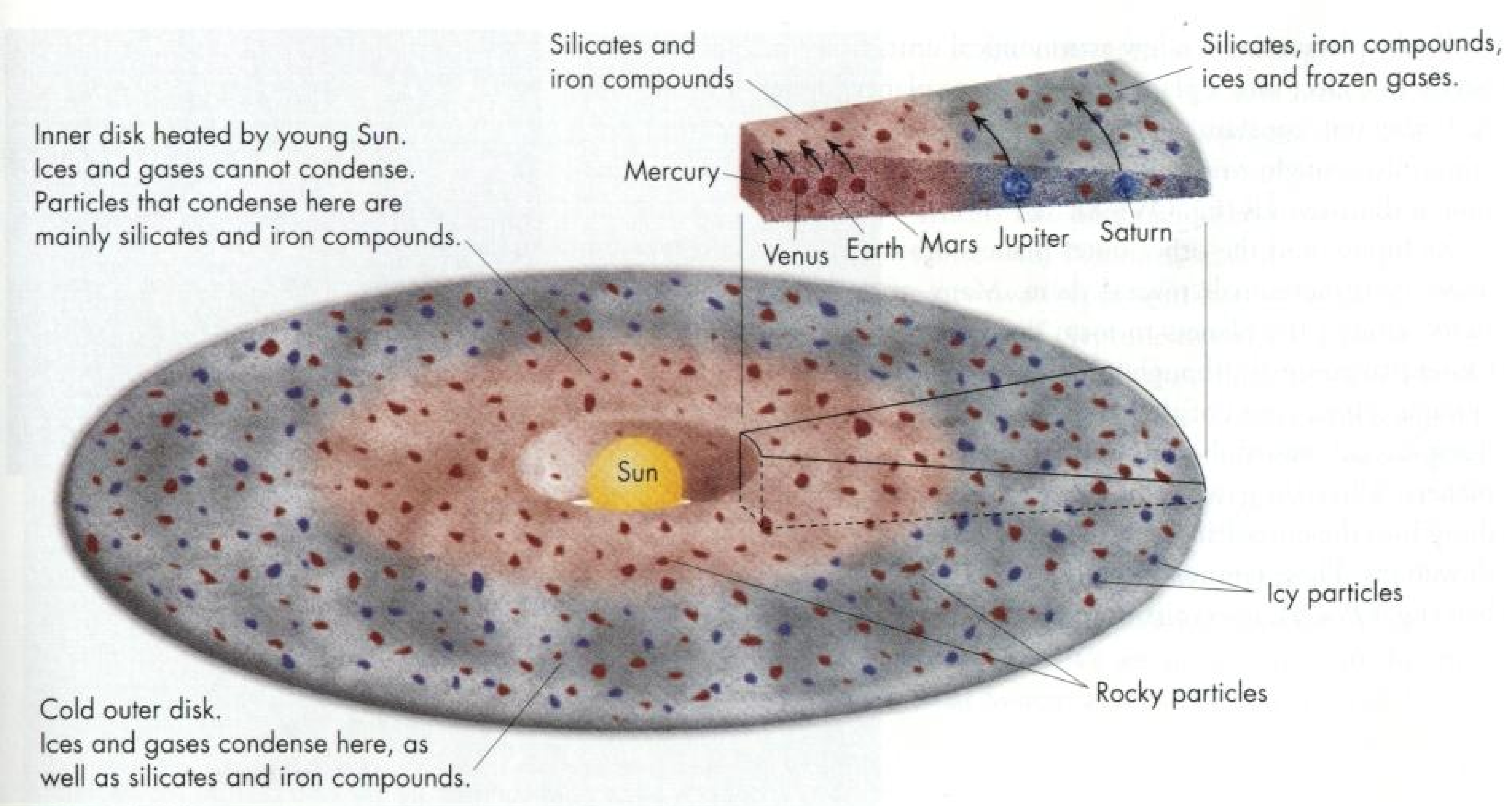
Planetary system dynamics

- Planetary migration
- Kozai resonance
- Apsidal resonance and secular theories
- Mean motion resonances
- Gravitational scattering

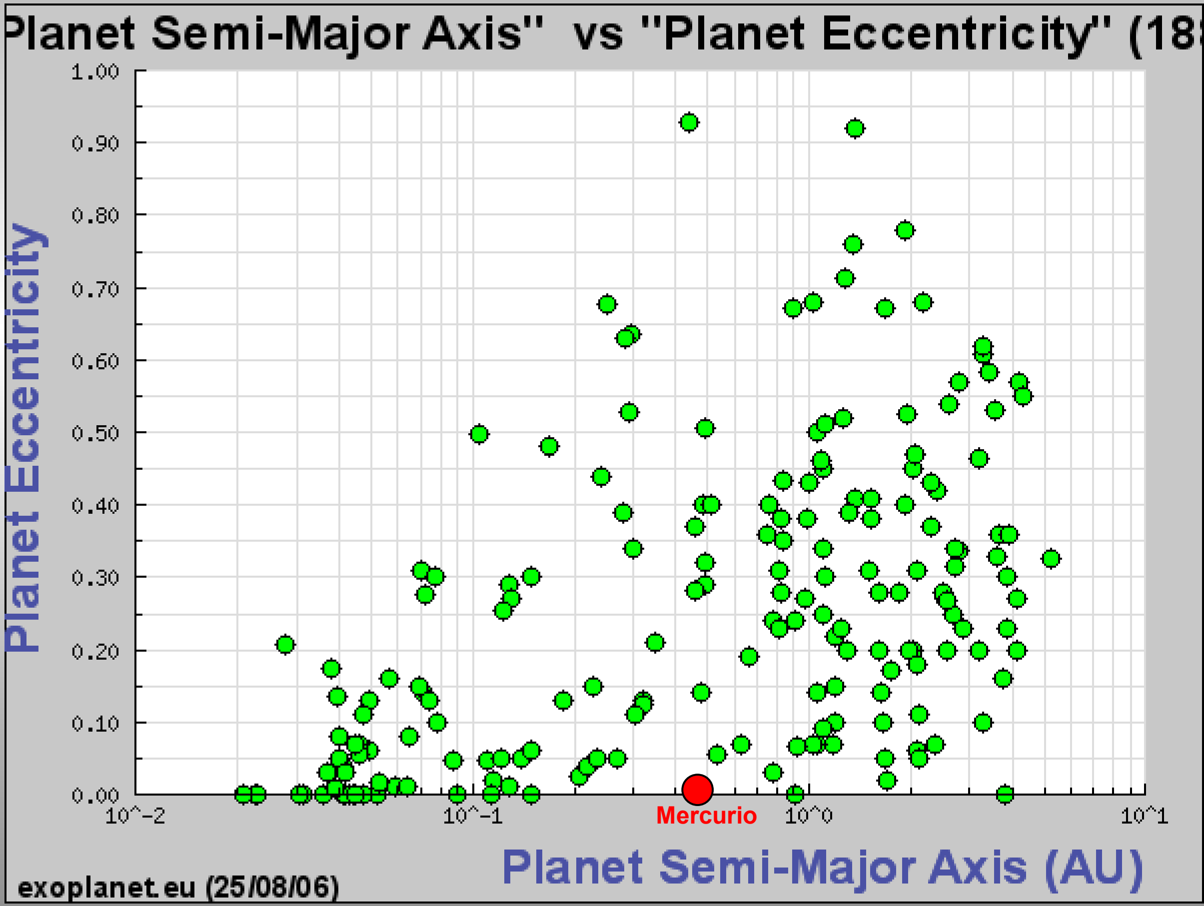
How should the planets of a 'typical' planetary system be distributed?

Following our experience with the Solar System…….



Within the 'frost line' terrestrial planets, giant planets only beyond. All low eccentric orbits!

Hot Jupiters: planets very close to their host star.



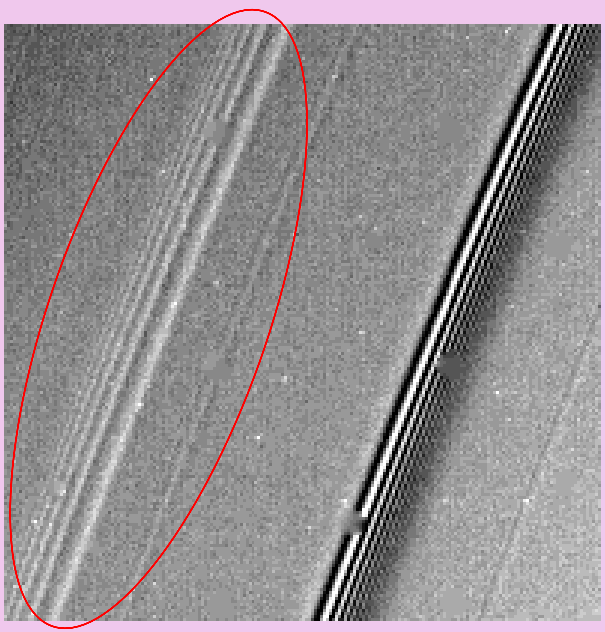
Type I migration – dynamical origin

- Lindblad resonances: $\varphi_{LD} = j\,\lambda' + (k+p+1-j)\,\lambda - k\,\varpi' - \varpi - p\Omega'$
- Corotation resonances: $\varphi_{co} = j\,\lambda' + (k+p-j)\,\lambda - k\,\varpi' - p\,\Omega'$

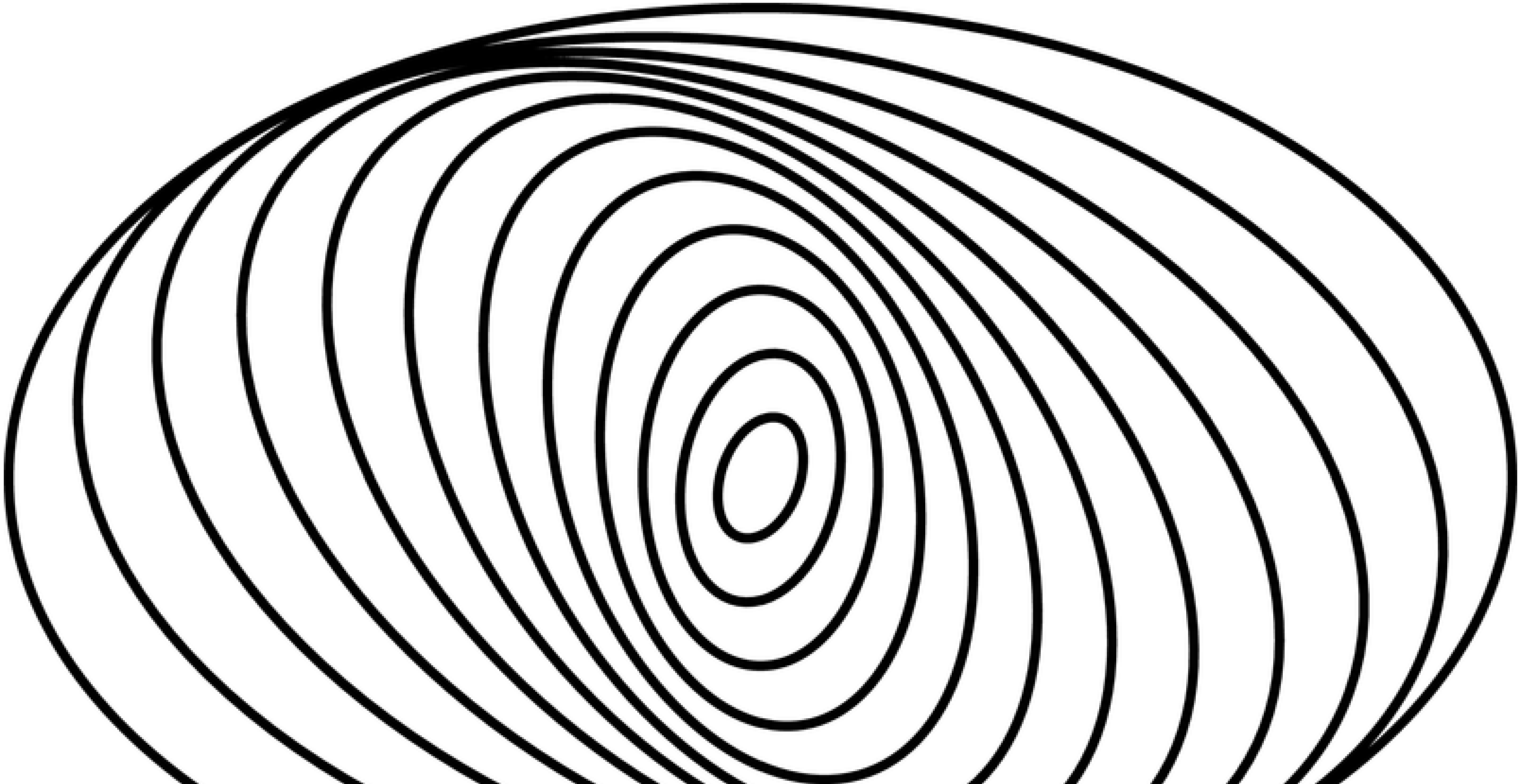
see Murray & Dermott.

$m(n-\Omega_p)=+/-\,\kappa$

$m\Omega_p = (m+k+p)n' - k\,\dot{\tilde{\omega}}' - p\dot{\Omega}'$ $\quad \kappa=n-\dot{\tilde{\omega}}$

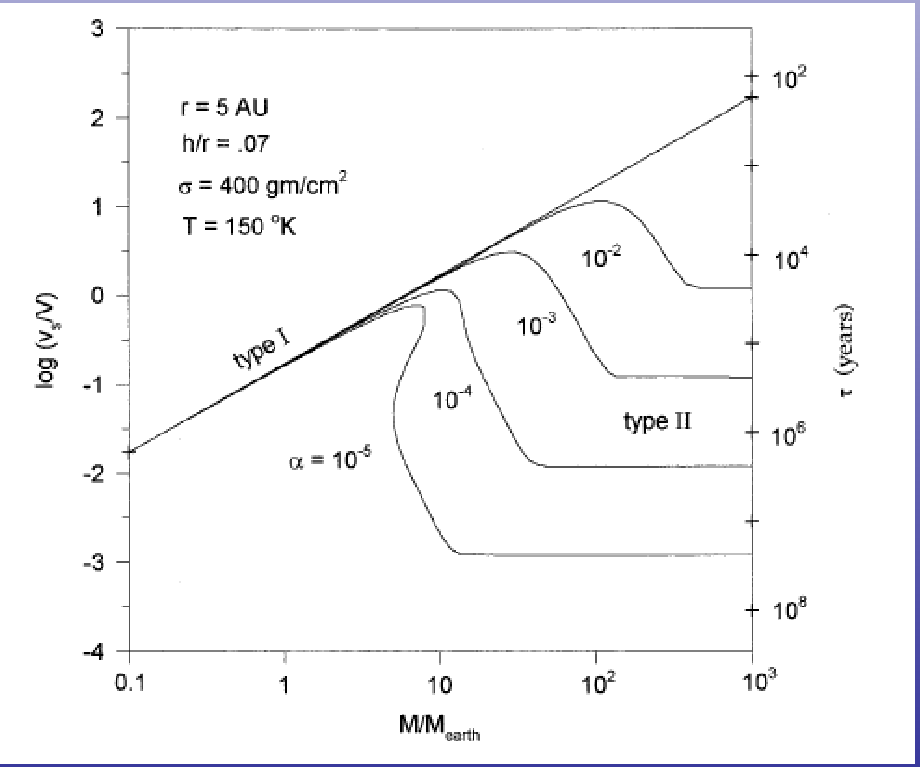


At the Lindblad resonances spiral density waves form (in the figure example from Saturn rings: 5:3 resonance with Mimas). Angular momentum transfer from disk to planet.

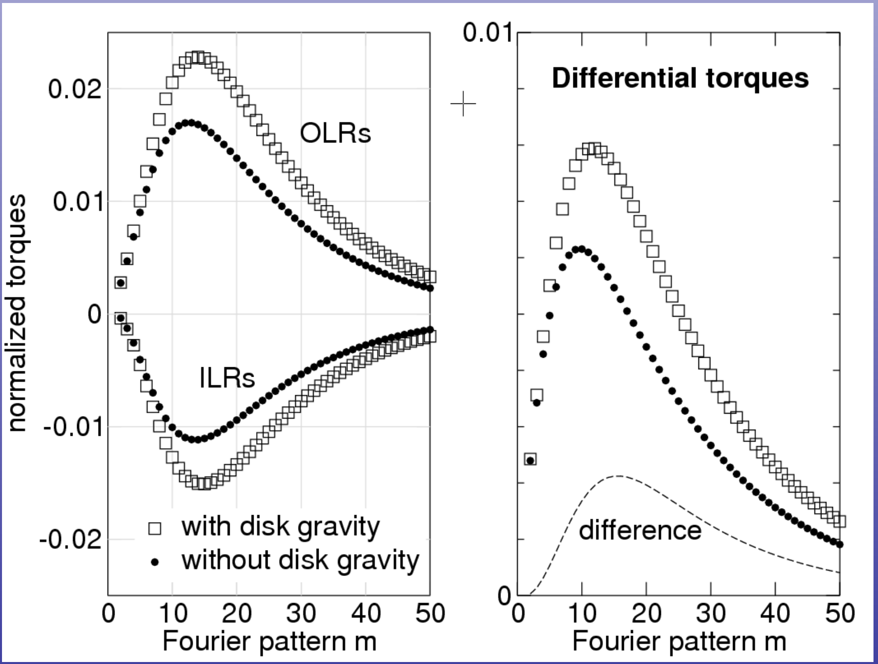


Troubles with type I migration: too fast!

- Including disk self-gravity torque increseas.
- Timescale: ~10^4 - 10^5 yrs



Self gravity effect



What can halt type I migration ?

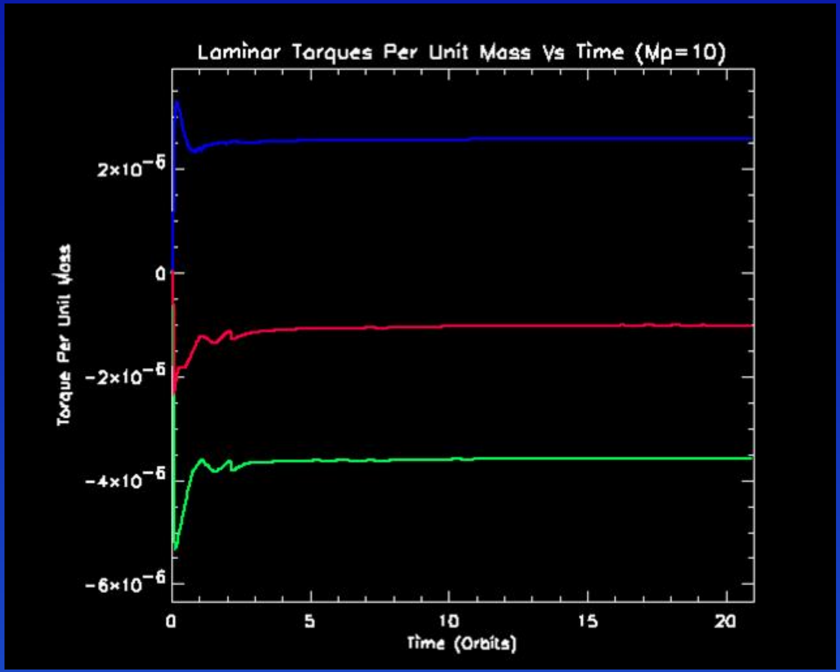
Far from the star:

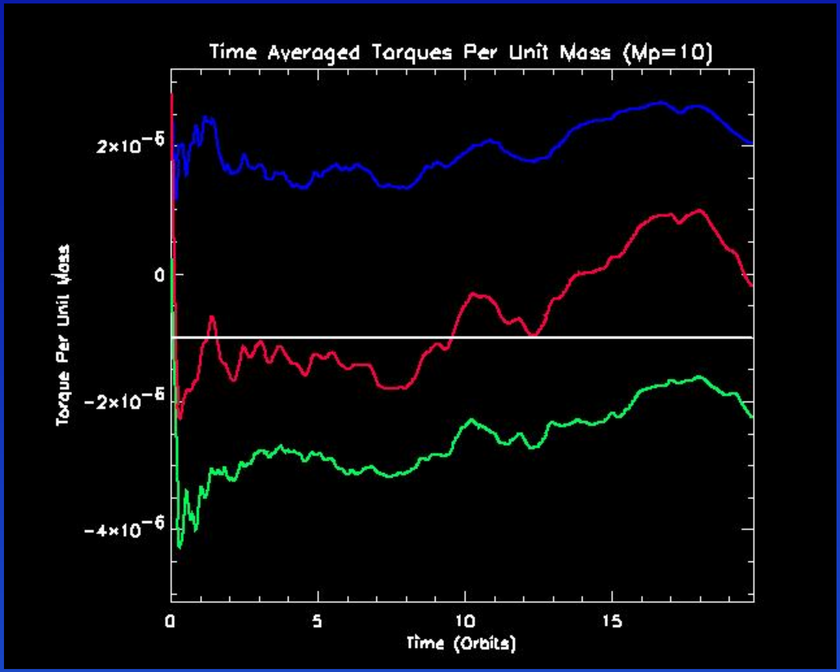
- Large scale turbulence
- Steep density changes on the disk .
- High eccentricity of the planet.
- Density gap formation (type II migration)

Close to the star:

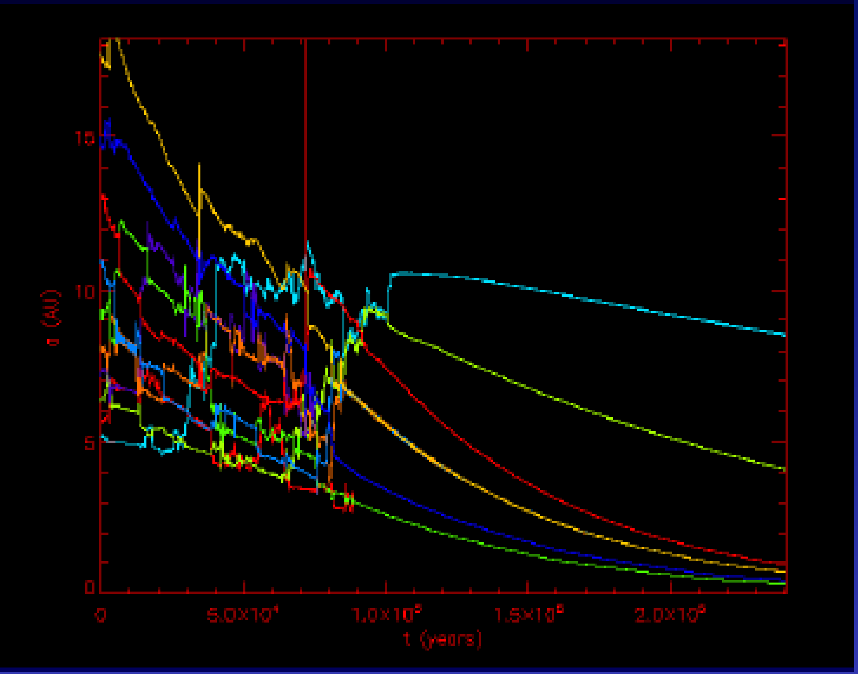
- Disk clearing in the proximity of the star caused by magnetic field
- Disk clearing induced by photoevaporation
- Tidal interaction with the star.
- PLanet-star mass exchange.

Large-scale turbulence in the disk: torque is randomized and planet can migrates outside if it has a mass lower than 30 Earth masses.



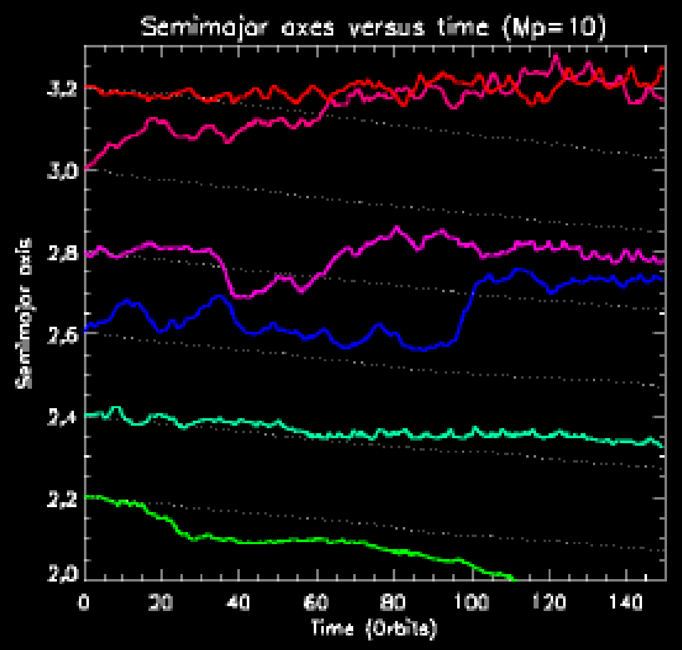


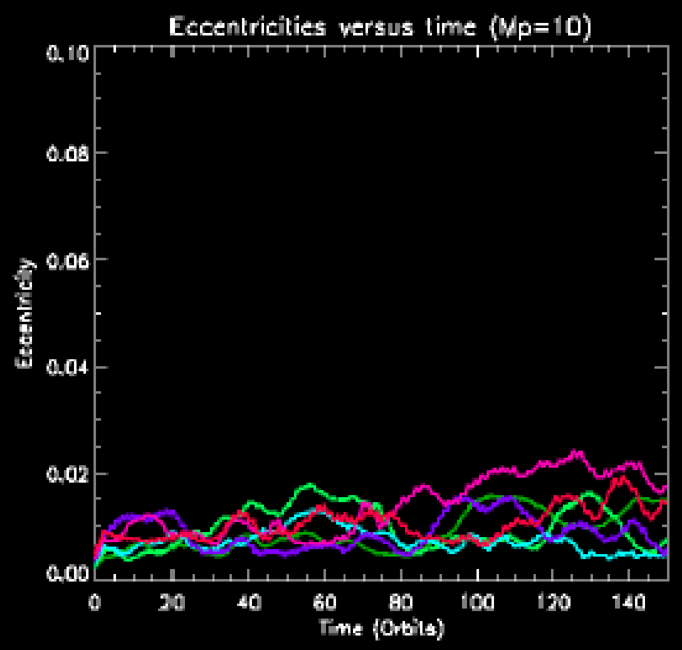
- Extended turbulence: large-scale changes of the disk density (MHD?)



Migration of a set of planets in a disk **without turbulence**.

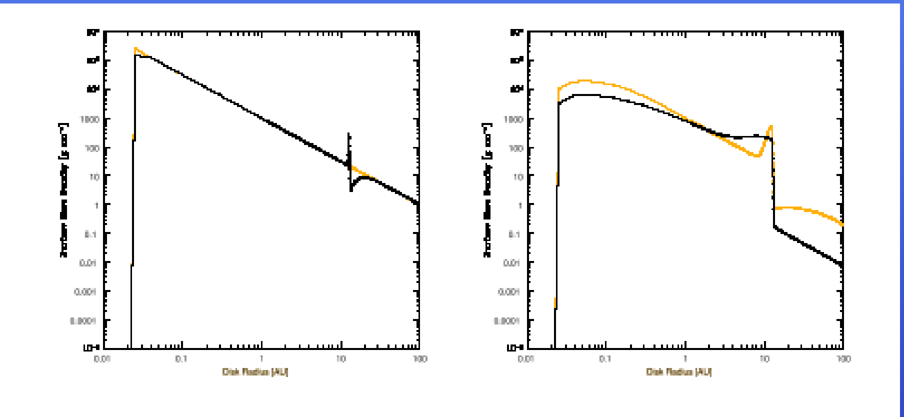
Random behaviour in **presence of turbulence.**

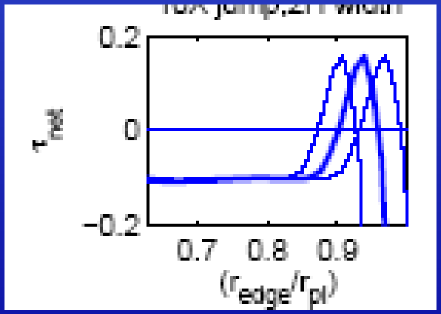




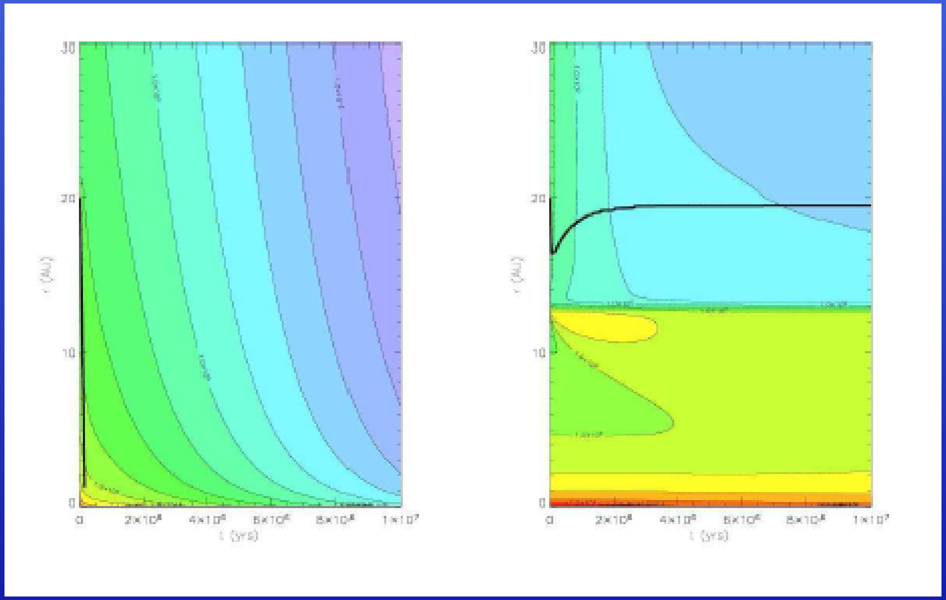
Halting and reversing type I migration in proximity of a 'dead zone' where viscosity is much lower ($\alpha \sim 10^{-5}$ in the dead zone, $\alpha \sim 10^{-2}$ outside)

Density changes dramatically at the border of the dead zone.



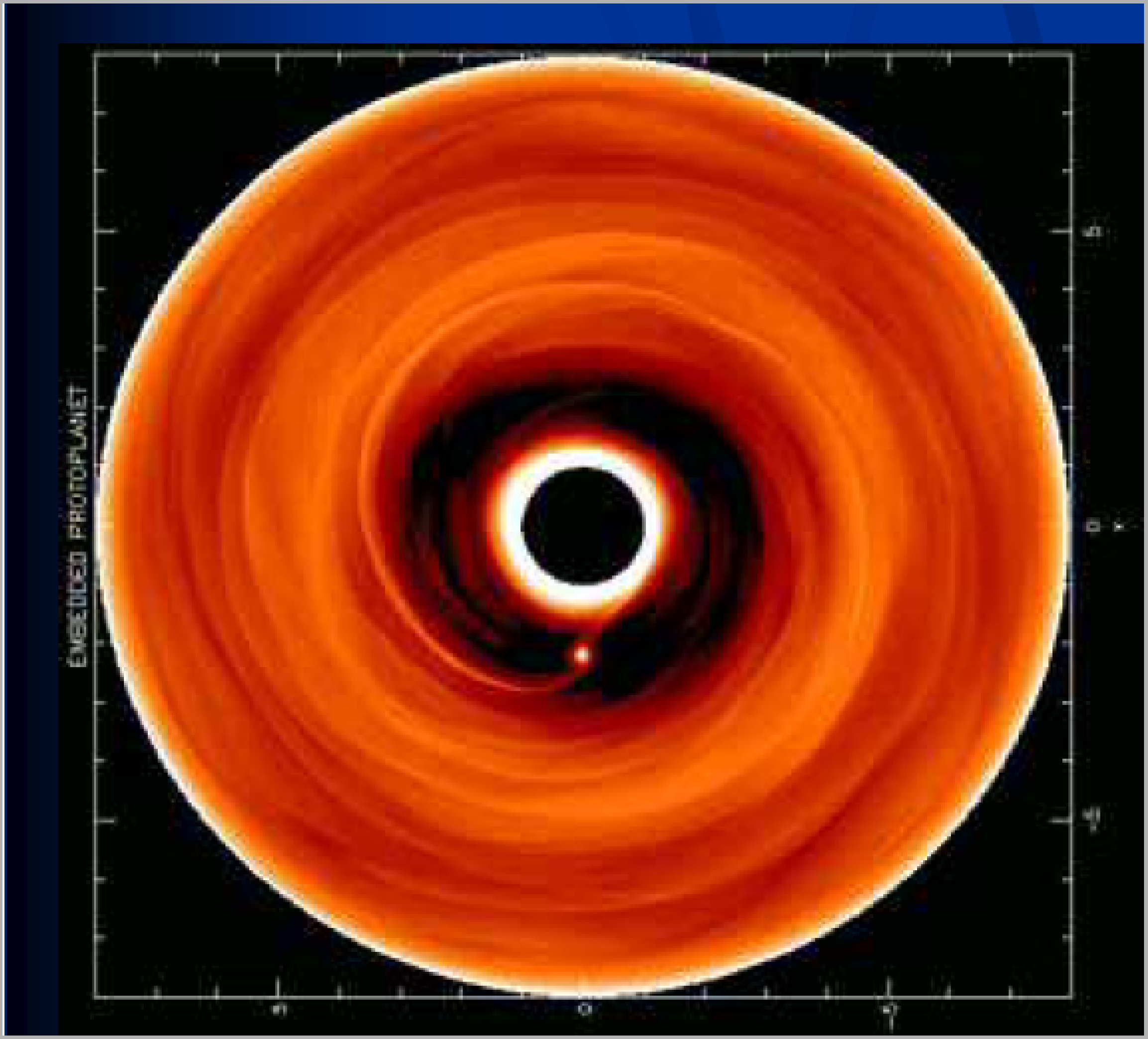


The torque by L-resonances is inverted.



because of the large amount of mass in the inner side of the border.

Gap formation: type II migration



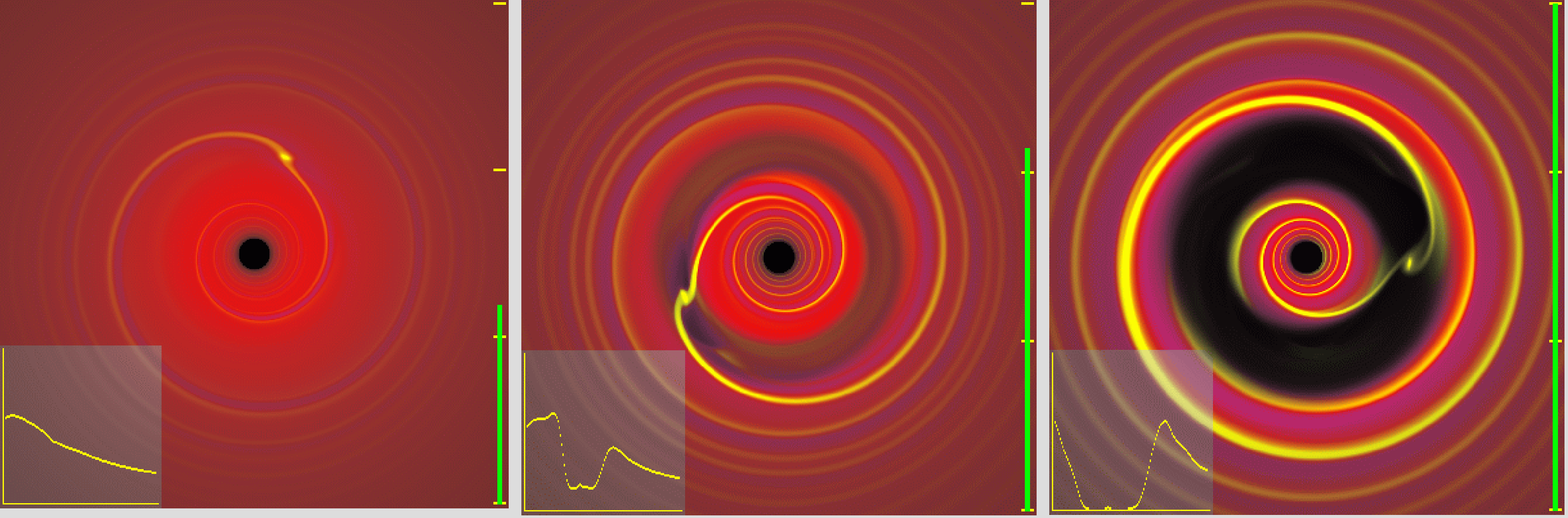
A gap forms in the disk around the planet orbit.

• Outer disk particles moves slower: get accelerated by planet.

• Inner particles move faster: they accelerate the planet.

• Equilibrium broken by viscous evolution of the disk. Outer part of the disk penetrates into the gap and pushes planet inwards.

• Asymmetric gap, planet migrates.



Type II migration is slower than type I which is turned off because of the gap: Lindblad resonances are within the now empty gap.

$$\tau_{II} = 3 \times 10^{5} \, (\alpha \; / \; 10^{-4})^{-1} \text{ yr}$$

When large-scale turbulence is included gap forms at a larger value of the planet mass and the density drops to a higher value than in the no-turbulence case.

Type II migration can be slowed down in presence of a *dead-zone*

Inside the dead-zone

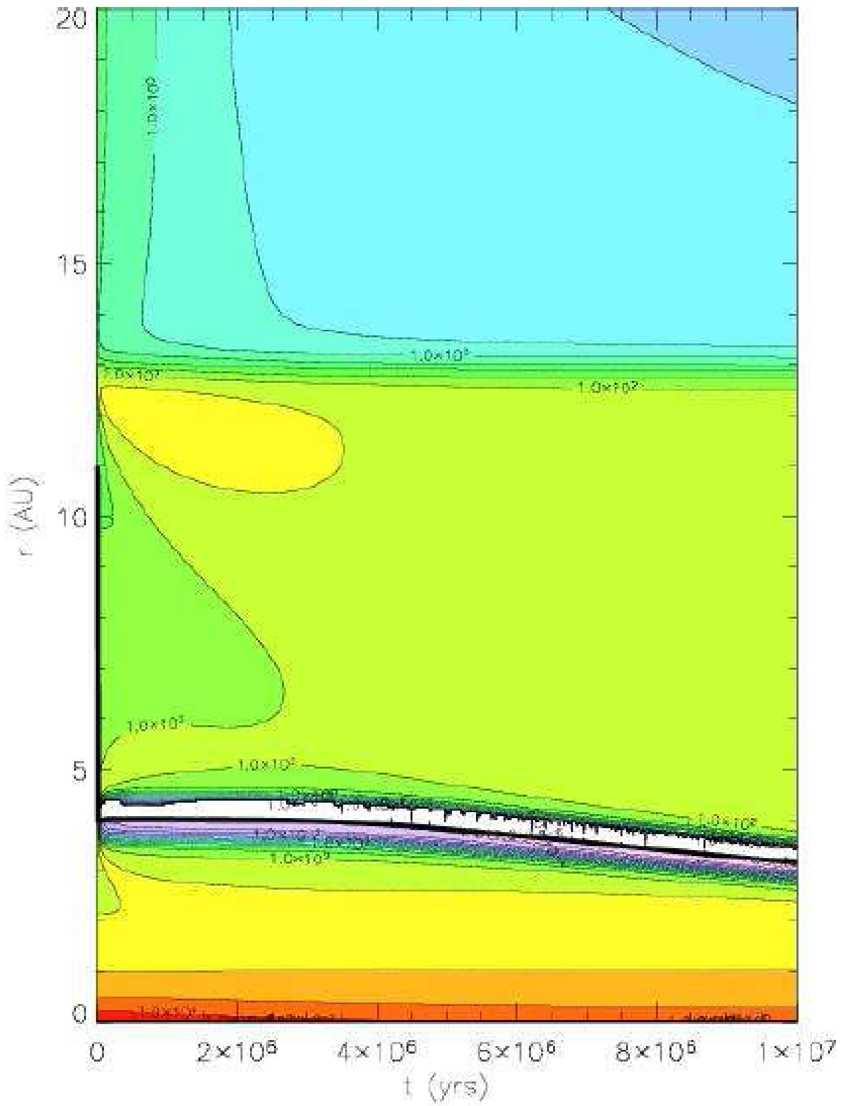
- **A gap opens up at a small planet mass.**
- **The viscous evolution is slower.**

$$\frac{M_p}{M_*} \gtrsim \sqrt{40\alpha \left(\frac{h_p}{r_p}\right)^5} ,$$

M_p = planet mass,

M_* =star mass,

h_p = scale height of the disk at r_p



. 7.— The same as Fig. 4, but the 10 M_E planet starts migrating from just inside the d zone, namely 11 AU. The planet migrates inward, and opens a gap at ~ 4 AU, which ees well with the estimate from Fig. 2 (~ 5 AU). After the gap-opening, its migration ed is significantly slowed down.

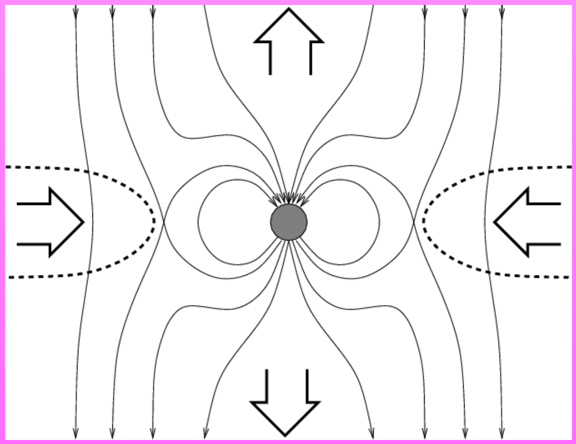
How can migration be stopped before the planet falls into the star?

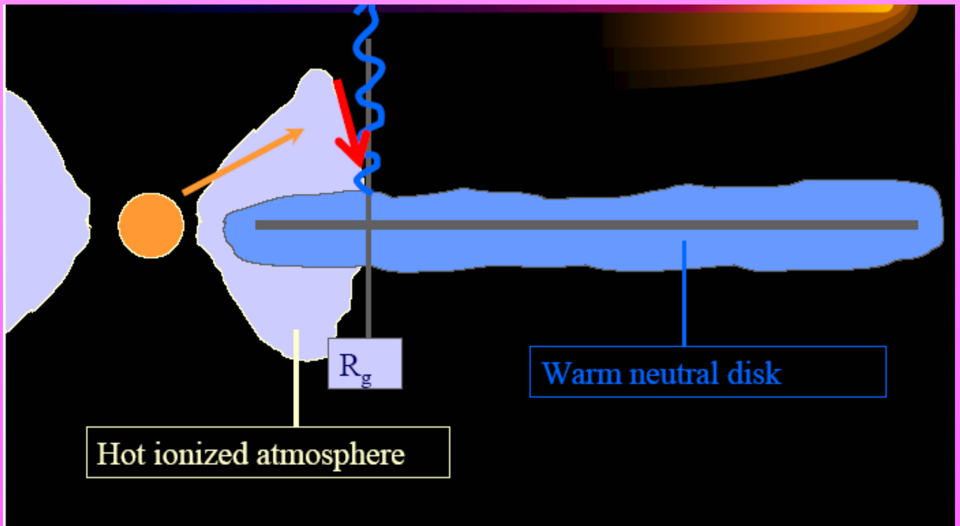
1) When the magnetic pressure is equal to the viscous pressure the disk is deviated to the poles. A cavity form in the proximity of the star where planet migration is halted. This occurs at the corotation angle where the Keplerian period of the disk is equal to the rotation rate of the star.

2) Tidal interaction with the star (like the Earth-Moon system) (

3) Mass exchange with the star through the Roche lobe.

4) Intense photoevaporation of the disk close to the star. Gap opening.





Why Jupiter and Saturn did not migrate in inside orbits?

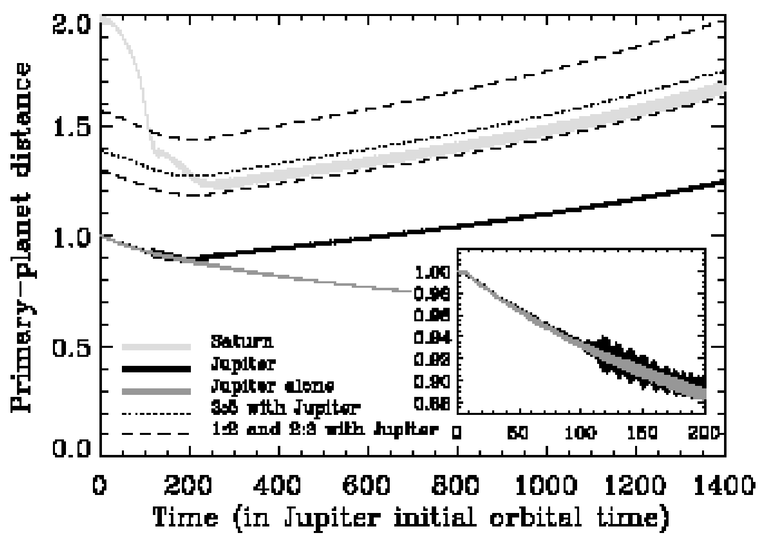


Figure 1. Primary–planet distances as a function of time. The outer dashed curve represents the nominal position of the 1:2 resonance with Jupiter, while the inner dashed curve is the nominal position of the 2:3 resonance. The zoomed plot enables one to closely compare Jupiter's orbital evolution against a test run without Saturn.

Masset & Snellgrove (2001).

Saturn forms close to Jupiter's gap. Type I migration (Saturn is smaller) takes it closer to Jupiter and it is trapped in a 2:3 mean motion resonance. While in resonance, the planets migrate outwards because external Lindblad resonances are proportional to M_S, while the internal ones are proportional to M_J (now the L-resonances are outside the gap since it includes both planets). This reverses migration.

Kozai resonance

and its effects on the eccentricity of planets in binary star systems.

16 Cyg B b and HD80606: planets in a Kozai resonance with the companion star of the primary?

(Holman, Touma, & Tremaine 1997, Wu & Murray 2003)

Planet (16 Cyg b B)

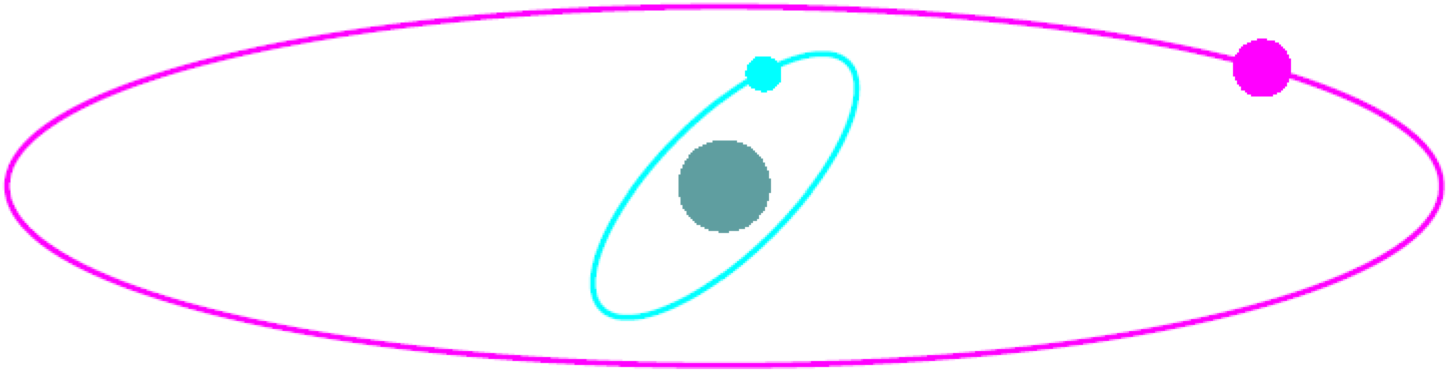
Period (d)	ecc	omega (deg)	Vel Amp, K (m/s)	Msini (M_jup)	a (AU)
798.938	0.67	84	51.24	1.69	1.67

Stella (16 Cyg b) Star separation about 840 AU?

Spectral Type	Mass (M_sun)	Apparent magnitude	Distance (pc)	P_rot (d)	[Fe/H]
G5V	1.01	6.25	21.41	26.20	0.09

Kozai resonance

Secular theory with disturbing function expansion to the second order in r/r' (for r'>>r) using Legendre polynomials. It works for highly inclined orbits (better than Laplace-Lagrange linear theory).



$$R = \frac{G_0 m' a^2}{8b'^3} \times [2 + 3e^2 - (3 + 12e^2 - 15e^2\cos^2\omega)\sin^2 I],$$

Con $b' = a'(1-e'^2)^{1/2}$

$$\frac{dI}{d\tau} = -\frac{15}{16\sqrt{1-e^2}}(e^2 \sin 2\omega \sin 2I),$$

$$\frac{de}{d\tau} = \frac{15}{8} e\sqrt{1-e^2} \sin 2\omega \sin^2 I,$$

$$\frac{d\omega}{d\tau} = \frac{3}{4\sqrt{1-e^2}}[2(1-e^2) + 5\sin^2\omega(e^2 - \sin^2 I)],$$

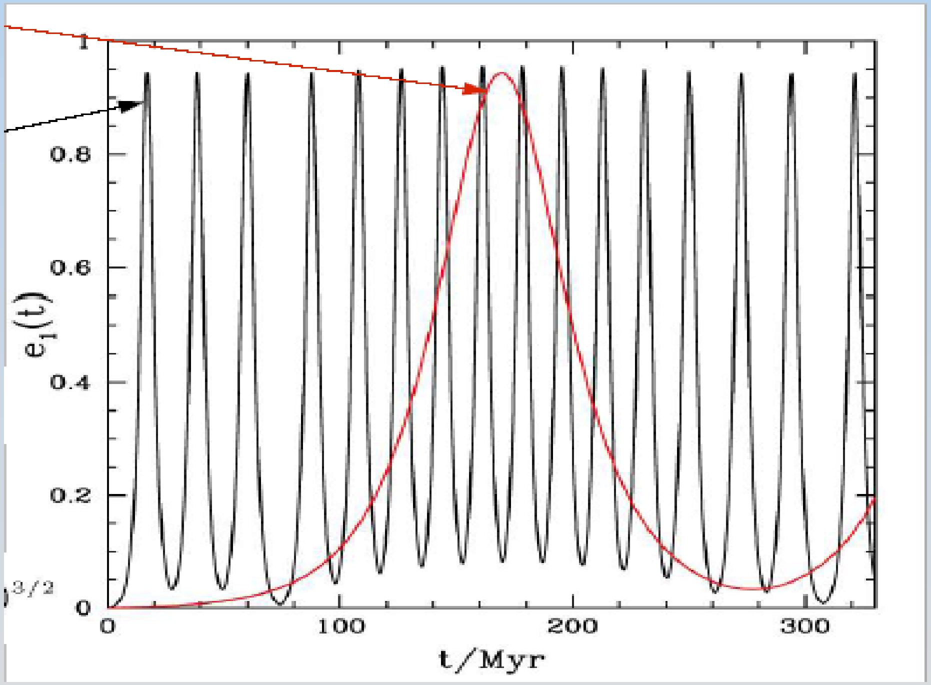
The sign of di/dt and de/dt are opposite. Anti-phased oscillations

$$\tau = G_0 m' t / n b'^3$$

$$R = -\frac{m'L^4}{8b'^3 G_0 m'}\left[-10 + \frac{3}{L^2}(3G^2 - 4H^2) + \frac{15H^2}{G^2} + 15\cos^2 g\left(1 - \frac{G^2}{L^2} - \frac{H^2}{G^2} + \frac{H^2}{L^2}\right)\right].$$

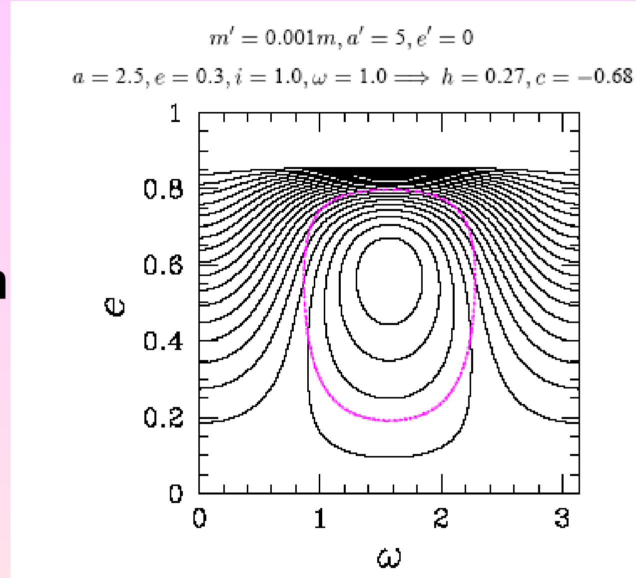
$$\Theta = (1 - e^2)\cos^2 I = H^2/L^2.$$

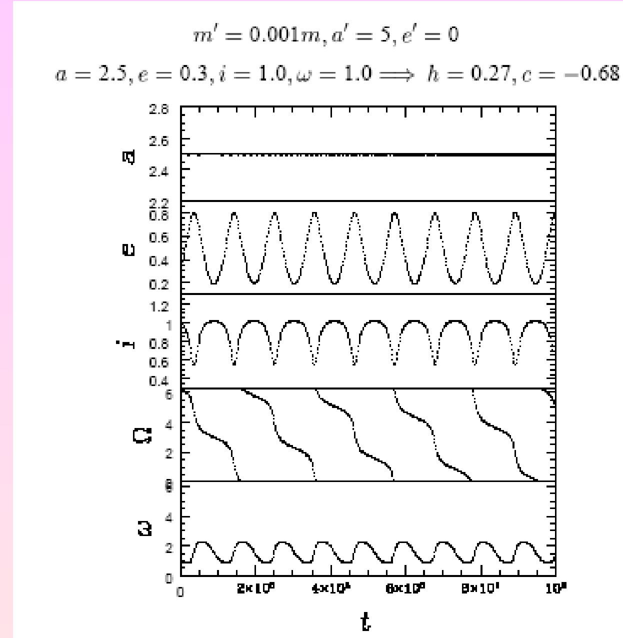
Kozai integral: It is constant -> anti-phased oscillations of e and i.



Oscillation period depends on the masses and binary semimajor axis.

- **Example of dynamical evolution while in Lozai resonance.**





- **Kozai migration (?): when the eccentricity is high tidal interaction with the star circularizes the orbit to a smaller semimajor axis. Repeated cycles causes migration.**

New secular theories of higher order.

The Laplace-Lagrange linear theory inadequate to describe extrasolar multiplat systems with high eccentric orbits.

The Laplace-Lagrange theory must be revised…

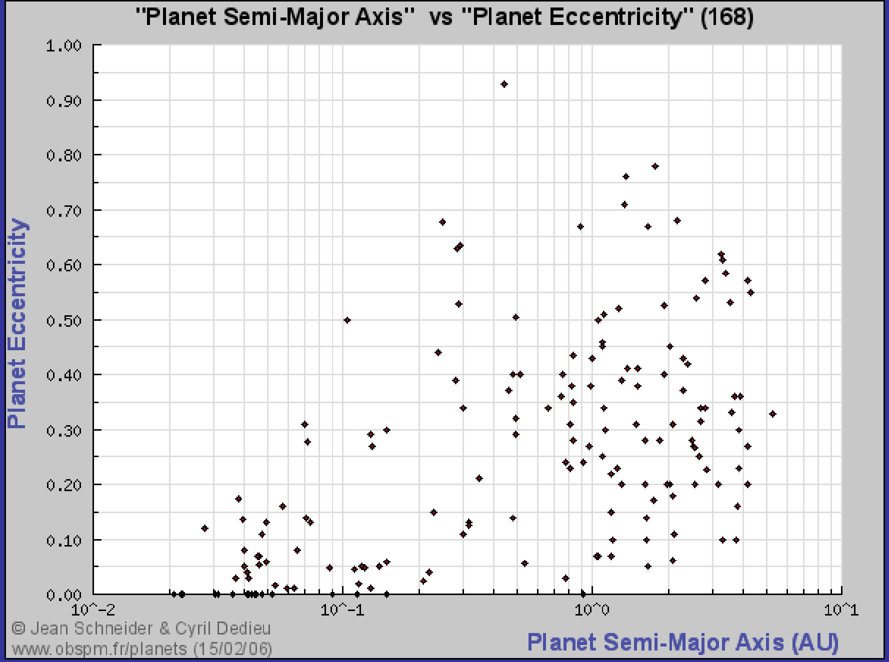
20 systems with more than one planet

2 with 4 (55 Cnc, HD 160691)

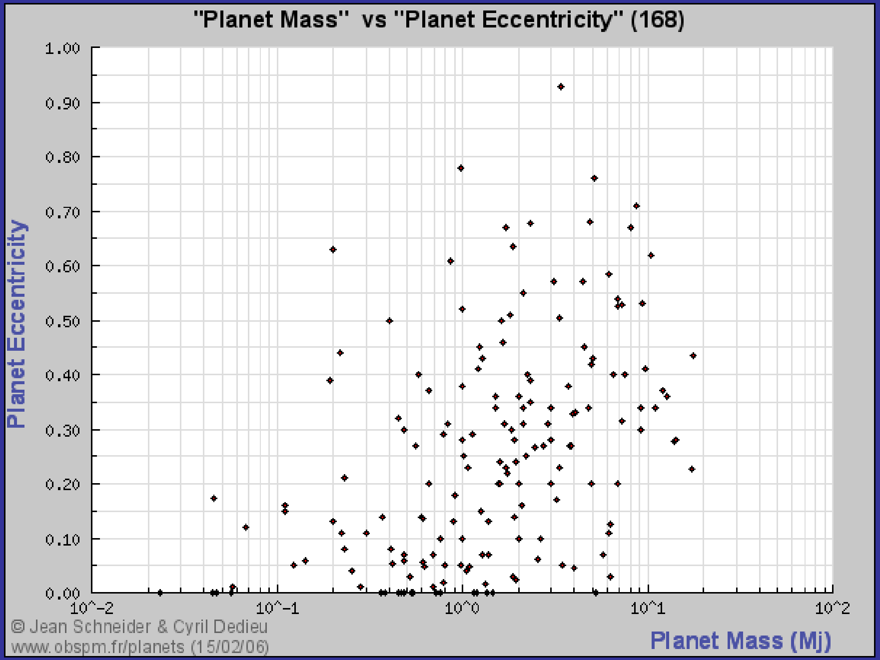
4 with 3 (Ups And, Gliese 876….)

14 with 2 (47 Uma, HD 8574 …..)

The L-L theory works for small e and *i* (like in the solar system) while many extrasolar systems have large e and i.



- HD74156 $e_1=0.636$ $e_2=0.583$
- HD202206 $e_1=0.435$ $e_2=0.267$
- HD12661 $e_1=0.350$ $e_2=0.20$
- HD128311 $e_1=0.25$ $e_2=0.17$
- Ups And $e_1=0.012$ $e_2=0.27$
- ....



Summary of the L-L theory for 2 planets. It is halted to the second order in e and *i*. THis is why it is inadequate to describe highly eccentric systems.

$$\mathcal{R}_j = n_j a_j^2 \left[\frac{1}{2} A_{jj} e_j^2 + A_{jk} e_1 e_2 \cos(\varpi_1 - \varpi_2) \right.$$
$$\left. \frac{1}{2} B_{jj} I_j^2 + B_{jk} I_1 I_2 \cos(\Omega_1 - \Omega_2) \right]$$

$$A_{jj} = +n_j \frac{1}{4} \frac{m_k}{m_c + m_j} \alpha_{12} \bar{\alpha}_{12} b_{3/2}^{(1)}(\alpha_{12})$$
$$A_{jk} = -n_j \frac{1}{4} \frac{m_k}{m_c + m_j} \alpha_{12} \bar{\alpha}_{12} b_{3/2}^{(2)}(\alpha_{12})$$
$$B_{jj} = -n_j \frac{1}{4} \frac{m_k}{m_c + m_j} \alpha_{12} \bar{\alpha}_{12} b_{3/2}^{(1)}(\alpha_{12})$$
$$B_{jk} = +n_j \frac{1}{4} \frac{m_k}{m_c + m_j} \alpha_{12} \bar{\alpha}_{12} b_{3/2}^{(1)}(\alpha_{12})$$

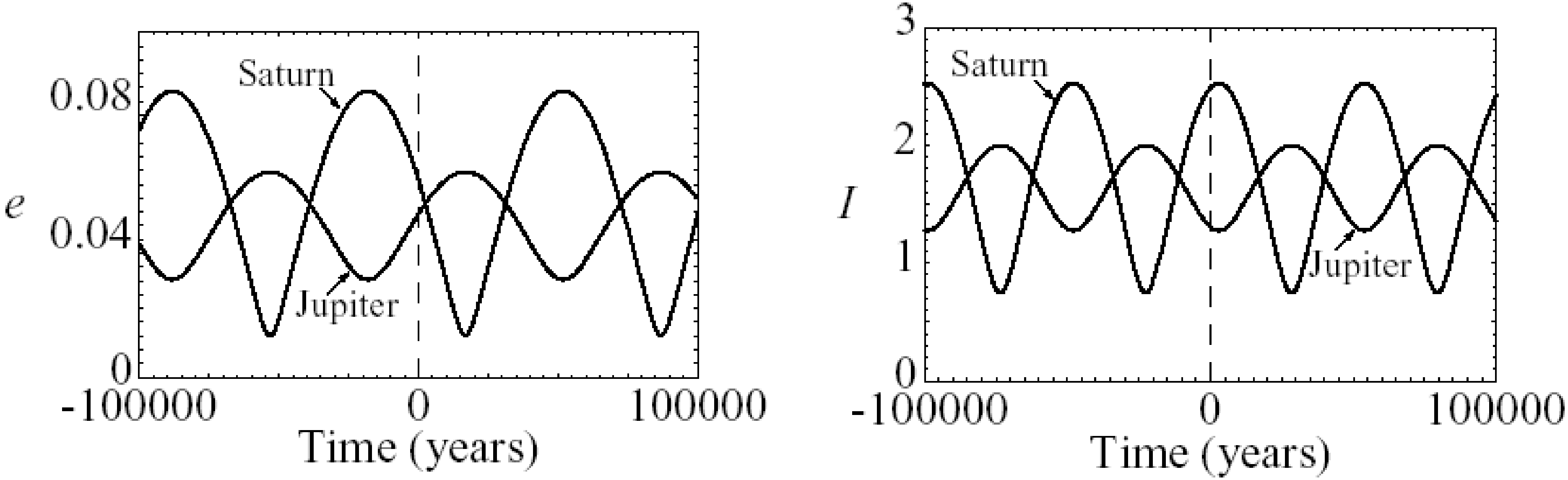
$$\dot{h}_1 = +A_{11} k_1 + A_{12} k_2 \qquad \dot{k}_1 = -A_{11} h_1 - A_{12} h_2$$
$$\dot{h}_2 = +A_{21} k_1 + A_{22} k_2 \qquad \dot{k}_2 = -A_{21} h_1 - A_{22} h_2$$
$$\dot{p}_1 = +B_{11} q_1 + B_{12} q_2 \qquad \dot{q}_1 = -B_{11} p_1 - B_{12} p_2$$
$$\dot{p}_2 = +B_{21} q_1 + B_{22} q_2 \qquad \dot{q}_2 = -B_{21} p_1 - B_{22} p_2$$

First order differential equations with constant coefficients.

$$h_j = \sum_{i=1}^{2} e_{ji} \sin(g_i t + \beta_i) \qquad k_j = \sum_{i=1}^{2} e_{ji} \cos(g_i t + \beta_i)$$

$$p_j = \sum_{i=1}^{2} I_{ji} \sin(f_i t + \gamma_i) \qquad q_j = \sum_{i=1}^{2} I_{ji} \cos(f_i t + \gamma_i)$$

3 frequencies



Eccentricity (and inclination) depends on a single frequency given by the difference between the two eigenvalues of matrix A.

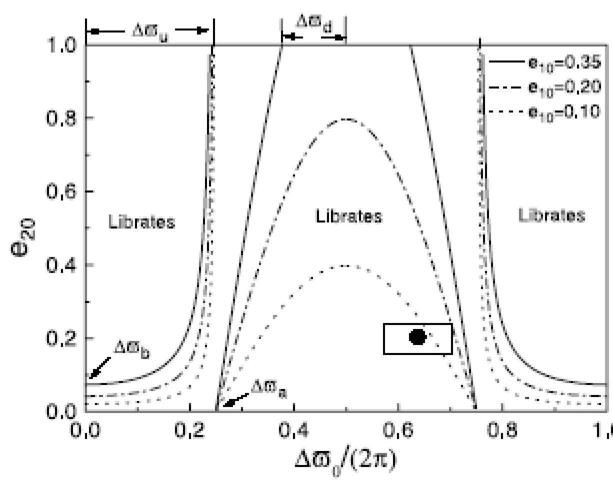
Apsidal corotation (g1~ g2)

$$e_1 e_2 \cos \Delta\varpi = (e_{11}e_{21} + e_{12}e_{22}) + (e_{11}e_{22} + e_{12}e_{21}) \cos(\psi_1 - \psi_2) \; .$$

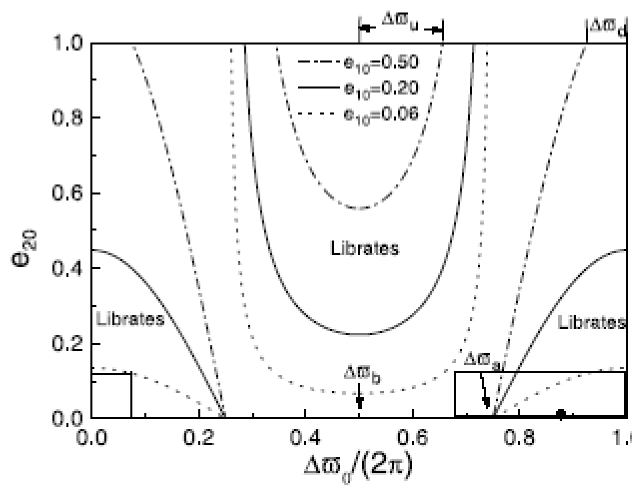
$$\psi_2 - \psi_1 = (g_2 t + \beta_2) - (g_1 t + \beta_1),$$

$$S = \left| \frac{e_{11}e_{22} + e_{12}e_{21}}{e_{11}e_{21} + e_{12}e_{22}} \right| \; .$$

Two-planets systems in apsidal corotation are more stable of those with circulation of the apses. (Lyapunov exponent calculations).



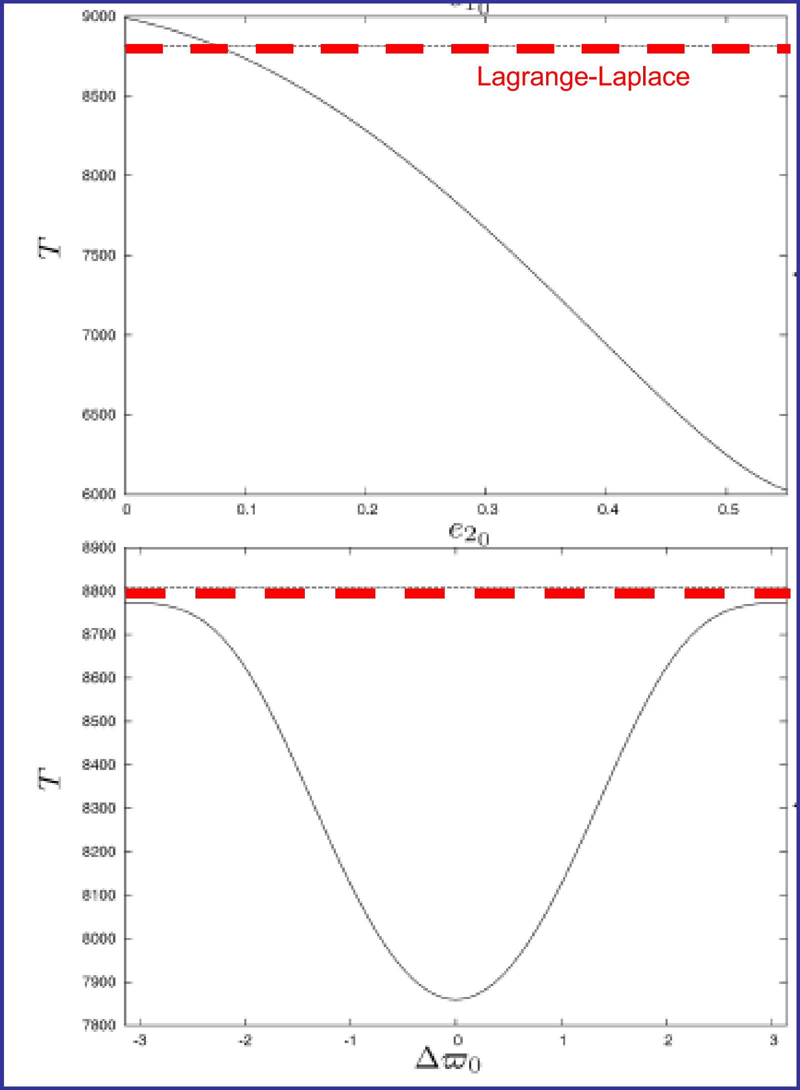
HD12661



47 Uma

Libert & Henrard (2006) Secular theory to order 12 in *e* and *i*.

$$-\frac{Gm_1m_2}{a_2}\sum_{k,i_1,i_2} C^k_{i_1,i_2} E_1^{k+2i_1} E_2^{k+2i_2} \cos k(p_1-p_2)$$

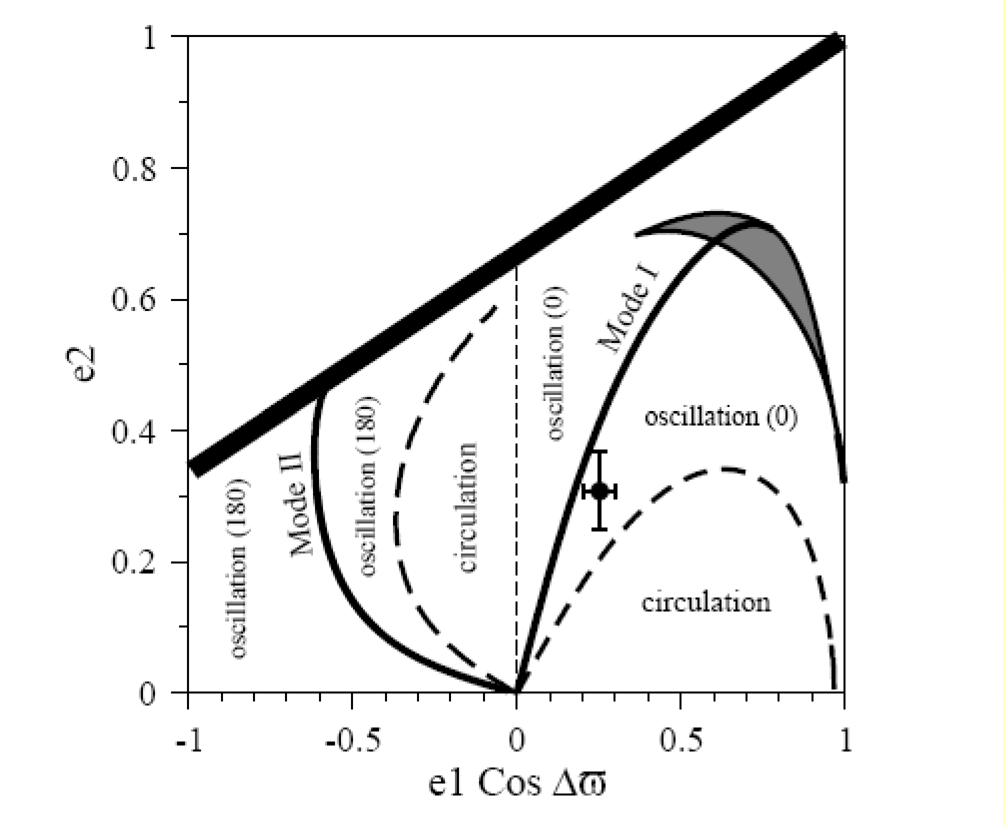


Comparison between the Libert & Henrard predictions with those of the L-L theory for the period of $\Delta\tilde{\omega}$

New secular theories …..

Michtchenko and Malhotra (2004), Michtchenko, Ferraz-Mello and Beaugè (2006), Lee and Peale (2003) ….

- **Better definition of the limits for which apsidal corotation occurs.**
- **Discovery of a new non-linear resonance.**

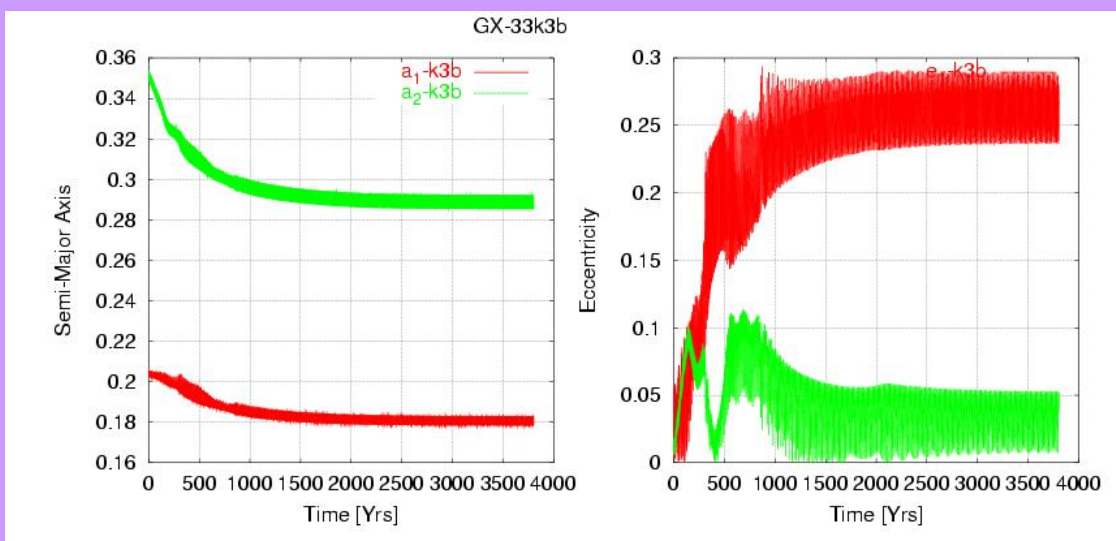


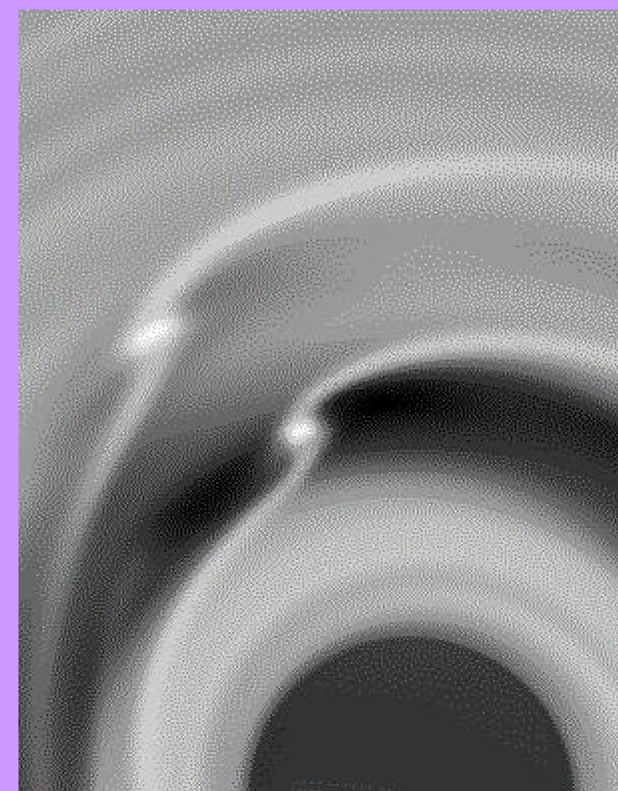
PLanets in mean motion resonances.

Migration favors capture in mean motion resonances

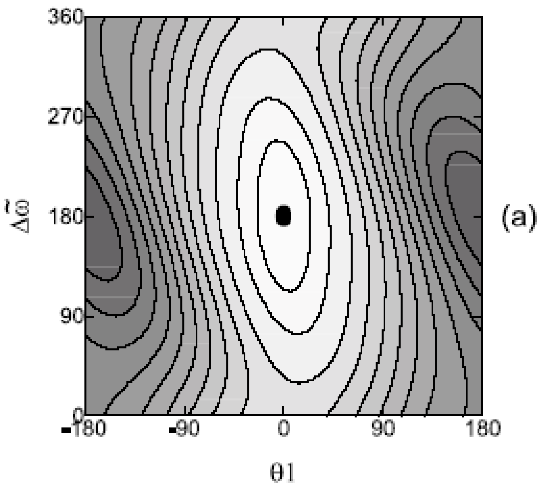
GJ876, HD82943, 55Cnc..

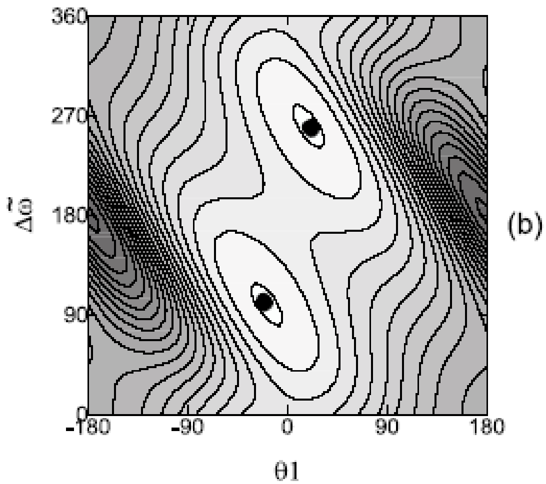
During migration, planets enter the resonance and evolve toghether.





Analytical approach..





$$q\sigma_1 = (p+q)\lambda_2 - p\lambda_1 - q\varpi_1 \ ,$$
$$q\sigma_2 = (p+q)\lambda_2 - p\lambda_1 - q\varpi_2 \ .$$

$$F_1 = \frac{Gm_1 m_2}{a_2} \sum_{j,k,l,u,s} R_{j,k,l,u,s}(\alpha-\alpha_0)^j \times e_1^k e_2^l \cos[uq\sigma_1 + s(\sigma_2-\sigma_1)] \ .$$

Development of the disturbing function.

Ad example the 2/1..:

- **$\sigma_1=2\lambda_2-\lambda_1-\tilde{\omega}_1$**
- **$\sigma_2=2\lambda_2-\lambda_1-\tilde{\omega}_2$**
- **$\Delta\tilde{\omega}=\tilde{\omega}_2-\tilde{\omega}_1$**

Beaugè, Ferraz-Mello & Michtchenko (2003, ..2006)

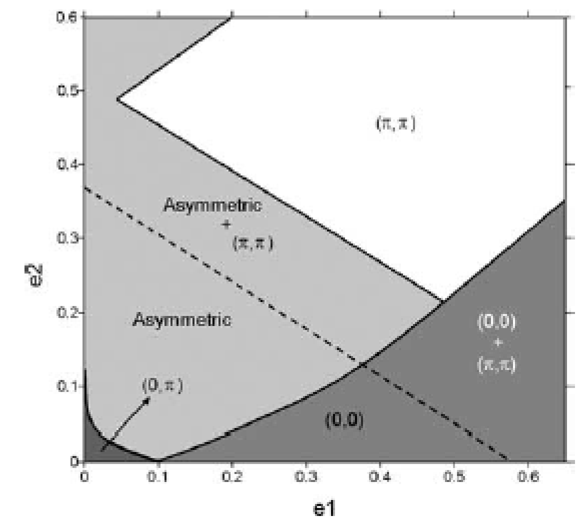


Figure 1. Domains of different types of exact corotational solutions in the 2/1 MMR, as seen in the plane of orbital eccentricities of both planets. See text for further explanations.

Numerical approach

Numerical integration of alarge number of systems and FMA analysis of stability.

Marzari, Scholl & Tricarico (2006)

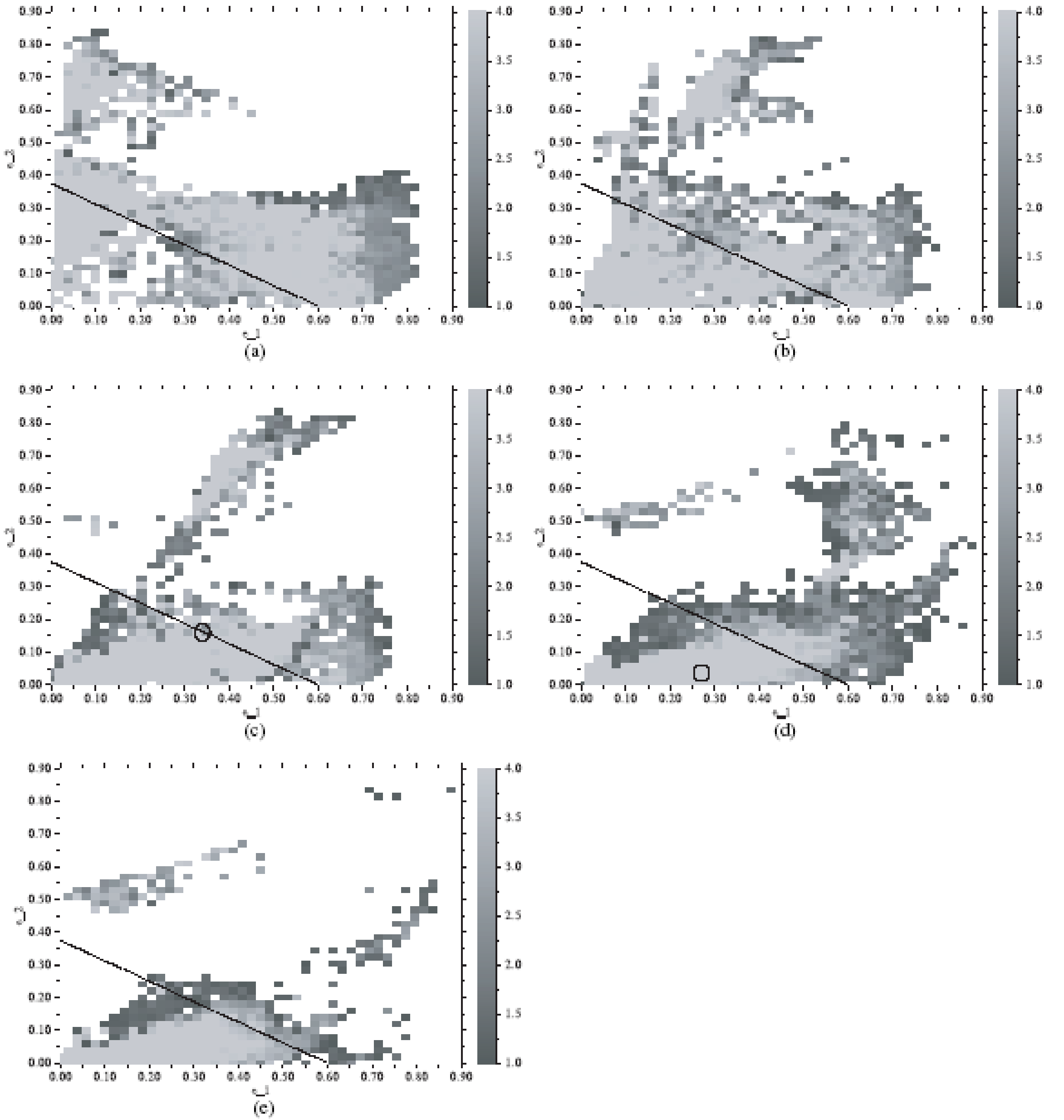


Fig. 1. Diffusion maps in the (e_1, e_2) plane for different planetary mass ratios μ. **a)** is for $\mu = 0.1$, **b)** for $\mu = 0.6$, **c)** for $\mu = 1.0$, **d)** for $\mu = 3.3$, and **e)** for $\mu = 10.0$. The grey levels correspond to different values of the diffusion speed measured by σ_{FMA}. Regions with large values of σ_{FMA} (light grey) are stable over a long timescale. The dashed line in each plot marks the boundary between crossing and non-crossing orbits. The empty circle in the plot for $\mu = 1.0$ shows the location in the map of the HD 82943 system, while that in the plot for $\mu = 3.3$ represents the GJ 876 system.

Planets in stable resonance not always are in apsidal corotation! Equal probability for both cvonfigurations. From an analytical point of view only corotating systems are studied.

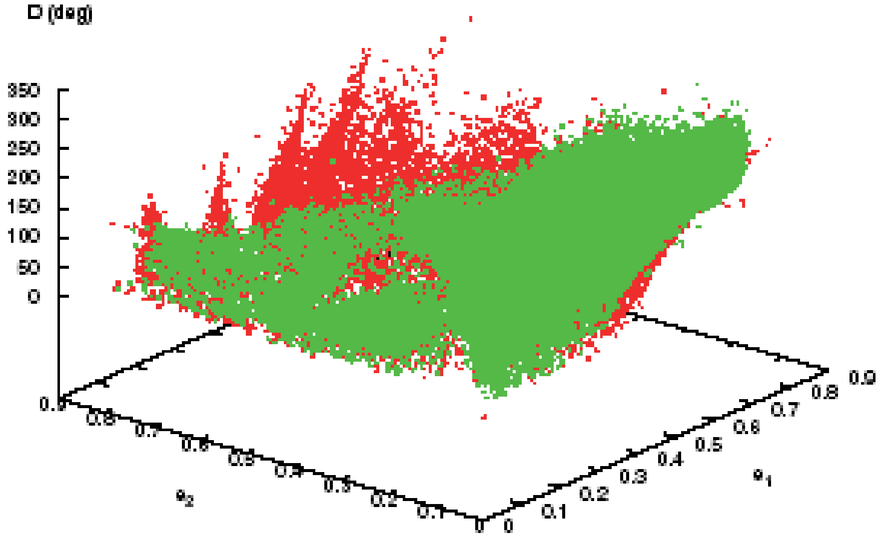
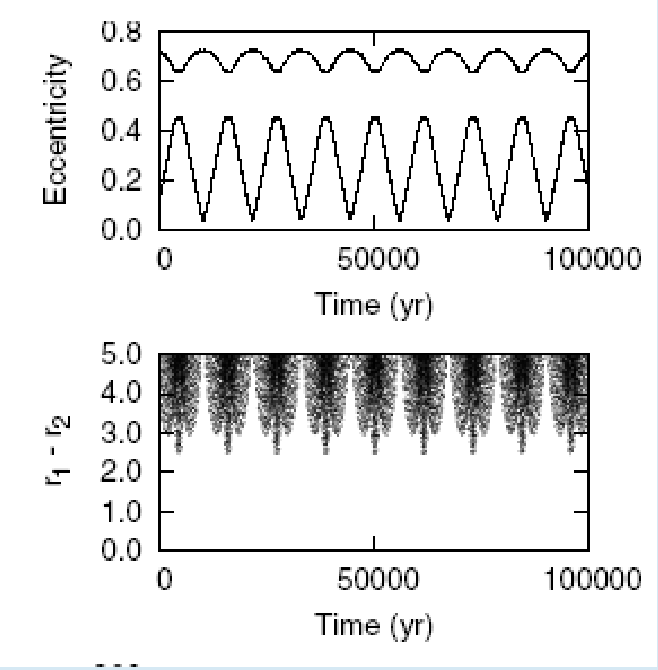


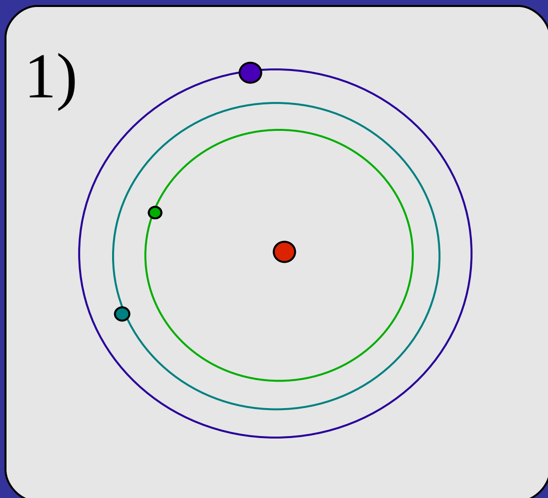
Fig. 4. Three-dimensional distribution of ACRs (red dots) and NACRs (green dots) in the (e_1, e_2, D) space. The regions of different apsidal behaviour are clearly separated.



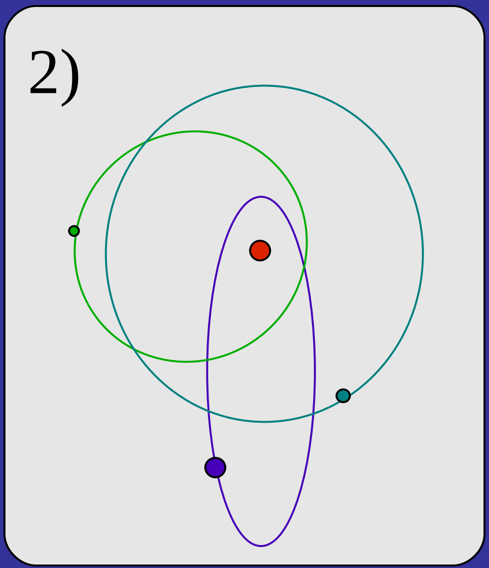
Protection mechanisms against mutual close encounters and collisions related to the resonance.

Gravitational interactions between planets
(The Jumping Jupiters)

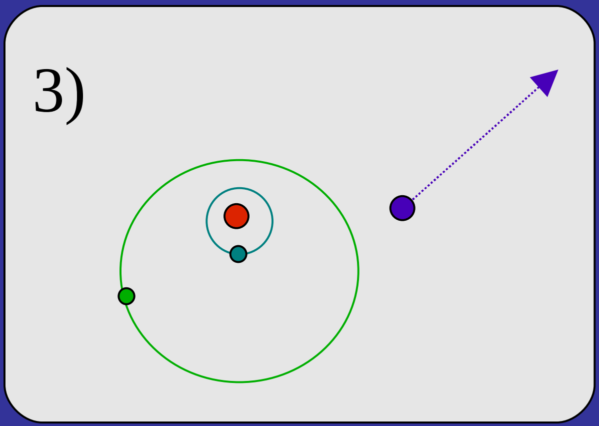
(Weidenschilling & Marzari 1996 ; Marzari & Weidenschilling 2002)



Giant planets form beyond the frost line according to the **standard model**.



Planetary orbits become unstable and have close encounters (**chaotic phase**).



Ejection of one planet on a hyperbolic orbit.
Injection of one planet in an internal and eccentric orbit.,

Tidal interaction with the star can circularize the orbit to the perihelion.

In conclusion.....

- New dynamical mechanisms are needed to explain extrasolar planetary systems.
- There are still problems with the migration scenarios, concerning in particular the migration speed and the eccentricity of planets.
- Resonances of any kind help to maintain stability on long timescales.
- Different mechanisms work in sinergy to produce observed systems (Morehead & Adams 2005).
- It is time to get rid of that..&@#!!%&.. observative bias that favors planets in close orbits to the stars.