Lecture 18 Inflation

Vittorio, chap. 4

Particle horizon characterizes the ability to SEE a galaxy, or a particle in general.

We want to characterize the ability to be causally connected with another region of the Universe.

e.g. thermalization requires mutual INFORMATION EXCHANGE

Causal connection happens within the Hubble horizon. Let's define the COMOVING HUBBLE HORIZON:

$$d_{cH} = \frac{1}{a}\frac{c}{H} = \frac{c}{\dot{a}} = \frac{1}{a} d_H$$

we formerly called $d_H = c/H$ the SLS.

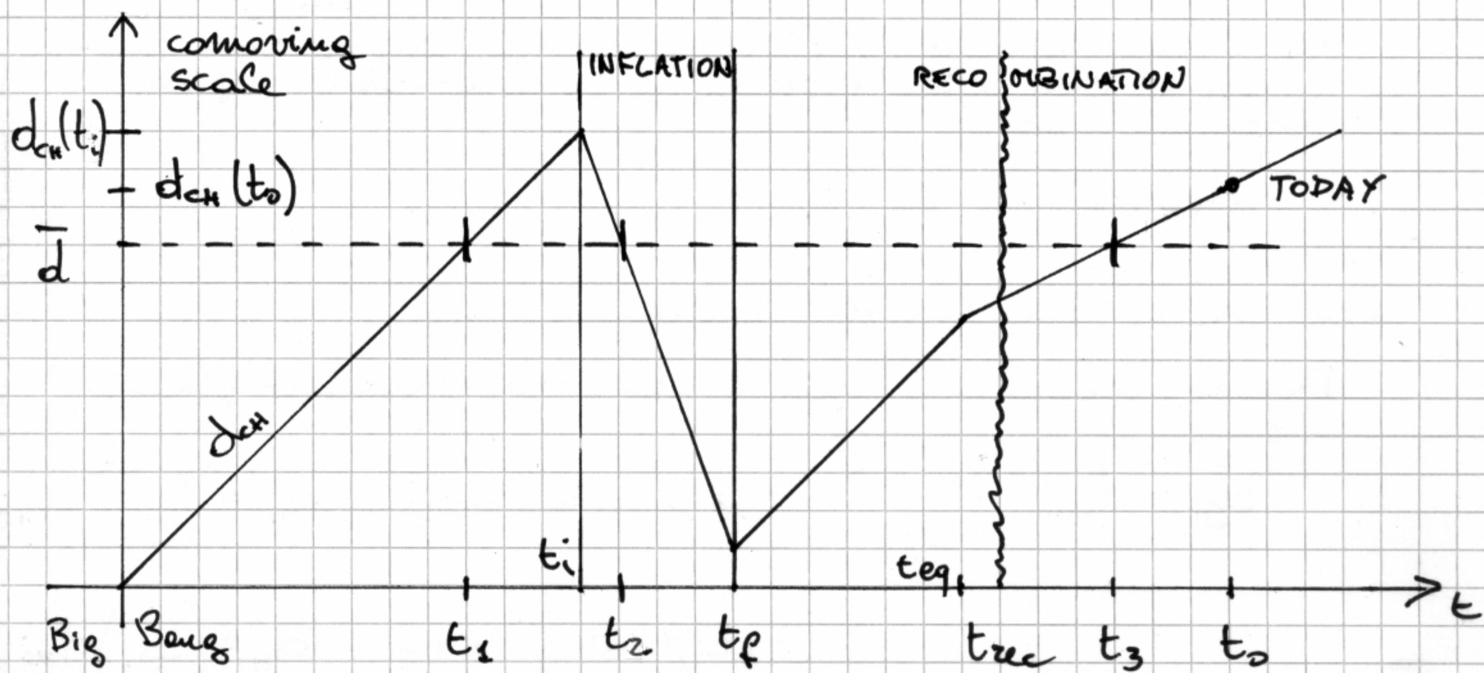
While the particle horizon always grows, d_{cH} can grow or decrease:

$$\dot{d}_{cH} = -\frac{\ddot{a}}{\dot{a}^2} c \quad , \quad \dot{d}_{cH} < 0 \quad \text{when } \ddot{a} > 0$$

We interpret d_{cH} as the distance below which causal connection, that is mutual information exchange, is possible.

Then, during inflation the comoving Hubble horizon shrinks

SCHEMATIC MODEL OF INFLATION



We follow in this plot a fixed comoving scale $\bar{d}$ and the comoving Hubble horizon d_{cH}.

- $t=0$: Big Bang, $d_{cH}=0$
- t_1 : $\bar{d} < d_{cH}$, particles on those scale can thermalize
- t_i : inflation starts, d_{cH} is at its maximum
- t_2 : $\bar{d} < d_{cH}$, particles lose causal contact on these scales
- t_f : inflation ends, d_{cH} is much smaller than $d_{cH}(t_i)$
- t_{eq} : matter-radiation equality
- t_{rec} : recombination makes the universe transparent; $d_{cH}(t_{rec})$ subtends ~ 3 deg, $\bar{d}$ is still beyond the horizon
- t_3 : $\bar{d}$ enters the horizon, causal processes are again possible on this scale
- t_0 : today

Exponential (de Sitter) inflation

This is the reference model for inflation, where expansion is driven by an effective cosmological constant

$$a(t) = a_i\, e^{H(t-t_i)} \quad , \quad H = \sqrt{\frac{\Lambda c^2}{3}} = \text{const}$$

The proper Hubble horizon $d_H = \frac{c}{H}$ is constant,

$$d_{cH} = d_{cH}(t_i)\, e^{-H(t-t_i)}$$

Particle horizon can only grow, but it stalls during de Sitter inflation:

$$d_{PH} = \int_0^t \frac{c\,dt}{a(t)} = d_{PH}(t_i) + \int_{t_i}^{t} \frac{c\,dt}{a_i}\, e^{-H(t-t_i)} =$$

$$= d_{PH}(t_i) + \frac{c}{a_i H}\left(1 - e^{-H(t-t_i)}\right)$$

then particle horizon becomes much larger than Hubble horizon; they will grow at a similar pace after inflation, so

$$d_{PH}(t_0) \gg d_{cH}(t_0)$$

Solution of the horizon problem

During radiation- and matter-dominated phases,

$$d_{cH} = d_{cHi}\left(\frac{a}{a_i}\right)^{\frac{1-\alpha}{\alpha}} \quad \Rightarrow \quad w=0,\ \alpha = \frac{2}{3},\ d_H = \left(\frac{a}{a_i}\right)^{1/2} d_{Hi}$$

$$\Rightarrow \quad w = \frac{1}{3},\ \alpha = \frac{1}{2},\ d_H = \left(\frac{a}{a_i}\right) d_{Hi}$$

We can then follow the evolution of the comoving Hubble horizon starting from $d_H(t_i)$, and

$$d_{cH}(t_0) = \underbrace{d_{cH}(t_i)}_{\text{initial horizon}} \times \underbrace{e^{-H(t_f-t_i)}}_{\text{inflation}} \times \underbrace{\frac{a_{eq}}{a_f}}_{\text{radiation era}} \times \underbrace{\sqrt{\frac{1}{a_{eq}}}}_{\text{matter era}}$$

We are neglecting here that the very last phase of evolution is driven by Λ.

Assuming $a_f \simeq 10^{-28}$ (SSB-driven inflation at $\sim 10^{15}$ GeV) and requiring

$$d_{cH}(t_0) < d_{cH}(t_i)$$

to solve the horizon problem, we call

$$N_e \equiv H(t_f - t_i)$$

and find that the growth of the scale factor during inflation must be, at least:

$$e^{N_e} > \frac{a_{eq}}{a_f}\sqrt{\frac{1}{a_{eq}}}$$

we know $a_{eq} = 1/(1+z_{eq})$, with $z_{eq} \simeq 1100$, so

$$N_e > \ln\left(\frac{a_{eq}}{a_f}\sqrt{\frac{1}{a_{eq}}}\right) \simeq 60$$

so inflation must let the scale factor grow by at least 60 e-folding

Duration of inflation: for a radiation-dominated phase,

$$H(t_i) = \frac{1}{2t_i}$$

assuming continuity, $H = \frac{1}{2t_i}$, so

$$t_f - t_i \simeq N_e 2t_i \simeq 10^{-34}\ \text{s}$$

still pretty small.

Inflation in a conformal diagram:

During inflation the conformal time grows by:

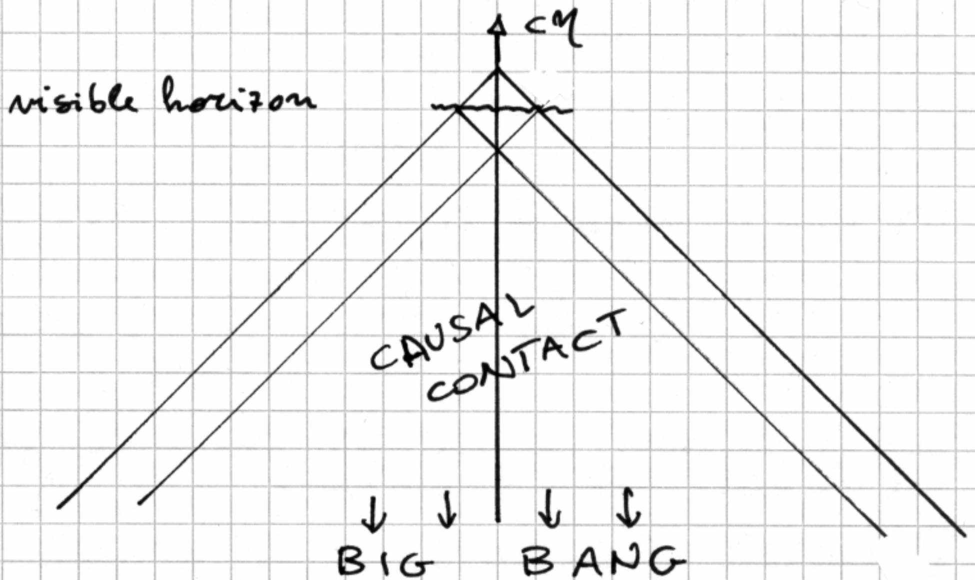
$$\Delta\eta = \int_{t_i}^{t_f} \frac{dt}{a(t)} = \frac{1}{H_i a_i}\left(1 - e^{-N_e}\right) \simeq \frac{1}{H_i a_i}$$

anchoring it to the time t_f, and assuming $H_i = H_f$

$$a_f = a_i e^{N_e} \Rightarrow \Delta\eta \simeq \frac{1}{H_f a_f} e^{N_e} \simeq \frac{1}{c} d_{cH}(t_f) e^{N_e}$$

so the duration of inflation is huge in terms of conformal time, as much as the particle horizon is huge with respect to the comoving Hubble horizon

The conformal diagram is thus extended below the visible horizon, making causal contact of CMB patches possible in the past.



Solution of the monopole problem:

Everything present before inflation is hugely diluted and pushed well beyond the visible horizon, including any topological defect.

Solution of the flatness problem:

If Friedmann equation is dominated by a Λ term,

$$\Omega_K(t) \simeq \frac{\Omega_{Ki}}{\Omega_{\Lambda i}} a^{-2}$$

This means that Ω_K gets very, very small at the end of inflation.

In other words, any curvature radius is inflated so much that it becomes much, much larger than the horizon

=> Inflation produces a flat Universe

So the flatness problem is solved because flatness is the result of a dynamic process

Origin of fluctuations

Quantum fluctuations of the "inflaton", the field that drives inflation, become classic and give rise to curvature fluctuations

=> QUANTUM DECOHERENCE

Origin of the Universe

An inflationary period causes a loss of memory ("cosmic no-hair theorem") of what was present before,

making the Big Bang, the singularity, much less essential for the model.

As a matter of fact, inflation is when the Universe begins.

End of inflation

Inflation must end, after inflation the inflaton re-couples with all the other fields, decaying into all possible particles

=> REHEATING

Predictions of inflation:

→ the Universe is flat ✓

→ a specific slope for the primordial power spectrum of fluctuations ✓

→ a background of gravitational waves ?

→ some non-gaussianity of perturbation ?

But beware: inflation is not a theory but a FAMILY OF THEORIES ~ a paradigm