Behaviour of Relativistic Black Hole Accretion Sufficiently Close to the Horizon

Paramita Barai[1], Ipsita Chakraborty[2], Tapas K Das[3,4], Paul J. Wiita[5,6]

[1] Departement de physique, Université Laval, Quebéc City, Canada (paramita.barai.1@ulaval.ca)
[2] Adamas Institute of Technology, Kolkata 700126, India (ipsita14@gmail.com)
[3] Harish Chandra Research Institute, Allahabad 211019, India (tapas@hri.res.in)
[4] Theoretical Institute for Advanced Research in Astrophysics, Taiwan (tapas@tiara.sinica.tw.edu)
[5] Institute for Advanced Study, Princeton University, USA (wiita@sns.ias.edu)
[6] Department of Physics and Astronomy, Georgia State University, USA (wiita@chara.gsu.edu)

Abstract

This work introduces a novel formalism to investigate the role of the spin of astrophysical black holes in determining the behaviour of matter falling onto such accretors. Equations describing the general relativistic hydrodynamic accretion flow in the Kerr metric are formulated, and stationary solutions for such flow equations are provided. The accreting matter may become multi-transonic, allowing a stationary shock to form for certain initial boundary conditions. Such a shock determines the disc geometry and can drive strong outflows. The properties of matter extremely close to the event horizon are studied as a function of the Kerr parameter, leading to the possibility of detecting a new spectral signature of black hole spin.

Flow Structure

- The energy momentum tensor $\Im^{\mu\nu}$ has been formulated (Novikov & Thorne [1973] gave the general most form of $\Im^{\mu\nu}$) in the Boyer-Lindquist co-ordinates, and its covariant derivative has been evaluated to obtain the general relativistic Euler equation and the equation of continuity, in the form of a set of spatio-temporal first order differential equations.
- Assuming axi-symmetry for the flow geometry (Fig. 1), the disk height (Abramowicz, Lanza & Percival [1997]):

$$h(r)=\sqrt{\frac{2}{\gamma+1}}r^2\left[\frac{(\gamma-1)c_c^2}{\left\{\gamma-\left(1+c_s^2\right)\right\}\left\{\lambda^2v_t^2-a^2(v_t-1)\right\}}\right]^{1/2}$$

- Two first integrals of motion for the system defined along streamlines:
  - ❑ Conserved specific flow energy:

$$\varepsilon=\left[\frac{(\gamma-1)}{\gamma-\left(1+c_s^2\right)}\right]\sqrt{\left(\frac{1}{1-u^2}\right)\left[\frac{Ar^2\Delta}{A^2-4\lambda arA+\lambda^2r^2\left(4a^2-r^2\Delta\right)}\right]}$$

  - ❑ Baryonic load rate: $\dot{M}=4\pi\Delta^{1/2}H\rho\frac{u}{\sqrt{1-u^2}}$
- And the corresponding entropy accretion rate:

$$\dot{\Xi}=\left(\frac{1}{\gamma}\right)^{1/(\gamma-1)}4\pi\Delta^{1/2}c_s^{2/(\gamma-1)}\frac{u}{\sqrt{1-u^2}}\left[\frac{(\gamma-1)}{\gamma-\left(1+c_s^2\right)}\right]^{1/(\gamma-1)}H(r)$$

- Differential solutions of the first integrals of motion provides the three velocity gradient (of the flow) as a first order autonomous dynamical system:

$$\frac{du}{dr}=\frac{\frac{2c_s^2}{(\gamma+1)}\left[\frac{r-1}{\Delta}+\frac{2}{r}-\frac{v_t\sigma\chi}{4\psi}\right]-\frac{\chi}{2}}{\frac{u}{\left(1-u^2\right)}-\frac{2c_s^2}{(\gamma+1)\left(1-u^2\right)u}\left[1-\frac{u^2v_t\sigma}{2\psi}\right]}$$

- Here:
  - r = Radial distance on the equatorial plane,
  - u = Three (flow) velocity,
  - c_s = Sound speed,
  - ρ = Density,
  - λ = Flow specific angular momentum,
  - γ = Flow adiabatic index,
  - a = Black hole spin (the Kerr parameter),
  - A, $\Delta = f(r, a)$,
  - $v_t = f(u, r, \lambda, a, A, \Delta)$.
- Other terms (esp. in du/dr) are functions of above quantities and of various metric elements as well as their space derivatives (details in Barai, Das & Wiita [2004]).

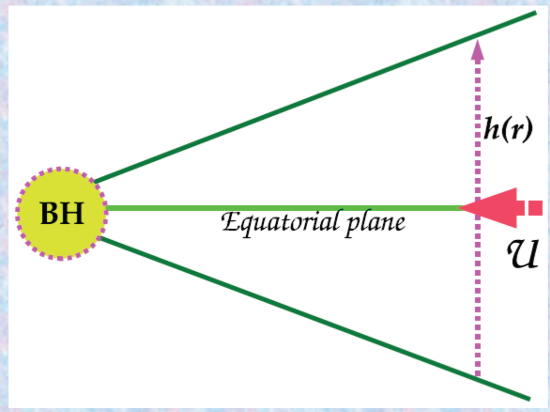

Fig. 1. --- The yellow circular patch with BH written inside represents the black hole, and the pink dashed boundary mimics the event horizon. The wedge shaped dark green lines represents the envelop of the accretion disc. The light green line centrally flanked by the two dark green disk boundaries, is the equatorial plane, on which all of the dynamical quantities (e.g., the advective three velocity u) are assumed to be confined. Any thermodynamic quantity (e.g., the flow density) is averaged over the local disc height $h(r)$.

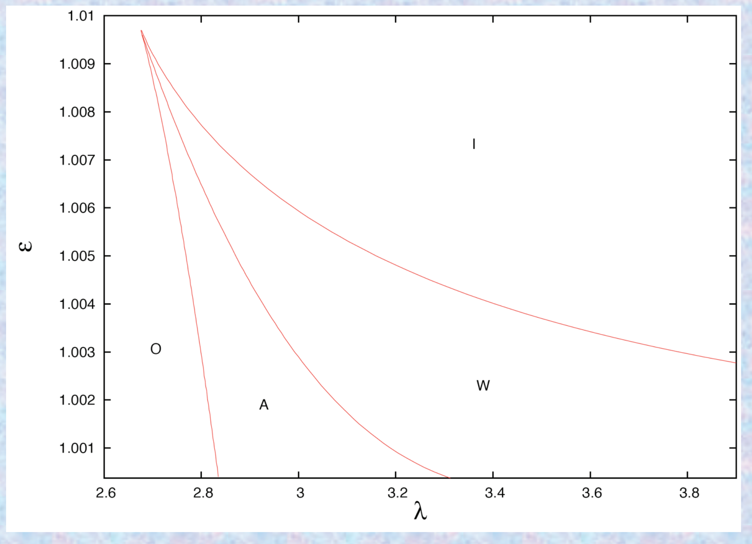

Fig. 2. --- Region of multi-transonicity for both accretion (marked A) and wind (marked W) in the [ε-λ] parameter space, with a=0.3 and γ=1.33. The labels I and O indicate regions with lone inner and outer critical points, respectively.

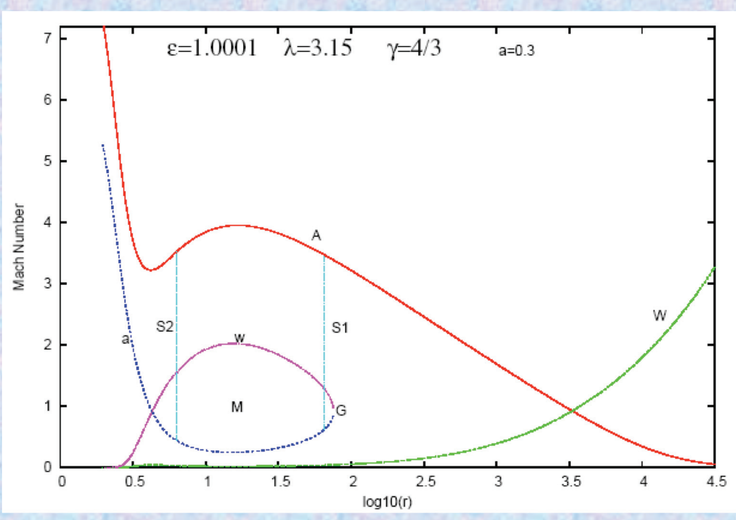

Fig. 3. --- Transonic flow (marked by 'A') through the outer sonic point encounters a stable stationary Rankine-Hugoniot shock transition marked by S1 (S2 being the other formal shock location, which is unstable and hence is not considered), produces post shock subsonic flow, which again becomes supersonic (segment marked by 'a') after passing through the inner sonic point; and finally plunges through the event horizon.

Transonicity, Shock and Outflow Generation

- The fixed point solution obtained from du/dr provides the number of critical points the flow can pass through, and thus provides the number of transonic flips.
- Fig. 2 shows an e.g. of multi- and mono-transonic regions.
- Multi-transonic (actually, bi-modal, since the middle sonic point does not support a steady solution that passes through it) accretion flows may encounter a stationary shock. The general relativistic shock conditions are:

$$\left[\left[\rho u\Gamma_u\right]\right]=0,\ \left[\left[\Im_{t\mu}\eta^\mu\right]\right]=\left[\left[(p+\varepsilon)v_tu\Gamma_u\right]\right]=0,$$
$$\left[\left[\Im_{\mu\nu}\eta^\mu\eta^\nu\right]\right]=\left[\left[(p+\varepsilon)u^2\Gamma_u^2+p\right]\right]=0$$

where, Γ_u = the Lorentz factor,
η_μ = the normal to the hypersurface Σ of discontinuity.

- The shock invariant S_h is:

$$S_h=c_s^{\frac{2\gamma+3}{\gamma-1}}\left(\gamma-1-c_s^2\right)^{\frac{3\gamma+1}{2(1-\gamma)}}u\left(1-u^2\right)^{-\frac{1}{2}}\left[\lambda^2v_t^2-a^2(v_t-1)\right]^{-\frac{1}{2}}\left[\frac{u^2\left(\gamma-c_s^2\right)+c_s^2}{c_s^2\left(1-u^2\right)}\right]$$

- Denoting the pre- and post-shock values of any flow variable by – and +, define:
  - ❑ Shock strength ≡ Ratio of Mach numbers = M_-/M_+
  - ❑ Entropy enhancement ratio (at the shock) ≡ Ratio of the entropy accretion rate = Ξ_+/Ξ_-
- Simultaneous solution of the equations provides the phase portrait of the shocked multi-transonic accretion (Fig. 3).
- As a consequence of the shock formation, the post shock flow has higher temperature, density, pressure and residence time compared to its pre-shock counterpart. This favours the generation of an optically thick halo (yellow coloured elliptic patches in Fig. 4, where the disc structure has been obtained by solving the equations) in the post shock region as a result of disc evaporation, and initiates the production of thermally and centrifugally driven cosmic outflows.
- Figs. 5a, 5b, 5c show various shock related quantities.
- Stronger shocks (higher value of (M_-/M_+)) are produced closer to the black hole, and is a result of maximum entropy production (measured by (Ξ_+/Ξ_-)) due to the shock (Fig. 5b).

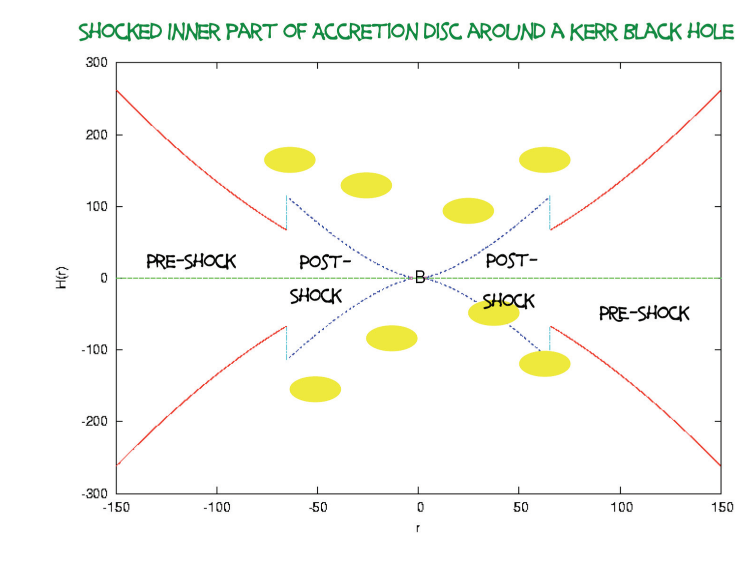

Fig. 4. --- Pre- and post-shock disc geometry with thermally driven optically thick halo.

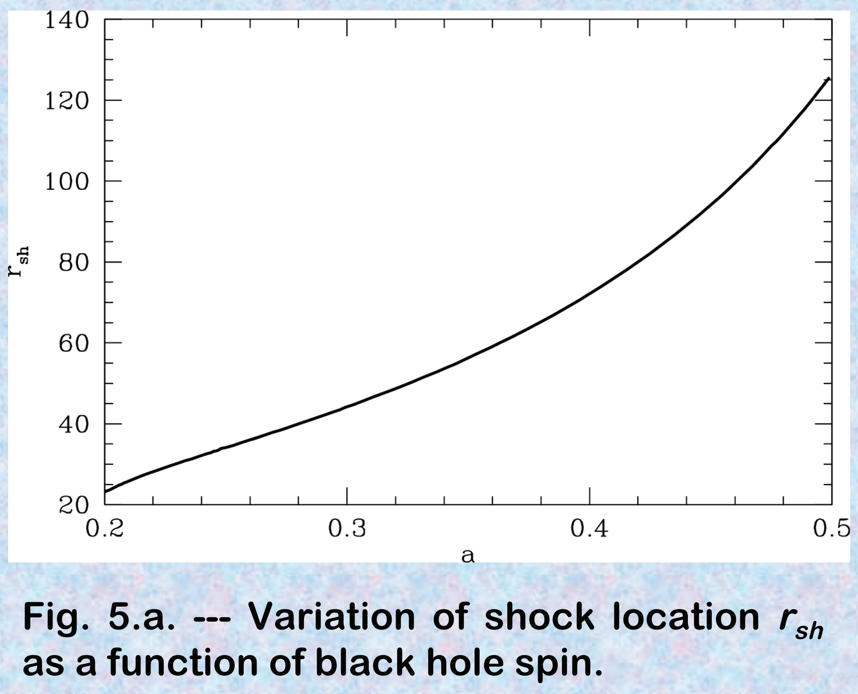

Fig. 5.a. --- Variation of shock location r_{sh} as a function of black hole spin.

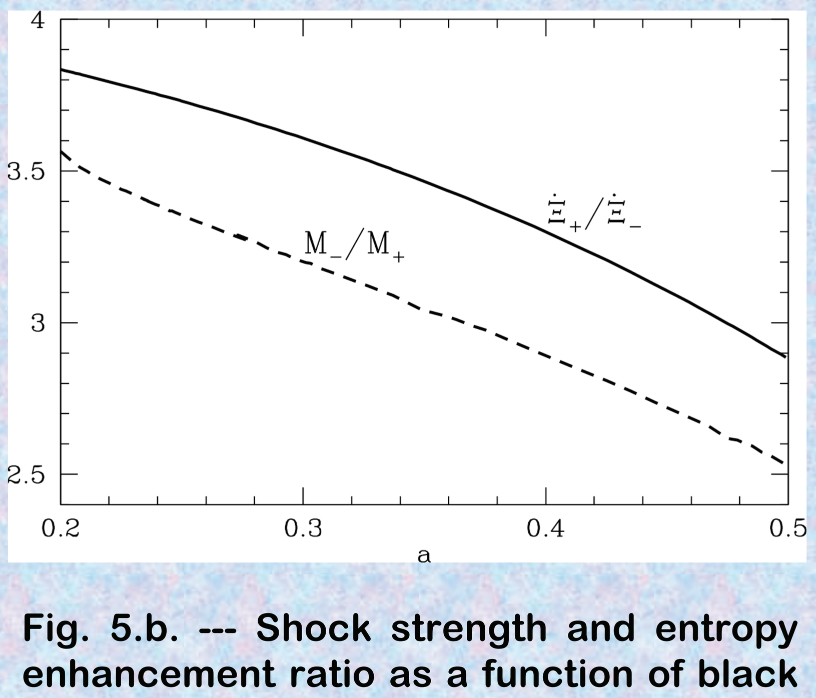

Fig. 5.b. --- Shock strength and entropy enhancement ratio as a function of black hole spin.

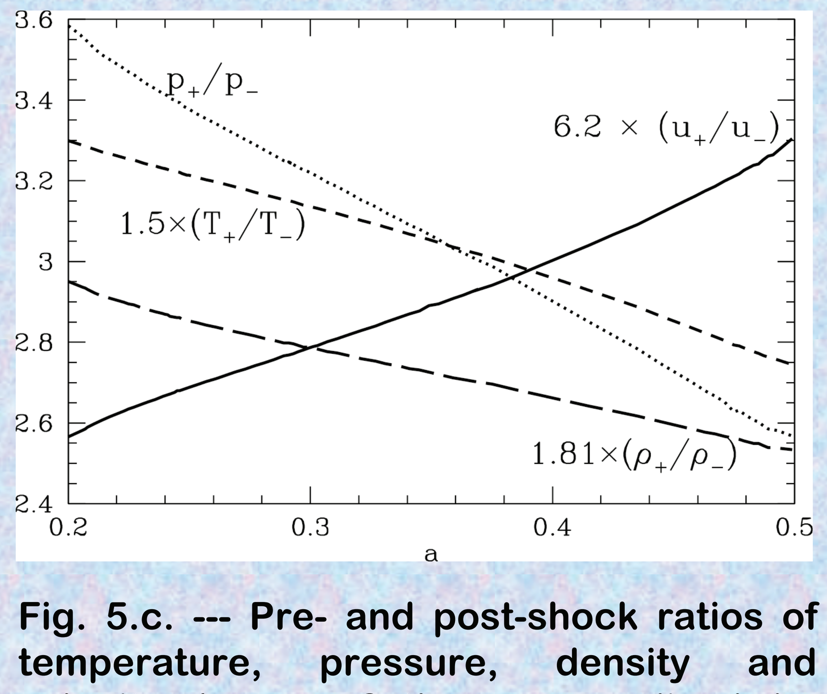

Fig. 5.c. --- Pre- and post-shock ratios of temperature, pressure, density and velocity (some of these normalized by appropriate numerical factors so as to fit in the same figure) vs. black hole spin.

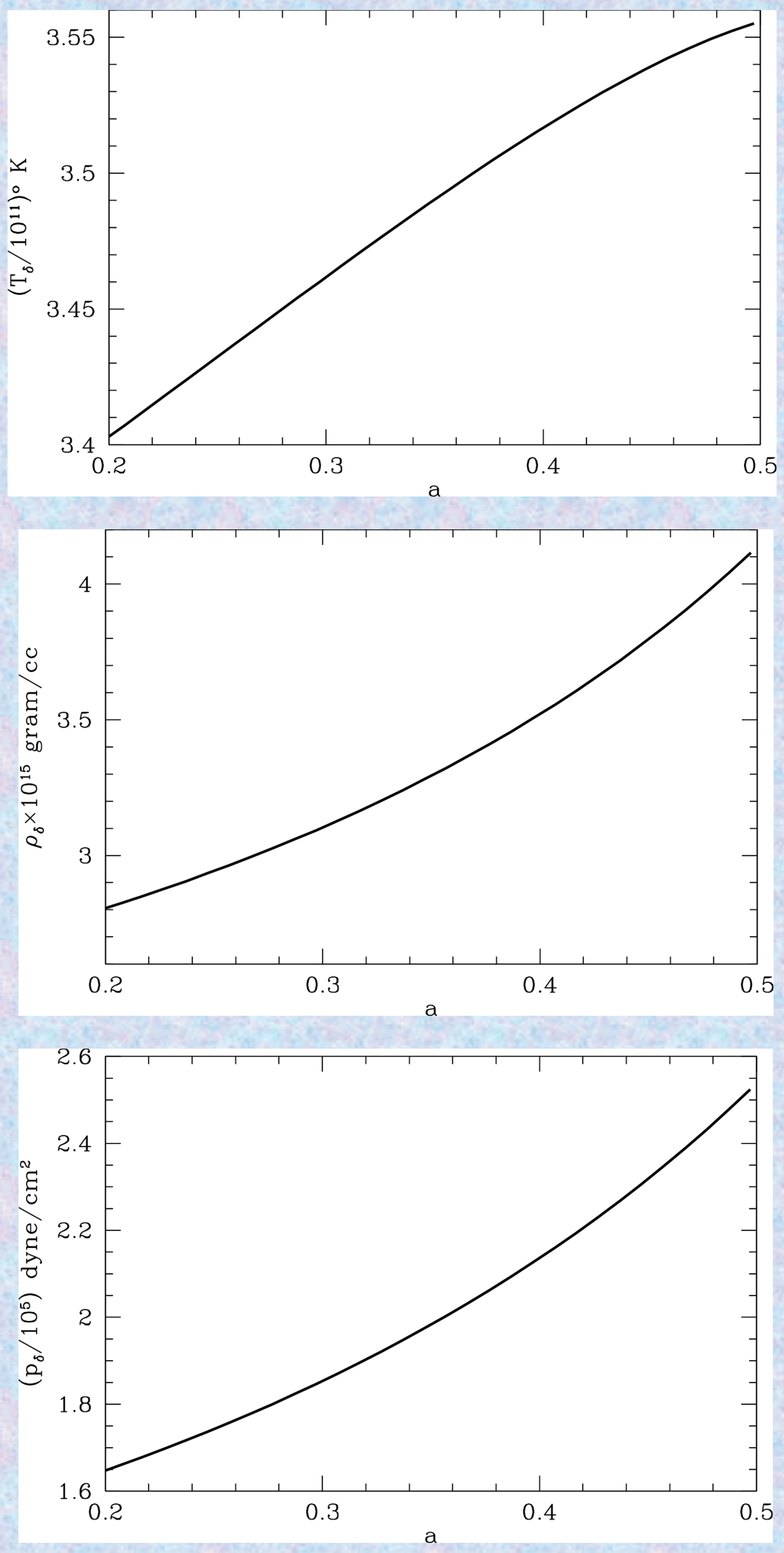

Fig. 6. --- Variation of the (normalized) quasi-terminal temperature (T_δ), density (ρ_δ), pressure (p_δ), as a function of the black hole spin.

Quasi-terminal Values

- Flow variables (calculated along the solution 'a') at a very close proximity

$$r_\delta=r_++\delta$$
$$\left(\text{where, } r_+=1+\sqrt{\left(1-a^2\right)},\text{ and }\delta=0.001r_g\right)$$

of the event horizon are termed as quasi-terminal values, and are distinguished with a subscript δ.
- Fig. 6 shows some quasi-terminal quantities, for a black hole with,

$$M_{BH}=3\times10^6M_{Sun},\ \text{Accretion rate}=4.29\times10^{-6}M_{Sun}\ \text{Yr}^{-1}$$

Conclusions

➢ For the first time, the shocked relativistic accretion in the Kerr metric has been studied at such a close proximity to the black hole event horizon.

➢ The effects of the black hole spin on the dynamical and thermodynamical properties of the accreting material have been made explicit.

➢ Accurate estimates of the flow temperature, velocity and density profile has been made. This leads to a possible formalism for the study of the spectral signature of the black hole spin (work in progress).

References

- Abramowicz, M. A., Lanza, A., & Percival, M. J. 1997, ApJ, 479, 179
- Barai P., Das T. K. & Witta P. J. 2004, ApJ, 613, L49
- Novikov, I. & Thorne, K. S. 1973, in Black Holes. ed. DeWitt, C. & DeWitt, B. Gordon & Breach. NY. p. 343