Lecture 10

Horizons
Vittorio, Chap 2
Ellis & Rotemman

The comoving distance in Einstein-de Sitter does not diverge when the integral is computed from $t_e = 0$.

This means that the distance traveled by a photon from the Big Bang to now is finite.

The PARTICLE HORIZON is a surface in ordinary 3-space that separates particles that may have been "seen" by the observer from particles that have not (yet).

It is different from an event horizon, that separates events that CAN or CANNOT communicate.

The COMOVING distance of the particle horizon is:

$$d_{PH} = d_c(t=0) = \int_0^{t_0} \frac{c\,dt}{a(t)}$$

For E-dS:

$$d_{PH} = 3ct_0$$

But a photon starting at $t \simeq 0$ travels a distance ct_0!

However, d_{PH} is the distance of a galaxy TODAY that emitted its photon at $t = \varepsilon \ll t_0$, the photon has NOT traveled from $3ct_0$

Suppose the universe is static: $a(t) = 1$

and suppose it started to exist at $t=0$:

$$\Rightarrow d_{PH} = ct_0$$

However, a static and eternal universe does not have a Big Bang:

$$\Rightarrow d_{PH} \to \infty$$ OLBER'S PARADOX

HORIZON AT TIME t

Its comoving distance would be (E-dS):

$$d_{PH}(t) = \int_0^t \frac{c\,dt}{a(t)} = ct_0 \int_0^{t/t_0} x^{-2/3}\,dx =$$

$$= 3ct_0\left(\frac{t}{t_0}\right)^{1/3} = 3ct\left(\frac{t}{t_0}\right)^{\frac{1}{3}-1} = \frac{3ct}{a(t)}$$

so its PROPER distance is:

$$d_{P,PH}(t) = 3ct$$

SUPERLUMINAL MOTION

A non-observable version of Hubble law is:

$$v = \dot{d}_P = H d_P$$

It can happen that $v > c$, for $d_P > \frac{c}{H}$

For $t = t_0$: $\frac{c}{H_0} = \frac{3}{2} ct_0$

This is called in cosmology HUBBLE HORIZON; for didactical purposes, and following Ellis & Rothman, we call it for the moment SPEED-OF-LIGHT SPHERE:

$$d_{SLS} = \frac{3}{2} ct$$

NB: superluminal motion here is NOT a violation of SR, that states that a body moving in a (local) inertial frame cannot be faster than c

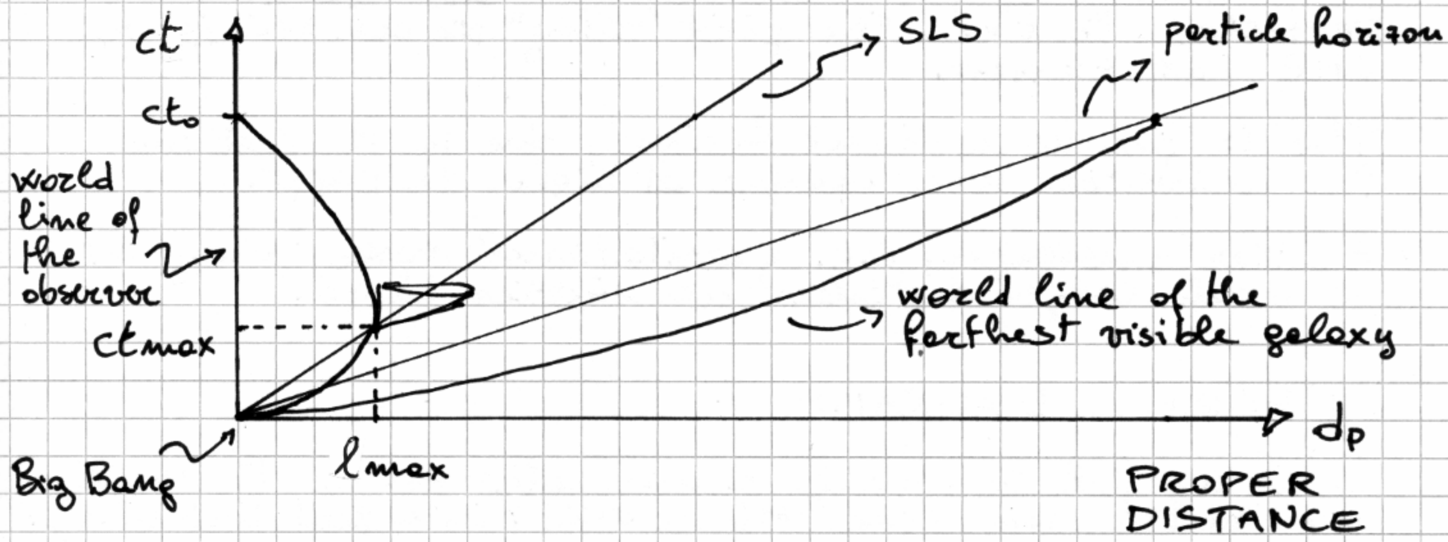
PAST LIGHT CONE

Let's take a photon emitted at t_e and observed at t_0. The proper distance from the observer's world line of the galaxy that emitted the photon is:

$$\ell(t_e) = a(t_e)\int_{t_e}^{t_0} \frac{c\,dt}{a(t)} = \left(\frac{t_e}{t_0}\right)^{2/3} ct_0 \int_{t_e/t_0}^{1} x^{-2/3}\,dx =$$

$$= \left(\frac{t_e}{t_0}\right)^{2/3} 3ct_0\left[1 - \left(\frac{t_e}{t_0}\right)^{1/3}\right] = 3c\left(t_e^{2/3} t_0^{1/3} - t_e\right)$$

By varying t_e we can reconstruct the past light cone of the observer at t_0:



The maximum of $\ell(t_e)$ is for: $t_{max} = \frac{8}{27} t_0$, $l_{max} = \frac{4}{9} ct_0$

$$a(t_{max}) = \frac{4}{9}, \quad z_{max} = \frac{5}{4}$$

It coincides with the redshift of the largest angular diameter distance.

It can be shown that the SLS passes through the largest distance event:

- → a point on the SLS recedes at speed c
- → a photon emitted toward the observer starts with zero speed
- → the slope of the past light cone is vertical at the maximum distance

So:
- + at $t = \varepsilon \ll t_0$ the farthest visible galaxy emits a photon
- + but it is well beyond the particle horizon at that time
- + so it is also beyond the SLS and recedes at speed $> c$
- + the direction of the future light cone is heavily bent (see E & R)
- + the photon travels toward the observer, reaching at most d_{max}
- + at $t = t_0$ the photon reaches the observer
- + in the meantime the galaxy is at $d_p = 3ct_0$

and:
- + any galaxy seen at $z < 1.25$ (in E-dS) was receding at $v < c$ at the emission time
- + higher-redshift galaxies were superluminal at the emission time

VISUAL HORIZON

As a matter of fact, we cannot "see" galaxies or particles to $t=0$. The first "thing" we can see is the CMB, at $z \simeq 1100$.

This is for us a visual horizon, due to the fact that the universe was opaque at higher redshift.

Another messenger, e.g. gravitational waves, may let us reach larger distances an earlier times.

CONFORMAL DIAGRAMS

It is useful to change coordinates so as to make the metric look like Minkowski (in a flat Universe):

$$ds^2 = a^2(t)\left[-c^2 d\eta^2 + dr^2 + r^2 d\Omega^2\right]$$

where $\eta = \frac{dt}{a}$ is the conformal time.

Expansion does not change angles, so the past light cone in conformal time and comoving distance looks like in Minkowski space

Using comoving distance and conformal time: