Lecture 17 — Problems of the hot Big Bang

Vittorio, chap 4

Monopole problem

In the '80s theorists were exploring a GUT based on

$$SU(5) \longrightarrow \underset{QCD}{SU(3)} \times \underset{WF}{SU(2)} \times \underset{EM}{U(1)}$$

This model is now ruled out because it predicts the unobserved proton decay.

SSB of SU(5) cannot produce the same vacuum state everywhere, in particular it cannot break in the same way beyond the horizon.

As for a ferromagnet cooling down, SSB produces domains in which the vacuum state is the same, the separations of different vacuum domains is called TOPOLOGICAL DEFECT.

Topological defects can have various topologies, depending on the broken symmetry: walls, strings, points → monopoles or textures.

SSB of SU(5) produces MAGNETIC MONOPOLES.

If topological defects decay there is no problem, but magnetic monopoles are stable.

There should be ~1 monopole for each domain, whose size ξ will be smaller than the horizon:

$$\xi < ct \simeq 3 \times 10^{-26}\ \text{cm}$$

This implies a number density of monopoles of:

$$n \sim \xi^{-3} > 3 \times 10^{76}\ \text{cm}^{-3} \quad \text{(physical)}$$

Transforming from physical to comoving coordinates:

$$n_0 > a^3 (3 \times 10^{76}\ \text{cm}^{-3}) \simeq 3 \times 10^{-8}\ \text{cm}^{-3}$$

This lower limit is just below the baryon number density, $n_{b0} \simeq 2.4 \times 10^{-7}\ \text{cm}^{-3}$

... but we don't see them!

We can assign an energy to magnetic monopoles, and because SSB takes place at $\sim 10^{15}$ GeV their equivalent mass would be $\gg m_p c^2$

$\Rightarrow$ ridiculous energy density $\Omega_{mono} \sim 10^{15}$

More in general, SSB produces topological defects, if they are stable they should be abundant

Why don't we see them?

Though SU(5) is now ruled out, this question triggered a discussion on other problems.

Flatness (curvature) problem

As seen in Lecture 11, if we plot $\Omega_k(t)$ versus $\log t$ and allow for many orders of magnitude to start from Planck time, and if there is no Λ, Ω_k has a transition from flat to Milne or Big Crunch.

If the Universe is not exactly flat, why do we see it as it is today? We can see $\Omega_k < 1$ only for a small time.

There is a better way to put this down.

$$\Omega_k(t) = -\frac{kc^2}{H^2 a^2 R_0^2} = \Omega_{k0} \frac{a^2}{\Omega_{r0} + \Omega_{m0} a + \Omega_{k0} a^2 + \Omega_{\Lambda 0} a^4}$$

Here Ω_{r0} is the Ω of radiation today.

When $a \ll 1$, we have:

$$\Omega_k(t) = a^2 \frac{\Omega_{k0}}{\Omega_{r0}}$$

Now, we want to set the initial conditions of our model at the Planck time:

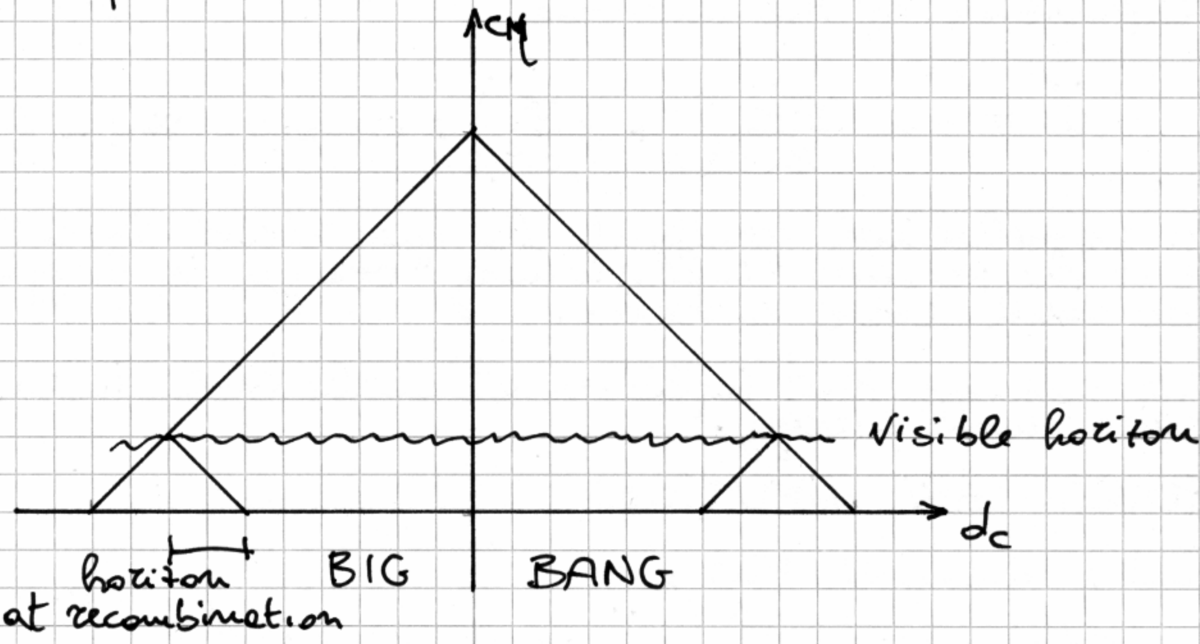
$$\Omega_k(t_P) = a_P^2 \frac{\Omega_{k0}}{\Omega_{r0}} \simeq 4 \times 10^{-64} \frac{\Omega_{k0}}{\Omega_{r0}}$$

So to have $\Omega_{k0} \sim 1$ today we need to set the initial conditions with an accuracy of 10^{-64}

FINE TUNING!

Why do we have to be so paranoically precise to have curvature today?

Horizon problem



Projected on the sky, the horizon at recombination subtends ~ 3 deg

So regions of the CMB that are more distant than 3 deg have not had time to communicate before recombination.

So why is the CMB temperature so uniform?

Fluctuations problem

What mechanism created the fluctuations, visible in the CMB, that give rise to the observed large-scale structure?

COBE has a resolution of ~7 deg, so CMB fluctuations seen by it are beyond the horizon.

How is it possible to create super-horizon fluctuations?

There is a mechanism that can solve these four problems together : INFLATION

Inflation is a limited period of accelerated expansion of the universe

$$\ddot{a} = -a\,\frac{4\pi G}{3}\left(\rho + 3\frac{P}{c^2}\right) > 0 \implies w < -\frac{1}{3}$$

A cosmological constant Λ can drive inflation, but the present "inflationary" stage is happening at much lower energies.

We assume that inflation at early times is unrelated with dark energy.

POWER-LAW INFLATION

Let's assume that:

$$a(t) = a_i\left(\frac{t}{t_i}\right)^{\alpha}, \quad \ddot{a} = \alpha(\alpha-1)\,\frac{a_i}{t_i^2}\left(\frac{t}{t_i}\right)^{\alpha-2}$$

where $\alpha = \dfrac{2}{3(w+1)}$

$$\ddot{a} > 0 \implies \alpha > 1 \implies -1 < w < -\frac{1}{3}$$

particle horizon (proper distance):

$$d_{p,PH}(t) = a(t)\int_0^t \frac{c\,dt'}{a(t')} = \frac{c\,t^{\alpha}}{1-\alpha}\left[t'^{\,1-\alpha}\right]_0^t$$

future horizon:

$$d_{p,future}(t) = a(t)\int_t^{\infty} \frac{c\,dt'}{a(t')} = \frac{c\,t^{\alpha}}{1-\alpha}\left[t'^{\,1-\alpha}\right]_t^{\infty}$$

For $\alpha > 1$: $\quad d_{PH} \to \infty \quad , \; d_{p,future} = \dfrac{1}{\alpha-1}\,ct = d_{EH}$

We found a similar behaviour in the case of de Sitter exponential expansion.

In an accelerated universe the particle horizon can be very large, while the future horizon converges.