Cosmology 1

2023/2024
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Proposed problem, lecture 18
Topic: Inflation.

A flat universe evolves as follows: (1) from $t = 0$ to $t = t_i = 10^{-35}$ s it evolves like a radiation-dominated universe, (2) at t_i an exponential (de Sitter) inflationary phase starts with Hubble parameter H, lasting for N_e e-folds up to $t = t_f$ (so that $H(t_f - t_i) = N_e$), (3) from t_f to $t_0 = 13.8$ Gyr it evolves again like a radiation-dominated universe (thus neglecting the matter dominated and Λ-dominated phases). In this evolution the scale factor $a(t)$ and the Hubble parameter $H(t)$ are continuous.

1. Find an analytic description of the scale factor $a(t)$ for the whole evolution of this universe.

2. What are the values, as a function of N_e, of the scale factors $a_i = a(t_i)$ and $a_f = a(t_f)$, assuming of course $a_0 = a(t_0) = 1$?

3. Compute the dimension of the Hubble comoving horizon $d_{cH} = c/\dot{a}$ at $t = t_i$ and at $t = t_0$. How many e-folds are needed to solve the horizon problem?

4. Compute the contributions $\Delta\eta$ to the conformal time in the three phases, as a function of N_e.