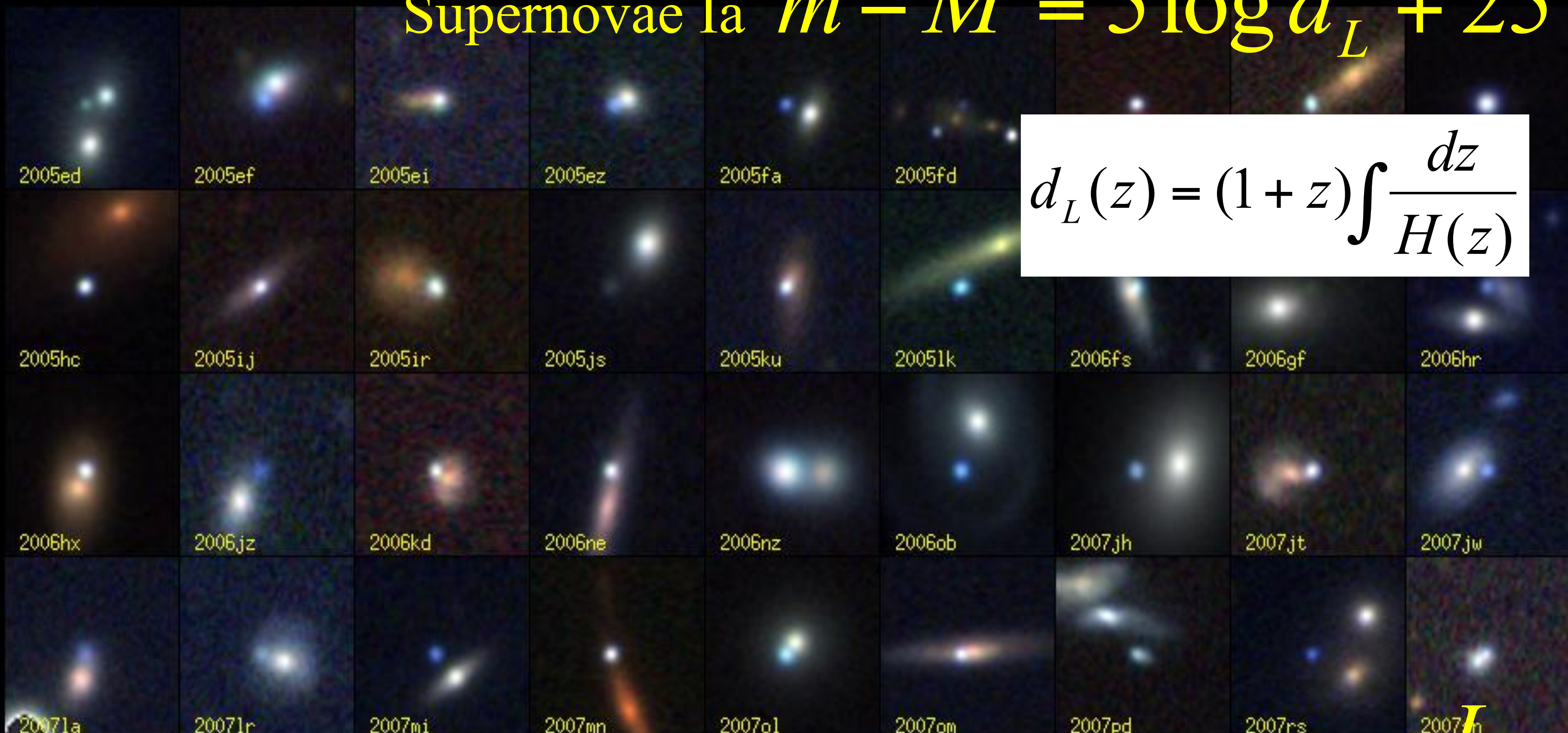
Luca Amendola

University of Heidelberg

Trieste 2014

Lighthouses in the dark

Supernovae Ia $m - M = 5\log d_L + 25$



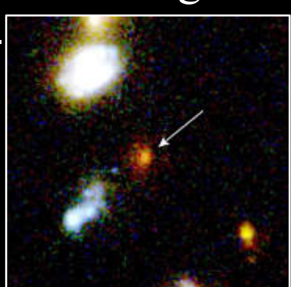
$$d_L(z) = (1+z)\int \frac{dz}{H(z)}$$

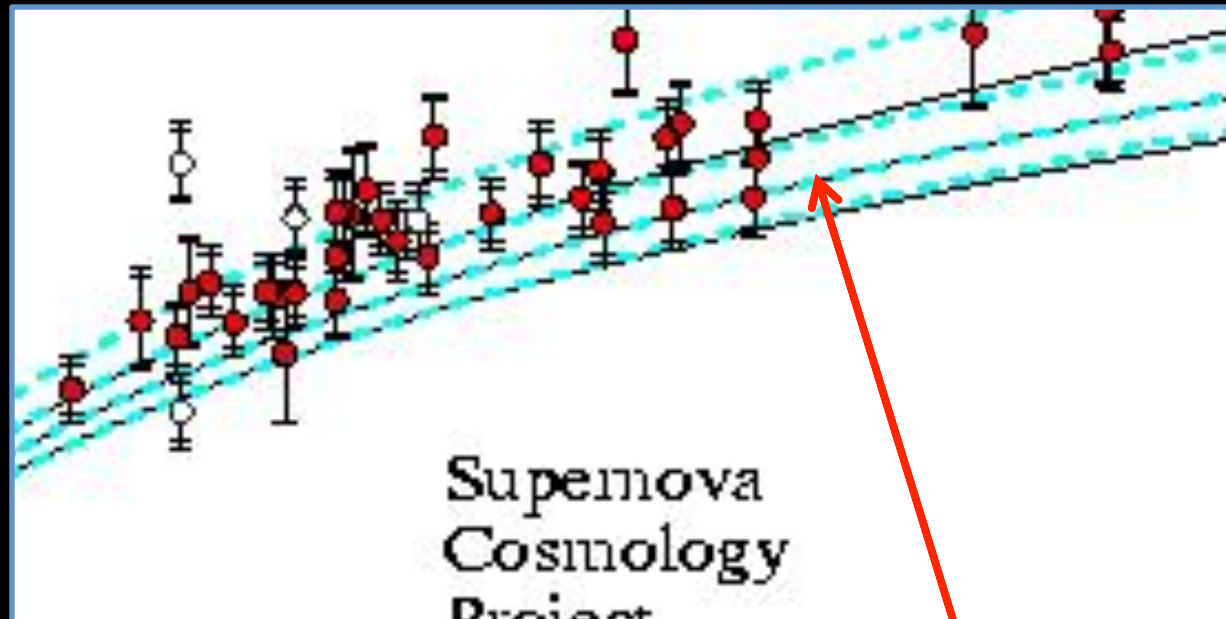
$$d^2 = \frac{L}{4\pi f}$$

Bug or feature?

Conclusion: SNIa are dimmer than expected in a matter universe !

BUT:

- Dependence on progenitors?
- Contamination?
- Environment?
- Host galaxy?
- Dust?
- Lensing?
- 



Ordinary matter

Cosmological explanation

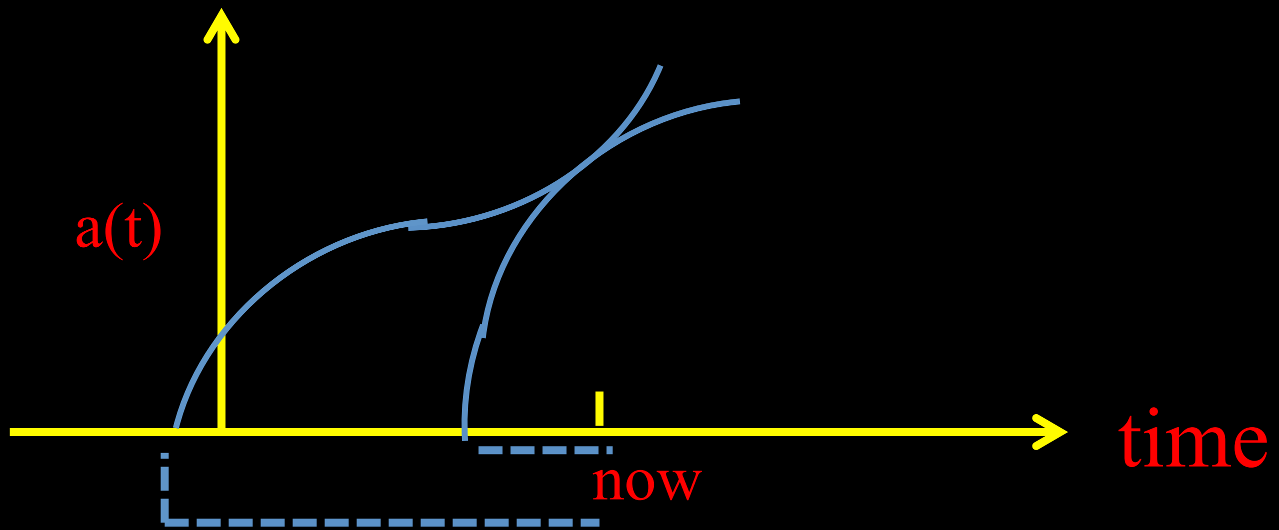
There is however a simple cosmological solution

Local Hubble law

$$r(z) = \frac{z}{H_0} \longrightarrow r(z) = \int \frac{dz}{H(z)}$$

Global Hubble law

If H(z) in the past is smaller (i.e. acceleration), then r(z) is larger: larger distances (for a fixed redshift) make dimmer supernovae

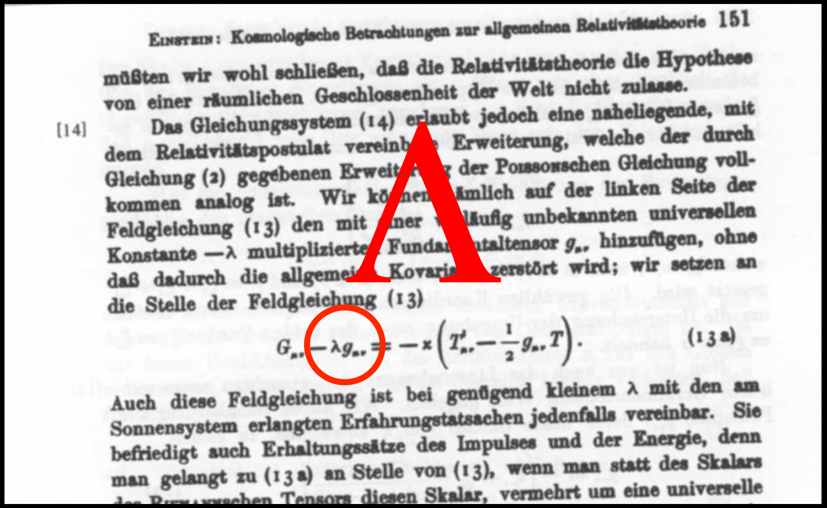


Cosmological constant

acceleration, in GR, can only occur if pressure is large and negative

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$p_\Lambda = -\rho_\Lambda$$


EINSTEIN: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie 151

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

[14] Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinb[illegible] Erweiterung, welche der durch Gleichung (2) gegebenen Erweit[illegible] der Poissonschen Gleichung vollkommen analog ist. Wir kö[illegible]nämlich auf der linken Seite der Feldgleichung (13) den mit [illegible]ner [illegible]läufig unbekannten universellen Konstante $-\lambda$ multiplizierte[illegible] Funda[illegible]taltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgeme[illegible] Kovari[illegible] zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right). \qquad (13a)$$

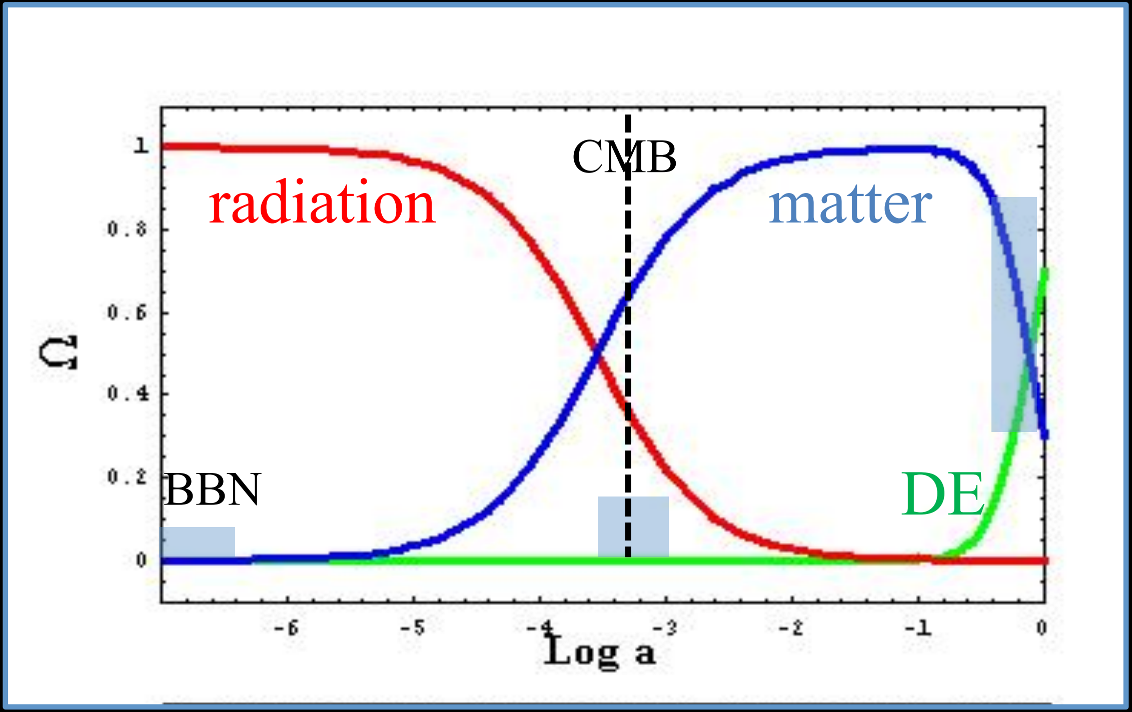
Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemannschen Tensors diesen Skalar, vermehrt um eine universelle

Λ

Einstein 1917

Time view

We know so little about the evolution of the universe!
We assumed for many years that there were just matter and radiation



Shall we repeat our mistake and think that there is just a Λ ?

Prolegomena zu einer jeden künftigen Dark Energy physik

©Kant

Isotropy

Abundance

Observational requirements

Slow evolution

Weak clustering

Classifying the unknown

1. Cosmological constant
2. Dark energy w=const
3. Dark energy w=w(z)
4. quintessence
5. scalar-tensor models
6. coupled quintessence
7. mass varying neutrinos
8. k-essence
9. Chaplygin gas
10. Cardassian
11. quartessence
12. quiessence
13. phantoms
14. f(R)
15. Gauss-Bonnet
16. anisotropic dark energy
17. brane dark energy
18. backreaction
19. degravitation
20. TeVeS
21. oops....did I forget *your* model?

The past ten years of dark energy models

$$\int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi)R + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

$$\int dx^4 \sqrt{-g}\left[f(\phi,\frac{1}{2}\phi_{,\mu}\phi^{,\mu})R + G_{\mu\nu}\phi^{,\nu}\phi^{,\mu} + K(\frac{1}{2}\phi_{,\mu}\phi^{,\mu}) + V(\phi) + L_{matter}\right]$$

The Horndeski Lagrangian

The most general 4D scalar field theory with second order equation of motion

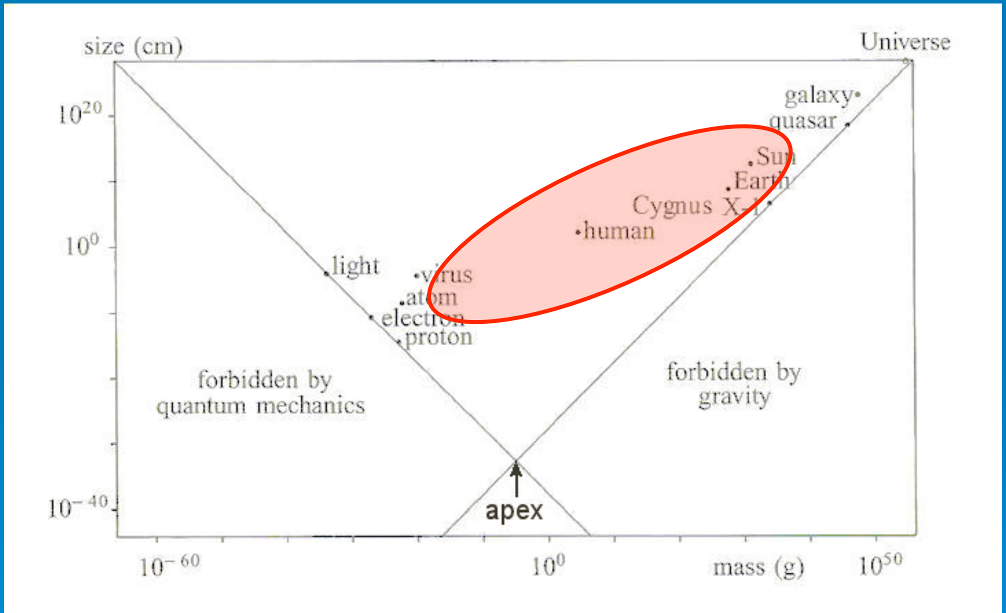
$$\int dx^4 \sqrt{-g}\left[\sum_i L_i + L_{matter}\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right].
\end{aligned}$$

- ✓ First found by Horndeski in 1975
- ✓ rediscovered by Deffayet et al. in 2011
- ✓ no ghosts, no classical instabilities
- ✓ it modifies gravity!
- ✓ it includes f(R), Brans-Dicke, k-essence, Galileons, etc etc etc

Why testing gravity?

we only directly test gravity within the solar system, at the present time, and with “baryons”



On Space and Time, Edited by Shahn Majid

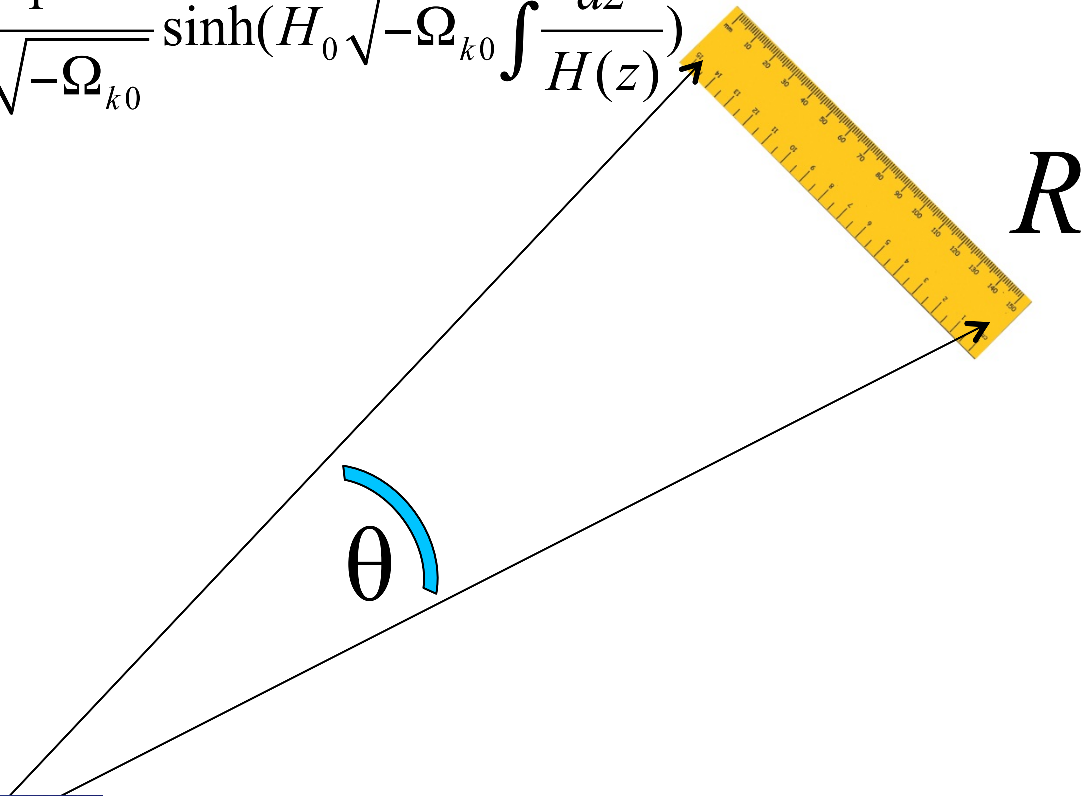
The next ten years of DE research

Combine observations of background, linear and non-linear perturbations to reconstruct as much as possible the Horndeski model

… or rule it out!

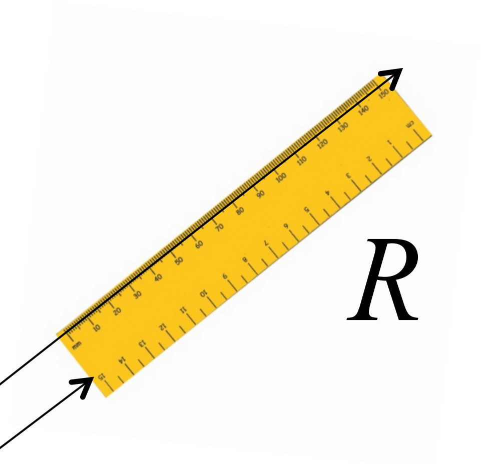
Standard rulers

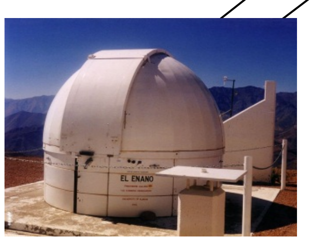
$$D(z) = \frac{R}{\theta} = \frac{1}{H_0\sqrt{-\Omega_{k0}}}\sinh\left(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)}\right)$$



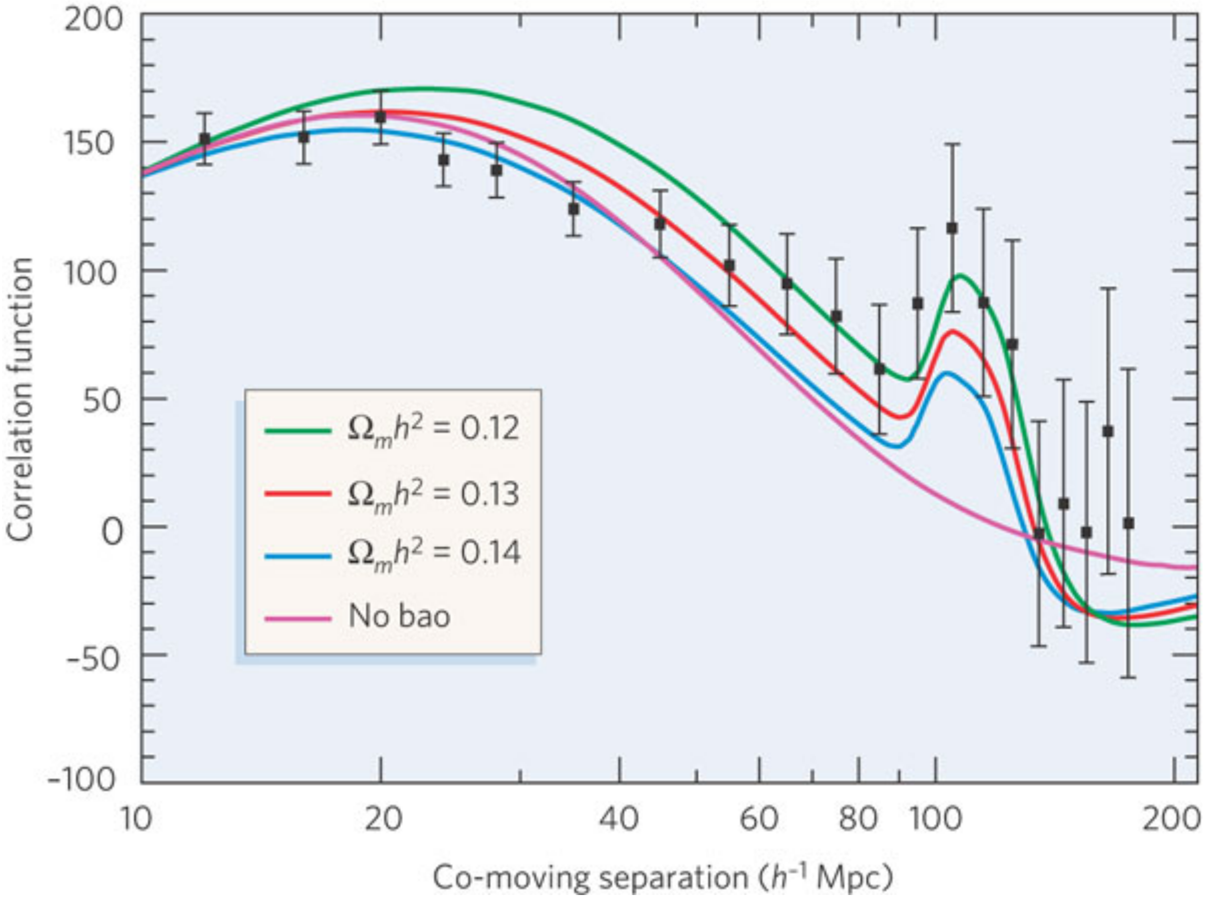
Standard rulers

$$H(z) = \frac{dz}{R}$$





BAO ruler



Charles L. Bennett

Nature 440, 1126-1131(27 April 2006)

Background: SNIa, BAO, …

Then we can measure H(z) and

$$D(z) = \frac{1}{H_0\sqrt{-\Omega_{k0}}}\sinh(H_0\sqrt{-\Omega_{k0}}\int\frac{dz}{H(z)})$$

and therefore we can reconstruct the full FRW metric

$$ds^2 = dt^2 - \frac{a(t)^2}{\left(1-\frac{\Omega_k H^2}{4}r^2\right)^2}(dx^2+dy^2+dz^2)]$$

Two free functions

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

At linear order we can write:

- Poisson equation

$$\nabla^2\Psi = 4\pi G a^2 \rho_m \delta_m$$

- zero anisotropic stress

$$1 = -\frac{\Phi}{\Psi}$$

Two free functions

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

At linear order we can write:

- modified Poisson equation

$$\nabla^2\Psi = 4\pi G a^2 Y(k,a)\rho_m\delta_m$$

- non-zero anisotropic stress

$$\eta(k,a) = -\frac{\Phi}{\Psi}$$

Modified Gravity at the linear level

- standard gravity

$$Y(k,a)=1$$
$$\eta(k,a)=1$$

- scalar-tensor models

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{2(F+F'^2)}{2F+3F'^2}$$
$$\eta(a)=1+\frac{F'^2}{F+F'^2}$$

Boisseau et al. 2000
Acquaviva et al. 2004
Schimd et al. 2004
L.A., Kunz &Sapone 2007

- f(R)

$$Y(a)=\frac{G^*}{FG_{cav,0}}\frac{1+4m\frac{k^2}{a^2R}}{1+3m\frac{k^2}{a^2R}},\quad \eta(a)=1+\frac{m\frac{k^2}{a^2R}}{1+2m\frac{k^2}{a^2R}}$$

Bean et al. 2006
Hu et al. 2006
Tsujikawa 2007

- DGP

$$Y(a)=1-\frac{1}{3\beta};\quad \beta=1+2Hr_cw_{DE}$$
$$\eta(a)=1+\frac{2}{3\beta-1}$$

Lue et al. 2004;
Koyama et al. 2006

- massive bi-gravity

$$Y(a)=....$$
$$\eta(a)=....$$

see F. Koennig and L. A. 2014

Modified Gravity at the linear level

In the quasi-static limit, every Horndeski model is characterized at linear scales by the two functions

$$\eta(k,a) = h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

$$Y(k,a) = h_1\left(\frac{1+k^2h_5}{1+k^2h_3}\right)$$

k = wavenumber

h_i = time-dependent functions

De Felice et al. 2011; L.A. et al.,arXiv:1210.0439, 2012

Modified Gravity at the linear level

$$h_1 \equiv \frac{w_4}{w_1^2} = \frac{c_{\rm T}^2}{w_1}, \qquad h_2 \equiv \frac{w_1}{w_4} = c_{\rm T}^{-2},$$

$$h_3 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 w_2 H - w_2^2 w_4 + 4w_1 w_2 \dot{w}_1 - 2w_1^2(\dot{w}_2 + \rho_{\rm m})}{2w_1^2},$$

$$h_4 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 H^2 - w_2 w_4 H + 2w_1 \dot{w}_1 H + w_2 \dot{w}_1 - w_1(\dot{w}_2 + \rho_{\rm m})}{w_1},$$

$$h_5 \equiv \frac{H^2}{2XM^2}\frac{2w_1^2 H^2 - w_2 w_4 H + 4w_1 \dot{w}_1 H + 2\dot{w}_1{}^2 - w_4(\dot{w}_2 + \rho_{\rm m})}{w_4},$$

$$\begin{aligned}
w_1 \equiv & 1 + 2\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi} XHG_{5,X}\right), \\
w_2 \equiv & -2\dot{\phi}\left(XG_{3,X} - G_{4,\phi} - 2XG_{4,\phi X}\right) + \\
& + 2H\left(w_1 - 4X\left(G_{4,X} + 2XG_{4,XX} - G_{5,\phi} - XG_{5,\phi X}\right)\right) - \\
& - 2\dot{\phi} XH^2\left(3G_{5,X} + 2XG_{5,XX}\right), \\
w_3 \equiv & 3X\left(K_{,X} + 2XK_{,XX} - 2G_{3,\phi} - 2XG_{3,\phi X}\right) + 18\dot{\phi} XH\left(2G_{3,X} + XG_{3,XX}\right) - \\
& - 18\dot{\phi} H\left(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2 G_{4,\phi XX}\right) - \\
& - 18H^2\left(1 + G_4 - 7XG_{4,X} - 16X^2 G_{4,XX} - 4X^3 G_{4,XXX}\right) - \\
& - 18XH^2\left(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2 G_{5,\phi XX}\right) + \\
& + 6\dot{\phi} XH^3\left(15G_{5,X} + 13XG_{5,XX} + 2X^2 G_{5,XXX}\right), \\
w_4 \equiv & 1 + 2\left(G_4 - XG_{5,\phi} - XG_{5,X}\ddot{\phi}\right).
\end{aligned}$$

De Felice et al. 2011; L.A. et al.,arXiv:1210.0439, 2012

The Yukawa correction

Every Horndeski model induces at linear level, on sub-Hubble scales, a Newton-Yukawa potential

$$\Psi(r) = -\frac{GM}{r}\left(1+\beta e^{-r/\lambda}\right)$$

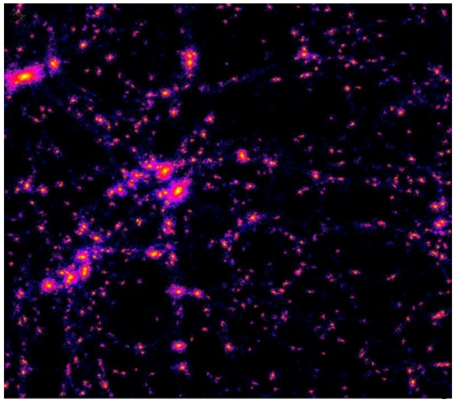
where β and λ depend on space and time

Reconstruction of the metric

$$ds^2 = a^2[(1+2\Psi)dt^2 - (1+2\Phi)(dx^2 + dy^2 + dz^2)]$$

Non-relativistic particles respond to Ψ

$$\delta_m'' + (1 + \frac{\mathcal{H}'}{\mathcal{H}})\delta_m' = -k^2\Psi$$

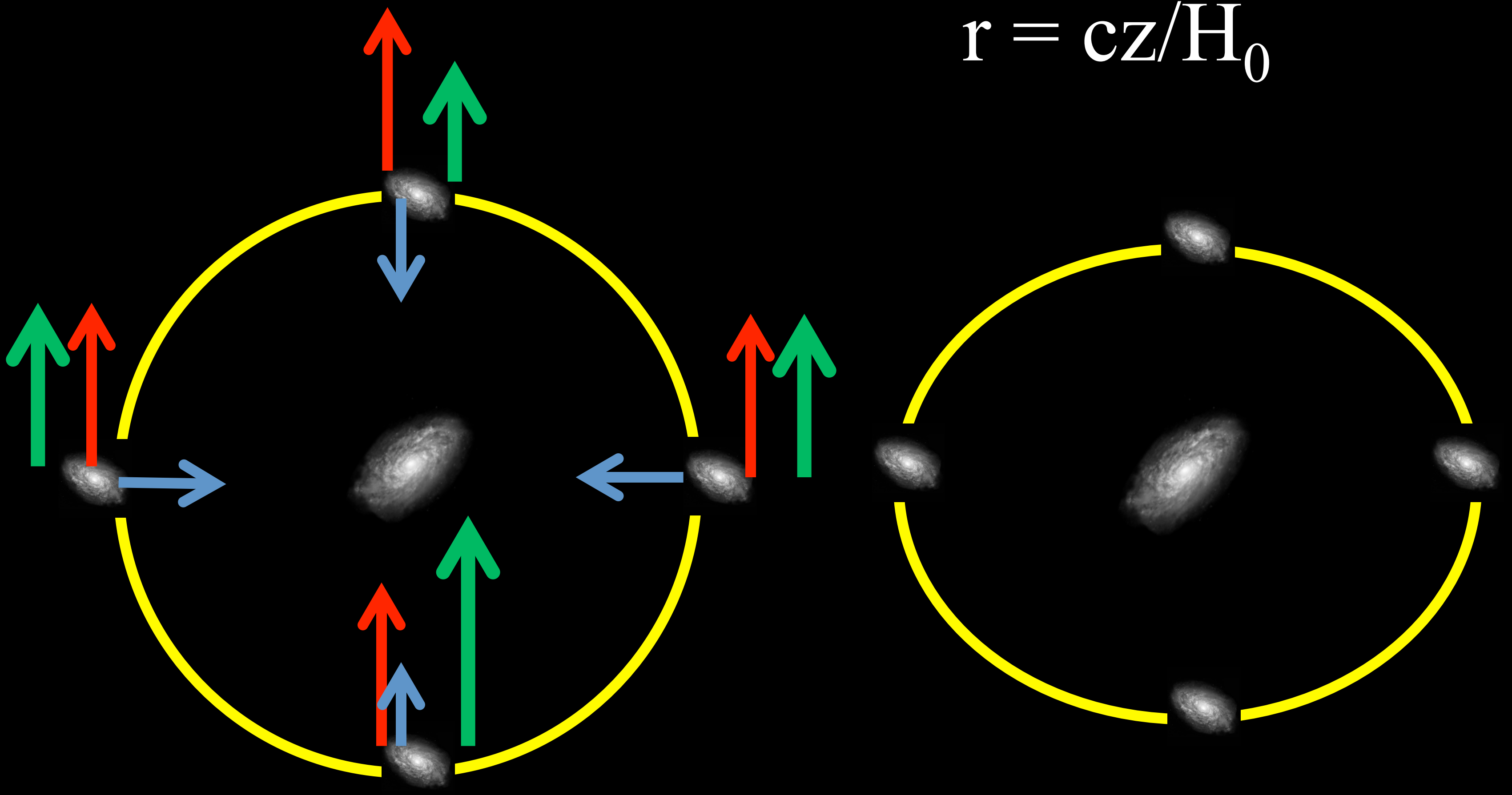


Relativistic particles respond to Φ-Ψ

$$\alpha = \int \nabla_{perp}(\Psi - \Phi)dz$$

Peculiar velocities

$r = cz/H_0$



All you can ever observe in linear Cosmology

Expansion rate
Amplitude of the power spectrum
Redshift distortion of the power spectrum
Weak lensing

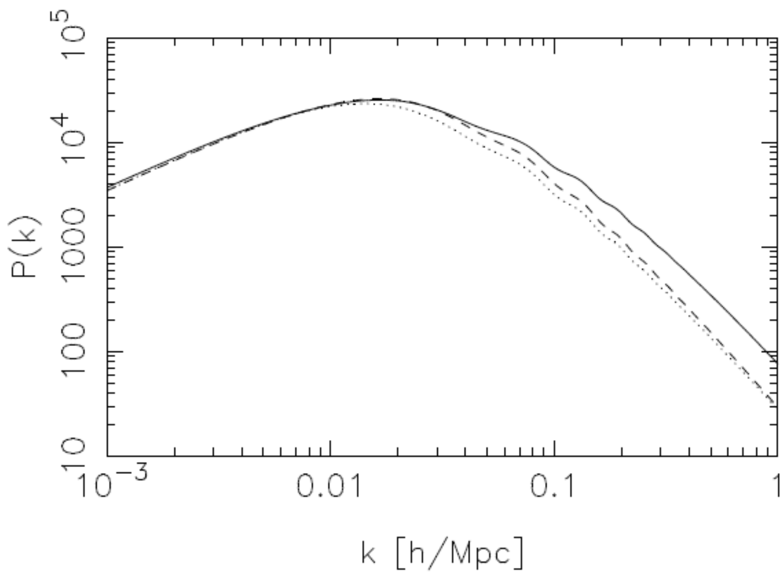
as function of redshift and scale!

How to combine observations to test the theory?

The two problem of initial conditions

How do we know if the shape of the power spectrum we observe is due to dark energy or to initial conditions?

$$P(k,z) = AP_{in}(k)T^2(k,z;DE,DM,etc)$$



Four model-independent observational quantities

Redshift distortion/Amplitude

$$P_1 = \frac{\text{redshift distortion}}{\text{amplitude}}$$

Lensing/Redshift distortion

$$P_2 = \frac{\text{lensing}}{\text{redshift distortion}}$$

Redshift distortion rate

$$P_3 = \text{rate of variation redshift dist}$$

Expansion rate

$$E = \text{expansion rate}$$

Four model-independent observational quantities

Redshift distortion/Amplitude

$$P_1 = \frac{R}{A} = \frac{f}{b}$$

Lensing/Redshift distortion

$$P_2 = \frac{L}{R} = \frac{\Omega_{m0} Y(1+\eta)}{f}$$

Redshift distortion rate

$$P_3 = \frac{R'}{R} = \frac{f'}{f} + f$$

Expansion rate

$$E = \frac{H}{H_0}$$

Conservation equation

Matter conservation equation independent of gravity theory

$$\delta_m'' + \left(1 + \frac{H'}{H}\right)\delta_m' = -k^2\Psi = \frac{3}{2}H^2\Omega_m Y\delta_m$$

Testing the entire Horndeski Lagrangian

A unique combination of model independent observables

$$\frac{3P_2(1+z)^3}{2E^2(P_3+2+\frac{E'}{E})}-1=\eta=h_2\left(\frac{1+k^2h_4}{1+k^2h_5}\right)$$

Observables

Theory

L.A. et al. 1210.0439

Combine lensing and galaxy clustering !

Euclid in a nutshell

Simultaneous (i) visible imaging (ii) NIR photometry (iii) NIR spectroscopy

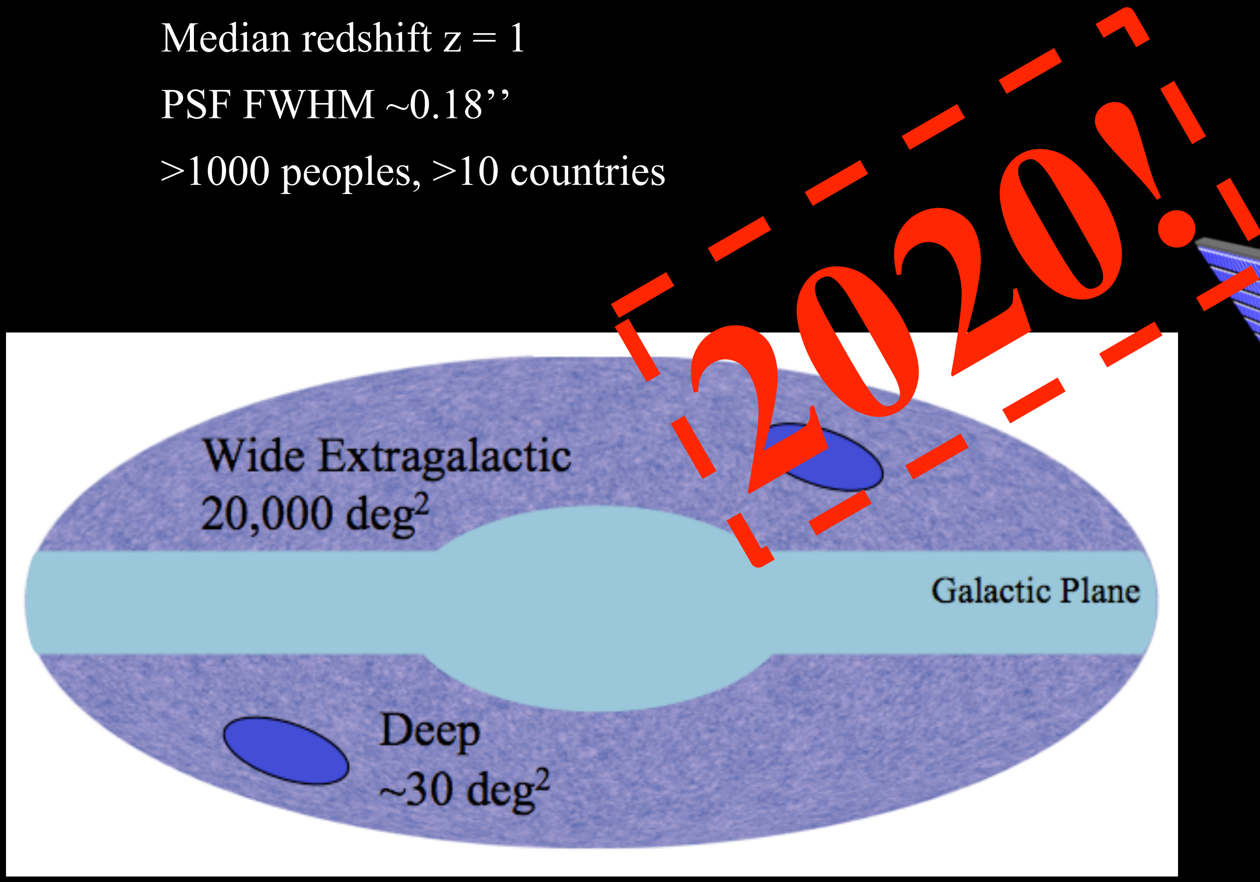
15,000 square degrees

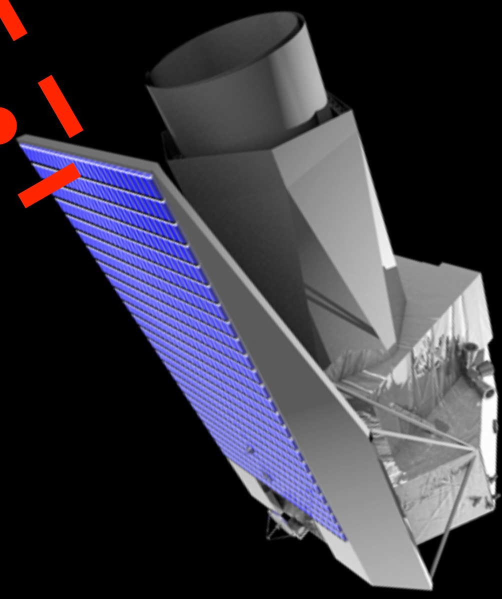
70 million redshifts, 2 billion images

Median redshift z = 1

PSF FWHM ~0.18''

>1000 peoples, >10 countries

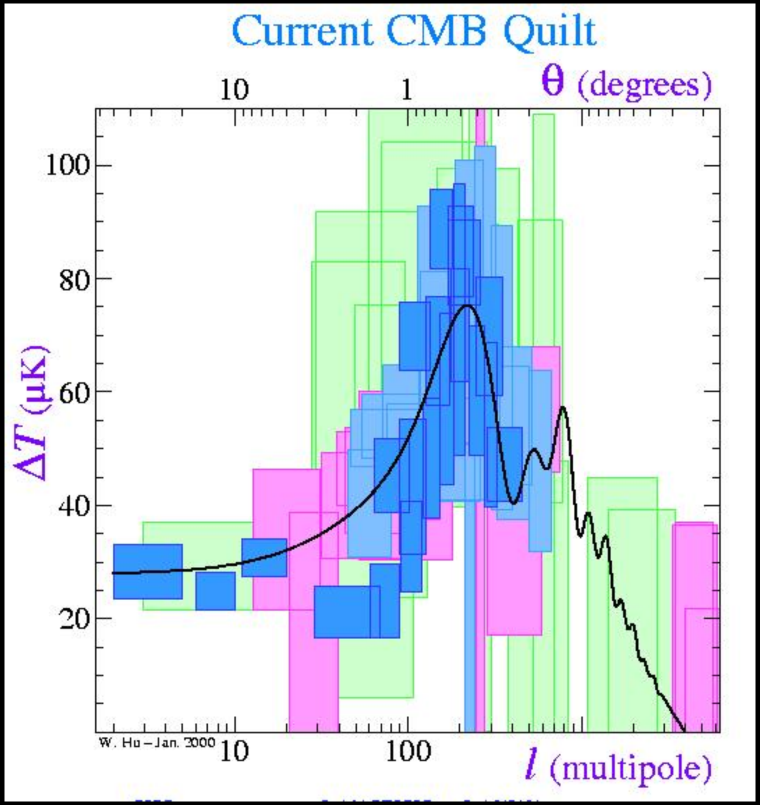




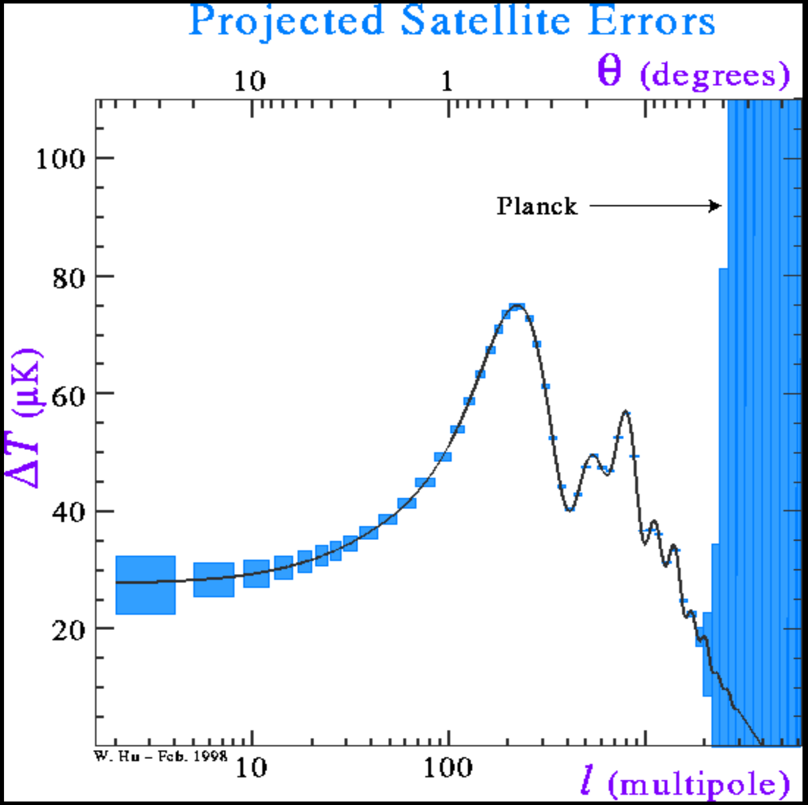
Euclid satellite

arXiv Red Book 1110.3193

History repeats itself...



1998

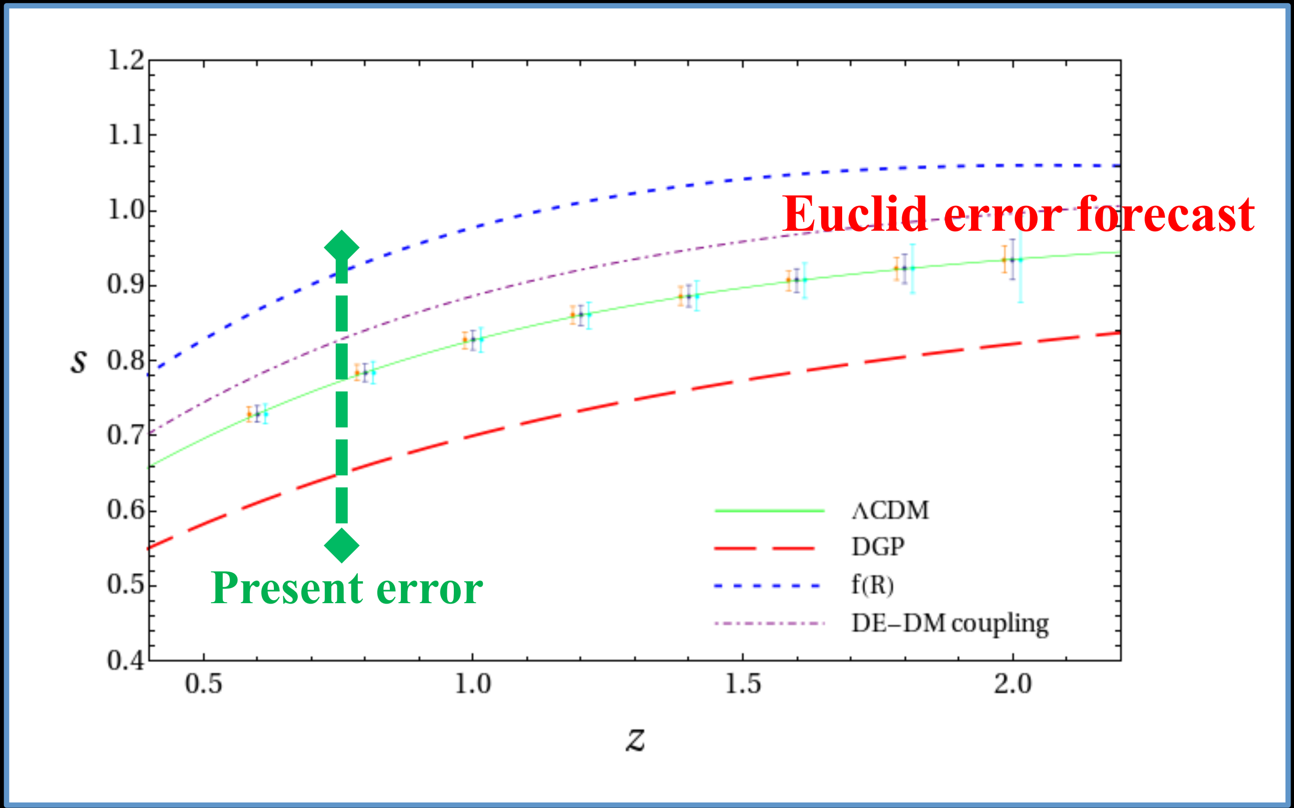


2011

Euclid's challenge

C. Di Porto & L.A. 2010

$$s \equiv \frac{d \log \delta}{d \log a}$$



Growth of matter fluctuations

Results...

$$\eta(k,a) = H_2\left(\frac{1+k^2H_4}{1+k^2H_5}\right)$$

Model 1: η constant for all z, k
Error on η around 1%

Model 2: η varies in z
Error on η

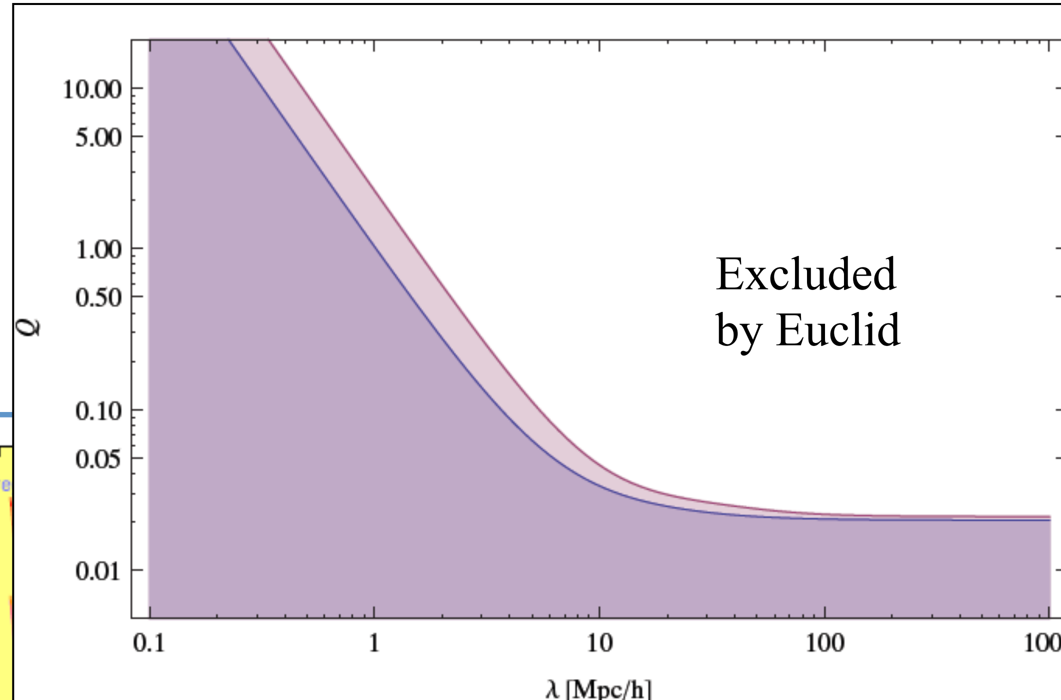
TABLE X. Fiducial values and errors for the parameters $P_1, P_2, P_3, E'/E$ and $\bar{\eta}$ for every bin. The last bin has been omitted since R' is not defined there.

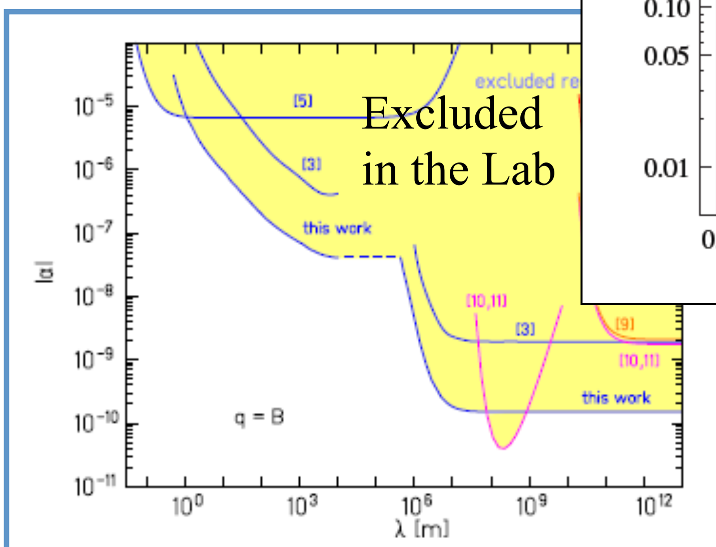
$\bar{z}$	P_1	ΔP_1	$\Delta P_1(\%)$	P_2	ΔP_2	$\Delta P_2(\%)$	P_3	ΔP_3	$\Delta P_3(\%)$	(E'/E)	$\Delta E'/E$	$\Delta E'/E(\%)$	$\bar{\eta}$	$\Delta\bar{\eta}$	$\Delta\bar{\eta}(\%)$
0.6	0.766	0.012	1.6	0.729	0.013	1.8	0.134	0.13	99	-0.920	0.022	2.4	1	0.11	11
0.8	0.819	0.010	1.2	0.682	0.011	1.6	0.317	0.12	38	-1.04	0.046	4.4	1	0.091	9.1
1.0	0.859	0.0093	1.1	0.650	0.011	1.7	0.460	0.12	26	-1.13	0.099	8.7	1	0.090	9.0
1.2	0.888	0.0092	1.0	0.628	0.014	2.3	0.569	0.13	23	-1.21	0.12	10	1	0.097	9.7
1.4	0.911	0.010	1.1	0.613	0.020	3.3	0.654	0.11	16	-1.26	0.09	7.1	1	0.073	7.3

L.A, M. Kunz, A. Vollmer,
A. Guarnizo, S. Fogli, 1311.4765, 2013

Cosmological exclusion plot

$$\Psi(r) = -\frac{GM}{r}(1+\alpha e^{-r/\lambda})$$





$$\Psi = -\frac{GM}{r}h_1(1+\frac{h_5-h_3}{h_3}e^{-r/\sqrt{h_3}}) = -\frac{\bar{G}M}{r}(1+Qe^{-r/\lambda})$$

$$\Phi = \frac{GM}{r}h_1 h_2(1+\frac{h_4-h_3}{h_3}e^{-r/\sqrt{h_3}}) = -\frac{\tilde{G}M}{r}(1+\tilde{Q}e^{-r/\lambda})$$

Three Messages

1

If DE is not a Horndeski field or massive gravity, then
I don't know what could be

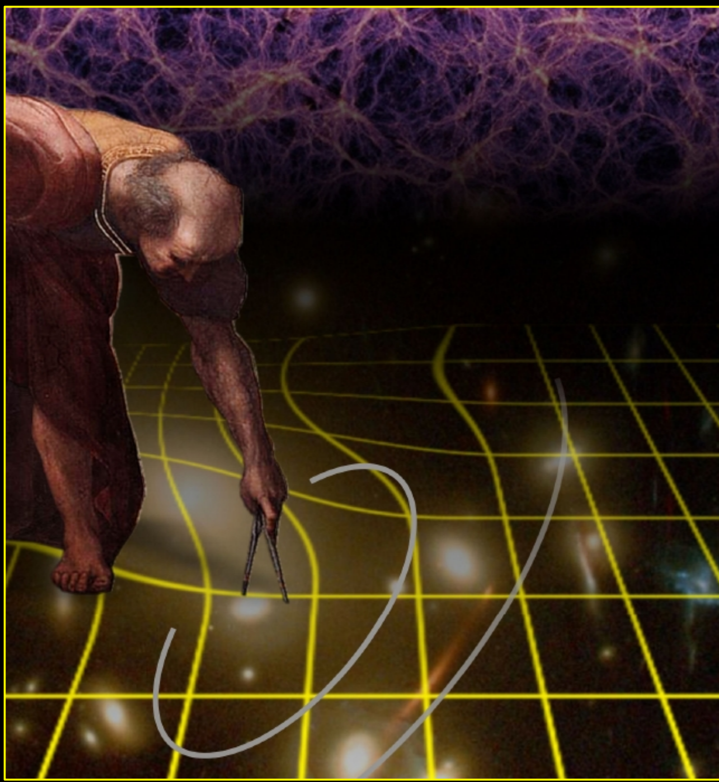
2

k-binned data are crucial!
e.g. growth factor, redshift distortion parameter

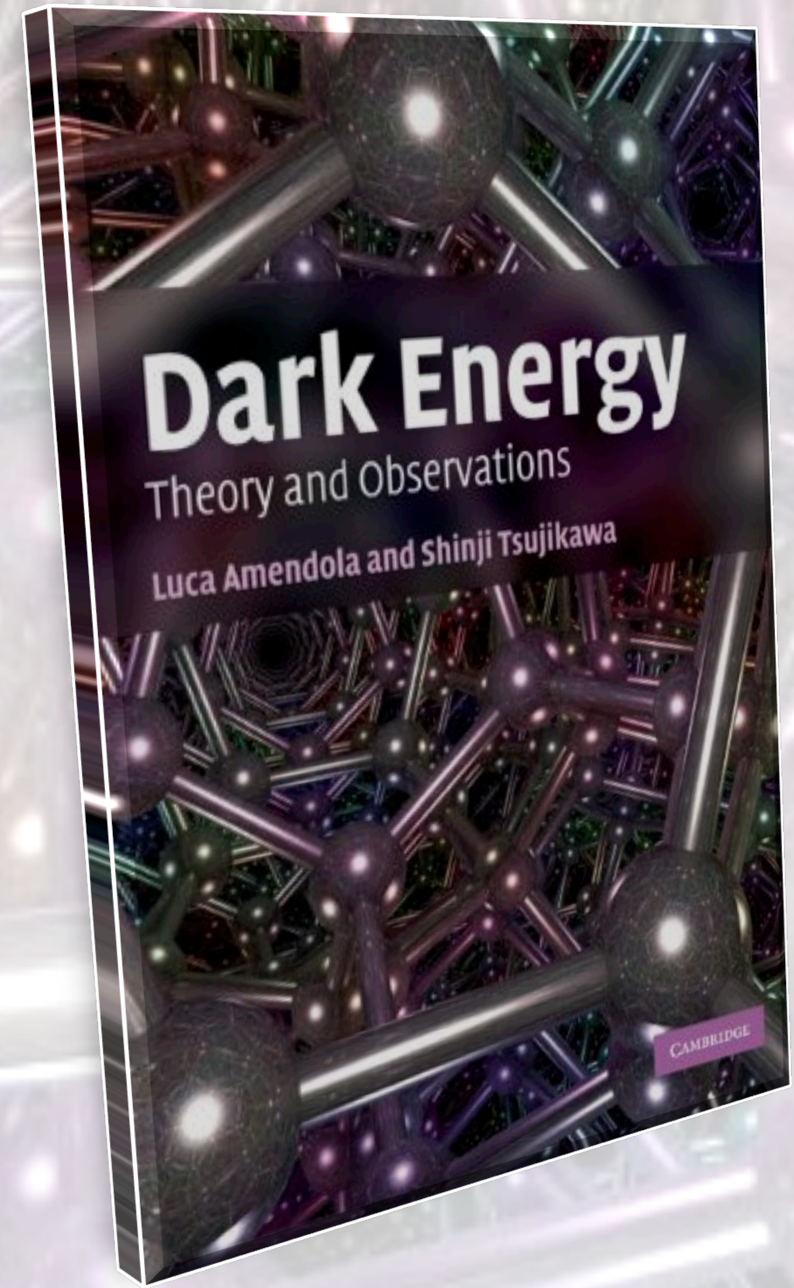
3

Only by combining galaxy clustering and lensing
can DE be constrained (or ruled out!) in a model-independent way

Summary: Euclid's challenge



Issue	Our Targets
Dark Energy	Measure the DE equation of state parameters w0 and wa to a precision of 2% and 10%, respectively, using both expansion history and structure growth.
Test of General Relativity	Distinguish General Relativity from the simplest modified-gravity theories, by measuring the growth factor exponent γ with a precision of 2%
Dark Matter	Test the Cold Dark Matter paradigm for structure formation, and measure the sum of the neutrino masses to a precision better than 0.04eV when combined with Planck.
The seeds of cosmic structures	Improve by a factor of 20 the determination of the initial condition parameters compared to Planck alone.



Cambridge
University
Press

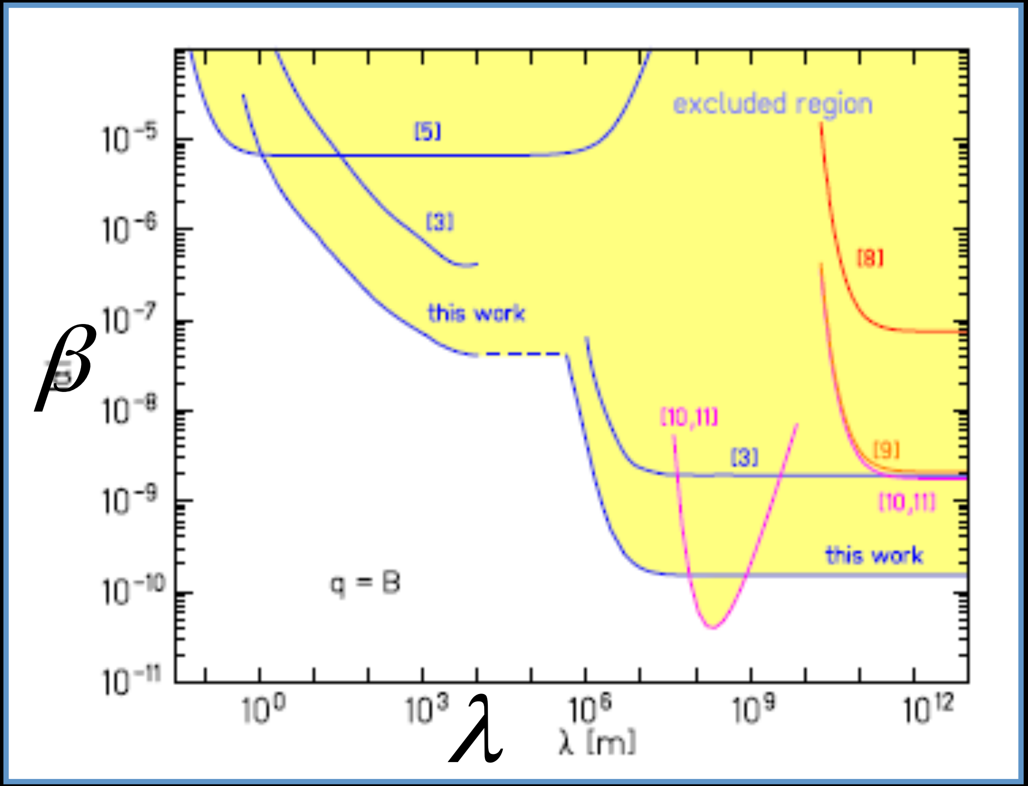
Dark Force

Limits on Yukawa coupling are strong but local!

$$\frac{GM}{r} \to \frac{GM}{r}\left(1+\beta e^{-r/\lambda}\right)$$

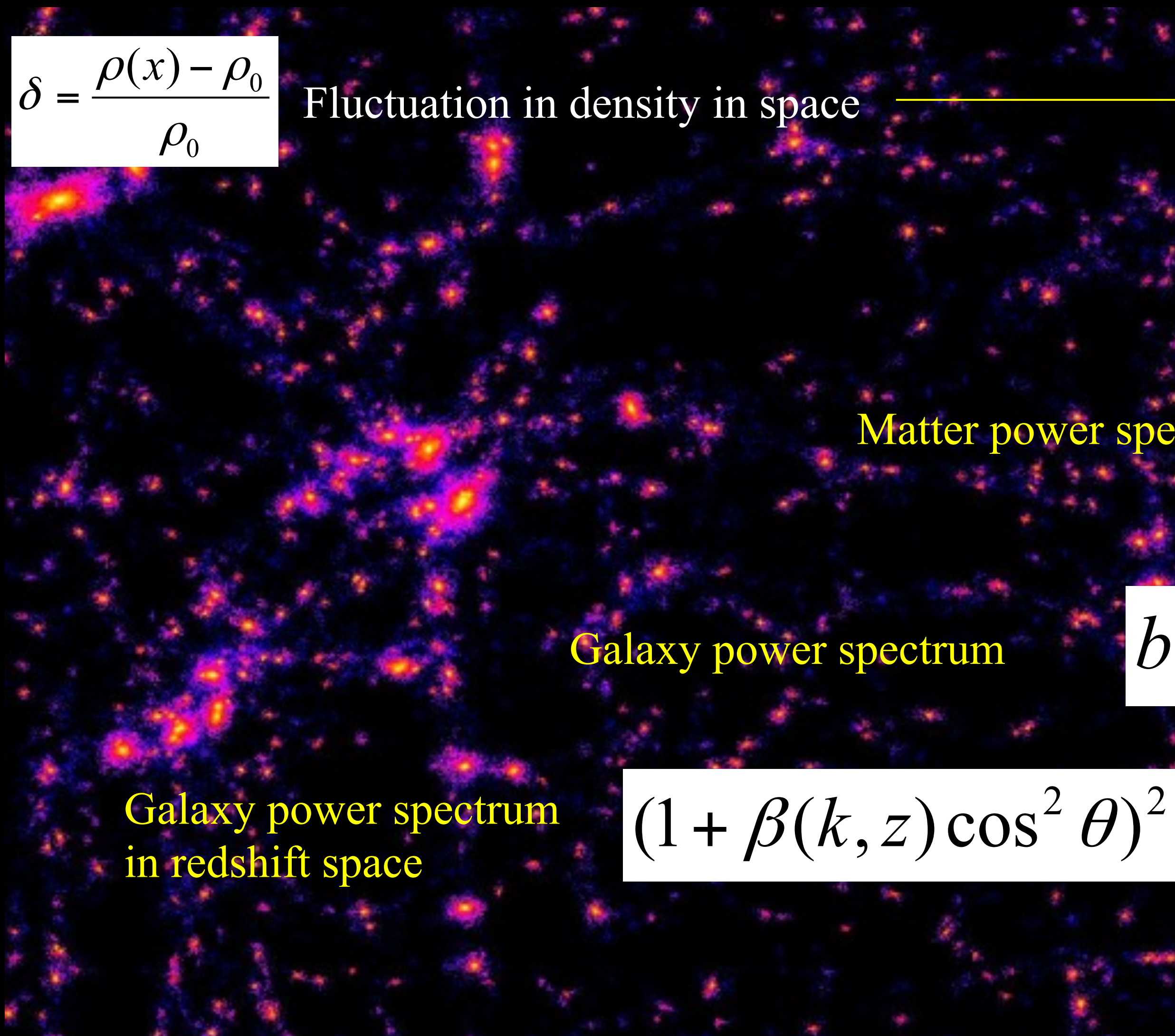
$$\lambda^2 = m_\varphi^{-2} = V''(\varphi)$$

$$\varphi = \varphi(r)$$



Schlamminger et al 2008

Reality check



$$\delta = \frac{\rho(x) - \rho_0}{\rho_0}$$

Fluctuation in density in space

$$\left\langle \delta_k{}^2 \right\rangle = P(k,z)$$

Matter power spectrum

$$P_{matter}(k,z)$$

Galaxy power spectrum

$$b^2(k,z)P_{matter}(k,z)$$

Galaxy power spectrum in redshift space

$$(1+\beta(k,z)\cos^2\theta)^2 b^2(k,z)P_{matter}(k,z)$$