Lecture 22 Recombination

Bonometto, chap VIII sec. 4 + A1

When BBN is over, the Universe is filled with a plasma of coupled γ, e^-, nuclei and decoupled neutrinos (and dark matter)

Photons and baryons/electrons are coupled though Thomson scattering of γ and e^- + Coulomb scattering of e^- and nuclei

Decoupling: assume photons are the particles and electrons the target

$$\tau = \frac{1}{n_e \sigma_T c}, \qquad n_e \simeq \frac{\rho_b}{m_p}$$

where we have neglected He (easy to insert)

Evaluated today:

$$\tau_0 = \frac{m_p}{\rho_{b0} \sigma_T c} = 1.92 \times 10^{20}\ \mathrm{s} = 6 \times 10^{12}\ \mathrm{yr}$$

so $\tau_0 \gg H_0^{-1}$ and $\tau_0 H_0 \simeq 420$

Clearly $\tau \propto a^{-3}$, while in matter domination $t \propto a^{3/2}$.

Decoupling would roughly be at:

$$\tau H = \tau_0 H_0 a^{3/2} = 1 \Rightarrow a_{dec} = (\tau_0 H_0)^{-2/3}$$

$$\Rightarrow z_{dec} \simeq 60$$

Apart all the approximation done, this computation assumes that all e^- are free.

As a matter of fact, photon-baryon decoupling is a result of (re)combination of e^- and nuclei to atoms.

EVOLUTION OF T FOR PHOTON-BARYON PLASMA

Let's start with Friedmann equation, adding baryon thermal pressure (no He again)

$$\rho c^2 = \rho_b c^2 + \frac{3}{2}\frac{\rho_b}{m_p} k_B T + a_r T^4$$

$$P = \frac{\rho_b}{m_p} k_B T + \frac{1}{3} a_r T^4$$

and $\rho_b a^3 = \mathrm{const}$ for massive particles third Friedmann equation:

$$d\left[\left(\rho_b c^2 + \frac{3}{2}\frac{\rho_b}{m_p}k_B T + a_r T^4\right)a^3\right] = -\left(\frac{\rho_b k_B T}{m_p} + \frac{a_r T^4}{3}\right)da^3$$

$$\frac{3}{2}\frac{\rho_b}{m_p}a^3 k_B dT + 4a_r T^3 a^3 dT + 3a_r T^4 a^2 da =$$

$$= -\frac{\rho_b}{m_p}k_B T\, 3a^2 da - a_r T^4 a^2 da$$

For photons the entropy density is:

$$\sigma = \frac{4}{3}a_r T^3$$

let's define: $\sigma_{rad} \equiv \frac{\sigma}{n_b k_B} = 3.60\,\eta^{-1}$, $n_b = \frac{\rho_b}{m_b}$

Dividing Friedmann equation by $3\frac{\rho_b}{m_p}k_B T a^3$:

$$\frac{1}{2}\frac{dT}{T} + \sigma_{rad}\frac{dT}{T} + \frac{da}{a} + \sigma_{rad}\frac{da}{a} = 0$$

$$\Rightarrow \frac{dT}{T} = -\frac{1+\sigma_{rad}}{\frac{1}{2}+\sigma_{rad}}\frac{da}{a}$$

but $\sigma_{rad} \gg 1$, so:

$$\frac{dT}{T} \simeq -\frac{da}{a} \Rightarrow T \propto a^{-1}$$

Because entropy is dominated by radiation, the evolution of temperature of the coupled plasma is the same as with radiation alone.

This is true even if we go into the matter-dominated phase (we need to add a term

$$d\left[\rho_{dm} c^2 a^3\right] = 0)$$

(RE)COMBINATION

Helium has higher ionization energy, so it will recombine before Hydrogen. We will ignore He.

H recombines with:

$$H^+ + e^- \rightleftarrows H + \gamma$$

As for BBN, the high value of n_γ wrt n_b makes it hard to accumulate neutral H until temperature drops well below $13.6\ eV \sim 1.5\times10^5\ K$

With H^+, e^- and H following Boltzmann statistics, one can work out a Saha equation ($H^+ \to p$)

$$\mu_p + \mu_e = \mu_H \quad , \quad \mu_\gamma = 0$$

$$\mathcal{N}_{sp} = \mathcal{N}_{se} = \mathcal{N}_{s\gamma} = 2 , \quad \mathcal{N}_{sH} = 4$$

$$m_p c^2 + m_e c^2 = m_H c^2 + B_H \quad , \quad B_H = 13.6 \text{ eV}$$

$$m_p \simeq m_H \quad , \quad m_e \ll m_p$$

Following the same steps as for BBN:

$$\frac{n_p n_e}{n_H n_\gamma} = \frac{\sqrt{\pi}}{2\,\zeta(3)} \left(\frac{m_e c^2}{2 k_B T} \right)^{3/2} e^{-\frac{B_H}{k_B T}}$$

We need to wait for $\quad \dfrac{n_\gamma (E \gtrsim 13.6 \text{ eV})}{n_\gamma} \sim \eta$

Let's call x the fraction of ionised Hydrogen:

$$n_p = x\, n_b = n_e \quad , \quad n_H = (1-x)\, n_b$$

$$\frac{n_p n_e}{n_\gamma n_H} = \frac{n_b}{n_\gamma} \frac{x^2}{1-x} = \eta \frac{x^2}{1-x}$$

Saha equation becomes:

$$\frac{x^2}{1-x} = \eta^{-1} \frac{\sqrt{\pi}}{2\,\zeta(3)} \left(\frac{m_e c^2}{2 k_B T} \right)^{3/2} e^{-\frac{B_H}{k_B T}}$$

Recombination is half-way when $x = \frac{1}{2}$, $\frac{x^2}{1-x} = \frac{1}{2}$

so Saha equation can be solved to find T_{rec}

This yields $T_{rec} \sim 3000 \text{ K} \ll 13.6 \text{ eV}$

However, this is not accurate enough, because the reaction goes out of equilibrium. A more accurate estimate gives (Planck cosmology)

$$z_{rec} \simeq 1100$$

Moreover, a residual $x \sim 2 \times 10^{-4}$ ionised fraction is predicted to remain to low z.

The reason why the reaction goes out of equilibrium is that Lyman photons, emitted at each recombination, are promptly absorbed by other atoms, and multiple absorpions lead to ionisation.

The reaction rate gets thus slower than the expension rate.

RECOMBINATION AND DECOUPLING

Rewriting the reaction time as:

$$\tau = \frac{1}{x\, n_b \sigma_T c}$$

it is clear that a drop of x causes a jump of τ, so that recombination is promptly followed by photon decoupling and last scattering at $z \sim 1060$.

Because matter and photons share the same temperature up to decoupling, all these processes do not change the black body spectrum of radiation.

It is very important that recombination takes place AFTER decoupling:

$$\frac{\rho_{r0}}{\rho_{m0}} = 2.48 \times 10^{-5} \left(\Omega_m h^2\right)^{-1} \times \left(1 + 0.227 N_\nu\right)$$

$$\Longrightarrow \quad 1 + z_{eq} \simeq \frac{40300}{1 + 0.227 N_\nu} \Omega_m h^2 \simeq 3400$$

$$> 1 + z_{rec}$$

EVOLUTION OF PHOTONS AFTER RECOMBINATION

The CMB spectrum is that of a black body:

$$I_\nu = \frac{1}{2\pi^2} \frac{(k_B T)^3}{(\hbar c)^2} \left[\frac{x^3}{e^x - 1}\right] , \qquad x = \frac{h\nu}{k_B T}$$

Both ν and T scale like a^{-1}, so the black body spectrum is preserved along universe expansion.

EVOLUTION OF BARYON TEMPERATURE

After recombination decoupled photons evolve like:

$$\rho c^2 = \rho_b c^2 + \frac{3}{2} \frac{\rho_b}{m_p} k_B T_b , \qquad \rho_b a^3 = \text{const}$$

$$P = \frac{\rho_b}{m_p} k_B T_b$$

again neglecting He. Third Friedmann equation gives:

$$d\left[\left(\rho_b c^2 + \frac{3}{2}\frac{\rho_b}{m_p} k_B T_b\right) a^3\right] = \frac{3}{2} \frac{\rho_b}{m_p} a^3 k_B \, dT_b =$$

$$= - \frac{\rho_b}{m_p} k_B T_b \, da^3$$

$$\Rightarrow \frac{dT_b}{T_b} = -2\frac{da}{a} \Rightarrow T_b = T_{bi}\left(\frac{a}{a_i}\right)^{-2}$$

This is consistent with the behaviour of a monoatomic gas:

$$T_b V^{\gamma-1} = \text{const}, \quad \gamma = \frac{5}{3}, \quad \gamma - 1 = \frac{2}{3}$$

$$T_b (a^3)^{2/3} = T_b a^2 = \text{const} \Rightarrow T_b \propto a^{-2}$$

So baryons cool faster than radiation after decoupling

COSMIC REIONISATION

At $z \sim 8$, radiation from the first astrophysical sources, galaxies and AGN, becomes sufficient to re-ionise the hydrogen in the Universe, but this is another story.