Lecture 16 — Planck time and phase transitions

Vittorio, chap 4
Bonometto, chap. III.9, XI + A2

Planck units

Take a particle of mass m:

- Compton radius $\quad l_c = \frac{\hbar}{mc}$
- gravitational radius $\quad l_G = \frac{Gm}{c^2}$

Usually $l_c \gg l_G$, e.g. for a proton:

$$l_c \sim 2\times 10^{-14}\ \text{cm}, \quad l_G \sim 2\times 10^{-52}\ \text{cm}$$

Let's define the PLANCK MASS as the mass of a particle for which $l_c = l_G$:

$$\frac{\hbar}{m_P c} = \frac{G m_P}{c^2} \implies m_P = \sqrt{\frac{\hbar c}{G}} = 2.2\times 10^{-5}\ \text{g}$$

Such a particle would be so massive that its effects on curvature would NOT be on scales well below l_c, where quantum effects become important.

Such particles would require QUANTUM GRAVITY to be described.

PLANCK ENERGY:

$$E_P = m_P c^2 = \sqrt{\frac{\hbar c^5}{G}} = 1.2\times 10^{19}\ \text{GeV}$$

PLANCK LENGTH: the Compton length or the gravitational radius of a Planck mass particle:

$$l_P = \sqrt{\frac{G\hbar}{c^3}} = 1.6\times 10^{-33}\ \text{cm}$$

It is easy to see that the gravitational binding energy of a Planck mass particle is E_P:

$$\frac{G m_P^2}{l_P} = m_P c^2 = E_P$$

PLANCK TIME : the light-crossing time of l_P:

$$t_P = \frac{l_P}{c} = \sqrt{\frac{G\hbar}{c^5}} = 5.4\times 10^{-44}\ \text{s}$$

No surprise that $E_p t_p = \hbar$

PLANCK DENSITY:

$$\rho_P = \frac{m_P}{\ell_p^3} = \frac{c^5}{\hbar G^2} = 5.2 \times 10^{93} \; g\,cm^{-3}$$

Again, no surprise that the free-fall time of the Planck density is the Planck time:

$$\frac{1}{\sqrt{G\rho_P}} = t_p$$

The Universe at the Planck time

We neglect here any numerical factor of $\mathcal{O}(1)$.

At Planck time t_p a radiative universe has:

horizon: $d_H = c t_p = \ell_P$

density: $\rho = \frac{1}{G t_p^2} = \rho_P$

mass within horizon: $m_H = \rho\, d_H^3 = \rho_p \ell_P^3 = m_p$

energy within horizon: $E_H = m_P c^2 = E_P$

We define the PLANCK TEMPERATURE as:

$$T_p = \frac{E_P}{k_B} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.4 \times 10^{32} \; K$$

In this context, $a_r = \frac{k_B^4}{\hbar^3 c^3}$, so: $\rho_P c^2 = a_r T_P^4$ as in a radiative universe.

The adimensional entropy within the horizon results:

$$\frac{1}{k_B} a_r T_p^3 \ell_p^3 = 1$$

This is consistent with having 1 particle (of Planck mass) within the horizon.

This means that we are really at the limit with a thermodynamical description of the Universe

Natural units

One could define a system of units of measure such that

$$c = \hbar = G = k_B = 1$$

Then all measurements would be given by pure numbers: masses in units of m_P, lengths in units of l_P etc.

$$\Rightarrow \quad G \sim \frac{1}{m_P^2}$$

Scale factor at Planck time

We can estimate $a(t_P)$ (given $a(t_0) = 1$) assuming that entropy was conserved across the expansion history:

$$g^{*\,1/3}\, a T = \text{const}$$

g^* is 2 today, and should be 2 for gravitons:

$$a_P \sim \frac{T_{CMB}}{T_P} = 2 \times 10^{-32}$$

How big was the Universe at that time?

The horizon length today is $d_{H0} \approx \frac{c}{H_0} \simeq 1.4 \times 10^{28}$ cm $\simeq 4400$ Mpc.

At Planck time the whole universe was compressed to:

$$a_P\, d_{H0} \simeq 2.7 \times 10^{-4} \text{ cm}$$

Conversely, the comoving size of the horizon at Planck time is:

$$\frac{c t_P}{a_P} \simeq 0.08 \text{ cm}$$

The age of phase transitions

Planck era should mark the separation of gravity from other unified forces:

QUANTUM GRAVITY → CLASSICAL GRAVITY

QUANTUM GRAVITY → G.U.T. (10^{19} GeV)

G.U.T. (10^{15} GeV) → … → STRONG FORCE

G.U.T. (10^{15} GeV) → … → ELECTROWEAK FORCE

G.U.T. might break at $T \simeq 10^{15}$ GeV

We can compute the time at which this happens assuming a radiative Universe after Planck time:

$$a = a_P \left(\frac{t}{t_P}\right)^{1/2} , \quad T = T_P \left(\frac{g^*}{2}\right)^{-1/3} \left(\frac{a}{a_P}\right)^{-1}$$

$$\Rightarrow \quad t = t_P \left(\frac{g^*}{2}\right)^{-2/3} \left(\frac{T}{T_P}\right)^{-2}$$

Assuming some high value for g^* at G.U.T. breaking, this should take place when:

$t \simeq 10^{-36}$ s		time since the Big Bang
$ct \simeq 3 \times 10^{-26}$ cm		proper horizon size
$a \simeq 10^{-28}$		scale factor
$ct/a \simeq 300$ cm		comoving horizon size
$a c t_0 \simeq 1.4$ cm		size of visible Universe at that time

Breaking of G.U.T. gives rise to Spontaneous Symmetry Breaking (SSB)

Let's consider the free energy $F = U - TS$

A system tends to the minimum of F.

In these two examples F evolves with T

$T > T_c$: $\phi = 0$ is a minimum

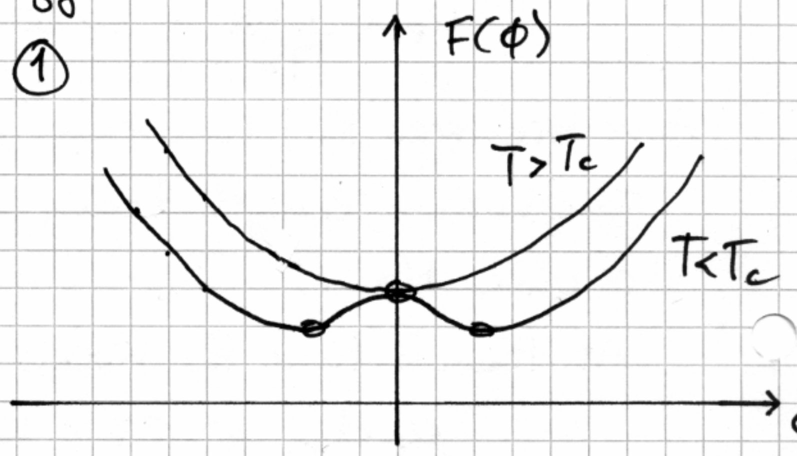
$T < T_c$: the minimum moves

① The motion of the minimum is continuous

$\Rightarrow$ second-order phase transition

② $\phi = 0$ becomes a local minimum, the system can tunnel from the false vacuum to the new configuration

$\Rightarrow$ first-order phase transition

①


②