Lecture 20

Thermal history of the early Universe

Bonometto, chap VI + A3
Vittorio, chap 3.

Starting from the end of inflation, the Universe is dominated by radiation.

In this phase we can use entropy conservation

$$a T g^{\star 1/3} = \text{const}$$

and the relation between age and energy density u, in terms of $\rho = u/c^2$, of a radiative Universe:

$$t = \sqrt{\frac{3}{32 \pi G \rho}}$$

The thermal history of the early Universe is determined by the events that take place as energy decreases

BARYOGENESIS

A perfect balance of matter and antimatter would leave a Universe without baryons.

Some mechanism in the early Universe, taking place after inflation, must have created a small excess of matter.

Let's define: $$\eta = \frac{n_{b0}}{n_{\gamma 0}}$$

$$n_{\gamma 0} = \frac{30\, \zeta(3)}{\pi^4} \frac{a_r T_{CMB}^4}{k_B} = 412 \text{ cm}^{-3}$$

$$n_{b0} = \frac{\Omega_b \rho_c}{m_p} = \frac{3\,(100 \text{ km s}^{-1} \text{Mpc}^{-1})}{8 \pi G m_p} \Omega_b h^2 = 1.10 \times 10^{-5} \text{ cm}^{-3} \Omega_b h^2$$

$$\eta = 2.68 \times 10^{-8} \Omega_b h^2 \simeq 6 \times 10^{-10}$$

In the relativistic limit, before annihilation of matter-antimatter pairs: $n_b \sim n_\gamma$

So η gives the magnitude of the needed imbalance.

We will assume that baryogenesis took place.

CHEMICAL POTENTIALS

In presence of reactions, the distribution of particle species i is:

$$f_i(p) = \frac{N_{s,i}}{(2\pi\hbar)^3} \frac{1}{\exp\left(\frac{E_i(p) - \mu_i}{kT}\right) \pm 1}$$

where μ_i is the chemical potential. If N is the number of particles in a volume V,

$$T\,dS = dU + p\,dV - \mu\,dN \quad , \quad \mu = -T\frac{\partial S}{\partial N}$$

Photons do not conserve their number, so

$$\mu_\gamma = 0$$

$\Rightarrow$ for a particle-antiparticle pair, $\mu_{\bar{x}} = -\mu_x$

If a reaction $1+2 \rightleftarrows 3+4$ is in equilibrium:

$$\mu_1 + \mu_2 = \mu_3 + \mu_4$$

It is possible to demonstrate that at early times, when all particles are relativistic, all μ's vanish.

EVOLUTION OF $g^\star$

$$g^\star(T) = \sum_{bos} N_{s,i} + \frac{7}{8}\sum_{fer} N_{s,i}$$

This can be computed at T by summing over the degrees of freedom of particles for which

$$m_i c^2 > k_B T$$

Even if a fraction η of the particles survive annihilation and is stable, it does not contribute to energy density

PARTICLE DECOUPLING

To be in thermal equilibrium, a particle must have an interaction time shorter than the age of the Universe

$$\tau = \frac{1}{n\sigma v} < t$$

A decoupled particle contributes to energy density but is not part of the thermal soup.

We start our account of thermal history from the

QUARK - HADRON TRANSITION, or QCD TRANSITION

This is considered as the last phase transition.

$T \simeq 150-200$ MeV $\simeq 2\times10^{12}$ K
$t \simeq 2\times10^{-5}$ s
$ct \simeq 6$ km
$a \simeq 4\times10^{-13}$ $(g^* \simeq 60)$
$act_0 \simeq 2\times10^{15}$ cm $\simeq 130$ AU — larger than the Kuiper belt in the Solar System

The QH transition brings g^* to a much lower value.
p and n have $mc^2 > k_B T$, so they annihilate
π's (mesons) have masses ~130 MeV, so they form
τ leptons have already annihilated, μ's mass is ~100 MeV
So g^* is:

$\pi^+\pi^-\pi^0$	mesons	$\longrightarrow$	$g^* = 3$
$e^+e^-, \mu^+\mu^-$	leptons	$\longrightarrow$	$g^* = 4\times2\times\frac{7}{8} = 7$
$\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$	leptons	$\longrightarrow$	$g^* = 6\times1\times\frac{7}{8} = 5.25$
γ	photons	$\longrightarrow$	$g^* = 2$
			$g^* = 17.25$

Equilibrium is obtained through electric and weak forces (nuclear forces have residual importance at the beginning)

As leptons give the largest contribution to g^*, and then to energy density, this phase is called LEPTON ERA

PIONS ANNIHILATION: $T \simeq 130$ MeV $\simeq 1.5\times10^{12}$ K, $t \simeq 4\times10^{-5}$ s

g^*: $17.25 \longrightarrow 14.25$

MUONS ANNIHILATION: $T \simeq 100$ MeV $\simeq 1.2\times10^{12}$ K, $t \simeq 7\times10^{-5}$ s

g^*: $14.25 \longrightarrow 10.75$

We are left with e^+e^-, ν's, γ, plus residual p and n still in thermal equilibrium, plus a relic, subdominant dark matter particle.

NEUTRINO DECOUPLING

Neutrinos interact with other particles through weak forces, with a cross section:

$$\sigma_w \simeq 2.6 \times 10^{-42} \left(\frac{T}{1\,\mathrm{MeV}}\right)^2 \mathrm{cm}^2$$

We can also assume that they are relativistic: $v \sim c$

The number density of targets (electrons) is

$$n_e = \frac{3\,\zeta(3)}{2\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$$

The collision time thus scales as:

$$\tau = \frac{1}{\sigma_w n_e c} \propto T^{-5} \propto a^5$$

Decoupling takes place when $\tau > t$, with $t \propto a^2$, so that the two time scales will cross at:

$$\tau = t \quad \Rightarrow \quad T_{\nu,dec} = 900\ \mathrm{keV} \qquad \text{(Bonometto)}$$

After decoupling neutrinos evolve in isolation, keeping the shape of their distribution. So their kinetic temperature evolves like $T_\nu \propto a^{-1}$.

For the thermal soup: $g^*: 10.75 \longrightarrow 5.5$

This happens at a time: $t_{\nu,dec} \simeq 0.9\,\mathrm{s}$

DEPENDENCE ON THE NUMBER OF NEUTRINOS

If N_ν is the number of neutrino families - the number of families of particles, then g^* before neutrino decoupling is:

$$g^* = 5.5 + \frac{7}{4} N_\nu$$

$$\Rightarrow \rho = \frac{1}{c^2}\frac{\pi^2}{30} g^* \frac{(k_B T)^4}{(\hbar c)^3} \Longrightarrow t = \sqrt{\frac{3}{32\pi G\rho}}$$

$\Rightarrow T_{\nu,dec}$ acquires a dependence on N_ν

$e^+ e^-$ annihilation

$T \simeq 0.5\ \text{MeV} \simeq 5.8 \times 10^9\ \text{K}$
$t \simeq 4\ \text{s}$
$ct \simeq 1.3 \times 10^6\ \text{km}$
$a \simeq 3.3 \times 10^{-10}$
$act_0 \simeq 4.3 \times 10^{18}\ \text{cm} \simeq 1.4\ \text{pc}$

$$g^* : 5.5 \longrightarrow 2$$

This marks the end of the lepton era and the beginning of the RADIATION ERA

The jump in energy due to this annihilation gives a jump in the photon temperature.

$$a T g^{*\,1/3} = \text{const}$$, assuming a does not change:

$$T_{after} = \left(\frac{g^*_{before}}{g^*_{after}} \right)^{1/3} T_{before}$$

The kinetic temperature of neutrinos will not be affected, so

$T_{before} \rightarrow T_\nu$
$T_{after} \rightarrow T_\gamma$

and their ratio remains constant later

$$\frac{g^*_{before}}{g^*_{after}} = \frac{5.5}{2} = \frac{11}{4} \implies T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 0.71\, T_\gamma$$

$$\implies T_{\nu 0} = 0.71\, T_{CMB} = 1.94\ \text{K}$$

number density:

$$n_\nu = N_\nu \times 2 \times \frac{3}{4} \times \frac{\zeta(3)}{\pi^2} \left(\frac{k_B T_\nu}{\hbar c}\right)^3$$

($N_s = 1$ for each particle)

$$n_\gamma = 2 \times \frac{\zeta(3)}{\pi^2} \left(\frac{k_B T_\gamma}{\hbar c}\right)^3 \implies \frac{n_\nu}{n_\gamma} = N_\nu \times \frac{3}{4} \times \frac{4}{11} = 0.27 N_\nu$$

energy density:

$$\rho_\nu c^2 = N_\nu \times 2 \times \frac{7}{8} \times \frac{1}{2} a_r T_\nu^4$$
$$\rho_\gamma c^2 = a_r T_\gamma^4$$

$$\implies \frac{\rho_\nu}{\rho_\gamma} = N_\nu \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} = 0.227\, N_\nu$$

Although they are decoupled, neutrinos contribute to the energy density of radiation, so that:

$$\rho_{rad} = \rho_\gamma \left(1 + 0.227 N_\nu\right) = \frac{1}{c^2} a_r T_\gamma^4 \times 1.681$$

if $N_\nu = 3$. This must be taken into account when computing matter-radiation equality.

As a matter of fact, neutrino decoupling was not complete at electron annihilation, this gives an effective number of (neutrino) relativistic species (beyond photons):

$$N_{eff} = 3.046$$