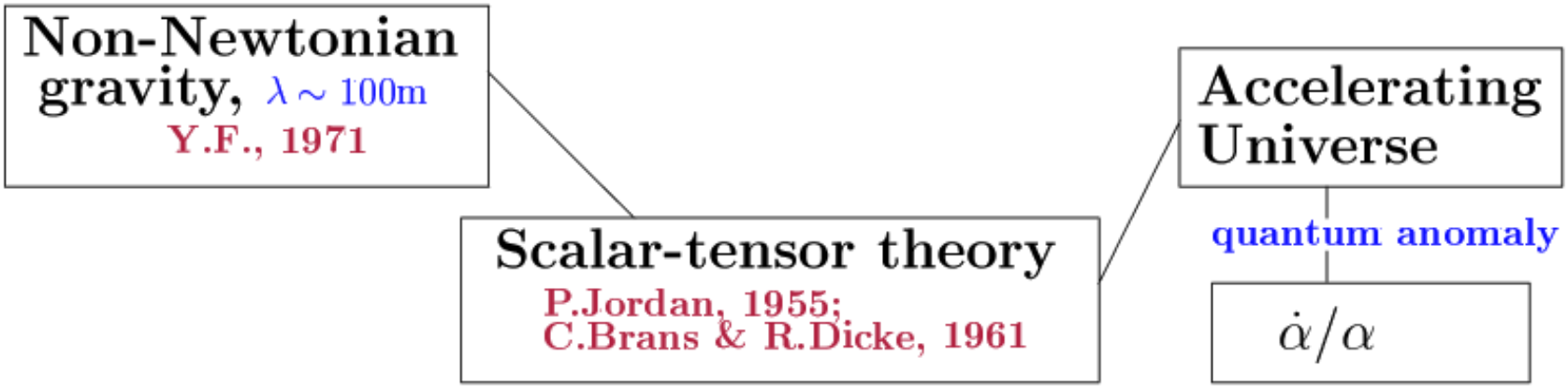
OATS, 5 Mar 07

Scalar-tensor theory, the accelerating universe and the time-dependent fine-structure constant, I

Yasunori Fujii

1 Introduction



Today's version of the cosmological constant problem

- **Fine-tuning problem:** $\Lambda_{\rm obs}/\Lambda_{\rm th} \sim 10^{-120}$ $\quad (\Lambda_{\rm th} \sim M_{\rm P}^4)$
- **Coincidence problem:** Is today special because the acceleration (mini-inflation) occurs only once during the whole history of the universe, if $\Lambda =$ const?

Y.F, & K.Maeda, *The scalar-tensor theory of gravitation*, Cambridge U Press, 2003; Y.F., PTP 110 (2003),433; Y.F., in *Gravity, Astrophysics and Strings at the Black Sea*, St.Kliment Ohridski U Press,2005, gr-qc/0410097

2 Scalar-tensor theory

Nonminimal coupling term (NM)

$$\mathcal{L}_{\rm STT} = \sqrt{-g}\left(\frac{1}{2}\xi\phi^2 R - \epsilon\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + L_{\rm matter}\right)$$

$$\mathcal{L}_{\rm EH} = \sqrt{-g}\frac{1}{16\pi G}R \quad \Rightarrow \quad \frac{1}{8\pi G_{\rm eff}} = \xi\phi^2$$

$$\xi > 0, \quad \epsilon = \pm 1, \quad 4\omega\xi = \epsilon = {\rm Sign}(\omega)$$

$\zeta^{-2} = 6 + \epsilon\xi^{-1} = 6 + 4\omega > 0$ for positive energy of diagonalized scalar field $\Rightarrow$ $\epsilon = -1$ implies a ghost ϕ, but not immediately a trouble

$\phi \left\{ \begin{array}{l} \text{coupled} \\ \text{decoupled} \end{array} \right\}$ to matter for $\left\{ \begin{array}{l} \text{J theory} \\ \text{BD model} \end{array} \right\} \Rightarrow$ WEP $\left\{ \begin{array}{l} \text{violated} \\ \text{valid} \end{array} \right.$

No mass term assumed $\sim \phi^2$

Reduced Planckian units $\quad c = \hbar = M_{\rm P}\left(= (8\pi G)^{-1/2}\right) = 1$

8.07×10^{-33}cm, 2.71×10^{-43}sec, 2.44×10^{18}GeV (t_0=13.7Gy $\approx 10^{60.21}$)

PPN approximation for solar-system experiment

$$g_{rr} \approx 1+\gamma\frac{2m_\odot}{r}, \quad \gamma = 1-4\zeta^2, \qquad \zeta^{-2} = 6+\epsilon\xi^{-1} = 6+4\omega > 0$$

Source	$\zeta^2 \lesssim$	$\xi \lesssim$	$\omega \gtrsim$	
BD (61)	**0.03**	**0.04**	**6**	
Viking (79)	2.5×10^{-4}	2.5×10^{-4}	**1,000**	Shapiro time delay
VLBI (99)	7×10^{-5}	7×10^{-5}	**3,600**	Deflection from QSO's
Cassini (03)	$.5\times10^{-6}$	$.5\times10^{-6}$	**500,000**	Shapiro time delay

unnaturally too small !? too large !?

$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ from Cassini

B.Bertotti et al, Nat,425(03)374

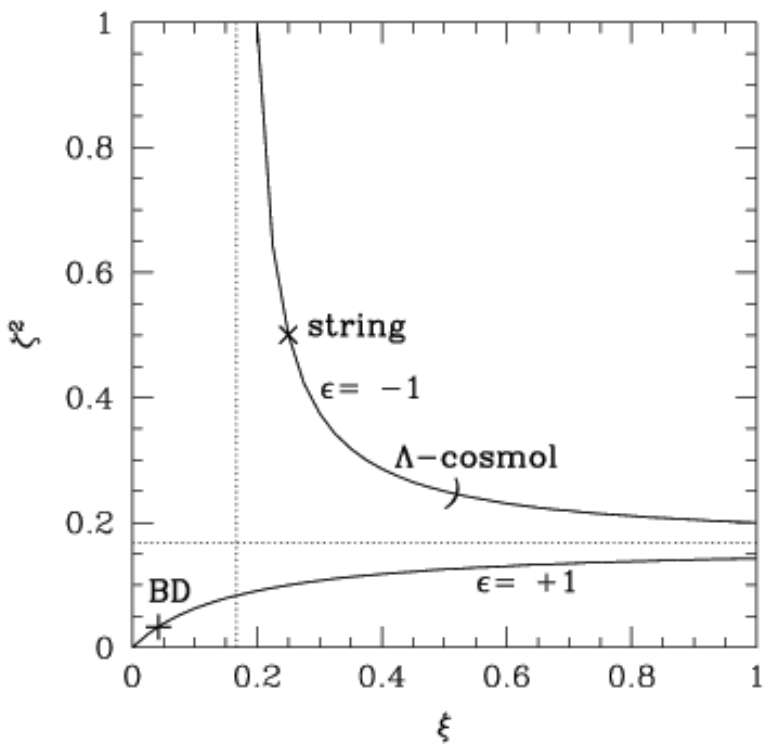
Is STT dying? But isn't σ massive? $\zeta^2 \to \zeta^2 e^{-r/\lambda}$

No immunity against acquiring a self-mass

$$\mu^2 \sim Gm^4 \sim (10^{-9}\mathrm{eV})^2 \Rightarrow \lambda = \mu^{-1} \sim 100\mathrm{m} \ll R_\odot \sim 10^9\mathrm{m}$$

ζ^2 no longer constrained by solar-system exp, saving STT

- **ξ can be larger and more natural**
- **ζ^2 can be larger than $1/6$, thus $\epsilon = -1$**



2 branches of

$$\zeta^2 = \left(6 + \epsilon\xi^{-1}\right)^{-1} > 0$$

If $\epsilon = -1$, $\xi > 1/6$

$\Rightarrow$ no GR limit $(\zeta^2 \to 0)$

No Least Coupling Principle

T.Damour &A.Polyakov, Nucl.Phys, B423(94)532

3 Lambda cosmology BD+Λ

A.D. Dolgov, In *The very early universe*, Cambridge U Press, 1982

$$\mathcal{L}_{\rm BD\Lambda} = \sqrt{-g}\left(\frac{1}{2}\xi\phi^2 R - \frac{1}{2}\epsilon g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \Lambda + L_{\rm matter}\right), \qquad \Lambda > 0$$

Conformal transformation (Weyl rescaling, Local change of units)

$$g_{\mu\nu} \to g_{*\mu\nu} = \Omega^2(x) g_{\mu\nu}, \qquad \textbf{or} \qquad ds^2 \to ds_*^2 = \Omega^2(x) ds^2$$

Then by choosing $\Omega = \xi^{1/2}\phi$

$$\mathcal{L}_{\rm BD\Lambda} = \sqrt{-g_*}\left(\frac{1}{2}R_* - \frac{1}{2}\epsilon_* g_*^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \underbrace{V(\sigma)}_{= \Omega^{-4}\Lambda = \Lambda e^{-4\zeta\sigma}} + L_{*\rm matter}\right), \quad \epsilon_* = {\rm Sign}(\zeta^2)$$

Moved to Einstein CF from Jordan CF

CF	J	E	$\Omega = \xi^{1/2}\phi = e^{\zeta\sigma}$
gravitational constant G	varies	constant	$G_* = G\Omega^2$
particle mass m	constant	varies	$m_* = m\Omega^{-1} \sim t_*^{-1/2}$
canonical scalar field	ϕ	σ	$\phi = \xi^{-1/2}e^{\zeta\sigma}$
potential	none	exponential	$V(\sigma) = \Lambda e^{-4\zeta\sigma}$
cosmic time	t	t_*	$t_* = t^2$
scale factor	a, const	a_*, expanding	$a_* = a\Omega \sim t_*^{1/2}$

- Prefer ECF as physical CF because of expanding a_*
- Call JCF theoretical CF (string CF) with large, constant Λ

In ECF ($k=0$ FRW, spatially uniform $\phi(t)$)

$3H_*^2 = \rho_\sigma + \rho_*$ where $\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma) = \Lambda_{\rm eff}$ = dark energy

$\Lambda_{\rm eff} = \rho_\sigma = \frac{3}{16}\zeta^{-2}t_*^{-2} \sim (10^{60})^{-2} \sim 10^{-120}$

Scenario of decaying cosmological constant

today's Λ is small because our universe is old

not due to fine-tuning

Y.F., PRD10(82)2580

$\rho_* = \frac{3}{4}\left(1 - \frac{1}{4}\zeta^{-2}\right)t_*^{-2} > 0 \quad \Rightarrow \quad \zeta^2 > \frac{1}{4} \quad \Rightarrow \quad \epsilon = -1$

Finite force-range inevitable?

Scaling behavior $\Rightarrow$ no crossing ($\Omega_\Lambda < 0.5$), no acceleration

$\Rightarrow$ Tracking behavior

4 Two issues

4.1 To use atomic clocks

Too much time-dependence $m_* \sim t_*^{-1/2}$ entailing

- Jeopardizes success on primoridial nucleosynthesis
- $a_* \sim t_*^{1/2}$ also for dust-dominance
- Inconsistency with the use of atomic clocks

The time standard of atomic clocks

$$\Delta E \sim m\alpha^2 \sim m$$

Change of its own standard never detected $\Rightarrow$ m is const
$\Rightarrow$ Using atomic clock implies living in CF with m const

Time depndence of m_* is rooted in $m =$ const in BD model in JCF; No ϕ in $L_{\rm matter}$. Revise this

dimensionless coupling const

$$L_\psi = -\sqrt{-g}\bar{\psi}\left(\partial\!\!\!/ + m\right)\psi \quad \Rightarrow \quad L'_\psi = -\sqrt{-g}\bar{\psi}\left(\partial\!\!\!/ + f\phi\right)\psi$$

$$\left(L'_\psi\right)_{\rm ECF} = -\sqrt{-g_*}\bar{\psi}_*\left(\partial\!\!\!/_* + f\xi^{-1/2}\right)\psi_*, \qquad \begin{cases} m_* = f\xi^{-1/2} = \text{const} \\ \psi_* = \Omega^{-3/2}\psi \end{cases}$$

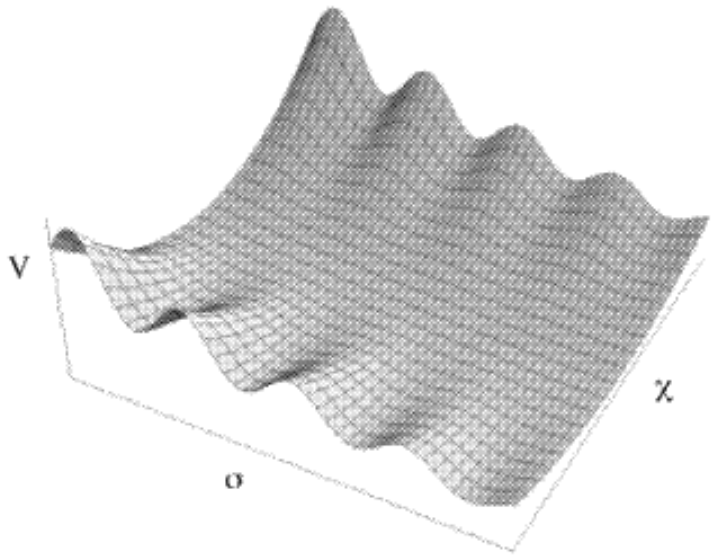
Scale-invariant model in JCF, a departure from BD model, against BD's premise $\Rightarrow$ WEP violation though not immediate

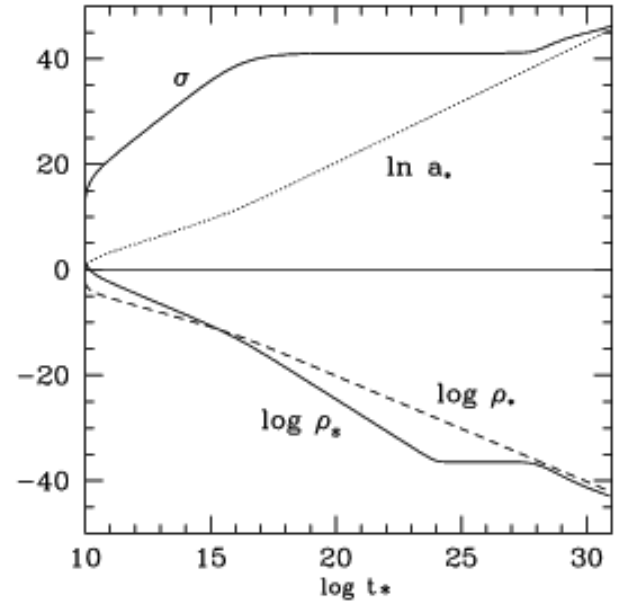
4.2 To accelerate the universe

To overcome scaling behavior, introduce another scalar field χ in ECF, at a phenomenological level

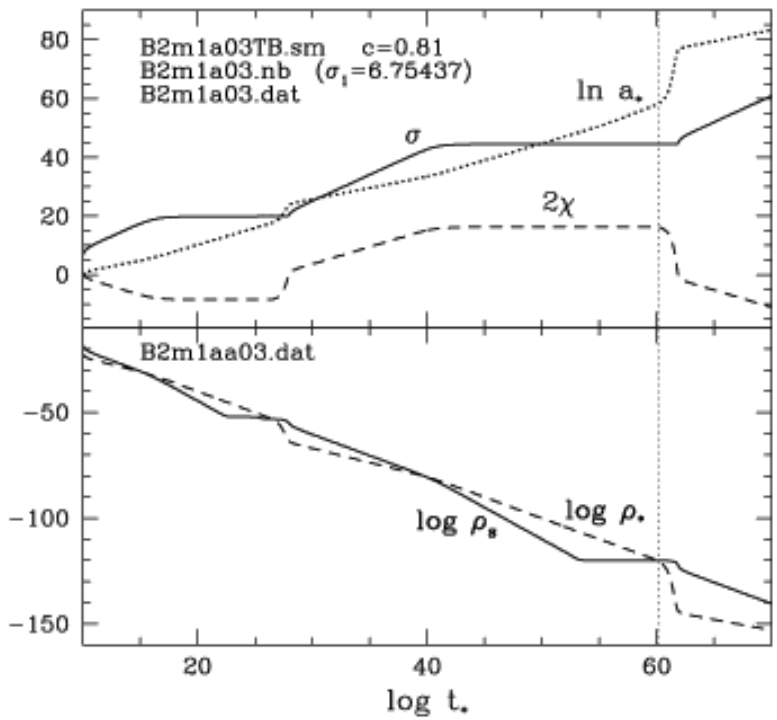
$$V(\sigma,\chi) = e^{-4\zeta\sigma}\tilde{V} = e^{-4\zeta\sigma}\left\{\Lambda + \frac{1}{2}m_\chi^2\chi^2\left[1+\gamma\sin(\kappa\sigma)\right]\right\}, \qquad |\gamma| < 1.$$

Y.F. & T.Nishioka, PLB 254(91)347, Y.F.,Astropart.Phys.5(96)133





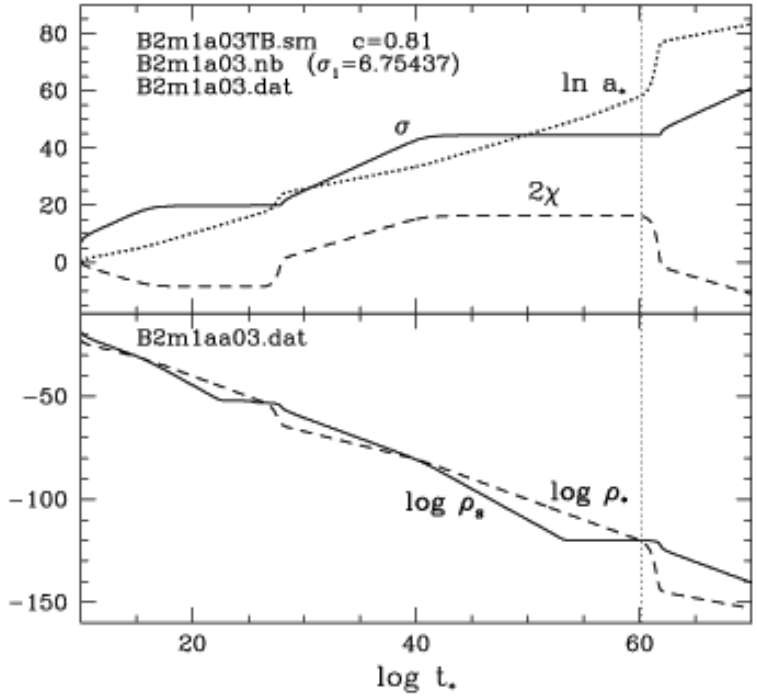
Plateau of ρ_σ **bent downward** avoiding **crossing**



Aided by a trapping barrier due to χ,

$$\rho_s = \Lambda_{\rm eff} = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\dot{\chi}^2 + V(\sigma,\chi)$$

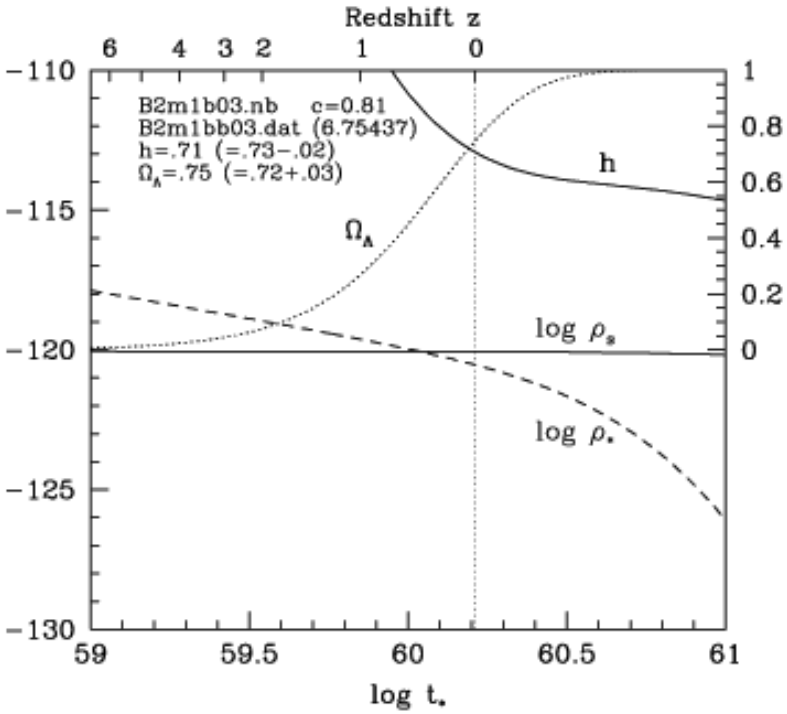
thrusts into ρ_*, crossing, $\Omega_\Lambda = 0.5$. Then σ goes down χ slope diminishing barrier to be released again

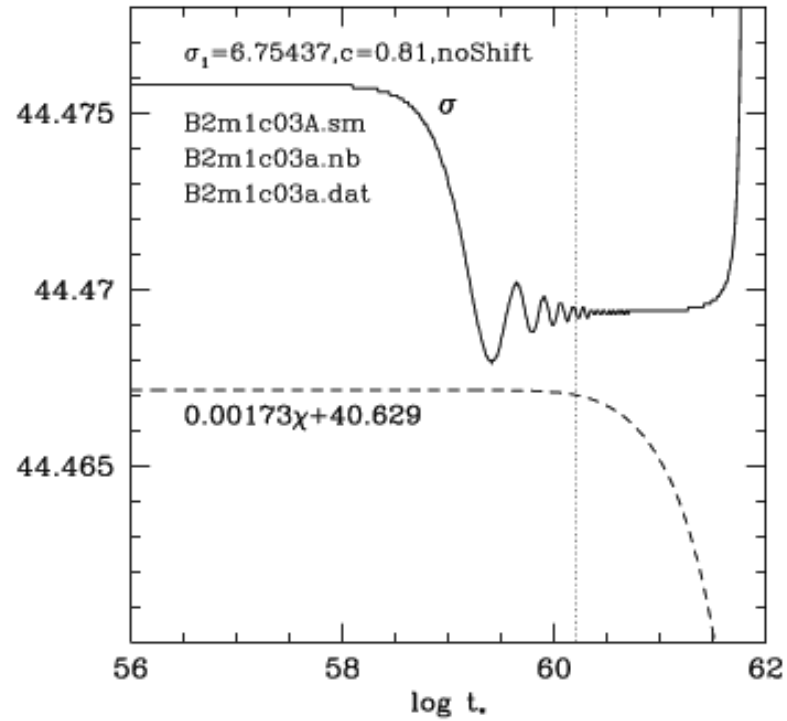


- **Repeated** occurrence of mini-inflations $\Rightarrow$ lessens gravity of **coincidence problem**
- **Interlacing** of ρ_s, ρ_* $\Rightarrow$ overall scenario of decaying Λ, solving **fine-tuning problem**

- Occasional **deviations** off the overall behavior $\Rightarrow$ Mini-inflation can occur at **any** time t_{*i}, always $\Lambda \sim t_{*i}^{-2}$, today is **not special** $\Rightarrow$ further weaken **coincidence problem**
- Can you find any trace of the far-past mini-inflation?
- ... ...

Fine-tuning to a **reasonable** extent





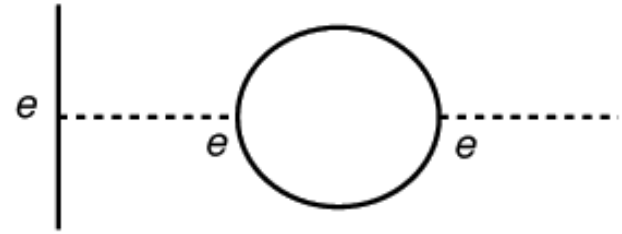
In the left plot, magnified around the present time, showing the fit to $h = 0.73 \pm 0.03, \Omega_\Lambda = 0.72 \pm 0.03$.

In the right plot, the vertical scale has been magnified nearly 10,000 times, showing how σ nearly at rest has been trapped oscillating, and then going to be released as the barrier due to χ disappears.

5 Time-dependent fine-structure constant

$$\Pi_{\mu\nu}(k) = \left(k_\mu k_\mu - \eta_{\mu\nu} k^2\right) C(k^2)$$

$$\delta e = e\frac{1}{2}C(0)$$



Missing the factor 1/2 in (6.182 -) in our book.

Divergences $\Rightarrow$ dimensional regularization in D dimensions, with D kept off 4 until the end of calculation

$$I_{mn} = \int d^D p \frac{(p^2)^{m-2}}{(p^2+\mathcal{M}^2)^n} = iV_D\left(\mathcal{M}^2\right)^{m-n+d-2}$$
$$\times \frac{\Gamma(m+d-2)\Gamma(n-m-d+2)}{2\Gamma(n)}, \quad d = \frac{D}{2}$$

divergence is represented by a pole $\Gamma(2-d) \sim (2-d)^{-1}$

$$C(k^2) = -\frac{\alpha}{12\pi}\Gamma(2-d)$$

$$(\mathcal{L}_{\rm mx})_{\rm 4dim} = -\frac{1}{4}\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\mu}F_{\rho\sigma}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$g_{\mu\nu} \to g_{*\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega = \xi^{1/2}\phi = e^{\zeta\sigma}$$

$$\sqrt{-g}g^{\mu\rho}g^{\nu\sigma} = (\sqrt{-g*}\Omega^{-4})(g_*^{\mu\rho}\Omega^2)(g_*^{\nu\sigma}\Omega^2) = \sqrt{-g_*}g_*^{\mu\rho}g_*^{\nu\sigma}$$

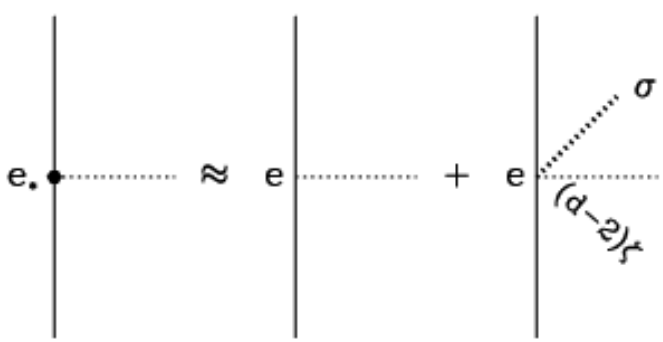
$$\Rightarrow \qquad A_{*\mu} = A_\mu$$

In D **dim,** $\sqrt{-g} = \sqrt{-g_*}\Omega^{-D}, \qquad A_{*\mu} = \Omega^{2-d}A_\mu$

From $eA_\mu = e_*A_{*\mu}$

$$e_* = \Omega^{d-2}e = e\exp\Big[(d-2)\zeta\sigma\Big] \approx e\Big[1 + (d-2)\zeta\sigma\Big]$$

In perturbation with respect to e_* **in ECF, we have**



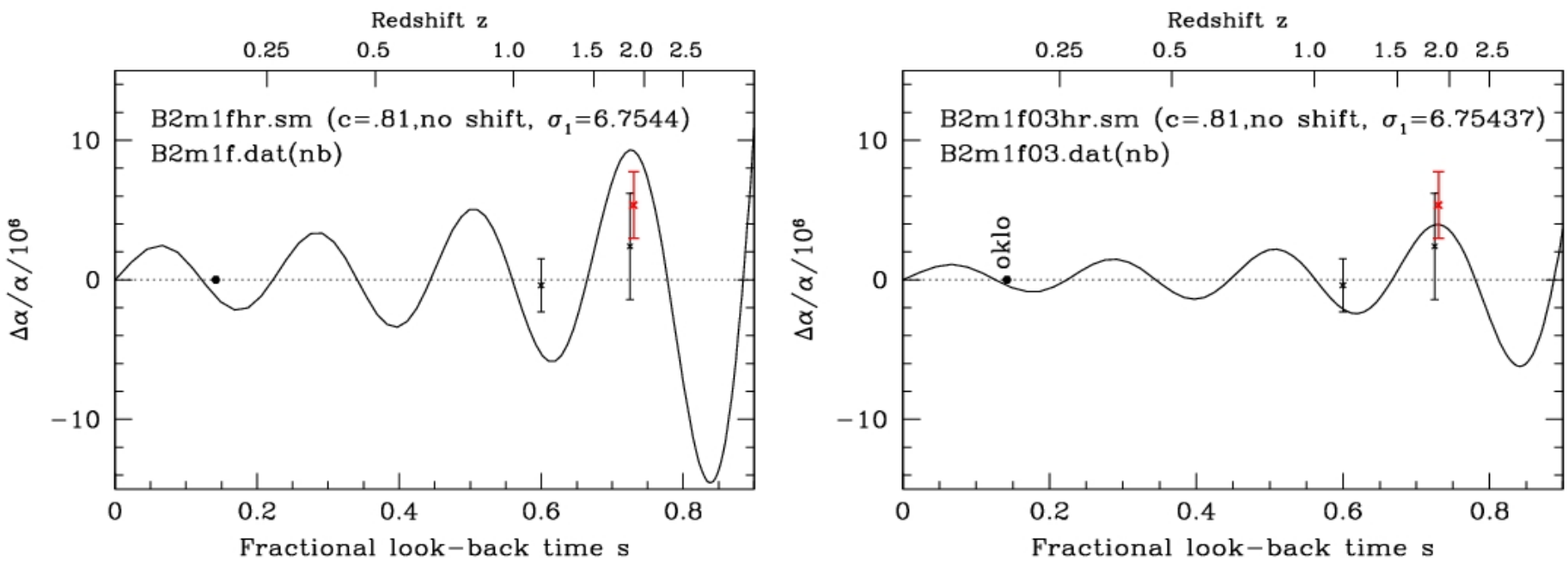
$(2-d)\Gamma(2-d) \approx 1$ **for** $d \approx 2$

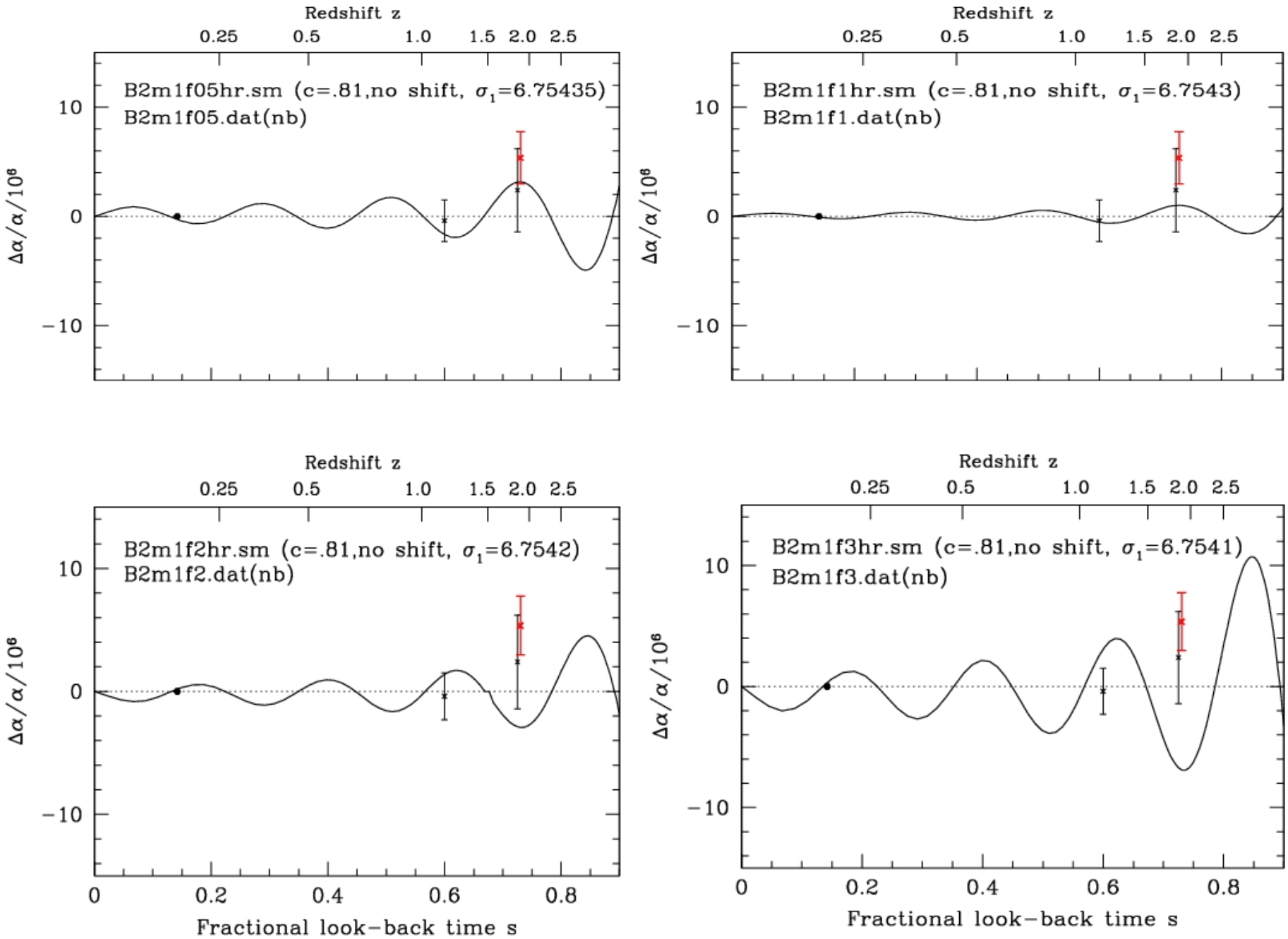
$$\Rightarrow \frac{\Delta\alpha_*}{\alpha_*} = \mathcal{Z}\frac{\alpha_*}{4\pi}\zeta\Delta\sigma, \qquad \mathcal{Z}\frac{\alpha_*}{4\pi}\zeta = 5\times 0.919\times 10^{-3} = 4.595\times 10^{-3}$$

Nonzero finite Quantum anomaly, violating results from classical invariance

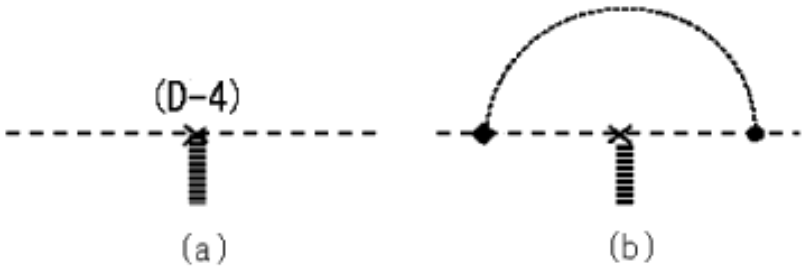
Compare with QSO observations

S.A.Levshakov, P.Molaro, et al, astro-ph/0611803




Redshift z
0.25
0.5
1.0
1.5
2.0
2.5
B2m1f05hr.sm (c=.81,no shift, σ_1=6.75435)
B2m1f05.dat(nb)
B2m1f1hr.sm (c=.81,no shift, σ_1=6.7543)
B2m1f1.dat(nb)
B2m1f2hr.sm (c=.81,no shift, σ_1=6.7542)
B2m1f2.dat(nb)
B2m1f3hr.sm (c=.81,no shift, σ_1=6.7541)
B2m1f3.dat(nb)
$\Delta\alpha/\alpha/10^6$
10
0
−10
0
0.2
0.4
0.6
0.8
Fractional look−back time s

Loop correction $\sim (D-4)^{-1}$
$(D-4) \times (D-4)^{-1} \Rightarrow$ **nonzero finite Quantum anomaly**

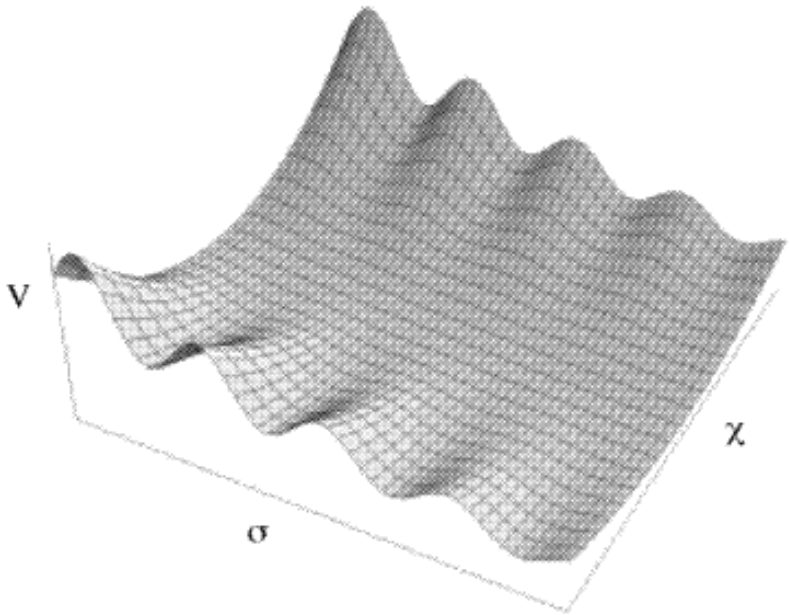


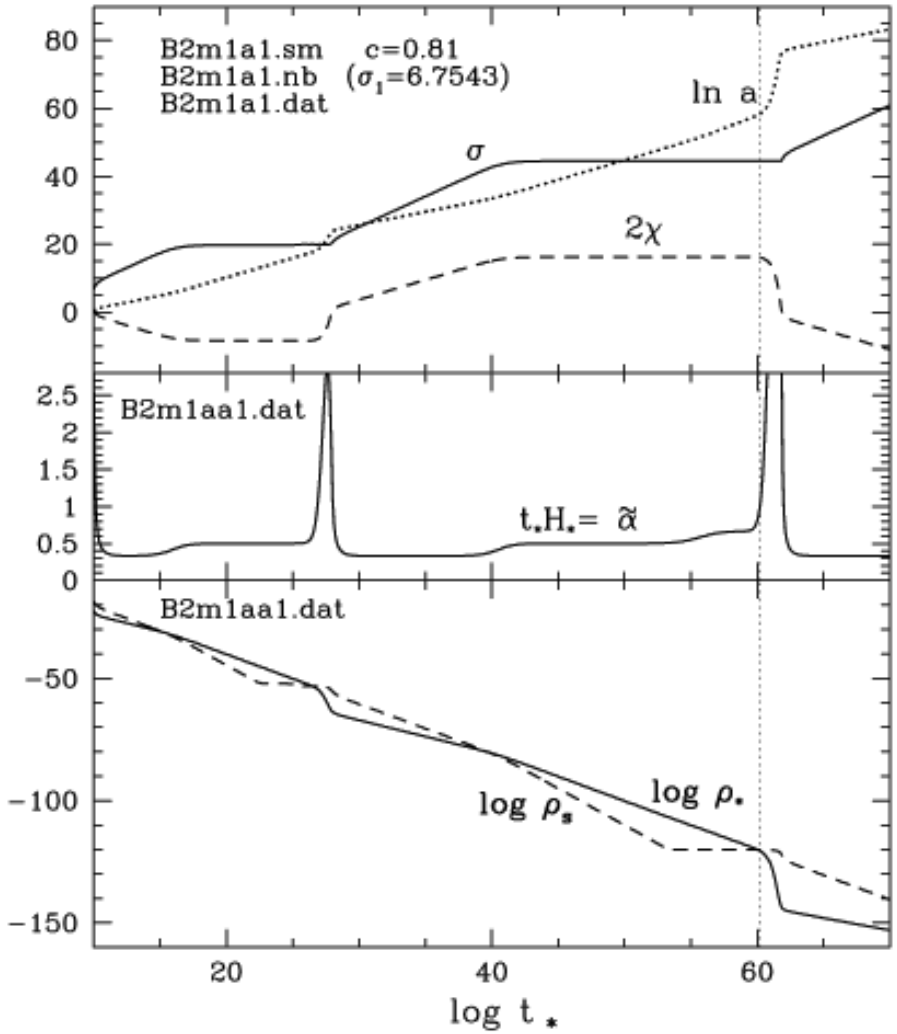
Time-dependence of m_* again $\Rightarrow$ another conf TR reaching PhysicalCF, though small difference from ECF

4.2 No acceleration for $a_* \sim t_*^{1/2}$

Also scaling behavior to be replaced by tracking behavior (nearly constant ρ_σ)
Decided to leave the simplest version of STT, by introducing another scalar field χ – 2 scalar model – with the potential in PhCR $(m, \gamma, \kappa \sim 1)$

$$V(\sigma, \chi) = e^{-4\zeta\sigma}\tilde{V} = e^{-4\zeta\sigma}\left\{\Lambda + \frac{1}{2}m^2\chi^2\left[1 + \gamma\sin(\kappa\sigma)\right]\right\}$$

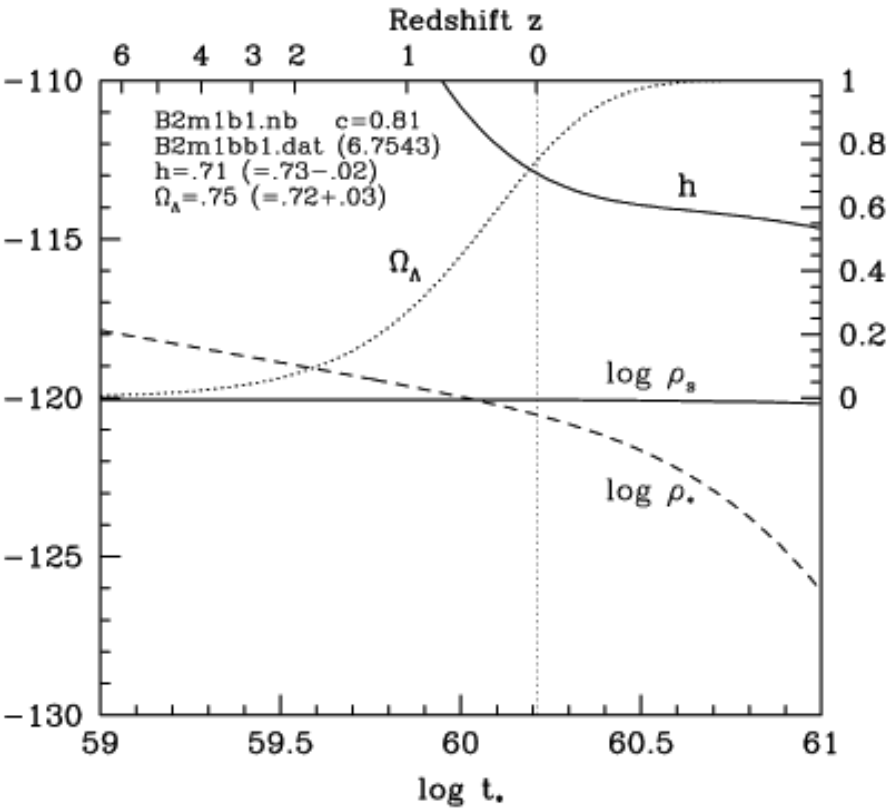


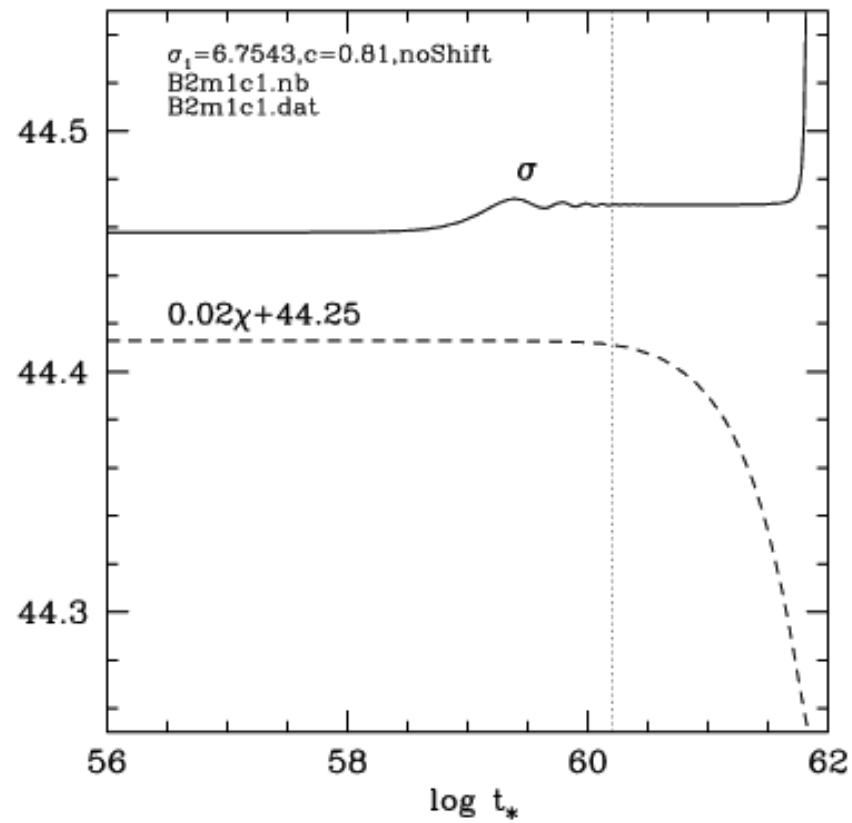


$$V(\sigma,\chi) = e^{-4\zeta\sigma} \times \left(\Lambda + \frac{1}{2}m^2\chi^2\left[1+\gamma\sin(\kappa\sigma)\right]\right)$$

$$\rho_s = \frac{1}{2}\dot{\sigma}^2 + \frac{1}{2}\dot{\chi}^2 + V(\sigma,\chi)$$

Y.F. & T.Nishioka, PLB254(91)347; Y.F., Astropart.Phys 5(96)133


Redshift z
6
4
3
2
1
0
B2m1b1.nb c=0.81
B2m1bb1.dat (6.7543)
h=.71 (=.73−.02)
Ω_Λ=.75 (=.72+.03)
h
Ω_Λ
log ρ_s
log ρ_*
−110
−115
−120
−125
−130
1
0.8
0.6
0.4
0.2
0
59
59.5
60
60.5
61
log t_*


σ_t=6.7543,c=0.81,noShift
B2m1c1.nb
B2m1c1.dat
σ
0.02χ+44.25
44.5
44.4
44.3
56
58
60
62
log t_*