Scalar-tensor theory, the accelerating universe and the time-dependent fine-structure constant, I

Yasunori Fujii

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3 Lambda cosmology
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1 Introduction

Non-Newtonian gravity, $\lambda \sim 100m$
Y.F., 1971

Scalar-tensor theory
P. Jordan, 1955;
C. Brans & R. Dicke, 1961

Accelerating Universe
quantum anomaly
$\dot{\alpha}/\alpha$

Today’s version of the cosmological constant problem

- **Fine-tuning problem:** $\Lambda_{\text{obs}}/\Lambda_{\text{th}} \sim 10^{-120}$ ($\Lambda_{\text{th}} \sim M_P^4$)
- **Coincidence problem:** Is today special because the acceleration (mini-inflation) occurs only once during the whole history of the universe, if $\Lambda = \text{const}$?

2 Scalar-tensor theory

Nonminimal coupling term (NM)

\[ \mathcal{L}_{\text{STT}} = \sqrt{-g} \left( \frac{1}{2} \xi \phi^2 R - \epsilon \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi + L_{\text{matter}} \right) \]

\[ \mathcal{L}_{\text{EH}} = \sqrt{-g} \frac{1}{16\pi G} R \quad \Rightarrow \quad \frac{1}{8\pi G_{\text{eff}}} = \xi \phi^2 \]

\( \xi > 0, \quad \epsilon = \pm 1, \quad 4\omega \xi = \epsilon = \text{Sign}(\omega) \)

\( \zeta^{-2} = 6 + \epsilon \xi^{-1} = 6 + 4\omega > 0 \) for positive energy of diagonalized scalar field \( \Rightarrow \epsilon = -1 \) implies a ghost \( \phi \), but not immediately a trouble

\( \phi \) \{ coupled \} to matter for \{ J theory \}

\{ decoupled \} \{ BD model \} \Rightarrow WEP \{ violated \}

No mass term assumed \( \sim \phi^2 \)

Reduced Planckian units \( c = \hbar = M_P \left( = (8\pi G)^{-1/2} \right) = 1 \)

\( 8.07 \times 10^{-33} \text{cm}, 2.71 \times 10^{-43} \text{sec}, 2.44 \times 10^{18} \text{GeV} (t_0 = 13.7 \text{Gy} \approx 10^{60.21}) \)
PPN approximation for solar-system experiment

$$g_{rr} \approx 1 + \gamma \frac{2m^\odot}{r}, \quad \gamma = 1 - 4\zeta^2, \quad \zeta^{-2} = 6 + \epsilon\zeta^{-1} = 6 + 4\omega > 0$$

<table>
<thead>
<tr>
<th>Source</th>
<th>$\zeta^2 \lesssim$</th>
<th>$\xi \lesssim$</th>
<th>$\omega \gtrsim$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD (61)</td>
<td>0.03</td>
<td>0.04</td>
<td>6</td>
</tr>
<tr>
<td>Viking (79)</td>
<td>$2.5 \times 10^{-4}$</td>
<td>$2.5 \times 10^{-4}$</td>
<td>1,000</td>
</tr>
<tr>
<td>VLBI (99)</td>
<td>$7 \times 10^{-5}$</td>
<td>$7 \times 10^{-5}$</td>
<td>3,600</td>
</tr>
<tr>
<td>Cassini (03)</td>
<td>$.5 \times 10^{-6}$</td>
<td>$.5 \times 10^{-6}$</td>
<td>500,000</td>
</tr>
</tbody>
</table>

unnaturally too small!? too large!?

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \quad \text{from Cassini}$$

B. Bertotti et al, Nat, 425(03)374

Is STT dying? But isn’t $\sigma$ massive? $\zeta^2 \rightarrow \zeta^2 e^{-r/\lambda}$

No immunity against acquiring a self-mass
\[ \mu^2 \sim Gm^4 \sim (10^{-9}\text{eV})^2 \Rightarrow \lambda = \mu^{-1} \sim 100\text{m} \ll R_\odot \sim 10^9\text{m} \]

\(\zeta^2\) no longer constrained by solar-system exp, saving STT

- \(\xi\) can be larger and more natural
- \(\zeta^2\) can be larger than 1/6, thus \(\epsilon = -1\)

2 branches of
\[ \zeta^2 = \left(6 + \epsilon \xi^{-1}\right)^{-1} > 0 \]

If \(\epsilon = -1\), \(\xi > 1/6\)

\(\Rightarrow\) no GR limit \((\zeta^2 \to 0)\)

No Least Coupling Principle

T. Damour & A. Polyakov, Nucl. Phys, B423(94)532
3 Lambda cosmology

A.D. Dolgov, In *The very early universe*, Cambridge U Press, 1982

\[ \mathcal{L}_{\text{BD}+\Lambda} = \sqrt{-g} \left( \frac{1}{2} \xi \phi^2 R - \frac{1}{2} \epsilon g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - \Lambda + L_{\text{matter}} \right), \quad \Lambda > 0 \]

Conformal transformation (Weyl rescaling, Local change of units)

\[ g_{\mu\nu} \to g_{*\mu\nu} = \Omega^2(x)g_{\mu\nu}, \quad \text{or} \quad ds^2 \to ds_*^2 = \Omega^2(x)ds^2 \]

Then by choosing \( \Omega = \xi^{1/2} \phi \uparrow = \Omega^{-4} \Lambda = \Lambda e^{-4\zeta \sigma} \)

\[ \mathcal{L}_{\text{BD}+\Lambda} = \sqrt{-g_*} \left( \frac{1}{2} R_* - \frac{1}{2} \epsilon_* g_*^{\mu\nu} \partial_{\mu} \sigma \partial_{\nu} \sigma - V(\sigma) + L_{*\text{matter}} \right), \quad \epsilon_* = \text{Sign}(\zeta^2) \]

Moved to Einstein CF from Jordan CF

<table>
<thead>
<tr>
<th>CF</th>
<th>J</th>
<th>E</th>
<th>( \Omega = \xi^{1/2} \phi = e^{\zeta \sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravitational constant ( G )</td>
<td>varies</td>
<td>constant</td>
<td>( G_* = G \Omega^2 )</td>
</tr>
<tr>
<td>particle mass ( m )</td>
<td>constant</td>
<td>varies</td>
<td>( m_* = m \Omega^{-1} \sim t_*^{-1/2} )</td>
</tr>
<tr>
<td>canonical scalar field ( \phi )</td>
<td>none</td>
<td>exponential</td>
<td>( \phi = \xi^{-1/2} e^{\zeta \sigma} )</td>
</tr>
<tr>
<td>potential</td>
<td>exponential</td>
<td>( V(\sigma) = \Lambda e^{-4\zeta \sigma} )</td>
<td></td>
</tr>
<tr>
<td>cosmic time</td>
<td>( t )</td>
<td>( t_* )</td>
<td>( t_* = t^2 )</td>
</tr>
<tr>
<td>scale factor</td>
<td>( a, \text{ const} )</td>
<td>( a_*, \text{ expanding} )</td>
<td>( a_* = a \Omega \sim t_*^{1/2} )</td>
</tr>
</tbody>
</table>
• Prefer ECF as physical CF because of expanding $a_*$

• Call JCF theoretical CF (string CF) with large, constant $\Lambda$

In ECF ($k = 0$ FRW, spatially uniform $\phi(t)$)

$$3H_*^2 = \rho\sigma + \rho_* \quad \text{where} \quad \rho\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma) = \Lambda_{\text{eff}} = \text{dark energy}$$

\[
\begin{align*}
\Lambda_{\text{eff}} &= \rho\sigma = \frac{3}{16} \zeta^{-2} t_*^{-2} \sim (10^{60})^{-2} \sim 10^{-120} \\
\text{Scenario of decaying cosmological constant} \\
\text{today's } \Lambda \text{ is small because our universe is old} \\
\text{not due to fine-tuning}
\end{align*}
\]

Y.F., PRD10(82)2580

\[
\begin{align*}
\rho_* &= \frac{3}{4} \left(1 - \frac{1}{4} \zeta^{-2}\right) t_*^{-2} > 0 \quad \Rightarrow \quad \zeta^2 > \frac{1}{4} \quad \Rightarrow \quad \epsilon = -1 \\
\text{Finite force-range inevitable?}
\end{align*}
\]

Scaling behavior $\Rightarrow$ no crossing ($\Omega_\Lambda < 0.5$), no acceleration $\Rightarrow$ Tracking behavior
4 Two issues

4.1 To use atomic clocks

Too much time-dependence \( m_* \sim t_*^{-1/2} \) entailing

- Jeopardizes success on primordial nucleosynthesis
- \( a_* \sim t_*^{1/2} \) also for dust-dominance
- Inconsistency with the use of atomic clocks

The time standard of atomic clocks

\[
\Delta E \sim m\alpha^2 \sim m
\]

Change of its own standard never detected \( \Rightarrow m \) is const
\( \Rightarrow \) Using atomic clock implies living in CF with \( m \) const

Time dependence of \( m_* \) is rooted in \( m = \text{const in BD model} \)
in JCF; No \( \phi \) in \( L_{\text{matter}} \). Revise this
\[ L_\psi = -\sqrt{-g\bar{\psi}} (\bar{\phi} + m) \psi \quad \Rightarrow \quad L'_\psi = -\sqrt{-g\bar{\psi}} (\bar{\phi} + f \phi) \psi \]

\[ (L'_\psi)_{\text{ECF}} = -\sqrt{-g_\ast \bar{\psi}_\ast} (\bar{\phi}_\ast + f \xi^{-1/2}) \psi_\ast, \quad \left\{ \begin{array}{l}
m_\ast = f \xi^{-1/2} = \text{const} \\
\psi_\ast = \Omega^{-3/2} \psi
\end{array} \right. \]

Scale-invariant model in JCF, a departure from BD model, against BD’s premise \( \Rightarrow \) WEP violation though not immediate

\[ 4.2 \quad \text{To accelerate the universe} \]

To overcome scaling behavior, introduce another scalar field \( \chi \) in ECF, at a phenomenological level

\[ V(\sigma, \chi) = e^{-4\zeta \sigma} \tilde{V} = e^{-4\zeta \sigma} \left\{ \Lambda + \frac{1}{2} m_\chi^2 \chi^2 [1 + \gamma \sin(\kappa \sigma)] \right\}, \quad |\gamma| < 1. \]

Y.F. & T.Nishioka, PLB 254(91)347, Y.F.,Astropart.Phys.5(96)133
Plateau of $\rho_\sigma$ bent downward avoiding crossing

Aided by a trapping barrier due to $\chi$,

$$\rho_s = \Lambda_{\text{eff}} = \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} \dot{\chi}^2 + V(\sigma, \chi)$$

thrusts into $\rho_*$, crossing, $\Omega_\Lambda = 0.5$. Then $\sigma$ goes down $\chi$ slope diminishing barrier to be released again
- Repeated occurrence of mini-inflations ⇒ lessens gravity of coincidence problem
- Interlacing of $\rho_s, \rho_*$ ⇒ overall scenario of decaying $\Lambda$, solving fine-tuning problem

- Occasional deviations off the overall behavior ⇒ Mini-inflation can occur at any time $t_{*i}$, always $\Lambda \sim t_{*i}^{-2}$, today is not special ⇒ further weaken coincidence problem
- Can you find any trace of the far-past mini-inflation?

Fine-tuning to a reasonable extent
In the left plot, magnified around the present time, showing the fit to $h = 0.73 \pm 0.03, \Omega_\Lambda = 0.72 \pm 0.03$.

In the right plot, the vertical scale has been magnified nearly 10,000 times, showing how $\sigma$ nearly at rest has been trapped oscillating, and then going to be released as the barrier due to $\chi$ disappears.
5 Time-dependent fine-structure constant

\[ \Pi_{\mu\nu}(k) = (k_\mu k_\mu - \eta_{\mu\nu} k^2) C(k^2) \]
\[ \delta e = e \frac{1}{2} C(0) \]

Missing the factor 1/2 in (6.182 - ) in our book.

Divergences \Rightarrow \textbf{dimensional regularization} in \( D \) dimensions, with \( D \) kept \textbf{off 4 until the end of calculation}

\[ I_{mn} = \int d^D p \frac{(p^2)^{m-2}}{(p^2 + \mathcal{M}^2)^n} = i V_D \left( \mathcal{M}^2 \right)^{m-n+d-2} \]
\[ \times \frac{\Gamma(m+d-2)\Gamma(n-m-d+2)}{2\Gamma(n)}, \quad d = \frac{D}{2} \]

\[ \text{divergence is represented by a pole} \quad \Gamma(2-d) \sim (2-d)^{-1} \]
\[ C(k^2) = -\frac{\alpha}{12\pi} \Gamma(2-d) \]
\[(\mathcal{L}_{\text{mx}})_{4\text{dim}} = -\frac{1}{4} \sqrt{-g} g^{\mu \rho} g^{\nu \sigma} F_{\mu \nu} F_{\rho \sigma}, \quad F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]
\[g_{\mu \nu} \rightarrow g_{\ast \mu \nu} = \Omega^2 g_{\mu \nu}, \quad \Omega = \xi^{1/2} \phi = e^{\zeta \sigma} \]
\[\sqrt{-g} g^{\mu \rho} g^{\nu \sigma} = (\sqrt{-g_{\ast} \Omega^{-4}})(g_{\ast}^{\mu \rho} \Omega^2)(g_{\ast}^{\nu \sigma} \Omega^2) = \sqrt{-g_{\ast} g_{\ast}^{\mu \rho} g_{\ast}^{\nu \sigma}} \]
\[\Rightarrow \quad A_{\ast \mu} = A_\mu \]

**In D dim**, \[\sqrt{-g} = \sqrt{-g_{\ast} \Omega^{-D}}, \quad A_{\ast \mu} = \Omega^{2-d} A_\mu \]

From \[e A_\mu = e_\ast A_{\ast \mu} \]
\[e_\ast = \Omega^{d-2} e = e \exp \left[ (d - 2) \xi \sigma \right] \approx e \left[ 1 + (d - 2) \xi \sigma \right] \]

In perturbation with respect to \[e_\ast\] in ECF, we have

\[e_\ast \approx e + e \left(2 - d\right) \Gamma(2 - d) \approx 1 \text{ for } d \approx 2\]
\[ \frac{\Delta \alpha_*}{\alpha_*} = \mathcal{Z} \frac{\alpha_* \zeta}{4\pi} \Delta \sigma, \quad \mathcal{Z} \frac{\alpha_* \zeta}{4\pi} = 5 \times 0.919 \times 10^{-3} = 4.595 \times 10^{-3} \]

Nonzero finite Quantum anomaly, violating results from classical invariance

Compare with QSO observations

B2m1f05hr.sm (c=.81, no shift, $\sigma_1=6.75435$)
B2m1f05.dat(nb)

B2m1f1hr.sm (c=.81, no shift, $\sigma_1=6.7543$)
B2m1f1.dat(nb)

B2m1f2hr.sm (c=.81, no shift, $\sigma_1=6.7542$)
B2m1f2.dat(nb)

B2m1f3hr.sm (c=.81, no shift, $\sigma_1=6.7541$)
B2m1f3.dat(nb)
Loop correction $\sim (D - 4)^{-1}$

$(D - 4) \times (D - 4)^{-1} \Rightarrow$ nonzero finite Quantum anomaly

Time-dependence of $m_*$ again $\Rightarrow$ another conf TR reaching Physical CF, though small difference from ECF

4.2 No acceleration for $a_* \sim t_*^{1/2}$

Also scaling behavior to be replaced by tracking behavior (nearly constant $\rho_\sigma$)

Decided to leave the simplest version of STT, by introducing another scalar field $\chi - 2$ scalar model – with the potential in PhCR $(m, \gamma, \kappa \sim 1)$

$$V(\sigma, \chi) = e^{-4\zeta \sigma} \tilde{V} = e^{-4\zeta \sigma} \left\{ \Lambda + \frac{1}{2} m^2 \chi^2 [1 + \gamma \sin(\kappa \sigma)] \right\}$$
\[ V(\sigma, \chi) = e^{-4\zeta\sigma} \times \left( \Lambda + \frac{1}{2} m^2 \chi^2 [1 + \gamma \sin(\kappa \sigma)] \right) \]
\[ \rho_s = \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} \dot{\chi}^2 + V(\sigma, \chi) \]
