MATTER-ANTIMATTER ASYMMETRY
In the Standard Model and Beyond

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Matter ⇔ Antimatter
Asymmetry
In the Standard Model and Beyond

- Antimatter in the Univers and $\mathbb{CP}$
- $\mathbb{CP}$, masses and weak couplings
- $\mathbb{CP}$ for Kaon and B mesons in the SM and beyond
- Conclusions and outlook
CP Violation was discovered about 37 years ago in $K^0 - \bar{K}^0$ mixing (weak interactions)
If not for C (Charge conjugation) and CP (C & Parity) violation, fundamental phenomena would be the same for matter & antimatter, thus we should have a universe filled with antimatter. Since antimatter annihilates matter producing an enormous quantity of energy, for example high energy photons, a diffused and massive presence of antimatter would have been already detected instead.

ALL ANTIMATTER PRODUCED IN OUR LABORATORIES DOES NOT EXCEED 10^{-12} GRAMS!!!
The second step of Armstrong on the moon shows that antimatter is negligible on planetary scales.

Antimatter from cosmic rays is about $1/10^5$ of matter.
THE ABSENCE OF VISIBLE EXPLOSIONS IN THE UNIVERSE EXCLUDES THE PRESENCE OF ANTIMATTER UP TO DISTANCES OF $O(20$ MEGAPARSECS) (ONE PARSEC $\sim 3.26$ LIGHT YEARS $\sim 3.1 \times 10^{18}$ cm)

$$\beta = \frac{N_B - \bar{N}_B}{N_\gamma} = 6 \times 10^{-10}$$

$N_\gamma = 412 \text{ /cm}^3$
WHEN AND WHY ANTIMATTER DISAPPEARED?
In 1967 Andrei Sakharov pointed out that, for the universe to evolve from the initial matter-antimatter fireball to the present matter antimatter asymmetric state, 4 conditions must be fulfilled:

1) Baryon number violation \( \Delta B \neq 0 \) (GUT ??)

\[
e^+ + d \rightarrow X \rightarrow u + u \quad (\Delta (B-L) = 0)
\]

Lepton number violation is possible but not necessary and could be zero because of the presence of a large number of antineutrinos

2) Charge symmetry violation \( \mathcal{C} \)

\[
\Gamma( e^+ + d \rightarrow X \rightarrow u + u ) \neq \Gamma( e^- + d \rightarrow X \rightarrow \bar{u} + \bar{u} )
\]

3) \( \mathcal{C}P \) violation: the number of left handed up quarks produced by \( X \) must be different from the number of right handed up antiquarks

4) The universe was not in equilibrium when this happened, otherwise if

\[
\Gamma( e^+ + d \rightarrow u + u ) > \Gamma( e^- + d \rightarrow \bar{u} + \bar{u} )
\]
then also

\[
\Gamma( u + u \rightarrow e^+ + d ) > \Gamma( \bar{u} + \bar{u} \rightarrow e^- + d )
\]
The amount of $CP$, discovered in 1964 in mixing (see below) is however too small to explain the scarcity of antimatter in the universe.
CP Violation in the Standard Model
\[ \mathcal{L}_{\text{quarks}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{yukawa}} + \mathcal{L}_{\text{weak int}} \]

Mass terms are forbidden by symmetries:

\[ \mathcal{q}_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \]

Hermiticity guarantees CP conservation for \( \mathcal{L}_{\text{weak int}} \):

\[ \mathcal{L}_{\mathcal{EC}}^{\text{weak int}} = \frac{g_W}{\sqrt{2}} \left( J^-_\mu W^+_{\mu} + J^+_{\mu} W^-_{\mu} \right) \]

\[ J^+_{\mu} = \bar{u} \gamma_{\mu} (1 - \gamma_5) d + \ldots \]

(\( u \to c,d \to s \) + (\( u \to t,d \to b \))

\[ \mathcal{C} \quad \bar{u} \gamma_{\mu} d \to - \bar{d} \gamma_{\mu} u \]

\[ \bar{u} \gamma_{\mu} \gamma_5 d \to - \bar{u} \gamma_{\mu} \gamma_5 d \]
In the Standard Model the quark mass matrix, from which the CKM Matrix and CP originate, is determined by the Yukawa Lagrangian which couples fermions and Higgs.

\[ \mathcal{L}_{\text{quarks}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{weak int}} + \mathcal{L}_{\text{yukawa}} \]

CP invariant

CP and symmetry breaking are closely related!
QUARK FAMILIES

1) $q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

$U_R = u_R$
$D_R = d_R$

2) $q_L \equiv \begin{pmatrix} c_L \\ s_L \end{pmatrix}$

$U_R = c_R$
$D_R = s_R$

3) $q_L \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix}$

$U_R = t_R$
$D_R = b_R$
QUARK MASSES ARE GENERATED BY DYNAMICAL SYMMETRY BREAKING

\[
H = \begin{pmatrix}
\phi^+ \\
\phi^0
\end{pmatrix}, \quad H^C = i\pi_2 H^*
\]

\[
\phi^+ \rightarrow 0 \quad \phi^0 \rightarrow \frac{V}{\sqrt{2}}
\]

\[\sqrt{\text{yukawa}} \equiv \sum_{i,k=1,N} \left[ Y_{i,k} (q^i_L H^C) U^k_R \\
+ X_{i,k} (q^i_L H) D^k_R \right] + \text{h.c.} \]

Charge +2/3

\[\sum_{i,k=1,N} \left[ m^u_{i,k} (\bar{u}^i_L u^k_R) \\
+ m^d_{i,k} (\bar{d}^i_L d^k_R) \right] + \text{h.c.} \]

Charge -1/3
It is easy to show the a necessary and sufficient condition for CP invariance is

\[ m_{u,d}^{i,k} = \text{real} \]

1) there is no compelling symmetry for \( m_{u,d}^{i,k} \) to be real
2) in field theory, all that may happen will happen [see below]
3) symmetries and accidental symmetries e.g. separate conservation of lepton and baryon numbers (it follows from gauge symmetry and renormalizability)
Diagonalization of the Mass Matrix

Up to singular cases, the mass matrix can always be diagonalized by 2 unitary transformations

\[ u^i_L \rightarrow U^{ik}_L u^k_L \quad u^i_R \rightarrow U^{ik}_R u^k_R \]

\[ M' = U^*_L M U_R \quad (M')^\dagger = U^*_R (M)^\dagger U_L \]

\[ \mathcal{L}^{\text{mass}} \equiv m_{\text{up}} (\bar{u}_L u_R + \bar{u}_R u_L) + m_{\text{ch}} (\bar{c}_L c_R + \bar{c}_R c_L) \]

\[ + m_{\text{top}} (\bar{t}_L t_R + \bar{t}_R t_L) \]

\[ L^{\text{weak int}}_{\text{CC}} = \frac{g_W}{\sqrt{2}} \left( J^- W^+_{\mu} + h.c. \right) \]

\[ \rightarrow \frac{g_W}{\sqrt{2}} \left( \bar{u}_L V^{CKM} \gamma_\mu d_L W^+_{\mu} + \ldots \right) \]
\[ \frac{N(N-1)}{2} \text{ angles and } \frac{(N-1)(N-2)}{2} \text{ phases} \]

- \(N=3\) 3 angles + 1 phase
- KM
- the phase generates complex couplings i.e. CP violation;
- 6 masses +3 angles +1 phase = 10 parameters

\[
\begin{array}{ccc}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{tb} & V_{ts} & V_{tb}
\end{array}
\]
<table>
<thead>
<tr>
<th>NO Flavour Changing Neutral Currents (FCNC) at Tree Level</th>
</tr>
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<tbody>
<tr>
<td>(FCNC processes are good candidates for observing NEW PHYSICS)</td>
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| CP Violation is natural with three quark generations (Kobayashi-Maskawa) |

| With three generations all CP phenomena are related to the same unique parameter (δ) |
Quark masses &
Generation
Mixing

\[ |V_{ud}| = 0.9735(8) \]
\[ |V_{us}| = 0.2196(23) \]
\[ |V_{cd}| = 0.224(16) \]
\[ |V_{cs}| = 0.970(9)(70) \]
\[ |V_{cb}| = 0.0406(8) \]
\[ |V_{ub}| = 0.00409(25) \]
\[ |V_{tb}| = 0.99(29) \]
(0.999)

\[ \frac{d\Gamma}{dq^2} \propto |V_{ij}|^2 f(q^2)^2 \]
\[c_{ij} = \cos \theta_{ij} \quad s_{ij} = \sin \theta_{ij} \quad c_{ij} \geq 0 \quad s_{ij} \geq 0\]

\[0 \leq \delta \leq 2\pi \quad |s_{12}| \sim \sin \theta_c\]

for small angles \[|s_{ij}| \sim |V_{ij}|\]
The Wolfenstein Parametrization

\[ \lambda \sim 0.2 \quad A \sim 0.8 \]

\[ \eta \sim 0.2 \quad \rho \sim 0.3 \]

\[ \sin \theta_{12} = \lambda \]
\[ \sin \theta_{23} = A \lambda^2 \]
\[ \sin \theta_{13} = A \lambda^3 (\rho - i \eta) \]
The Bjorken-Jarlskog Unitarity Triangle

$|V_{ij}|$ is invariant under phase rotations

\[ a_1 = V_{11} V_{12}^* = V_{ud} V_{us}^* \]
\[ a_2 = V_{21} V_{22}^* \]
\[ a_3 = V_{31} V_{32}^* \]

\[ a_1 + a_2 + a_3 = 0 \]
\[ (b_1 + b_2 + b_3 = 0 \text{ etc.}) \]

Only the orientation depends on the phase convention.
Gluons and quarks

The QCD Lagrangian:

\[ \mathcal{L}_{\text{strong}} = -\frac{1}{4} G^A_{\mu\nu} G_A^{\mu\nu} + \sum_{f=\text{flavour}} \bar{q}_f \left( i \gamma_\mu D_\mu - m_f \right) q_f \]

GLUONS

QUARKS ( & GLUONS)

\[ G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_0 f^{ABC} G^B_\mu G^C_\nu \]

\[ q_f \equiv q^a_f (x) \quad \gamma_\mu \equiv (\gamma_\mu)^{\alpha\beta} \quad D_\mu \equiv \partial_\mu I + i g_0 t^A_{ab} G^A_\mu \]
STRONG CP VIOLATION

\[ \mathcal{L}_\theta = \theta \tilde{\mathcal{G}}^{\mu \nu a} G^a_{\mu \nu} \]
\[ \tilde{G}^a_{\mu \nu} = \epsilon^{\mu \nu \rho \sigma} G^a_{\rho \sigma} \]

\[ \mathcal{L}_\theta \sim \theta \vec{E}^a \cdot \vec{B}^a \]

This term violates CP and gives a contribution to the electric dipole moment of the neutron

\[ e_\eta < 6.3 \times 10^{-26} \text{ e cm} \]

\[ \theta < 10^{-9} \quad \text{which is quite unnatural}!! \]
Neutron electric dipole moment in SuperSymmetry

\[ \Delta F = 0 = -i/2 \, C_e \bar{\psi} \sigma_{\mu \nu} \gamma_5 \psi \, F^{\mu \nu} \]
\[ -i/2 \, C_c \bar{\psi} \sigma_{\mu \nu} \gamma_5 \, t^a \psi \, G^{\mu \nu a} \]
\[ -1/6 \, C_g \, f_{abc} \, G^a_{\mu \rho} \, G^{b \rho \nu} \, G^c_{\lambda \sigma} \, \varepsilon_{\mu \nu \lambda \sigma} \]

\( C_{e,c,g} \) can be computed perturbatively
We may find states which are simultaneously eigenstates of $S$ and of the Energy.

$$[S, \mathcal{H}] = 0 \rightarrow |E, p, s\rangle$$

Consequences of a Symmetry

$CP / K_1^0 \rangle = + / K_1^0 \rangle$

$CP / K_2^0 \rangle = - / K_2^0 \rangle$

$\langle \pi\pi / K_1^0 \rangle \neq 0$

$\langle \pi\pi / K_2^0 \rangle = 0$

if $CP$ is conserved either $\alpha=0$ or $\beta=0$
Violation in the Neutral Kaon System

Expanding in several “small” quantities

\[ \eta^{00} = \frac{\langle \pi^0 \pi^0 / H_W / K_L \rangle}{\langle \pi^0 \pi^0 / H_W / K_S \rangle} \sim \varepsilon - 2 \varepsilon' \]

\[ \eta^{+-} = \frac{\langle \pi^+ \pi^- / H_W / K_L \rangle}{\langle \pi^+ \pi^- / H_W / K_S \rangle} \sim \varepsilon + \varepsilon' \]

Conventionally:

\[ \langle K_S \rangle = \langle K_1 \rangle_{CP=+1} + \varepsilon \langle K_2 \rangle_{CP=-1} \]

\[ \langle K_L \rangle = \langle K_2 \rangle_{CP=-1} + \varepsilon \langle K_1 \rangle_{CP=+1} \]
Indirect CP violation: mixing

\[ |K_L\rangle = |K_2\rangle_{\text{CP} = -1} \]

\[ \text{CP} = +1 \]

\[ \Delta S = 2 \]

Complex $\Delta S = 2$ effective coupling

Box diagrams:
They are also responsible for $B^0 - \bar{B}^0$ mixing

\[ \Delta m_{d,s} \]
$B^0 - B^0$ mixing

$\mathcal{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$

$\mathcal{H}_{\text{eff}}^{\Delta B=2}$

$\propto \left( \bar{d} \gamma_\mu (1 - \gamma_5) b \right)^2$

$\Delta m_{d,s} = \frac{G_F^2 M_W^2}{16 \pi^2} A^2 \lambda^6 F_{tt} \left( \frac{m_t^2}{M_W^2} \right)$

$\text{CKM}$

$\Delta B=2$ Transitions

Hadronic matrix element
**Direct CP violation: decay**

\[ |K_L\rangle = |K_2\rangle_{CP=-1} \]

Complex $\Delta S=1$ effective coupling

$DP_i(q_3, q_2, q_1; B, M_1, M_2)$
\[ \sum_{CP} = \sum_{\Delta F=0} + \sum_{\Delta F=1} + \sum_{\Delta F=2} \]

\( \Delta F=0 \quad d_e < 1.5 \times 10^{-27} \text{ e cm} \quad d_{\Lambda} < 6.3 \times 10^{-26} \text{ e cm} \)

\( \Delta F=1 \quad \varepsilon'/\varepsilon \)

\( \Delta F=2 \quad \varepsilon \quad \text{and} \quad B \rightarrow J/\psi \ K_s \)
<table>
<thead>
<tr>
<th></th>
<th>Exp</th>
<th>Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>$2.271 \pm 0.017 \times 10^{-3}$</td>
<td>$\eta (1-\rho) B_K$</td>
</tr>
<tr>
<td>$\varepsilon'$ / $\varepsilon$</td>
<td>$17.2 \pm 1.8 \times 10^{-4}$</td>
<td>$-7 \div 30 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Delta M_s / \Delta M_d$</td>
<td>$17.77 \pm 0.12$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.507 \pm 0.005 \text{ ps}^{-1}$</td>
<td>$[(1-\rho)^2 + \eta^2]^{-1} \xi$</td>
</tr>
<tr>
<td>BR(B $X_s \gamma$)</td>
<td>$3.11 \pm 0.39 \times 10^{-4}$</td>
<td>$3.50 \pm 0.50 \times 10^{-4}$</td>
</tr>
<tr>
<td>BR(K$^+ \pi^+ \nu\nu$)</td>
<td>$1.5 \pm 3.4 \pm 1.2 \times 10^{-10}$</td>
<td>$0.8 \pm 0.3 \times 10^{-10}$</td>
</tr>
</tbody>
</table>
Visualization of the unitarity of the CKM matrix

Unitarity Triangle in the \((\rho-\eta)\) plane

From
A. Stocchi
ICHEP 2002
For details see:
UTfit Collaboration

hep-ph/0501199
hep-ph/0509219
hep-ph/0605213
hep-ph/0606167

http://www.utfit.org

\[ Q^{\text{EXP}} = V_{CKM} \times \langle H_F | \hat{O} | H_I \rangle \]
\[ \mathcal{A}_{J/\psi K_s} = \frac{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) - \Gamma(B_d^0 \rightarrow J/\psi K_s, t)}{\Gamma(B_d^0 \rightarrow J/\psi K_s, t) + \Gamma(B_d^0 \rightarrow J/\psi K_s, t)} \]

\[ \mathcal{A}_{J/\psi K_s} = \sin 2\beta \sin (\Delta m c \Delta t) \]
DIFFERENT LEVELS OF THEORETICAL UNCERTAINTIES (STRONG INTERACTIONS)

1) First class quantities, with reduced or negligible theoretical uncertainties

\[ A_{CP}(B \rightarrow J/ψ K_s) \quad γ \quad \text{from } B \rightarrow D K \]
\[ K^0 \rightarrow π^0 ν\bar{ν} \]

2) Second class quantities, with theoretical errors of \( O(10\%) \) or less that can be reliably estimated

\[ ε_K \quad \Delta M_{d,s} \]
\[ \Gamma(B \rightarrow c, u) \quad K^+ \rightarrow π^+ ν\bar{ν} \]

3) Third class quantities, for which theoretical predictions are model dependent (BBNS, charming, etc.)

In case of discrepancies we cannot tell whether is \textit{new physics or we must blame the model}

\[ B \rightarrow K π \quad B \rightarrow π^0 π^0 \]
\[ B \rightarrow φ K_s \]
Unitary Triangle

$K^0 - \bar{K}^0$ mixing

$B^0_{d,s} - \bar{B}^0_{d,s}$ mixing

$B_d$ Asymmetry

$\sin2\beta$
Classical Quantities used in the Standard UT Analysis

\[ \frac{V_{ub}}{V_{cb}} \quad \varepsilon_K \quad \Delta m_d \quad \Delta m_d/\Delta m_s \]

Inclusive vs Exclusive Opportunity for lattice QCD see later

before only a lower bound
New Quantities used in the UT Analysis

Several new determinations of UT angles are now available, thanks to the results coming from the B-Factor experiments.

<table>
<thead>
<tr>
<th>(\sin 2\beta)</th>
<th>(\cos 2\beta)</th>
<th>(\alpha)</th>
<th>(\gamma)</th>
<th>(\sin(2\beta + \gamma))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B \to J/\Psi K^0)</td>
<td>(B \to J/\Psi K^{*0})</td>
<td>(B \to \pi\pi, \rho\rho)</td>
<td>(B \to D^{(*)}K)</td>
<td>(B \to D^{(*)}\pi, D\rho)</td>
</tr>
</tbody>
</table>

New Constraints from B and K rare decays (not used yet)

New bounds are available from rare B and K decays. They do not still have a strong impact on the global fit and they are not used at present.

| \(K \to \pi \nu \bar{\nu}\) | \(B \to \tau \nu\) | \(\frac{B \to \rho(\omega \gamma)}{B \to K^*\gamma}\) |
THE COLLABORATION

M. Bona, M. Ciuchini, E. Franco, V. Lubicz,
G. Martinelli, F. Parodi, M. Pierini,
P. Roudeau, C. Schiavi, L. Silvestrini,
V. Sordini, A. Stocchi, V. Vagnoni

Roma, Genova, Annecy, Orsay, Bologna

2006 ANALYSIS

- New quantities e.g. B -> DK included
- Upgraded exp. numbers (after ICHEP)
- CDF & Belle new measurements

www.utfit.org
Results for $\rho$ and $\eta$ & related quantities

With the constraint from $\Delta m_s$

- Contours @ 68% and 95% C.L.

- $\rho = 0.147 \pm 0.029$
- $\eta = 0.342 \pm 0.016$
- $\alpha = (91 \pm 8)^0$
- $\sin 2\beta = 0.690 \pm 0.023$
- $\gamma = (66.7 \pm 6.4)^0$
A closer look to the analysis:

1) Predictions vs Postdictions
2) Lattice vs angles
3) $V_{ub}^{\text{inclusive}}$, $V_{ub}^{\text{exclusive}}$ vs $\sin 2\beta$
4) Experimental determination of lattice parameters
**V_{UB} PUZZLE**

**Inclusive:** uses non perturbative parameters most **not** from lattice QCD (fitted from the lepton spectrum)

\[
\bar{\Lambda} \quad \lambda_1 \sim \frac{\bar{b}D^2 b}{2m_b} \quad \lambda_2 \sim \frac{\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b}{2m_b}
\]

**Exclusive:** uses non perturbative form factors from LQCD and QCDSR

\[
f^+(q^2) \quad V(q^2) \quad A_{1,2}(q^2)
\]
Tension between inclusive $V_{ub}$ and the rest of the fit

**Inclusive** $V_{ub} = (43.1 \pm 3.9) \times 10^{-4}$

Model dependent in the threshold region (BLNP, DGE, BLL)

But with a different modelling of the threshold region [U. Aglietti et al., 0711.0860] $V_{ub} = (36.9 \pm 1.3 \pm 3.9) \times 10^{-4}$

**Exclusive** $V_{ub} = (34.0 \pm 4.0) \times 10^{-4}$

Form factors from LQCD and QCDSR
### $V_{UB}$ PUZZLE

**Recent $|V_{ub}|$ determinations from $B \to \pi l\nu_l$**

| ref.          | $f_{B\pi}^+(q^2)$ calculation     | $f_{B\pi}^+(q^2)$ input   | $|V_{ub}| \times 10^3$   |
|---------------|-----------------------------------|---------------------------|-------------------------|
| Okamoto et al.| lattice ($n_f = 3$)                | -                         | $3.78 \pm 0.25 \pm 0.52$|
| HPQCD         | lattice ($n_f = 3$)                | -                         | $3.55 \pm 0.25 \pm 0.50$|
| Arnesen et al.| -                                 | lattice⊕SCET              | $3.54 \pm 0.17 \pm 0.44$|
| BecherHill    | -                                 | lattice                   | $3.7 \pm 0.2 \pm 0.1$   |
| Flynn et al.  | -                                 | lattice⊕LCSR              | $3.47 \pm 0.29 \pm 0.03$|
| Ball, Zwicky  | LCSR                              | -                         | $3.5 \pm 0.4 \pm 0.1$   |
| this work     | LCSR                              | -                         | $3.5 \pm 0.4 \pm 0.2 \pm 0.1$ |
$|V_{ub}|$ crisis (about to be resolved?)

- $|V_{ub}| f^{B \pi}_+(0) = (9.1 \pm 0.6 \pm 0.3) \times 10^{-4}$ from semileptonic $B \to \pi \ell \nu$ spectrum + form factor extrapolation (Ball, 2006)
  
  Also lattice results (HFQCD) tend to small values.

- $|V_{ub}| f^{B \pi}_+(0) = (8.1 \pm 0.4 (??)) \times 10^{-4}$ from $B \to \pi^+ \pi^-, \pi^+ \pi^0, \pi \rho, \ldots +$ factorization (M3, Neubert, 2003; Arsene et al, 2005; MB, Jager, 2005)

$\implies |V_{ub}| \approx 3.5 \times 10^{-4}$, in contrast to determination from moments of inclusive $b \to u \ell \nu$ decay, which was $|V_{ub}| \approx (4.5 \pm 0.3) \times 10^{-4}$.

But, according to (Neubert, LPQCD) $|V_{ub}| \approx (3.7 \pm 0.3) \times 10^{-4}$ after reevaluation of $m_b$ input and omitting $B \to X_s \gamma$ moments!
1) Predictions vs Postdictions
2) Lattice vs angles
3) $V_{ub}$ inclusive, $V_{ub}$ exclusive vs $\sin 2\beta$
4) Experimental determination of lattice parameters
IMPACT of the NEW MEASUREMENTS on LATTICE HADRONIC PARAMETERS

\[ f_{B_s} \hat{B}_{B_s}^{1/2} \xi \hat{B}_K \]
\[ f_{B_s} \sqrt{B_{B_s}} = 265 \pm 4 \text{ MeV} \]

\[ \xi = 1.25 \pm 0.06 \quad \text{UTA} \]

\[ B_K = 0.75 \pm 0.07 \]

\[ f_{B_s} \sqrt{B_{B_s}} = 270 \pm 30 \text{ MeV} \quad \text{lattice} \]

\[ \xi = 1.21 \pm 0.04 \quad \text{lattice} \]

SPECTACULAR AGREEMENT
(EVEN WITH QUENCHED LATTICE QCD)

V. Lubicz and C. Tarantino
0807.4605
beyond the Standard Model
CP beyond the SM (Supersymmetry)

- **Spin 1/2**
  - Quarks: \( q_L, u_R, d_R \)
  - Leptons: \( l_L, e_R \)

- **Spin 0**
  - SQuarks: \( Q_L, U_R, D_R \)
  - SLeptons: \( L_L, E_R \)

- **Spin 1**
  - Gauge bosons: \( W, Z, \gamma, g \)

- **Spin 0**
  - Higgs bosons: \( H_1, H_2 \)

- **Spin 1/2**
  - Gauginos: \( w, z, \gamma, g \)

- **Spin 1/2**
  - Higgsinos: \( \tilde{H}_1, \tilde{H}_2 \)
Only tree level processes $V_{ub}/V_{cb}$ and $B \to D K$(*)

CP VIOLATION PROVEN IN THE SM !!
\[ Q^{EXP} = V_{CKM} \langle F | \hat{O} | I \rangle \]

\[ Q^{EXP} = \sum_i C_{SM}^i(M_W, m_t, \alpha_s) \langle F | \hat{O}_i | I \rangle + \sum_{i'} C_{Beyond}^{i'}(\tilde{m}_\beta, \alpha_s) \langle F | \hat{O}_{i'} | I \rangle \]
In general the mixing mass matrix of the SQuarks (SMM) is not diagonal in flavour space analogously to the quark case. We may either Diagonalize the SMM or Rotate by the same matrices

the SUSY partners of the u- and d- like quarks

\[(Q^j_L) = U^{ij}_L \times Q^j_L\]
In the latter case the Squark Mass Matrix is not diagonal

\[(m^2_Q)_{ij} = m^2_{\text{average}} \ 1_{ij} + \Delta m^2_{ij} \]

\[\delta_{ij} = \frac{\Delta m^2_{ij}}{m^2_{\text{average}}}\]
New local four-fermion operators are generated

\[ Q_1 = (\bar{s}_L^A \gamma_\mu d_L^A) (\bar{s}_L^B \gamma_\mu d_L^B) \quad \text{SM} \]
\[ Q_2 = (\bar{s}_R^A d_L^A) (\bar{s}_R^B d_L^B) \]
\[ Q_3 = (\bar{s}_R^A d_L^B) (\bar{s}_R^B d_L^A) \]
\[ Q_4 = (\bar{s}_R^A d_L^A) (\bar{s}_L^B d_R^B) \]
\[ Q_5 = (\bar{s}_R^A d_L^B) (\bar{s}_L^B d_R^A) \]
+ those obtained by \( L \leftrightarrow R \)

Similarly for the \( b \) quark e.g.
\[ (\bar{b}_R^A d_L^A) (\bar{b}_R^B d_L^B) \]
$B_s$ mixing, a road to New Physics (NP)?

The Standard Model contribution to CP violation in $B_s$ mixing is well predicted and rather small.

- $\sin 2\beta_s = 0.037 \pm 0.002$ (SM or MFV)
- $\sin 2\beta_s = 0.041 \pm 0.004$ (Arbitrary NP)

The phase of the mixing amplitudes can be extracted from $B_s \rightarrow J/\Psi \phi$ with a relatively small th. uncertainty. A phase very different from 0.04 implies NP in $B_s$ mixing.
Main Ingredients and General Parametrizations

\[ H_{\Delta F=2} = \hat{m} - \frac{i}{2} \hat{\Gamma} \quad A = \hat{m}_{12} = \langle \tilde{M} | \hat{m} | M \rangle \quad \Gamma_{12} = \langle \tilde{M} | \hat{\Gamma} | M \rangle \]

Neutral Kaon Mixing

\[ ReA_K = C_{\Delta m_K} ReA_{K}^{SM} \quad ImA_K = C_{\epsilon} ImA_{K}^{SM} \]
$B_d$ and $B_s$ mixing

\[
A_q e^{2i\phi_q} \equiv C_{Bq} e^{2i\phi_{Bq}} \times A_q^{SM} e^{2i\phi_q^{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) \times A_q^{SM} e^{2i\phi_q^{SM}}
\]

\[
C_{Bs} e^{2i\phi_{Bs}} = \frac{A_s^{SM} e^{-2i\beta_s} + A_s^{NP} e^{2i(\phi_s^{NP} - \beta_s)}}{A_s^{SM} e^{-2i\beta_s}} = \frac{\langle \bar{B}_s | H_{eff}^{full} | B_s \rangle}{\langle \bar{B}_s | H_{eff}^{SM} | B_s \rangle}
\]

\[
\Gamma_{12}^q = -2\frac{\kappa}{C_{Bq}} \left\{ e^{i2\phi_{Bq}} \left( n_1 + \frac{n_6B_2 + n_{11}}{B_1} \right) - \frac{e^{i(\phi_q^{SM} + 2\phi_{Bq})}}{R_i^q} \left( n_2 + \frac{n_7B_2 + n_{12}}{B_1} \right) \right. \\
\left. + \frac{e^{i(\phi_q^{SM} + \phi_{Bq})}}{R_i^q} \left( n_3 + \frac{n_8B_2 + n_{13}}{B_1} \right) + e^{i(\phi_q^{Pen} + 2\phi_{Bq})} C_q^{Pen} \left( n_4 + \frac{n_9B_2}{B_1} \right) \right. \\
\left. - e^{i(\phi_q^{SM} + \phi_q^{Pen} + 2\phi_{Bq})} C_q^{Pen} \frac{R_i^q}{R_i^q} \left( n_5 + \frac{n_{10}B_2}{B_1} \right) \right\}
\]

$C_q^{Pen}$ and $\phi_q^{Pen}$ parametrize possible NP contributions to $\Gamma_{12}^q$ from $b \to s$ penguins
Physical observables

\[ \Delta m_s = |A_s| = C_{B_s} \Delta m_s^{SM} \]

\[ 2\phi_s = -\arg A_s = 2(\beta_s - \phi_{B_s}) \]

\[ A_{SL}^s = \frac{\Gamma(\bar{B}_s \to l^+X) - \Gamma(B_s \to l^-X)}{\Gamma(\bar{B}_s \to l^+X) + \Gamma(B_s \to l^-X)} = \text{Im} \left( \frac{\Gamma_{12}^s}{A_s} \right) \]

\[ A_{\mu\mu}^s_{SL} = \frac{f_d \chi_{d0} A_{SL}^d + f_s \chi_{s0} A_{SL}^s}{f_d \chi_{d0} + f_s \chi_{s0}} \]

\[ \frac{\Delta \Gamma_s}{\Delta m_s} = \text{Re} \left( \frac{\Gamma_{12}^s}{A_s} \right) \]

\[ \tau_{B_s}^{FS} = \frac{1}{\Gamma_s} \frac{1 + (\Delta \Gamma_s/2 \Gamma_s)^2}{1 - (\Delta \Gamma_s/2 \Gamma_s)^2} \]
Utfit 0707.0636
The two solutions for $\phi_s$ correspond to two regions for $A_s^{NP}$ and $\phi_s^{NP}$:

\[ A_s^{NP}/A_s^{SM} = 0.6 \pm 0.4 \quad \text{&} \quad \phi_{NP} = (123 \pm 10)^\circ \]  
\[ A_s^{NP}/A_s^{SM} = 1.8 \pm 0.1 \quad \text{&} \quad \phi_{NP} = (100 \pm 3)^\circ \]  
requires NP with new sources of CP violation!
* Chirality-flipping mass insertions are strongly bounded by $b \rightarrow s \gamma$: they are too small to produce the measured $\phi_s$

**Case #1**: single mass insertion, e.g. $(\delta_{23})_{LL}$

* Large MI needed for $\phi_s$: tension with $b \rightarrow s \gamma$

* MI saturates at 1:
  - Upper bound $\tilde{m} < O(1\ TeV)$

* Huge effect in $b \rightarrow s$ penguins

---

![Graph showing $\Delta S_K K_s$]
**case #2:** double mass insertion, \((\delta_{23})_{LL} \& (\delta_{23})_{RR}\)

* no need of large MIs: \((\delta_{23})_{LL} \sim (\delta_{23})_{RR} \sim 3-4 \cdot 10^{-2}\)

\(b \rightarrow s \gamma\) is no longer a problem

* large effects in \(b \rightarrow s\) penguins still possible
  (larger if LR MIs are also switched on)
When SUSY is broken at a scale larger than $M_{\text{GUT}}$ SQuark and SLepton masses unify including the non-diagonal coupling $(\delta_{ij})_{LL}$, $(\delta_{ij})_{RR}$

The following relations holds at $M_Z$

(Ciuchini et al. hep-ph/0307191)

\[
\begin{align*}
(\delta^d_{ij})_{RR} & \approx \frac{m^2_L}{m^2_D} (\delta^l_{ij})_{LL} \\
(\delta^u_{ij})_{RR} & \approx \frac{m^2_E}{m^2_U} (\delta^l_{ij})_{LL} \\
(\delta^d_{ij})_{LR} & \approx \frac{m^2_{L_{\text{ave}}}}{m^2_Q_{\text{ave}}} \frac{m_b}{m_\tau} (\delta^l_{ij})^{*}_{RL} \\
(\delta^u_{ij})_{LR} & \approx \frac{m^2_{E_{\text{ave}}}}{m^2_Q_{\text{ave}}} \frac{m_b}{m_\tau} (\delta^l_{ij})^{*}_{RL}
\end{align*}
\]
$b \rightarrow s \& \tau \rightarrow \mu \gamma$ in SUSY GUTS

mass insertion analysis in a SUSY-GUT scheme

* RG-induced $\left( \delta_{23} \right)_{LL}$
* explicit $\left( \delta_{23} \right)_{RR}$

$\Delta M_s, m_{sq} = 500$ GeV

Limits from Belle and Babar $< 4.5 \& 6.8 \times 10^{-8}$

In the UTfit range for the $B_s$ mixing phase:

$BR(\tau \rightarrow \mu \gamma) > 3 \times 10^{-9}$ !!
CONCLUSIONS: THANKS TO EXPERIMENTAL MEASUREMENTS AND IMPROVED LATTICE CALCULATIONS.
UTA in the SM: 2007 vs 2015

\[ \sigma(\bar{\rho}) / \bar{\rho} = 20\% \]

\[ \sigma(\bar{\eta}) / \bar{\eta} = 4.7\% \]

\[ \sigma(\bar{\rho}) / \bar{\rho} = 1.3\% \]

\[ \sigma(\bar{\eta}) / \bar{\eta} = 0.8\% \]
The evidence (strong suggestion, hint, ..) of a large Bs mixing phase survives to a second run of measurements.

The upgraded UTFit analysis gives a 2.9 \( \sigma \) deviation from the SM (new CDF measurements still to be included).

In this framework MFV ruled out; MSSM could work with LL and RR insertions without conflict with b \( \rightarrow \) s \( \gamma \).

Within SUSY GUT a large BR(\( \tau \) \( \rightarrow \) \( \mu \gamma \)) is expected.