“Hydrogen is a light, odorless gas, which, given enough time, turns into people.”

Edward Robert Harrison
Measuring the Jeans Scale of the Intergalactic Medium with Close Quasar Pairs

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OUTLINE

- Structure formation, reionization and thermal history of the IGM
- Probing the IGM with the Lyman-\(\alpha\) forest
- Statistics and degeneracies in the Ly-\(\alpha\) forest: a parameter study
- A new statistic: phase differences in quasar pairs
- Properties and constraining power of phase statistic
- Preliminary results at \(z=2\) and \(z=3\)
The Epoch(s) of Reionization

Cold, neutral IGM

Hot, ionized IGM

Z ~ 30 - 6: H and He singly ionized

Z~2-4: He II ionized

Ionizing photons from first galaxies

Hard photons (> 4 Ry) from quasars

13.7 BILLION YEARS OF AGE

13

11

PRESENT
Density Distribution in the Universe

- Cold IGM: $T=0$ K
- Hot IGM: $T=10^4$ K
The Jeans Scale

- Self-gravitating structure of size $\lambda$ collapses if free-fall time $(G\rho)^{-1/2} < \lambda c_s$ sound-crossing time
  
  critical $\lambda$ is called "Jeans scale"

  $$\lambda_J = \frac{c_s}{1+z} \sqrt{\frac{\pi}{G\rho}} \approx 1.1 \text{ Mpc} \left(\frac{T}{10^4 \text{K}}\right)^{1/2} \left(\frac{1+z}{1+3}\right)^{-1/2}$$

- Small-scale-structure growth and Galaxy formation suppressed below the Filtering mass:

  $$M_F = \frac{4}{3} \pi \bar{\rho} \lambda_J^3$$

- Jeans filtering scale depends on the whole thermal history (Gnedin & Hui, 1998): sound-crossing time $\lambda_j/c_s$ is comparable to the Hubble time

  $$\lambda_j^2(t) = \int_0^t f(T(t'))dt'$$
Factors Governing Reionization and Thermal History

- Population and spectrum of ionizing sources
- Escape fraction
- Clumping factor (also related to Jeans scale!)
- Adiabatic cooling due to expansion
- Perturbations growth

Under the assumption of ionization equilibrium:

\[ T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{\gamma-1} \]

“Equation of state” of the IGM (Hui and Gnedin, 1997)
The Instantaneous temperature of the IGM provides little information about reionization heating occurred in the distant past.

\[ T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{\gamma^{-1}} \]

Reionization at
\[ z = 4 \]
\[ z = 2 \]

Hui and Haiman 2003, measurements from Zaldarriaga et al. 2001
Galaxies

• Reionize Hydrogen and Helium in the IGM

• Energetic photons deposit energy: higher IGM temperature

• Pressure increase changes the Jeans scale of the IGM

• Fuel for galaxy formation

• Pressure determines the minimum-mass galaxy that can collapse

• Clumps absorb ionizing photons and slow down reionization
Neutral hydrogen clouds

Low redshift:
- Lower density
- Higher ionized fraction

High redshift:
- Higher density
- Lower ionized fraction

Probing the IGM: the Lyman $\alpha$ Forest
Physics of the Lyman $\alpha$ Forest

- Photoionization equilibrium
  \[ n_{HII} \approx n_H^2 \frac{\alpha(T)}{\Gamma} \]
- Equation of state
  \[ T \propto (1 + \delta)^{\gamma^{-1}} \]
- Fluctuating Gunn-Peterson approximation
  \[ \tau \propto (1 + \delta)^{2^{-0.7(\gamma-1)}} \]

- Physics relatively easy to understand
- Not biased by light-to-mass assumptions
- Probes smaller scales than CMB
Relation Forest – Thermal State

Credit: R. Cen
Line broadening must be connected to temperature → 

Several statistical tools used to constrain temperature:
- Power spectrum
- Flux Probability distribution
- Wavelet analysis
- Line fitting
- Curvature

The Jeans scale also contributes to line width, and must be assumed or modeled in these measurements.
There are ~ 300 known pairs at $1.6 < z < 4.3$ ar $r < 1$ Mpc

(Hennawi et al. 2004, 2006, 2009)

- Close separation (380 comoving kpc)
- High degree of coherence

We are observing the Jeans scale at $z=4$
Key Questions

cold and steady IGM: Lyman-α forest traces density exactly.

Real IGM: **thermal broadening** and **redshift-space distortion** introduce degeneracies among density distribution and thermal properties of the IGM.

- To which extent does pressure affect the statistics of the Lyman-α forest lines?
- Which is the optimal statistic for measuring the Jeans scale with pairs?
- What additional constraints do quasar pairs put on the parameters governing the IGM thermal state?
Methodology: Simulations and Parameter Study

1. Dark matter only simulation, snapshot at $z=3$ ($Box=50 \ Mpc/h$, $n=1500^3$)
2. Mimic pressure support by a smoothing density with a kernel of radius $\lambda_j$.
3. Definition of the equation of state

$$T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{-\gamma^{-1}}$$

We calculate 500 different thermal models, defined by $T_0$, $\gamma$ and $\lambda_j$. 

$r = 130 \ kpc$
Sensitivity of the LOS Power Spectrum to 1D Smoothing

\[ z = 3 \]

\[ F \]

\[ v \text{ [km/s]} \]

\[ k \text{ [s/km]} \]

\[ P(k) \propto |F(k)|^2 \]

\[ T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{\gamma-1} \]

\[ T_0 \text{ [10}^3 \text{ K]} \]

\[ \gamma \]

\[ \lambda_r \text{ [kpc]} \]
Sensitivity of the LOS Power Spectrum to 1D Smoothing

\[ z = 3 \]

\[ T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{\gamma^{-1}} \]

\[ P(k) \propto |F(k)|^2 \]
Sensitivity of the LOS Power Spectrum to 3D Smoothing

$z = 3$

$P(k) \propto |F(k)|^2$

$T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{\gamma^{-1}}$
1D Degeneracy Broken by Pairs

\[ T(\rho) = T_0 \left( \frac{\rho}{\rho_0} \right)^{\gamma^{-1}} \]

Line-of-sight power spectrum

\[ P(k) \propto |F(k)|^2 \]

Cross power spectrum

\[ \pi_{1,2}(k, r_{1,2}) \propto \Re\{\tilde{F}_1^*(k)\tilde{F}_2(k)\} \]
Cross Power: Definition

- Fourier transformed of the cross-correlation
- Cross-product of Fourier modes
- Can be expressed in terms of moduli and phases:

\[ F_j (k) = a_j (k) \exp[i\Theta_j (k)] \]

\[ \pi_{1,2} (k) = a_1 (k) a_2 (k) \cos(\Theta_1 (k) - \Theta_2 (k)) \]
Cross Power: Sensitivity to 1D smoothing

\[ P(k) = \pi(k, r=0) \]

\[ \pi_{1,2}(k, r_{1,2}) \propto \Re\{\tilde{F}_1^*(k)\tilde{F}_2(k)\} \]

\[ z = 3 \]
Cross Power: Sensitivity to 1D smoothing

\[ P(k) = \pi(k, r=0) \]

\[ \pi_1,2(k, r_{1,2}) \propto \Re \{ \tilde{F}_1^*(k) \tilde{F}_2(k) \} \]
Cross Power: Sensitivity to 3D smoothing

\[ P(k) = \pi(k, r = 0) \]
Isolating the 3D information: Phase Differences

Rorai et al., 2013

\[ \pi_{1,2}(k, r_{1,2}) \propto \Re\{\tilde{F}_1^*(k)\tilde{F}_2(k)\} \]

In terms of moduli and phases:

\[ \tilde{F}(k) = \rho(k)e^{i\theta(k)} \]

\[ \pi(k, r_{1,2}) \propto \rho_1(k)\rho_2(k)\cos\theta_{12}(k) \]

The amplitudes of the modes depend on the LOS power:

\[ P(k) \sim \langle \rho_1 \rho_2 \rangle \]

Angular part retains the new 3D information of pairs

\[ \theta_{12}(k) = \arccos \left( \frac{\Re[\tilde{F}_1^*(k)\tilde{F}_2(k)]}{\sqrt{|\tilde{F}_1(k)|^2|\tilde{F}_2(k)|^2}} \right) \]

Problem: what is the best estimator for the angular component?
Phase Probability Distributions

Transverse separation: $r = 333$ kpc

Fourier mode:
- $k = 9.4 \times 10^{-2}$ s/km
- $\lambda = 2.2$ Mpc

Thermal model:
- $\lambda_j = 50$ kpc
- $\lambda_j = 200$ kpc

Rorai et al., 2013
Phase PDF: Fit with Wrapped Cauchy

\( r = 33.3 \text{ kpc} \)

\( k = 9.4 \times 10^{-2} \text{ s/km} \)
\( \lambda = 2.2 \text{ Mpc} \)

\( p(\theta) \)

\( \lambda_j = 50 \text{ kpc} \)
\( \lambda_j = 200 \text{ kpc} \)

Rorai et al., 2013
Phase PDF: Dependencies

\[ k = 3.7 \times 10^{-2} \text{ s/km} \]
\[ \lambda = \frac{2\pi}{Hak} = 2.2 \text{ Mpc} \]

\[ k = 9.4 \times 10^{-2} \text{ s/km} \]
\[ \lambda = \frac{2\pi}{Hak} = 0.9 \text{ Mpc} \]
Bayesian Inference from Phase PDF

1 - Quasar pair data set → ensemble of phase differences \( \{\Theta(k_i, r_j)\} \)

2 – Grid of simulated thermal models → Prediction of phase probability distributions as a function of thermal parameters \((Likelihood\ function)\)

\[
L(\{\theta\}| T_0, \gamma, \lambda_j) = \prod_{i,j} P(\theta(k_i, r_j)| T_0, \gamma, \lambda_j)
\]

3 - Via MCMC techniques, we do a Bayesian analysis of parameter space, estimating:

- **Degeneracies** between parameters
- **Accuracy** of a measurement
Jeans scale measurements are independent on the equation of state
We can achieve a precision up to 5% with only 20 pairs.

Phase difference statistic is by far more efficient than LOS power and cross-power.

Noise decreases the precision by few %.

Rorai et al., 2013
Cross-power spectrum (and thus cross-correlation) contains line-of-sight information.

Phases differences represent genuine 3D information.

Phase Probability functions follow with good approximation a general functional form.

Phase statistics is very sensitive to the Jeans scale and independent on the EOS.
Pair Sample: Overlapping Forest Distribution in $z$ and $r$
Real Phases: PDF and Dependencies

Typical scale = 4.4 Mpc

Typical scale = 1.4 Mpc

2.4<z<3.0
2.0<z<2.4
Forward-Modeling of Simulation

Real Pair

Simulated Noise

r=190 kpc, APOJ1622+0702
Conclusions

- The Jeans scale of the IGM is of key importance in cosmology:
  - Threshold for galaxy formation
  - Record of the thermal history

- Close quasar pairs open new possibilities to probe the small-scale structures of the universe
  - Phase analysis will provide a first precise measurement of the Jeans Scale
  - Phase statistic insensitive to the equation of state of the IGM
  - Combinations with other Ly-α forest statistics can break degeneracies and improve constraints on the thermal history

- Small Jeans scale?
  - Preliminary results point toward $\lambda_J < 60-40$ kpc at $z=2-3$
  - This would imply an abundance of small-scale structures
  - Comparison with hydro simulations will be made in the future
Thank you!
A filament produces absorption features in two close spectra.

Phase differences are driven by the orientation of these filaments.

If the orientation $\phi$ is uniformly distributed, then the phase $\Theta$ follows a wrapped Cauchy distribution:

$$P_{WC}(\theta; \zeta) = \frac{1}{2\pi} \frac{1 - \zeta^2}{1 + \zeta^2 - 2\zeta \cos(\theta)}$$

Defines a 1-parameter family of functions.

Rorai et al., 2013
Nice Properties of Phase Differences

- Invariant under convolution with symmetric Kernel $W$

$$W(k) = \int W(x) \cos(kx) + i \int W(x) \sin(kx) = \text{real}$$

- Thermal Broadening has very small effect
- Resolution does not need to be precisely modeled

- Sensitive to the Jeans scale also at low $k$
  - High-resolution spectra are not required if we resolve the Jeans scale in the transverse dimension

$$P(k) = \int_k^\infty 2\pi k' P_{3D}(k') dk'$$

- Invariant under rescaling of the normalized flux

$$\theta_{12}(k) = \arccos \left( \frac{\Re[\tilde{F}_1^*(k)\tilde{F}_2(k)]}{\sqrt{|\tilde{F}_1(k)|^2|\tilde{F}_2(k)|^2}} \right)$$

- Robustness to uncertainties in continuum fitting
Sensitivity to the Equation-of-State Parameters

Phases are almost independent of the equation of state.

\[ k = 3.7 \times 10^{-2} \text{s/km} \]
\[ \lambda = \frac{2\pi}{Hak} = 2.2 \text{Mpc} \]

\[ k = 9.4 \times 10^{-2} \text{s/km} \]
\[ \lambda = \frac{2\pi}{Hak} = 0.9 \text{Mpc} \]

\[ T_0 = 0.7 \times 10^4 \text{K, } \gamma = 1.7 \]
\[ T_0 = 2.4 \times 10^4 \text{K, } \gamma = 1.2 \]
\[ T_0 = 1.4 \times 10^4 \text{K, } \gamma = 0.8 \]
The Cross-Power is partially affected by the line-of-sight degeneracies.
Bias tests

Repeating our “experiment” for 400 different mock dataset shows that our method is not biased.

Overestimating the S/N by 20% does not significantly modify the outcome of the measurement.
Sensitivity to the Jeans scale

The wrapped Cauchy function provides an excellent fit to the simulation results!

Phase difference distribution for DENSITY, in real space

\( r = 142 \text{ kpc} \)

\( k = 3.7 \times 10^{-2} \text{ s/km} \)
\( \lambda = 2.2 \text{ Mpc} \)

\( r = 333 \text{ kpc} \)

\( k = 9.4 \times 10^{-2} \text{ s/km} \)
\( \lambda = 0.9 \text{ Mpc} \)

\( \lambda_J = 50 \text{ kpc} \)
\( \lambda_J = 100 \text{ kpc} \)
\( \lambda_J = 200 \text{ kpc} \)
Are Phases Independent?

Correlation factors are at most of the order of 3%
The Lyman α forest can be used to constrain the thermal state of the IGM.
Sensitivity to the Jeans scale

Why does pressure smoothing affects large-scale modes of the forest?

High-k 3D modes projects into low-k modes in 1D spectra