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# The collapse of dark energy

arXiv:0811.0827 [astro-ph] (JCAP)
with G. D'Amico, J. Noreña and F. Vernizzi

arXiv:0911.0701 [astro-ph]
with G. D'Amico, J. Noreña, L. Senatore and F. Vernizzi

# The Universe accelerates

In 1998 the Universe started accelerating…

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad w \equiv p/\rho$$

- Data are converging towards w ≈ -1

- Λ is the simplest explanation: w = -1

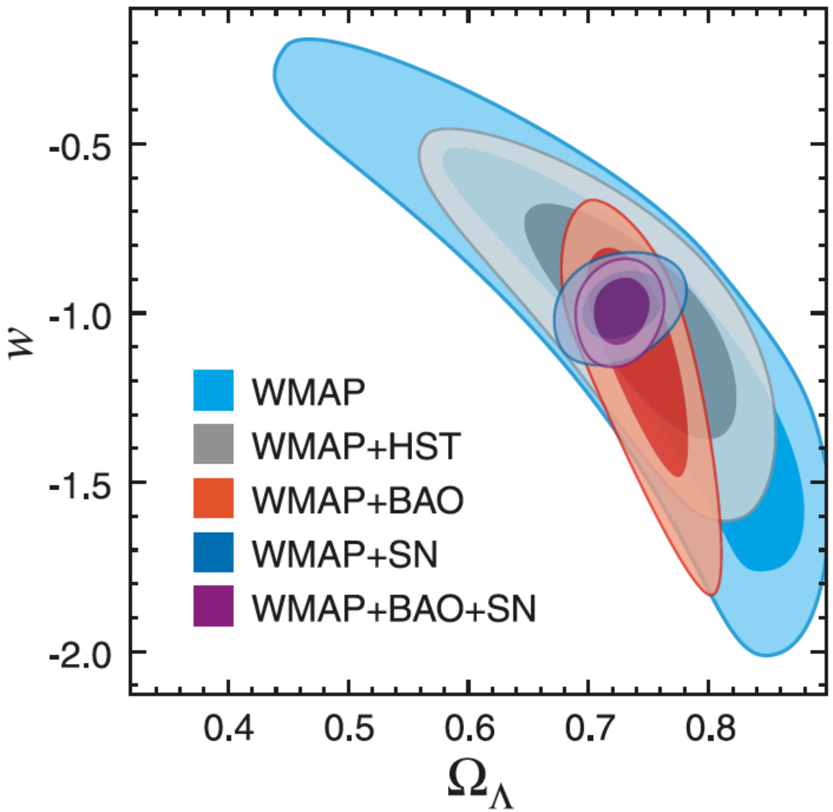
- **Quintessence**
  (here a general single field dark energy)

$$w_Q(z) \neq -1$$

Not spatially homogeneous

Compelling evidence from supernovae
+ other observations

Komatsu etal 2008



# Outline

- Study the most general theory of single field quintessence
- $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$ gives $w_Q > -1$. Is there life for $\mathbf{w_Q < -1}$?
- Theoretical constraints on the **quintessential plane**: $(1+w_Q)\,\Omega_Q$ vs $c_s^2$
- Motivation to study **zero speed of sound** quintessence
- Phenomenology of clustering quintessence
- **Spherical collapse model** and mass function

# Building up the action

K-essence: $$S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X) \;, \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Let us expand around: $$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad \phi = \phi_0(t)$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad \rho_Q = 2X_0 P_X - P_0 \;, \qquad p_Q = P_0$$

Action for perturbations, making explicit the **background dependence**

Convenient parametrization: $$\boxed{\phi(t,\vec{x}) = \phi_0(t + \pi(t,\vec{x}))}$$

$$\begin{aligned} \phi(t,\vec{x}) &= \phi_0 + \dot{\phi}_0\pi + \frac{1}{2}\ddot{\phi}_0\pi^2 + \ldots \;, \\ X(t,\vec{x}) &= X_0 + \dot{X}_0\pi + \frac{1}{2}\ddot{X}_0\pi^2 + 2X_0\dot{\pi} + 2\dot{X}_0\pi\dot{\pi} + X_0\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + \ldots \end{aligned}$$

$$S = \int \mathrm{d}^4x\, a^3 \left[P_0 + \dot{P}_0\pi + \frac{1}{2}\ddot{P}_0\pi^2 + 2P_X X_0\dot{\pi} + 2\,(P_X X_0)^{\cdot}\pi\dot{\pi} + P_X X_0\left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + 2P_{XX}X_0^2\dot{\pi}^2\right]$$

# The action for perturbations

... integrating by parts + using background EOM

Metric perturbations in synchronous gauge: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j$

$$S = \int \mathrm{d}^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi\right]$$

$M^4 \equiv P_{XX}X_0^2$

**Perturbations cannot be switched off if $\rho_Q + p_Q \neq 0$**

$(\rho_Q + p_Q)(t)$ and $M^4(t)$ are completely unconstrained

One can always find $P(\phi, X)$:

$$P(\phi, X) = \frac{1}{2}(p_Q - \rho_Q)(\phi) + \frac{1}{2}(\rho_Q + p_Q)(\phi)X + \frac{1}{2}M^4(\phi)(X-1)^2$$

$\phi = t$ and the correct $\rho_Q(t)$ and $p_Q(t)$

No field redefinition ambiguities: $\phi \to \tilde{\phi}(\phi)$

# No ghost!

We require a positive definite time kinetic term

$$\frac{1}{2}(\rho_Q + p_Q + 4M^4)\dot{\pi}^2 > 0$$

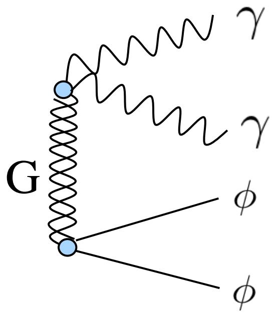
E.g. a minimal ghost field: $\mathcal{L} = +\frac{1}{2}(\partial\phi)^2 + V(\phi)$ **$w_Q < -1$ !!**

- **Classically**. Hamiltonian not bounded. Possibility of exchanging energy between positive and negative energy sectors.

  No pathology until linear theory remains valid.

- **Quantum mechanically**. Vacuum is unstable.

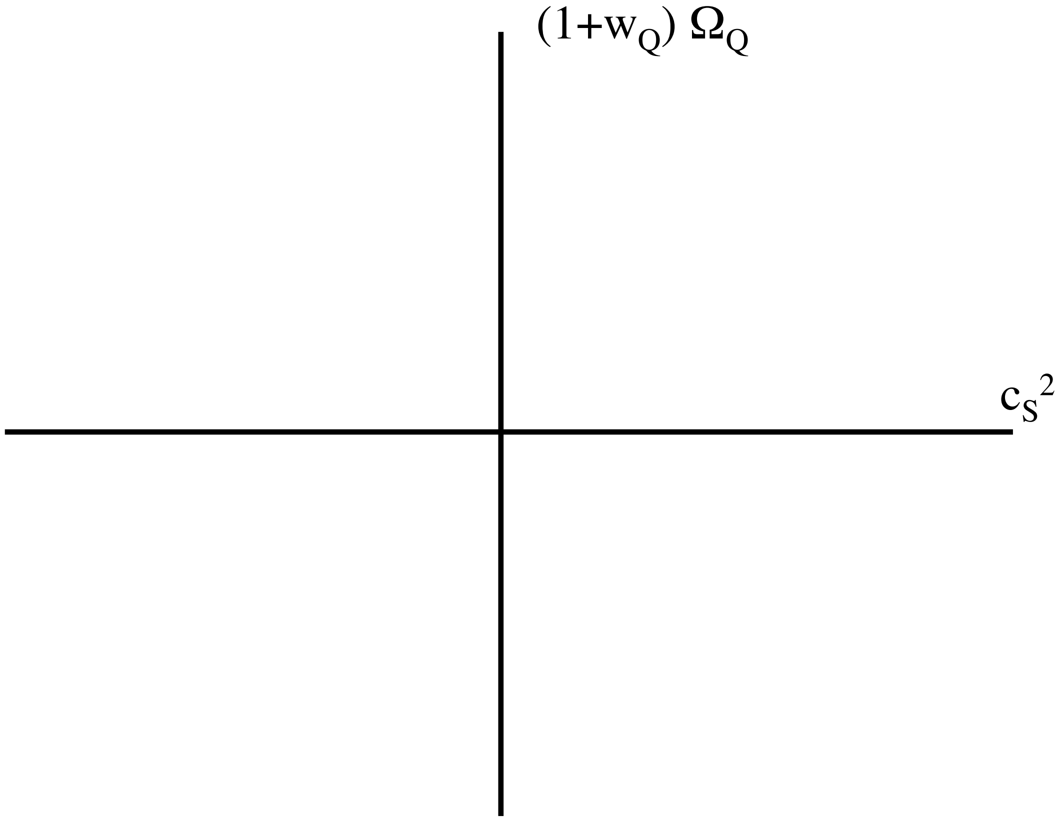
  Decay rate is infinite in any Lorentz invariant theory.




$$\Gamma \sim \frac{\Lambda^8}{M_P^4}$$

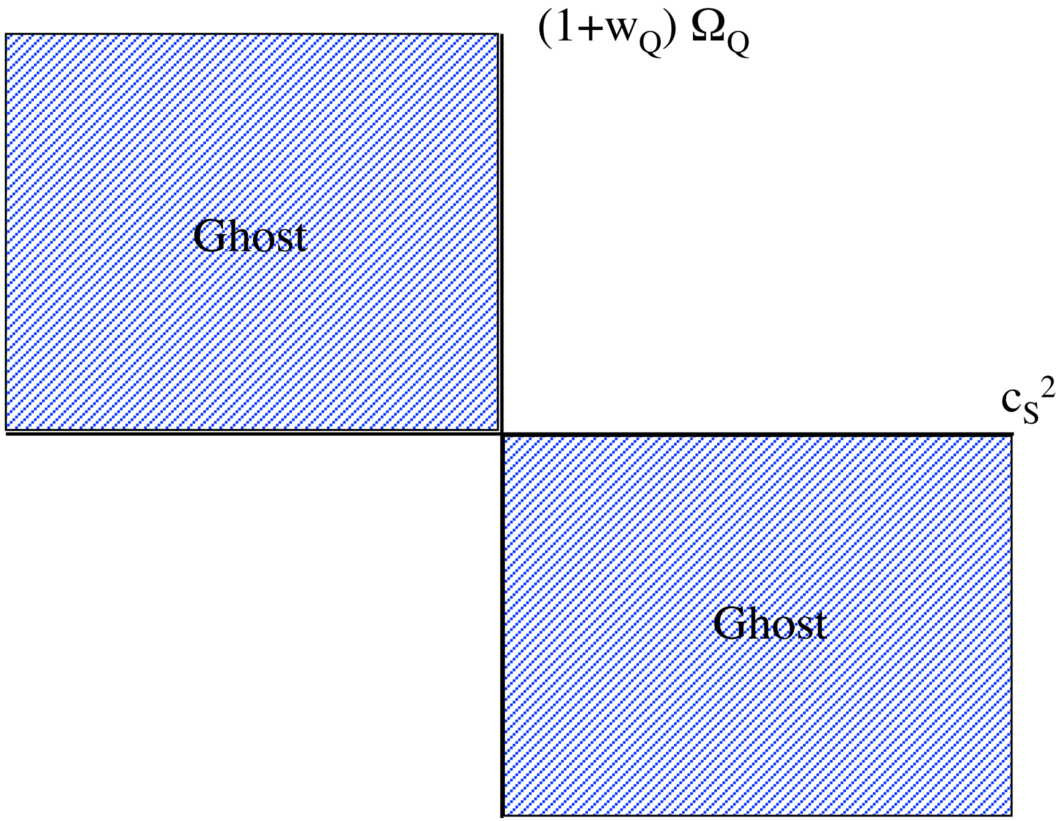
Cline, Jeon, Moore 03

# Quintessential plane




Let us study the different theoretical constraints on quintessence

# No ghost and $c_S^2$

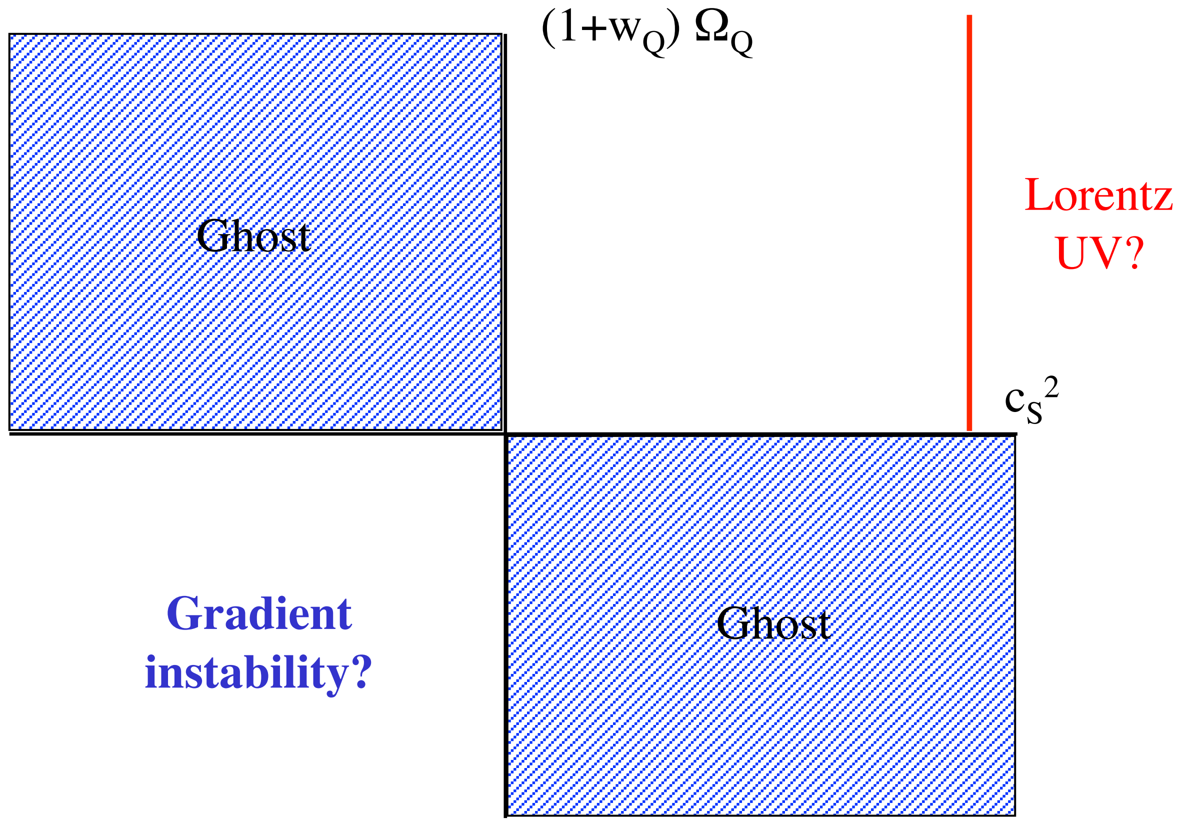



$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

**$c_S^2$ has the same sign of $1+w_Q$**

# w < -1 and gradient instabilities

Wise etal 04
Rattazzi etal 05




$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

It is difficult to violate the Null Energy Condition: $T_{\mu\nu}n^\mu n^\nu \geq 0$

# Small $c_S^2$ limit

Instability rate: $\omega = i\, c_s\, k$.

**If $c_S^2$ is very small instabilitites, $\omega > H$, only at short scales.**

Yes but short scales are still unstable…

$$S = \int d^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi\right]$$

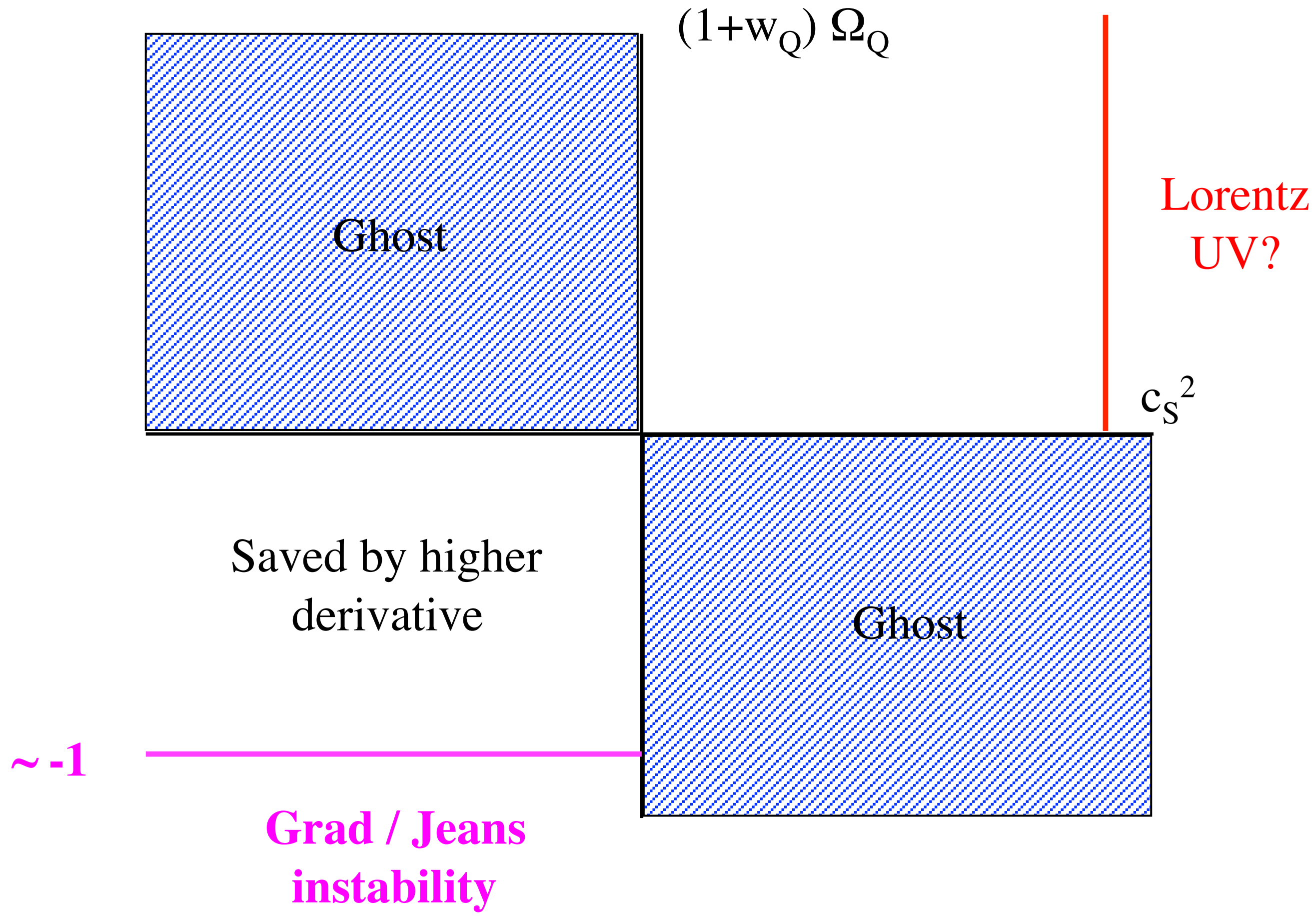
Consider the limit $\rho_Q + p_Q = 0$, no spatial kinetic term.

Higher derivative terms become relevant: $S \supset -\frac{\bar{M}^2}{2}\left(\frac{\nabla^2\pi}{a^2}\right)^2$

The stability analysis gets more complicated. Stability is possible for $w < -1$.

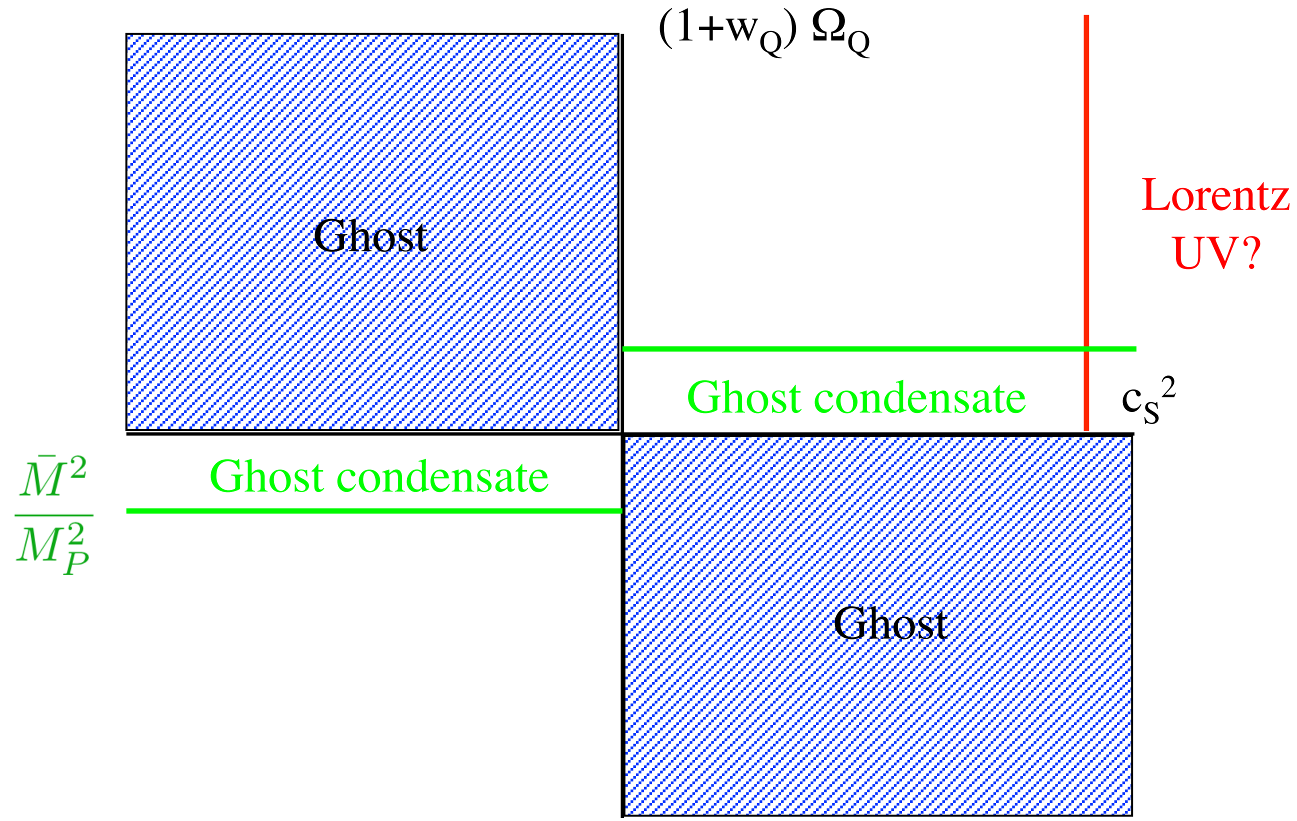
P.C. Luty, Nicolis and Senatore 05

# Back to the plane




This limit is very conservative and anyway pheno irrelevant

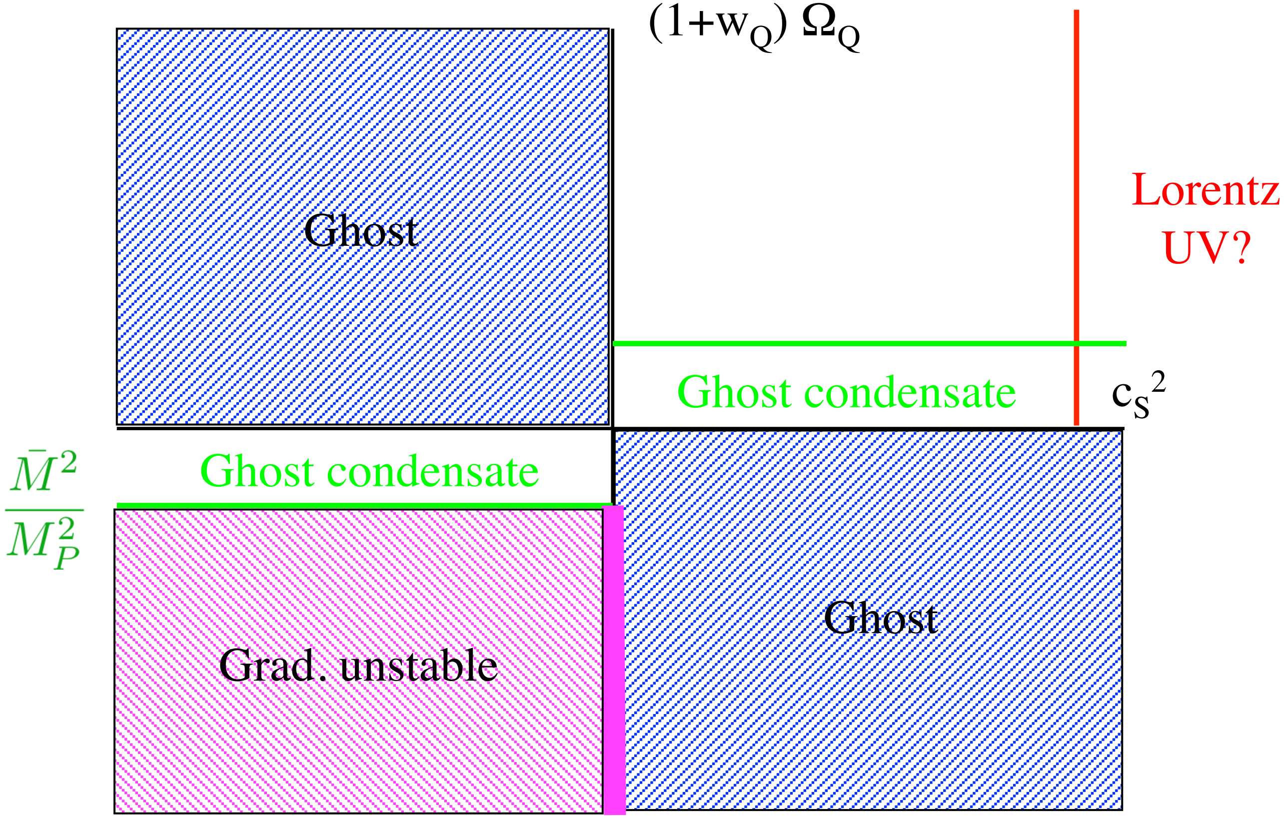
# Higher derivative in the codes?




$$S_Q \supset \frac{1}{2}\int d^3x dt\, a^3 \left[-(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} - \bar{M}^2\left(\frac{\nabla^2\pi}{a^2}\right)^2\right]$$

Cosmo modes k/a~H are dominated by $\omega = c_s$ k for: $\left|(1+w_Q)\Omega_Q\right| \gg \frac{\bar{M}^2}{M_P^2}$

# Small $c_s^2$: how small?




$$S_Q \supset \frac{1}{2}\int d^3x dt\, a^3 \left[4M^4\dot{\pi}^2 - (\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} - \bar{M}^2\left(\frac{\nabla^2\pi}{a^2}\right)^2\right] \qquad \omega_{\rm inst.} \simeq (1+w_Q)\Omega_Q \frac{M_P^2 H^2}{M^2 \bar{M}}$$

$$\omega_{\rm inst.} \ll H \quad \Rightarrow \quad c_s^2 \ll \frac{H\bar{M}}{M^2}$$

The scales M are the cutoff of my theory

M > (.1mm)$^{-1}$ --> $|c_s^2| < 10^{-30}$!!

# The phantom divide

**- What happens to perturbations when $w_Q = -1$?**

Fluid equations:

e.g. Bean, Doré 03

$$\dot{\delta} = -(1+w)\left\{\left[k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)\right]\frac{\theta}{k^2} + \frac{\dot{h}}{2}\right\} - 3\mathcal{H}(c_s^2 - w)\delta$$

$$\frac{\dot{\theta}}{k^2} = -\mathcal{H}(1-3c_s^2)\frac{\theta}{k^2} + \frac{c_s^2}{1+w}\delta \, .$$

$$\theta \equiv i k^j v_j \qquad c_a^2 \equiv \dot{p}/\dot{\rho} = w - \frac{1}{3H}\frac{\dot{w}}{1+w}$$

$$c_s^2 \equiv \delta\hat{p}/\delta\hat{\rho} \qquad T_i^0 = 0$$

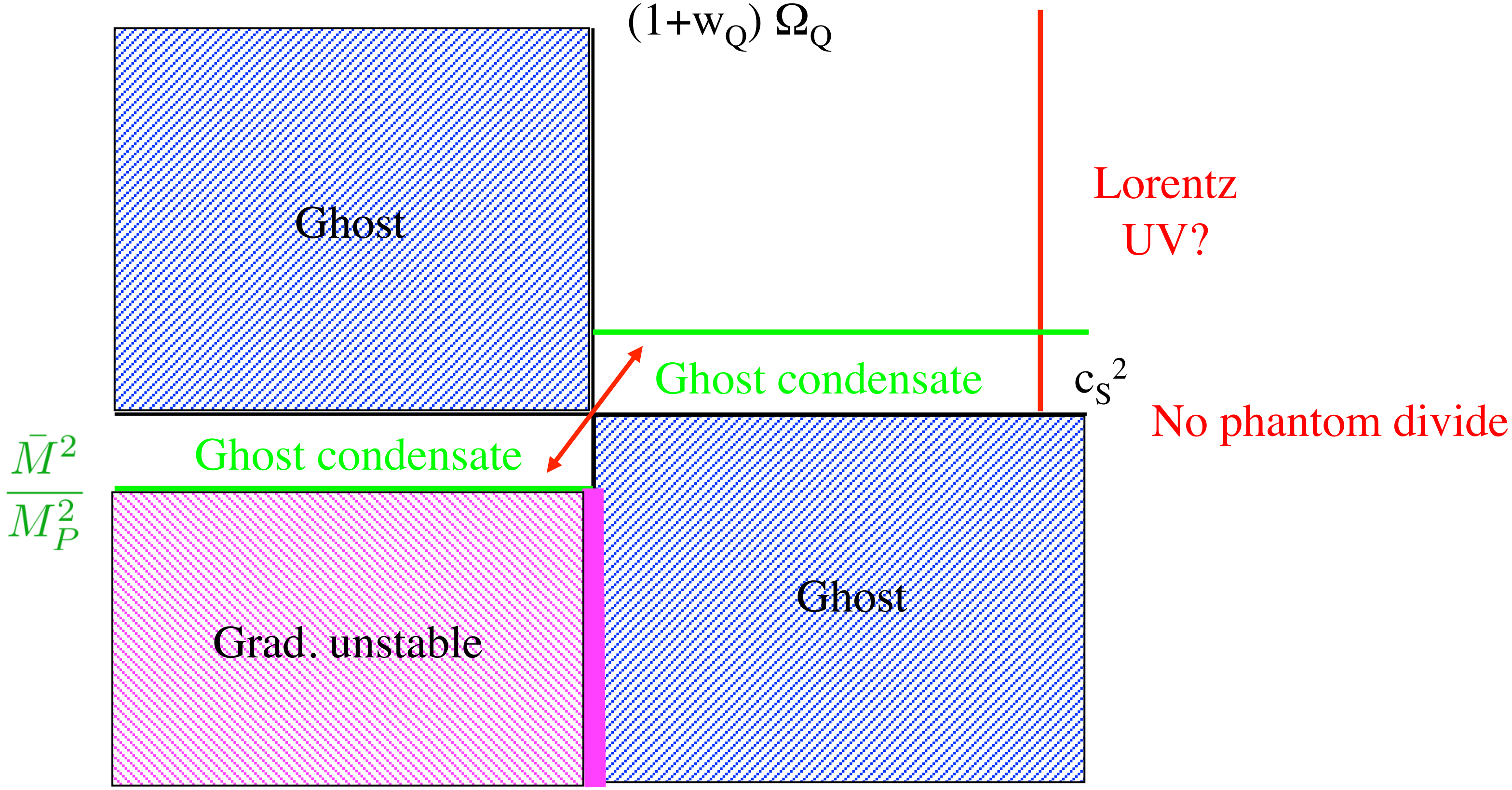
The one given by scalar kinetic term

**- The phantom psychosis:**

- 1[st] divergence: $c_a^2 \to \infty$ [Hu 04] So what?
- 2[nd] divergence: in $\theta$ equation [Caldwell, Doran 05] $c_s^2 \to 0$ at the crossing
- Instability: $c_s^2 \to 0 \Rightarrow c_s^2 < 0$ Higher derivative terms

[Vikman 04, Caldwell Doran 05, Kunz Sapone 06]

# The phantom divide is … a phantom




$$S_Q = \frac{1}{2}\int d^3x dt\, a^3 \Big[4M^4\dot{\pi}^2 + \cancel{(\rho_Q + p_Q)}\dot{\pi}^2 - \cancel{(\rho_Q + p_Q)}\frac{(\nabla\pi)^2}{a^2} + 3\dot{H}\cancel{(\rho_Q + p_Q)}\pi^2$$

$$-\cancel{(\rho_Q + p_Q)}\dot{h}\pi - \bar{M}^2\Big(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\Big)^2\Big]$$

Nothing strange happens when you cross $w_Q = -1$

# Clustering quintessence: $c_S \sim 0$

What does it mean that it has pressure but negligible $c_S$?

Euler equation: $$u^\mu \nabla_\mu u^\nu = -\frac{1}{(\rho_Q + p_Q)}(g^{\nu\sigma} + u^\nu u^\sigma)\nabla_\sigma p_Q$$

Scalar field is not barotropic: p(X, $\phi$)

$$c_s^2 = \frac{p_{,X}}{\rho_{,X}} \ll 1$$

For $c_s = 0$ pressure gradient orthogonal to the fluid 4-velocity vanishes

**Geodesic motion**
Quintessence remains comoving with DM

But pressure is not negligible!

Continuity equation: $$\dot{\rho} + \vec{\nabla} \cdot [(\rho + p)\vec{v}] = 0$$

# Phenomenology of $c_S \sim 0$ quintessence

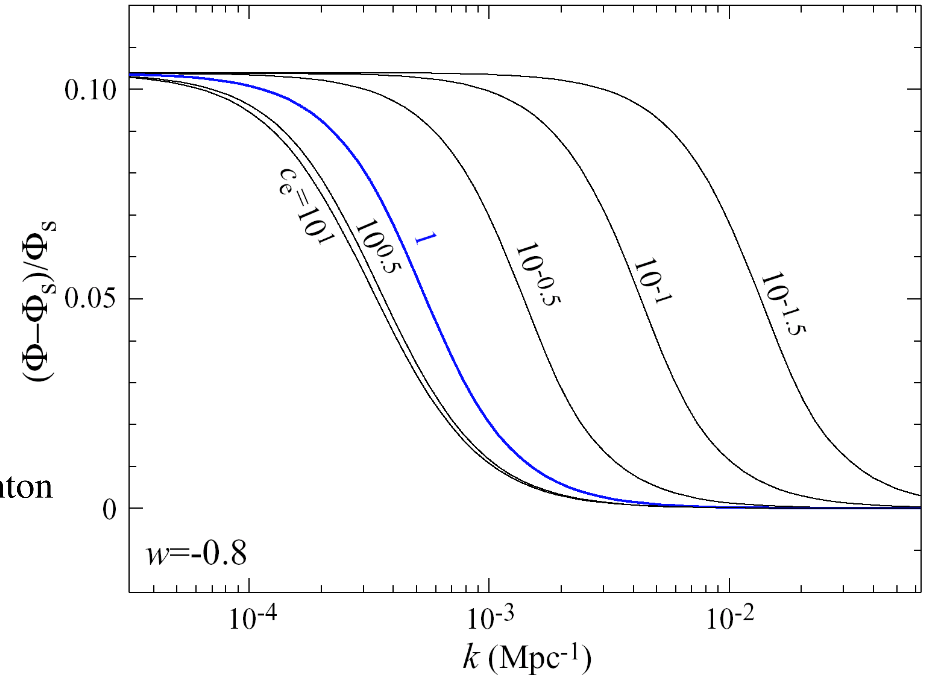
$$\dot{\delta}_Q - 3Hw\delta_Q + (1+w)\frac{1}{a}\vec{\nabla}\cdot\vec{u} = 0 \longrightarrow \delta_Q \simeq \frac{1+w}{1-3w}\delta_{\rm DM}$$

Comoving
with DM

Clusters on scales larger than sound horizon: $1/k_{\rm DE\ s.h.} = a\int_0^t \frac{c_s}{a}dt \simeq 2c_s H_0^{-1}$

Fractional difference in Φ
between smooth and clustered
quintessence

Hu, Scranton
04



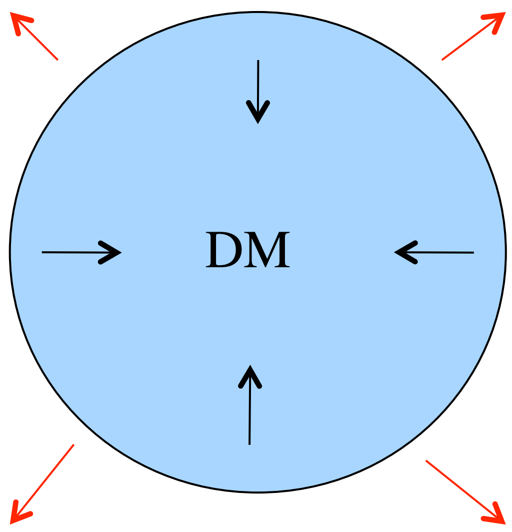
# Non-linear clustering

What happens at very short scales? For $c_S^2 = 0$ quintessence clusters at all scales.

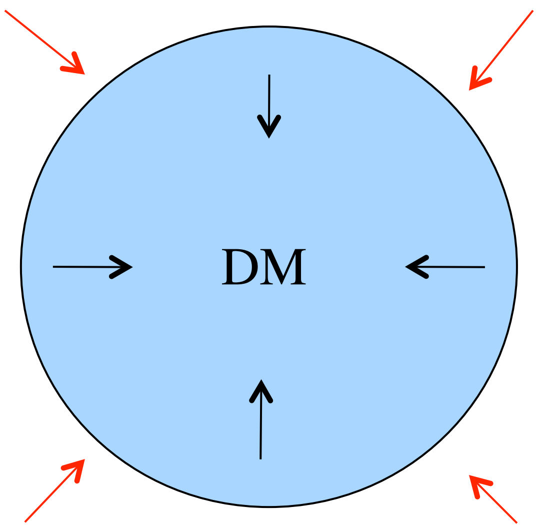
Effect on **non-linear structure formation**

**Spherical collapse**

$c_S = 1$




$c_S = 0$

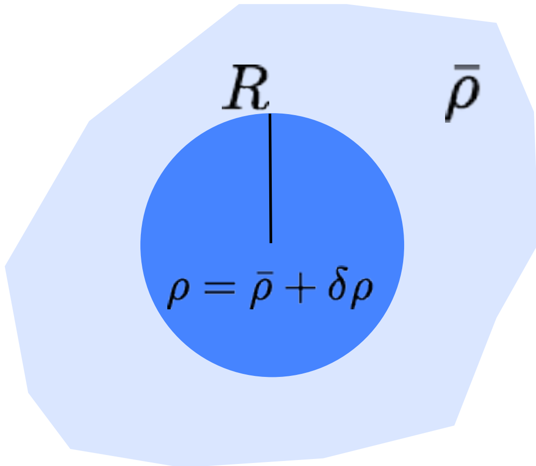


# Spherical collapse model

Simplest model for growth of non-linear structures

In EdS and ΛCDM, **Birkhoff's theorem implies (closed) FRW universe inside**

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{1}{R^2}$$




In EdS analytical solution:

$$R(\eta) = \frac{R_{\rm ta}}{2}(1 - \cos\eta)$$
$$t(\eta) = \frac{R_{\rm ta}}{2}(\eta - \sin\eta)$$

**Can linearize and compute linear δ at collapse time** $t_c = \pi R_{\rm ta}$

$$R(t) \simeq \frac{GM}{2}\left(\frac{6t}{GM}\right)^{\frac{2}{3}}\left[1 - \frac{1}{20}\left(\frac{6t}{GM}\right)^{\frac{2}{3}}\right] \quad \Rightarrow \quad \delta_c = \frac{3}{20}(12\pi)^{\frac{2}{3}} = 1.686$$

# Almost Minkowski

Spherical collapse occurs on scales << $H^{-1}$: **tiny deformation of Minkowski**

Corrections will be suppressed by (powers of) $H^2x^2$

Start from FRW metric: $$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + a^2(\tau)\left(1+\frac{K}{4}\vec{y}^2\right)^{-2}\mathrm{d}\vec{y}^2$$

Transform coordinates: $$\tau = t - \frac{1}{2}H(t)x^2\,, \quad \vec{y} = \frac{\vec{x}}{a(t)}\left(1+\frac{1}{4}H^2(t)x^2\right)$$

In t, x we get Minkowski + perturbations $$\mathrm{d}s^2 = -\left(1+2\Phi\right)\mathrm{d}t^2 + \left(1-2\Psi\right)\mathrm{d}\vec{x}^2$$

$$\Phi = -\frac{1}{2}\left(\dot{H}+H^2\right)x^2\,, \quad \Psi = \frac{1}{4}\left(H^2+\frac{K}{a^2}\right)x^2$$

A perfect fluid will have a velocity v ≃ H x. Continuity and Euler equations are:

$$\dot{\rho} + \vec{\nabla}\cdot[(\rho+p)\vec{v}] = 0 \qquad \dot{\vec{v}} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{1}{\rho+p}\left[\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right] - \vec{\nabla}\Phi$$

# $c_s = 1$ vs $c_s = 0$

Dark matter flow is different inside and outside the halo

$$\vec{v}_{m,\mathrm{out}} = H\vec{x} \qquad \vec{v}_{m,\mathrm{in}} = \frac{\dot{R}}{R}\vec{x}$$

Euler equation for matter: $$\frac{\ddot{R}}{R}\vec{x} = -\vec{\nabla}\Phi$$

$c_s = 1$

Quintessence does not cluster inside Hubble radius

It keeps following the external Hubble flow: $\vec{v}_Q = H\vec{x}$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho_m + \bar{\rho}_Q + 3\bar{p}_Q\right)$$

Wang, Steinhardt 98

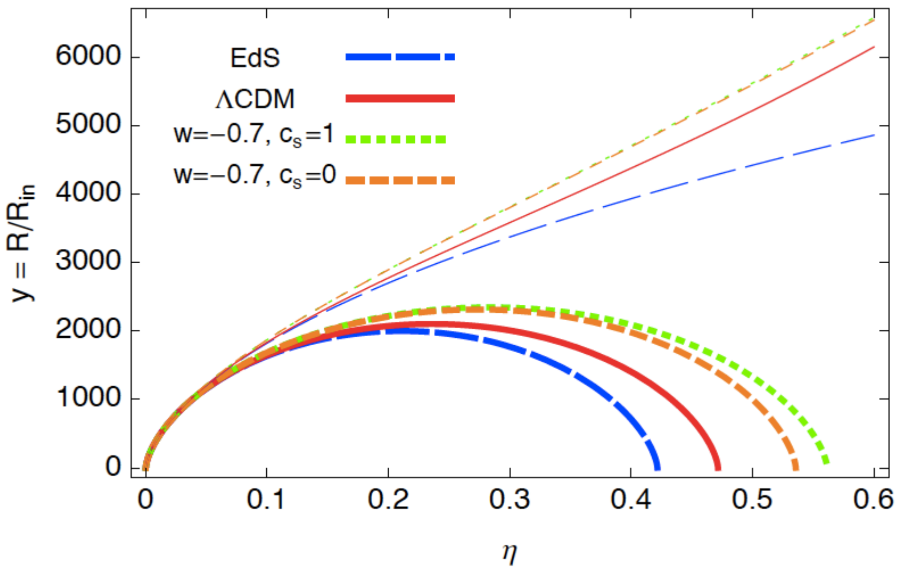
$c_s = 0$

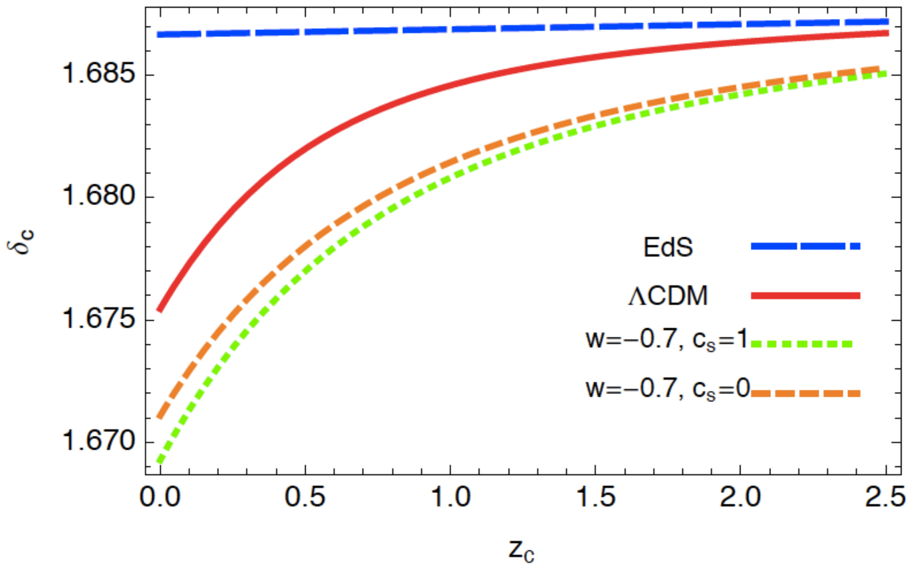
Quintessence clusters on all scales

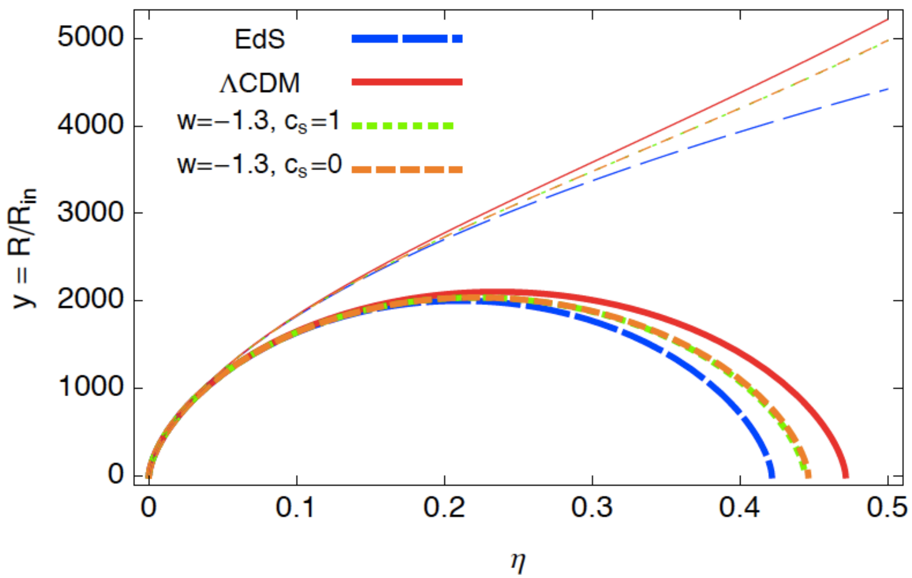
It follows the dark matter flow: $\vec{v}_{Q,\mathrm{out}} = H\vec{x}\,,\ \vec{v}_{Q,\mathrm{int}} = \frac{\dot{R}}{R}\vec{x}$

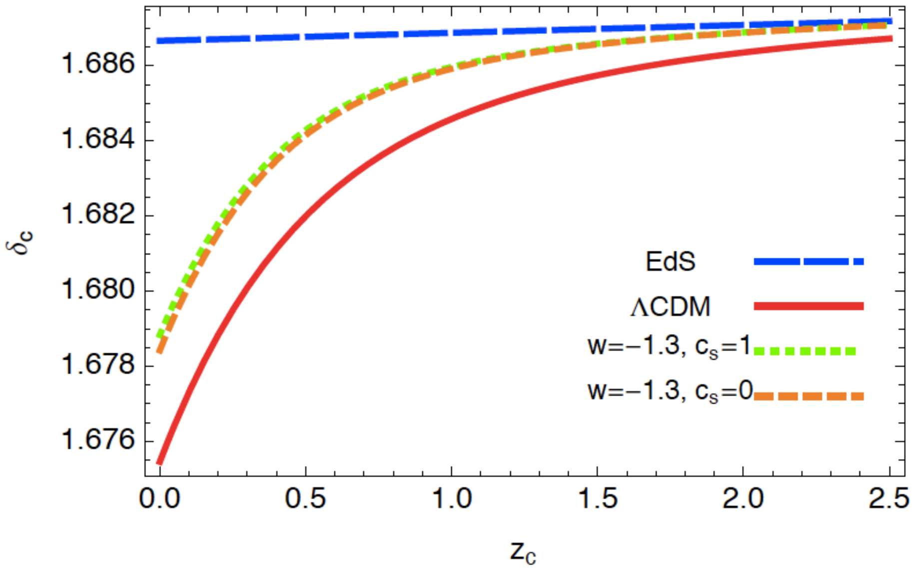
$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho_m + \rho_Q + 3\bar{p}_Q\right)$$

# Evolution and threshold



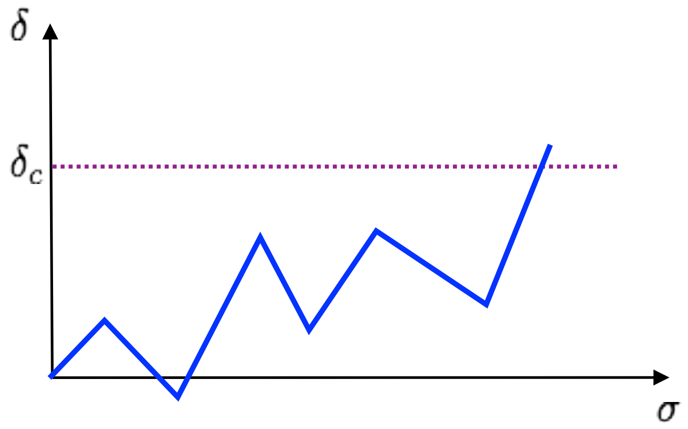



# Press – Schechter model

How to estimate the halo mass function?

1. Smooth the overdensity field on radius R:

$$\delta_R(\vec{x}) = \int \mathrm{d}^3x' W_R(\vec{x} - \vec{x}')\delta(\vec{x}')$$




2. Calculate smoothed variance:

$$\sigma_R^2 = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\, k^2\, \tilde{W}_R^2(k)P(k)$$

3. As R decreases, $\delta_R$ follows a random walk for every point.

Associate the point to a halo of radius R **when $\delta_R$ first upcrosses $\delta_c$**

$$\frac{\mathrm{d}n}{\mathrm{d}M}(M,z) = -\sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}}{M^2}\,\frac{\delta_c(z)}{D(z)\sigma_M}\exp\left[-\frac{\delta_c^2(z)}{2D^2(z)\sigma_M^2}\right]\frac{\mathrm{d}\log\sigma_M}{\mathrm{d}\log M}$$

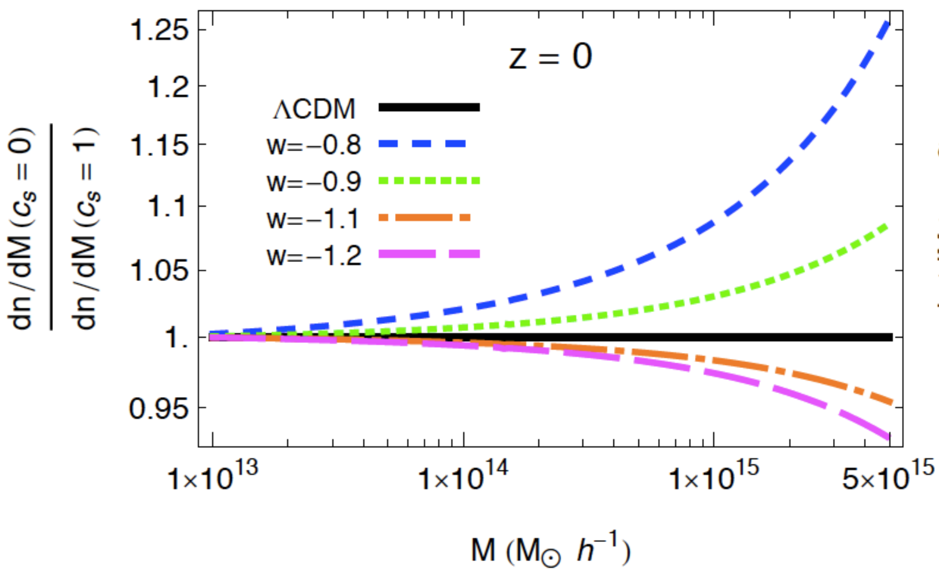
# Mass function

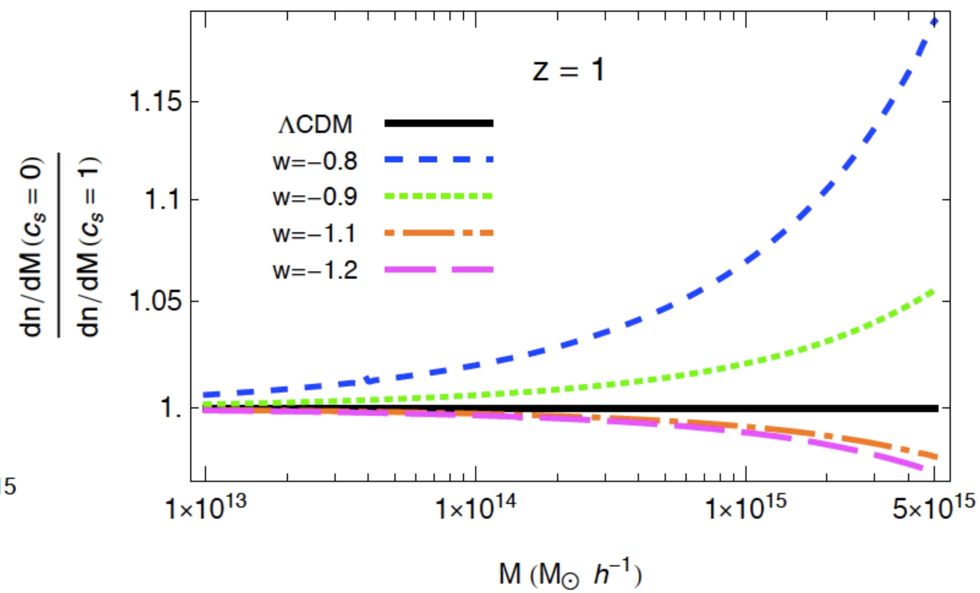
$$\frac{\mathrm{d}n_{PS}}{\mathrm{d}M}(M,z) = -\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c(z)}{D(z)\sigma_M}\exp\left[-\frac{\delta_c^2(z)}{2D^2(z)\sigma_M^2}\right]\frac{\mathrm{d}\log\sigma_M}{\mathrm{d}\log M}$$

$\delta_c$ dependence on z is very mild.

With a good approximation, **only dependence on $c_s^2$ is through linear cosmology**.

Small effect...



# Quintessence mass

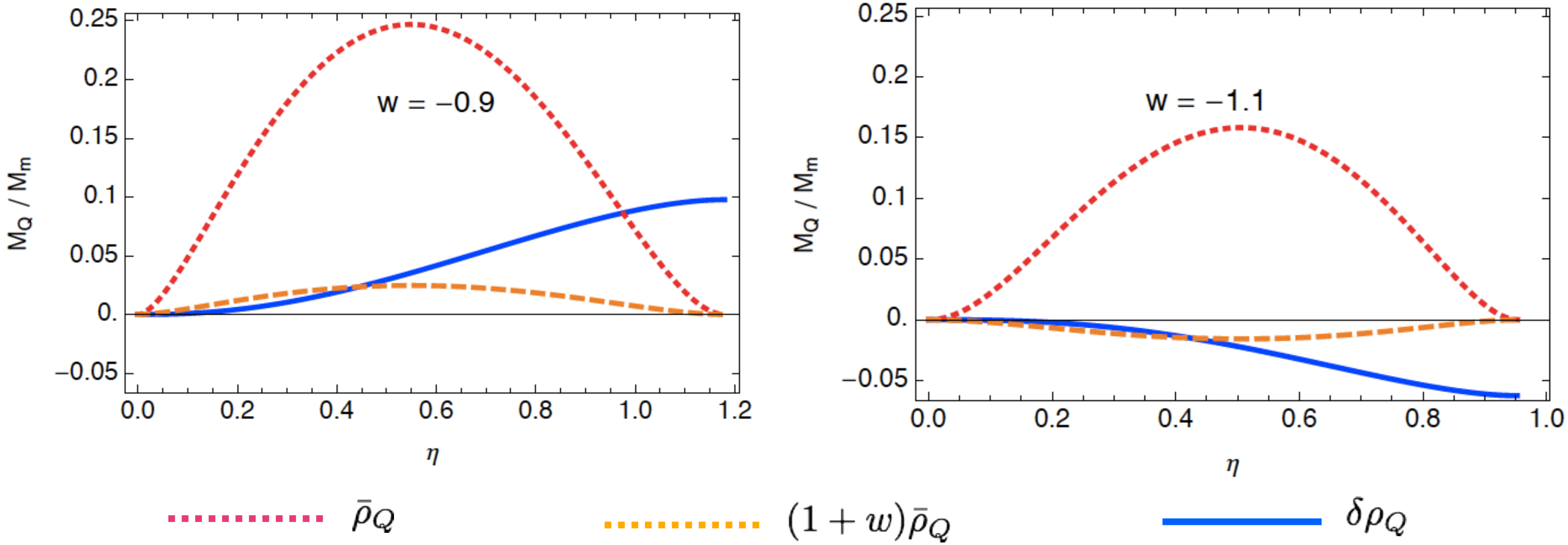
We forgot clustering quintessence mass
(negative for w < -1!)

$$M_Q = \int \mathrm{d}^3x\, \delta\rho_Q = \frac{4\pi}{3} R^3 \delta\rho_Q$$

It makes sense only: $\left|\frac{\dot{M}_Q}{M_Q}\right| \ll H$

Valid when: $|\delta_Q| \gg |1+w|$

$$\frac{M_Q}{M_m} \sim (1+w)\frac{\Omega_Q}{\Omega_m}$$



# Correcting the mass function

Formation of a DM halo of mass M at z will result in a halo of total mass

$$M(1+\varepsilon(z)) \qquad \varepsilon(z) \equiv \frac{M_{Q,\mathrm{vir}}}{M_{DM,\mathrm{vir}}}$$

**Tough**: should follow object evolution as it merges and accretes quintessence.

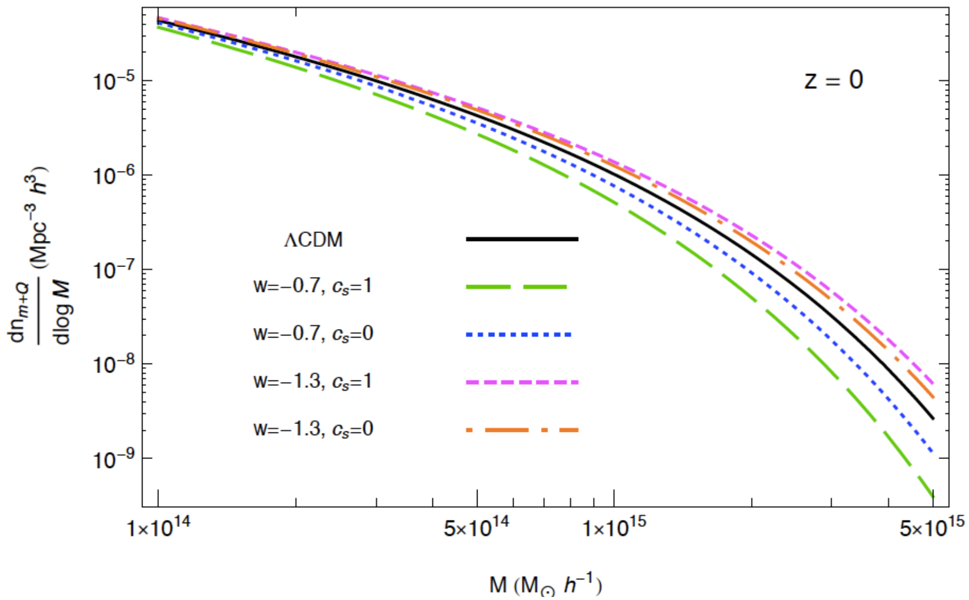
But ε is relevant only at low z, when large objects form.
Their formation rate (negligible merging) can be approximated

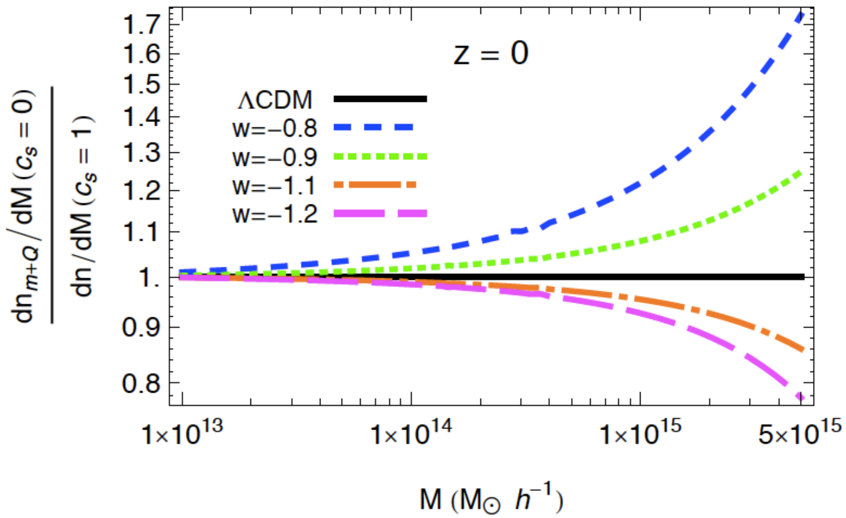
$$\left[-\frac{\partial}{\partial z}\frac{\mathrm{d}n_{PS}}{\mathrm{d}\log M}\right]^{+}$$

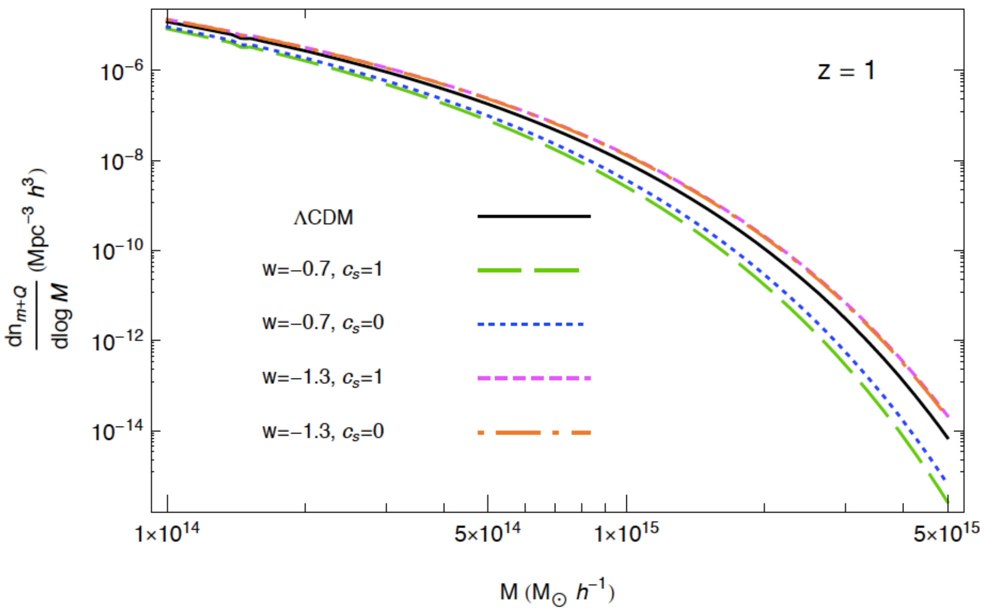
The corrected mass function is thus approximated as

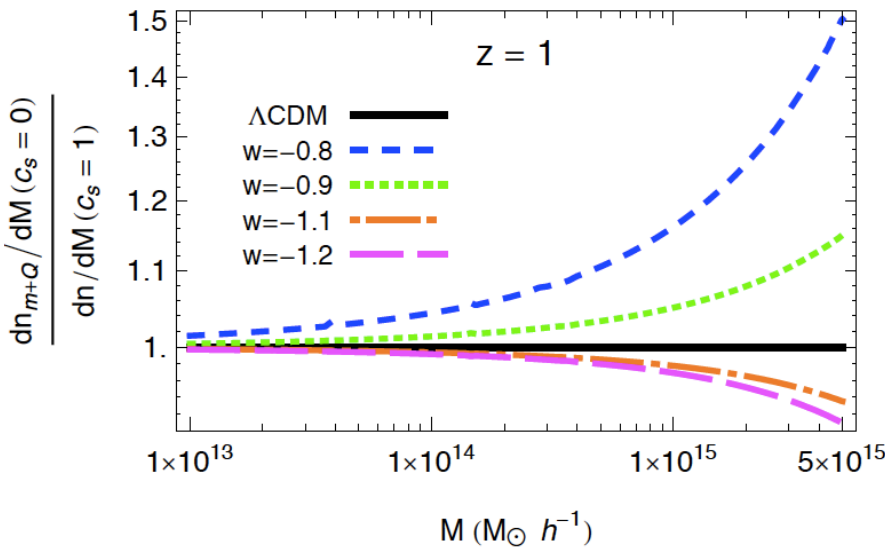
$$\frac{\mathrm{d}n_{PS,Q}}{\mathrm{d}\log M}(M,z) = \frac{\mathrm{d}n_{PS}}{\mathrm{d}\log M}(M,z) + M\frac{\partial}{\partial M}\int_{z_{in}}^{z} dz\,\epsilon(z)\left[-\frac{\partial}{\partial z}\frac{\mathrm{d}n_{PS}}{\mathrm{d}\log M}(M,z)\right]^{+}$$

# Corrected mass function



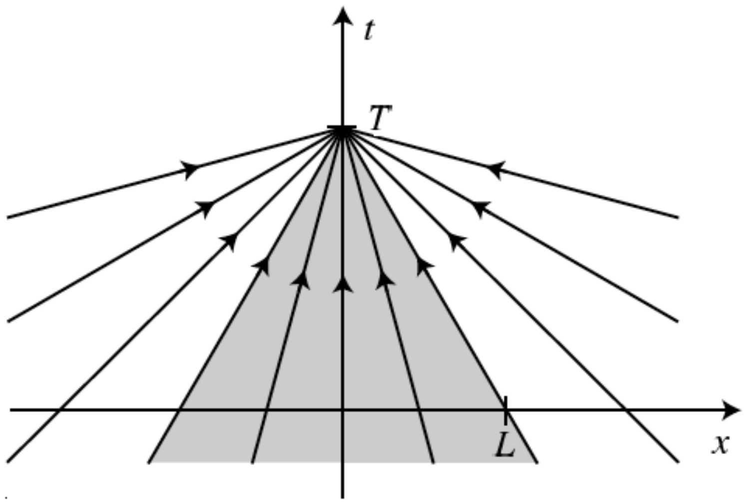



# Virialization and extra mass

- Particle trajectories tend to cross: the scalar field develops caustics. What next?

- Caustics can be resolved on short scales,
- allowing $c_s^2$ to become large at large density

- **How does this fluid distribute?**




- Clusters: Baryon mass + dark matter + quintessence mass

- Baryon fraction $f_{gas}$ should be sensitive to clustering quintessence

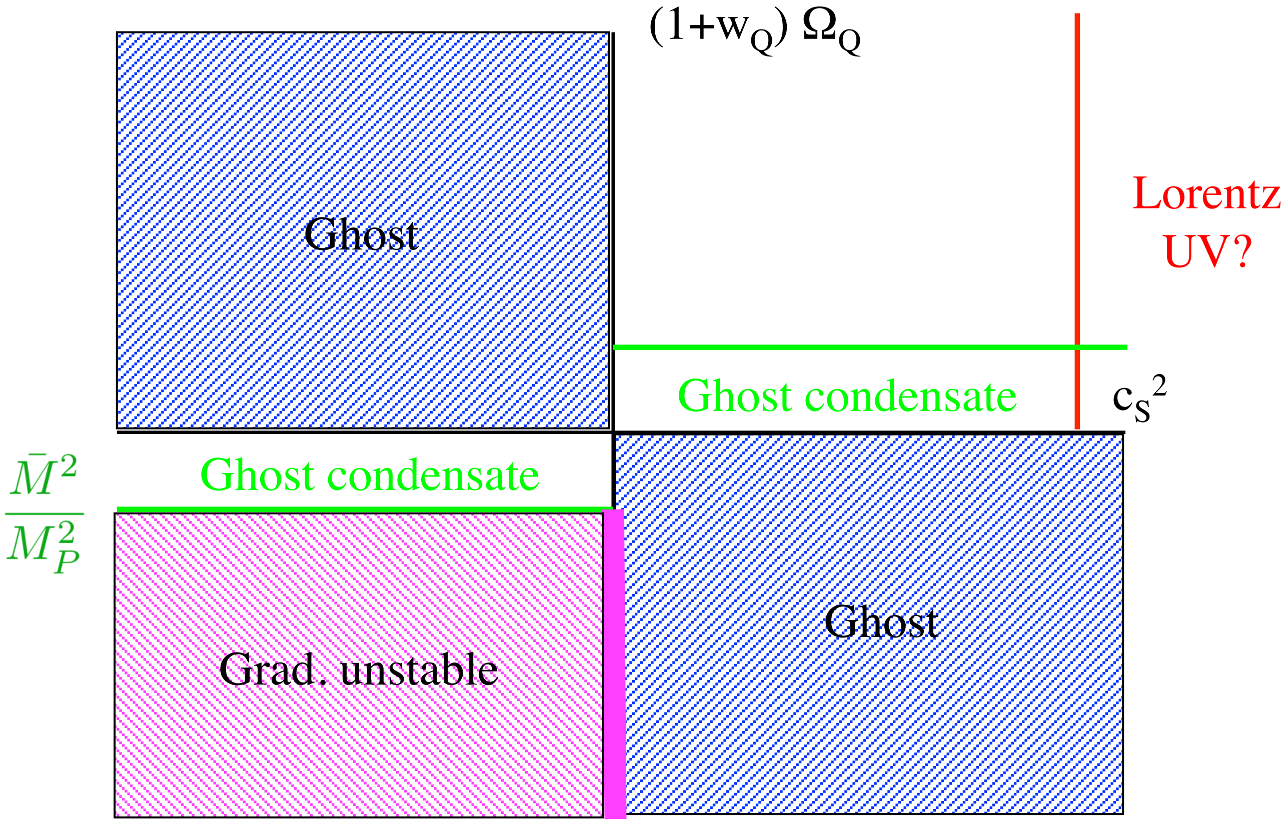
- Important signature of clustering quintessence is its **strong redshift dependence**

# Conclusions

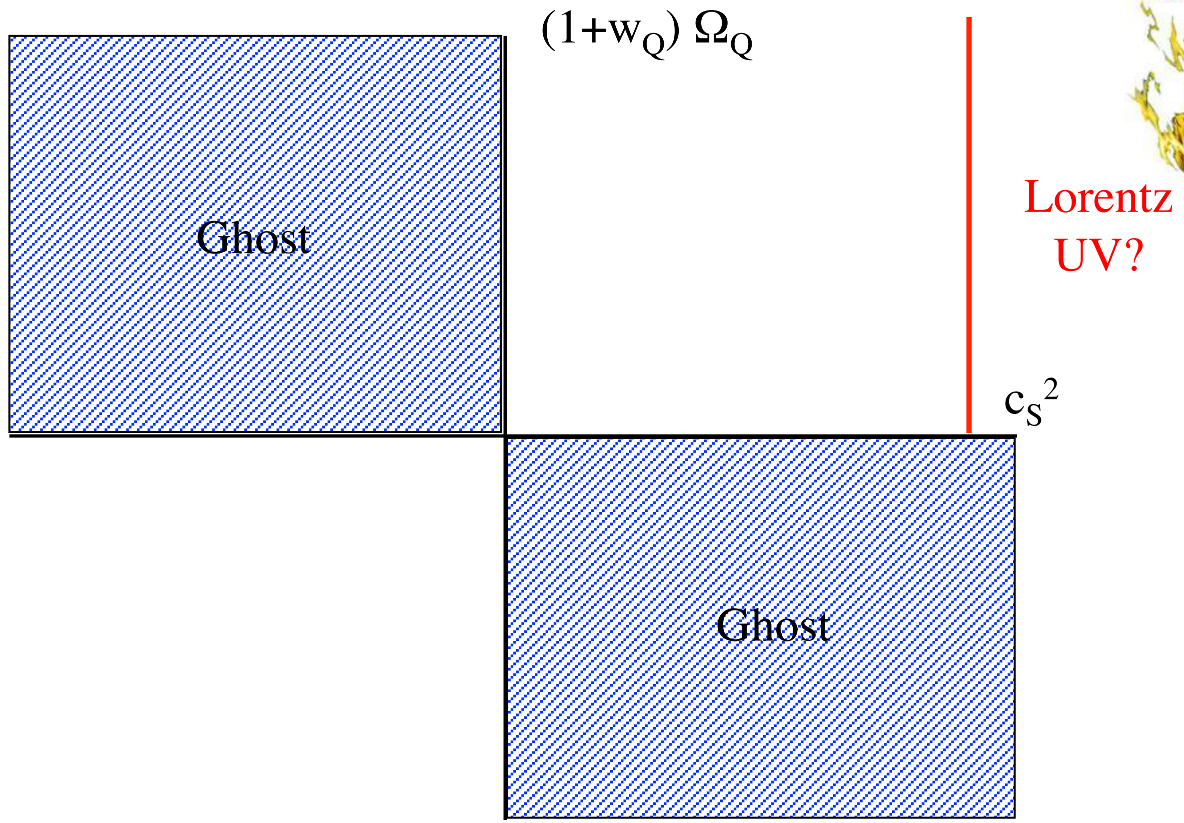
- General framework to study single field quintessence models
- $w_Q < -1$ region can be stable for $\mathbf{c_S^2 = 0}$
- Phenomenology of models with $c_S^2 = 0$ vs $c_S^2 = 1$ must be further explored
- Spherical collapse for $c_S^2 = 0$ and correction to the mass function
- Accretion of quintessence mass

**We do not anything about the clustering properties of dark energy!**

# Quintessential plane



# Faster than light?




$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

$c_S^2 > 1$ ($M^4 < 0$) implies a non-Lorentz invariant UV completion

Arkani-Hamed etal '06
Babichev etal '07

# Higher derivative

We have to consider higher derivative operators

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}(\Box\phi + 3H(\phi))^2$$

It does not change the background evolution. Only perturbations.

$$\mathcal{L}_{\bar{M}} = -\frac{\bar{M}^2}{2}\left(\ddot{\pi} + 3H\dot{\pi} - 3\dot{H}\pi - \frac{\nabla^2\pi}{a^2}\right)^2$$

Leading spatial derivative term

$\ll M^4\dot{\pi}^2$

Higher time derivative terms can be neglected for ω < M
No additional degrees of freedom

In the ghost condensate limit:
$\omega \propto k^2$

The Ghost Condensate is a point of enhanced symmetry.

A small breaking of the shift symmetry (and thus a small $c_s^2$ ) is **technically natural**

# Stability analysis

**Gradient instability:** $\left(\rho_Q + p_Q + 4M^4\right)\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$

$$\omega^2_{\rm grad} \simeq -\frac{(\rho_Q + p_Q)^2}{\bar{M}^2 M^4} \longrightarrow -\frac{\rho_Q + p_Q}{\bar{M}M^2} \lesssim H$$

**Jeans instability:** taking into account the mixing with gravity gives rise to a sort of Jeans like instability

$$S = \int {\rm d}^4x \left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2}\left(\frac{\dot{h}}{2} - \nabla^2\pi\right)^2\right] \longrightarrow \ddot{\pi} + \frac{\bar{M}^2}{4M^4}\nabla^4\pi = \frac{\bar{M}^2}{8M^4}\nabla^2\dot{h}$$

Solving for h: $\ddot{\pi} + \frac{\bar{M}^2}{4M^4}\nabla^4\pi = -\frac{\bar{M}^2}{2M^2_{\rm Pl}}\nabla^2\pi \longrightarrow \omega^2_{\rm Jeans} \simeq -\left(\frac{\bar{M}M^2}{M^2_{\rm Pl}}\right)^2$

**Stability window**

$$-(1+w_Q)\Omega_Q \lesssim \frac{\bar{M}M^2}{HM^2_{\rm Pl}} \lesssim 1$$

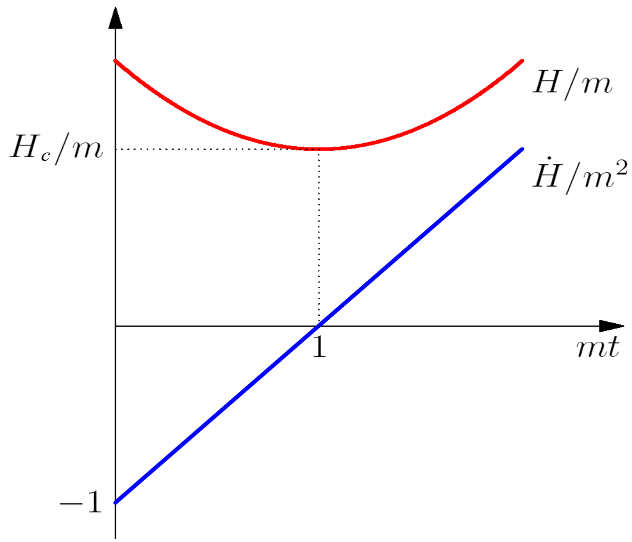
P.C. Luty, Nicolis and Senatore 05

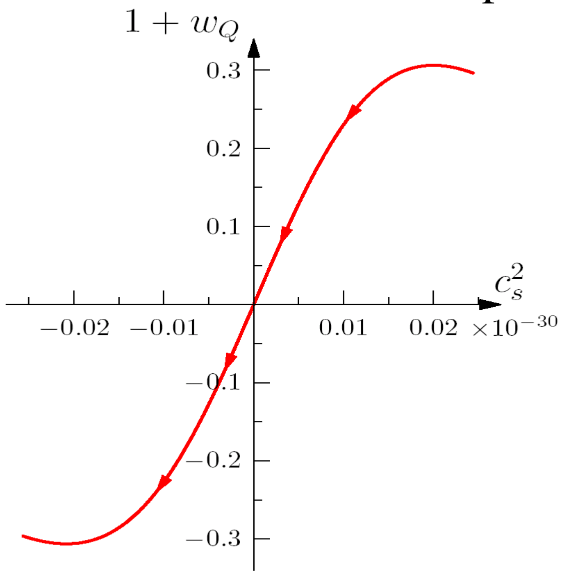
Very conservative...

# For example...

$$P(X,\phi) = -3M_{\rm Pl}^2 H^2(\phi) - M_{\rm Pl}^2 \dot{H}(\phi)(X+1) + \frac{1}{2}M^4(\phi)(X-1)^2$$

No other energy components




- The GC strip is very tiny. Effectively **$w_Q$= -1 is crossed by a k-essence with $c_S^2 = 0$**
- Numerical recipe. When comparing with data $w_Q(z)$ going through $w_Q$= -1, set $c_S^2 = 0$

# Quintessence ~ Λ

Ghost condensate limit

For cosmo scales: $\omega \sim k^2$ $\quad (1+w_Q)\Omega_Q \ll \frac{\bar{M}^2}{M_P^2}$

$$S_Q = \frac{1}{2}\int d^3x dt\, a^3 \left[4M^4\dot{\pi}^2 - \bar{M}^2\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2\right]$$

The scalar degree of freedom does not disappear even for $1+w_Q=0$

$$\ddot{\pi} + 3H\dot{\pi} = -\frac{\bar{M}^2}{12M^4M_P^2}\frac{\nabla^2\delta\rho_{\rm DM}}{Ha^2}$$

The driving of DM is not suppressed by $1+w_Q$ in this limit

$$\delta\rho_Q = 4M^4\dot{\pi} \sim \frac{\bar{M}^2}{M_P^2}\delta\rho_{\rm DM} \lll \delta\rho_{\rm DM}$$

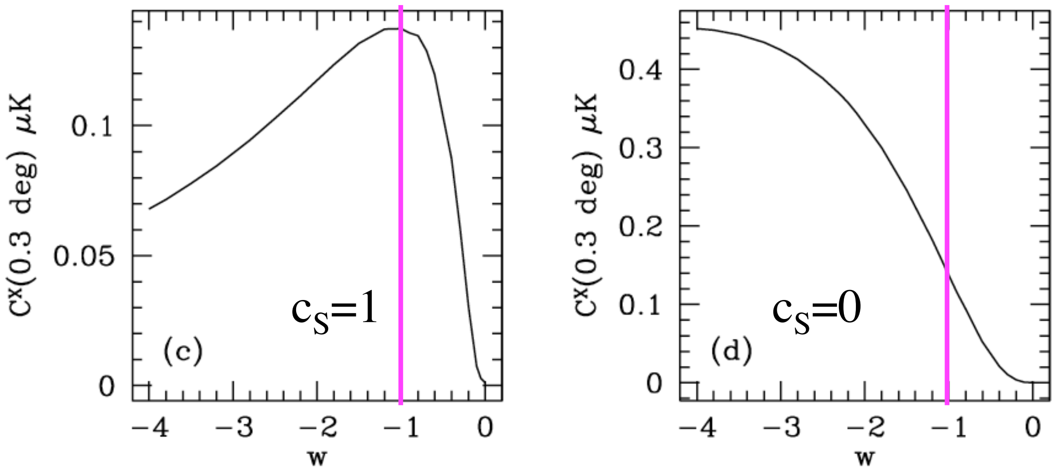
**No relevant perturbation!**

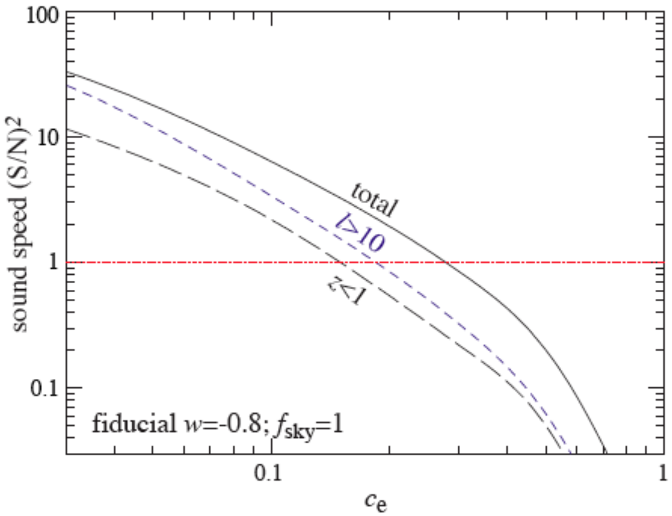
**The ghost condensate is a modification of gravity, but only on very short scales**
**Irrelevant cosmologically**

# ISW- galaxy correlation

**Is it possible to exp distinguish $c_s$=0 from $c_s$=1? Until which value of $1+w_Q$?**

Corasaniti, Giannantonio, Melchiorri 05



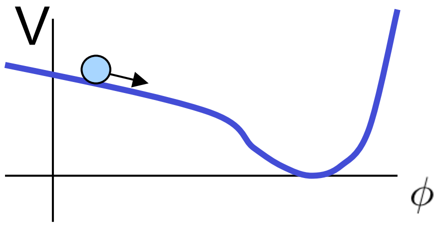




Hu, Scranton 04

Distinction possible for $|1+w_Q| \gtrsim 0.05$ ?

Forecasts done only for w > -1...

# A more general approach




Usual approach to quintessence/inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu \phi, \Box\phi \ldots)$
2. Solve EOM of the scalar + FRW. Find an accelerating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \qquad ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$

3. Study perturbations around this solution to work out predictions

We want to **focus directly on the theory of perturbations** around the accelerating solution

- Time diffeomorphisms are broken: $t \to t + \xi^0(t, \vec{x}) \qquad \delta\phi \to \delta\phi + \dot{\phi}_0(t)\xi^0$
- In unitary gauge $\phi(t, \vec{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:

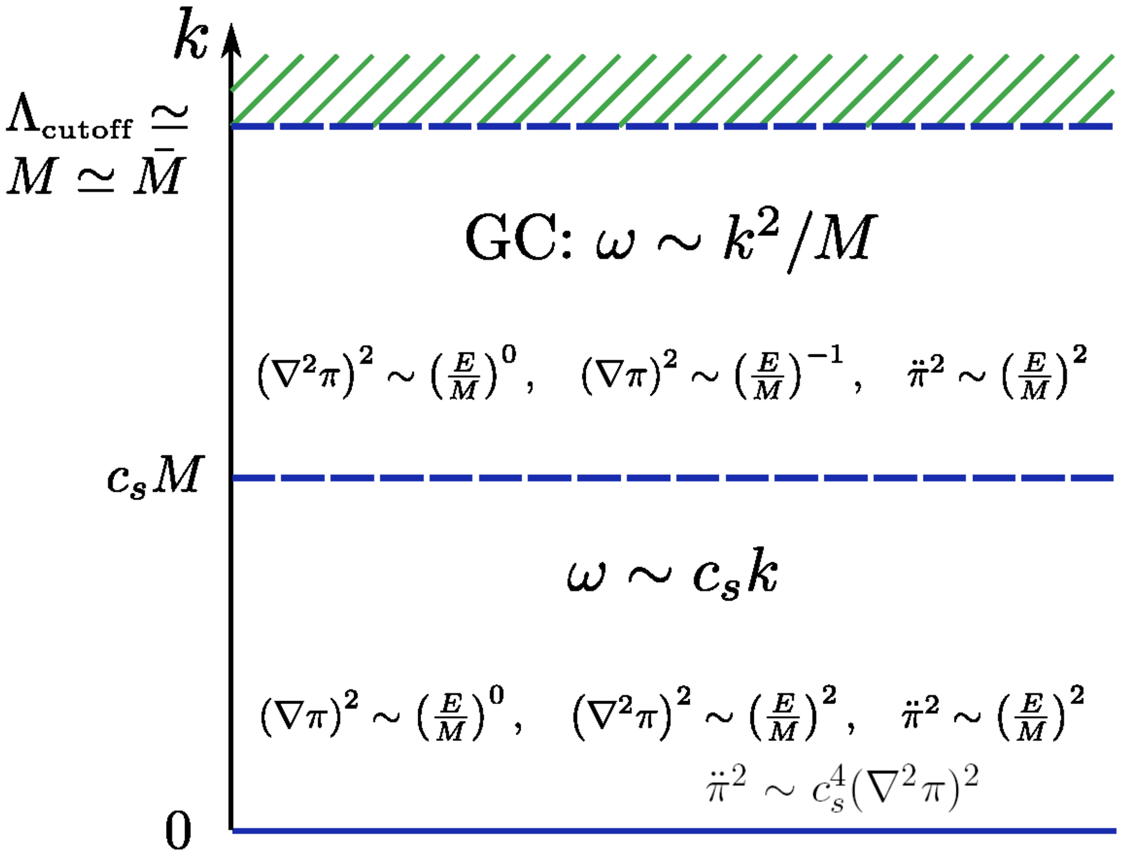
  3 degrees of freedom. Like in a broken gauge theory.
- The most generic action in unitary gauge

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_m + p_Q - \frac{1}{2}(\rho_Q + p_Q)(g^{00}+1) + \frac{M^4(t)}{2}(g^{00}+1)^2 - \frac{\bar{M}^2(t)}{2}\delta K^2 - \frac{\hat{M}(t)^3}{2}\delta K(g^{00}+1)\right].$$

# Scaling in EFT

Arkani-Hamed et al ‘03, Simon ’91, Weinberg ‘08

$$S = \frac{M^4}{2}\int d^3x dt \left[\dot{\pi}^2 - c_s^2(\nabla\pi)^2 - \frac{(\nabla^2\pi)^2}{M^2} + \ldots\right.$$




- scaling transformations:

$$E \to sE\,, \qquad t \to s^{-1}t\,,$$
$$x \to s^{-1/2}x\,, \qquad \pi \to s^{1/4}\pi$$

$$E \to sE\,, \qquad t \to s^{-1}t\,,$$
$$x \to s^{-1}x\,, \qquad \pi \to s^{1}\pi$$