# Lecture 21 Big Bang nucleosynthesis

Bonometto, chap VI + A3

## WHY BBN ?

Stars like the sun have 25% of their mass in Helium:

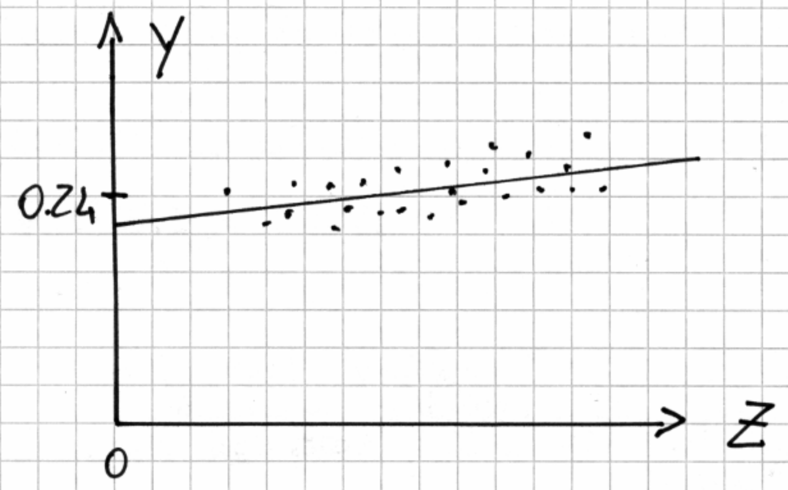
$$X \equiv \frac{M_H}{M_{TOT}} \simeq 0.73 \,, \quad Y \equiv \frac{M_{He}}{M_{TOT}} \simeq 0.25 \,, \quad Z \equiv \frac{M_{met}}{M_{TOT}} \simeq 0.02$$

Here we call "metals" all elements more massive than He

It is believed that metals were synthetised in stars

But stars are ~5% of baryons, it's hard to have a ~25% of mass in He with stellar nucleosynthesis.

Helium is present also in stars with very low metallicity Z




So even a "primordial" star, with $Z=0$, would have Helium:

$$Y_p = 0.238$$

Fusion of H to He produces photons :

$$4p + 2e \longrightarrow {}^4He + 28.3\,\text{MeV in } \gamma$$

so $\frac{\Delta m}{m} = 0.75\%$. To produce $Y = 0.24$ one would transform 0.18% of baryons to photons

$$\Omega_{\gamma, He} = \Omega_b \times 0.0018 \simeq 9 \times 10^{-5} \simeq \Omega_{\gamma 0}$$

this radiation is not observed.

So 23.8 % of baryons should be processed to He in the early Universe.

# WHY ONLY He (AND LIGHT ELEMENTS)?

To go beyond He one has to trigger the $3\alpha$ reaction:

$$3\,{}^4He \rightarrow {}^{12}C$$

but this requires a higher temperature, while the temperature in cosmology keeps decreasing.

Heavier elements can only be synthetised inside stars, where gravitational contraction makes temperature increase

# FREEZING OF n/p RATIO

p and n appear after the Q-H transition, and are kept in equilibrium by:

$$n + \nu_e \rightleftarrows p + e^-$$

$$n + e^+ \rightleftarrows p + \bar{\nu}_e$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Their number densities require chemical potentials to be known.

But their number ratio is (demonstrated in Vittorio):

$$\frac{n_n}{n_p} = \exp\left(-\frac{\Delta m c^2}{k_B T}\right)\left(\frac{m_n}{m_p}\right)^{3/2}, \quad \Delta m c^2 = 1.3\ \mathrm{MeV}$$

When $\nu$'s decouple at $T_{\nu dec} = 900$ keV, the first two channels become ineffective, while neutron decay has a time scale of 15 min, $\gg 0.9$s that is the age of the Universe then

$\Rightarrow$ $n_n/n_p$ freezes

Let's define:
$$X_n \equiv \frac{n_n}{n_b}, \quad X_{n,fr} = \frac{1}{1 + \left.\frac{n_p}{n_n}\right|_{fr}}$$

$$\left.\frac{n_n}{n_p}\right|_{fr} = \exp\left(-\frac{1.3\ \mathrm{MeV}}{0.9\ \mathrm{MeV}}\right) \simeq 0.23$$

$$X_{n,fr} \simeq 0.18, \quad X_n = X_{n,fr}\exp\left(-\frac{t - t_{\nu,dec}}{\tau_\beta}\right)$$

where $\tau_\beta = 887\ s \sim 1000\ t_{\nu,dec}$

# THE DEUTERIUM BOTTLENECK

The first reaction that takes place is:

$$p + n \rightleftarrows {}^2H + \gamma$$

This reaction destroys $^2H$ if $\gamma$ has an energy

$$E_\gamma \geq 2.2 \text{ MeV}$$

This energy corresponds to a temperature of $\approx 2.5 \times 10^{10}$ K

Being $> 0.5$ MeV, this should happen in the lepton era.

But $n_\gamma \gg n_b$ because $\eta \ll 1$

Note: after $e^- e^+$ annihilation $n_\gamma$ and $n_b$ both evolve $\propto a^{-3}$, so $\eta$ is constant.

If the reaction is in balance:

$$n_p \times n_n \simeq n_d \times n_\gamma$$

As an order of magnitude: $n_p \sim n_n \sim n_b$

$$\Rightarrow \quad \frac{n_d}{n_b} \sim \frac{n_b}{n_\gamma} \sim \eta \ll 1$$

so as long as many photons can participate to the reaction destroying $^2H$, deuterium cannot accumulate

To overcome this deuterium bottleneck we need to wait that

$$\frac{n_\gamma (E \geq 2.2 \text{ MeV})}{n_\gamma} \lesssim \eta$$

## SAHA EQUATION

p, n and d ($\equiv {}^2H$) are non-relativistic, so:

$$n_i = \mathcal{N}_{si} \left( \frac{m_i k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-\frac{m_i c^2 - \mu_i}{k_B T}} \qquad i = p, n, d$$

$$\mu_p + \mu_n = \mu_d \quad , \quad \mu_\gamma = 0$$

$$\mathcal{N}_{sn} = \mathcal{N}_{sp} = \mathcal{N}_{s\gamma} = 2 \quad , \quad \mathcal{N}_{sd} = 4$$

$$m_p c^2 + m_n c^2 = m_d c^2 + B_d \quad , \qquad B_d = 2.2 \text{ MeV}$$

Then:

$$\frac{n_d}{n_p n_n} = \left(\frac{m_d}{m_p m_n}\right)^{3/2} \left(\frac{k_B T}{2\pi \hbar^2}\right)^{-\frac{3}{2}} e^{\frac{B_d}{k_B T}}$$

Using

$$n_\gamma = \frac{2\,\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \quad \text{and} \quad m_p \sim m_n \sim \frac{1}{2} m_d$$

and calling:

$$R(T) = \frac{n_p n_n}{n_d n_\gamma}$$

we obtain Saha equation:

$$R(T) = \frac{\sqrt{\pi}}{2\,\zeta(3)}\left(\frac{m_p c^2}{4 k_B T}\right)^{3/2} e^{-\frac{B_d}{k_B T}}$$

The reaction is at its half when $n_d \sim n_n$:

$$\Rightarrow \frac{n_p n_n}{n_d} \sim n_p \sim n_b$$

$$\Rightarrow n_\gamma R(T) \sim n_b \quad \Rightarrow R(T) \sim \eta$$

such a low value is driven by $\exp(-B_d/k_B T)$

$$\Rightarrow T_{\text{bottleneck}} \simeq 100\,\text{keV}, \quad t_{\text{bot}} \simeq 130\,\text{s}$$

So nuclear reactions have to wait ~2 min to start, and are over in ~5 min

Clearly $t_{bot} \sim \tau_\beta$, so some neutrons will have decayed by then.

So the important parameters for BBN are

$T_{\nu,dec} \rightarrow N_\nu$

$\eta \rightarrow \Omega_b h^2$

## FROM $^2H$ TO $^4He$

$^4He$ is the most stable nucleus, so most neutrons will end up in $^4He$ nuclei

Reactions:

$$d + d \to {}^3He + n \quad \text{and} \quad {}^3He + d \to {}^4He + p$$
$$d + d \to {}^3H + p \quad \text{and} \quad {}^3H + d \to {}^4He + n$$
$$d + d \to {}^4He + \gamma \quad \text{(rare)}$$
$$p + d \to {}^3He + \gamma \quad \text{and} \quad {}^3He + d \to {}^4He + p$$
$$n + d \to {}^3H + \gamma \quad \text{and} \quad {}^3H + p \to {}^4He + \gamma$$

The resulting abundance of $^4He$ depends only weakly on $\eta$, but depends on $N_\nu$

$\Rightarrow N_\nu = 3$, well before CERN results

The abundance of intermediate products, $d$ and $^3He$, depends strongly on $\eta$

$\Rightarrow \Omega_b h^2 \simeq 0.022$, in agreement with CMB

But $\Omega_m \simeq 0.3$ and $h \simeq 0.7$, so $\Omega_b < \Omega_m$

This is one of the strongest pieces of evidence of non-baryonic dark matter

This thermal history was laid down even before detecting the CMB, see the paper by Dicke, Peebles, Roll and Wilkinson of 1963, linked in the web page

## END OF BBN

Reactions stop at

$T \simeq 50$ keV
$t \simeq 600$ s
$ct \simeq 2 \times 10^{13}$ cm $\simeq 2$ AV
$a \simeq 4.7 \times 10^{-9}$
$act_0 \simeq 6.1 \times 10^{19} \simeq 20$ pc