Cosmology 1

2019/2020
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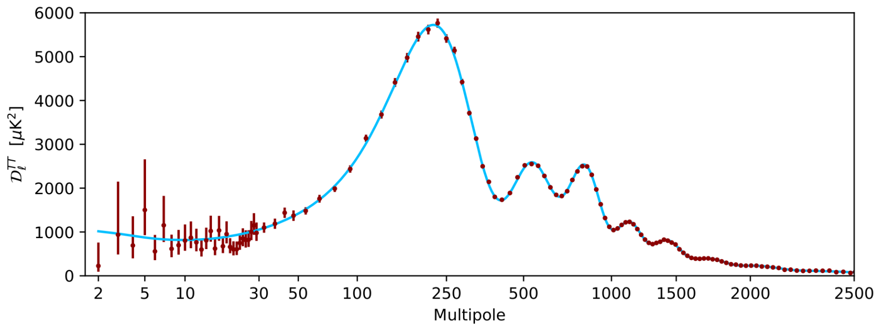
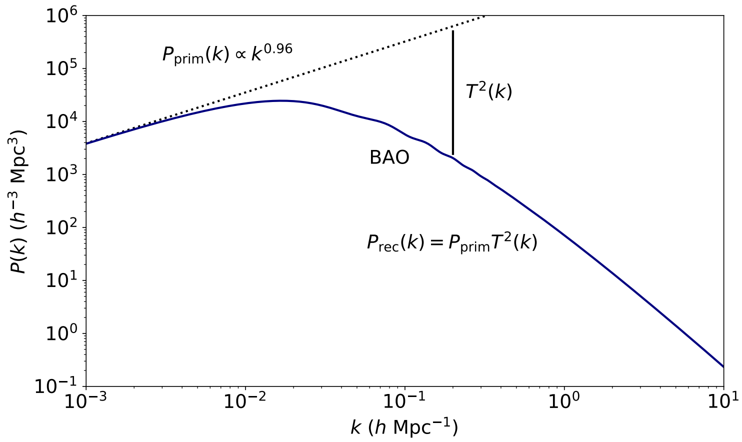
Second intermediate test
Topic: FRW models. Deadline: May 28, 13:00





A very useful standard ruler is given by the sound horizon at recombination. Before recombination, baryons and photons are tightly coupled in a highly ionised plasma, whose sound speed is $c_s = c/\sqrt{3}$; this is easily obtained from $c_s^2 = \partial p/\partial \rho$ when pressure is dominated by photons. Sound waves propagating in this plasma up to recombination imprint a spatial scale $r_{\rm sh}$, the sound horizon, that is visible as a feature both in the power spectrum of temperature fluctuations of the CMB (the acoustic peaks, see the upper figure above) and in the power spectrum of matter at later times (the Baryonic Acoustic Oscillations, BAOs, see the lower figure above). After recombination this scale expands with the scale factor.

The Planck satellite, whose cosmological parameters are given in slide 10 of Lecture 18, gives a recombination redshift of $z_{\rm rec} = 1060$ and a sound horizon comoving length of $r_{\rm sh} = 147$ Mpc.

(1) Consider a model that contains matter, radiation, cosmological constant and curvature. Compute the comoving distance and the diameter distance

as a function of redshift; numerical integration is the most obvious choice for this step. To do this, show that the comoving distance $d_c(z)$ (or $L_{\rm los}(z)$) of an object emitting light at redshift z can be written as:

$$d_c(z) = \int_{t(z)}^{t_0} \frac{cdt'}{a(t')} = \int_0^z \frac{cdz'}{H(z')}$$

Recall that the second Friedmann equation can be written as:

$$\frac{H(z)^2}{H_0^2} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda + \Omega_K(1+z)^2$$

with $\Omega_K = 1 - \Omega_m - \Omega_r - \Omega_\Lambda$.

Hint: It is a very good idea to test the numerical result against analytic calculations for the cases in which these are available.

Describe the numerical approach adopted, and report the diameter distance (in Mpc) as a function of z, in the range $z \in [0, 10]$, for Planck cosmological parameters, assuming a flat Universe, and for the same parameters but adding ± 0.1 to Ω_m, so as to have non-flat models. *Hint: the parameters given in slide 10 of Lecture 18 assume indeed that the universe is flat, with the exception of Ω_k. We will not need parameter errorbars here. Do not forget radiation, but fix the density parameters so as to keep the universe flat.*

(2) The sound horizon at recombination can be estimated as $1/\sqrt{3}$ times the particle horizon at the recombination redshift $z_{\rm rec} = 1060$, assumed to be constant. Use the tools developed above to compute the sound horizon for the Planck cosmological parameters. Report its physical (at $z_{\rm rec}$) and comoving values in Mpc; how much does the latter differ from the 147 Mpc comoving value found by Planck? This difference is mainly due to the fact that the sound speed c_s evolves in time, details are explained in Chapter 7 of the Vittorio textbook. *Hint: you can correct for this difference by rescaling the sound horizon (i.e. multiplying it by a constant) to be* 147 *Mpc for the Planck flat cosmology.*

(3) Suppose astronomers report that, at redshift $z = 0.3$, the BAO scale $r_{\rm sh}$ subtends an angle of $\theta = 0.1166 \pm 0.0023$ rad. Keeping H_0 and Ω_r fixed to their Planck values, what region of the $\Omega_m - \Omega_\Lambda$ parameter space is consistent with this measurement at 1 and 2 σ? Look carefully at your result, and comment it, in the light of our need to get the tightest constraints on cosmological parameters. *Hint: do not bother with negative values of Ω_Λ, but do not restrict either to flat universes.*

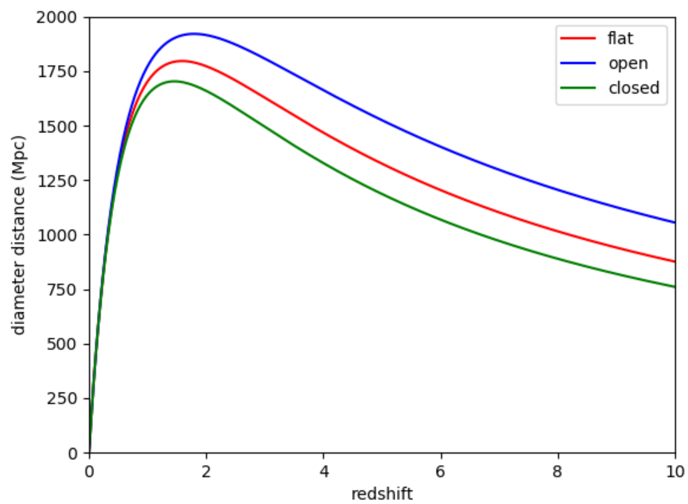
(4) The $r_{\rm sh}$ scale is best constrained by CMB observations. Suppose that it subtends an angle of $\theta* = 0.01041 \pm 0.00031$ rad (it's actual errorbar is 100 times smaller than this!). What region of the $\Omega_m - \Omega_\Lambda$ parameter space is consistent with this measurement at 1 and 2 σ? Look carefully at your result and argue what parameter is most accurately constrained by this observation.

Solution

(1) The numerical integration of the comoving distance amounts to computing a rather standard integral of a relatively well-behaved function (the integrand diverges at infinity in a way that is easy to handle), it can be easily done by any integration library. A good practice is to use directly the complete $H(z)$ as given above, but checking results against known analytic solutions, starting from the simple Einstein-de Sitter Universe.

The angular diameter distance is readily computed from the comoving distance. Calling $\mathcal{R}_0 = c/H\sqrt{|\Omega_K|}$, we have that the $d_D = d_c/(1+z)$ for a flat universe, while $d_D = \mathcal{R}_0 \sin(d_c/\mathcal{R}_0)/(1+z)$ or $d_D = \mathcal{R}_0 \sinh(d_c/\mathcal{R}_0)/(1+z)$ for a closed or open universe. To have an exactly flat Planck cosmology, the three parameters Ω_m, Ω_Λ and Ω_r must add to 1, but it is clear from the numbers given in the slides that the first two add to 1 while the third is simply smaller than the other parameters' errorbars. I have solved the issue by subtracting the value of Ω_r to the Ω_Λ value reported in the table.

The three requested diameter distances are reported below.



(2) The comoving sound horizon at recombination can be estimated solving a very similar integral as the comoving distance:

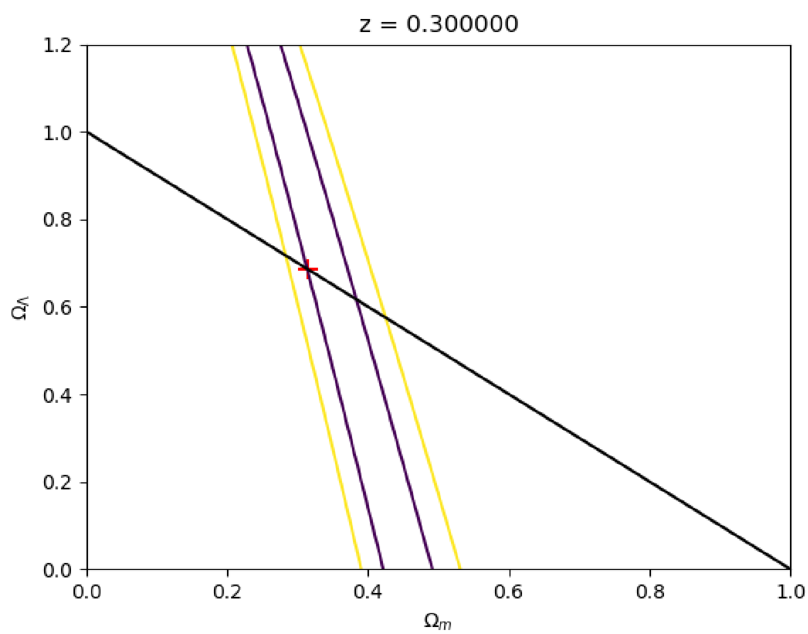
$$d_{\rm sh,c}(z_{\rm rec}) = \frac{1}{\sqrt{3}} \int_{z_{\rm rec}}^{\infty} \frac{c dz'}{H(z')} = 166 \text{ Mpc}$$

(the numerical values refers to what I have obtained for flat Planck cosmology) while the physical (proper) value at recombination is

$$d_{\rm sh,p}(z_{\rm rec}) = d_{\rm sh,c}(z_{\rm rec})/(1 + z_{\rm rec}) = 0.156 \text{ Mpc}$$

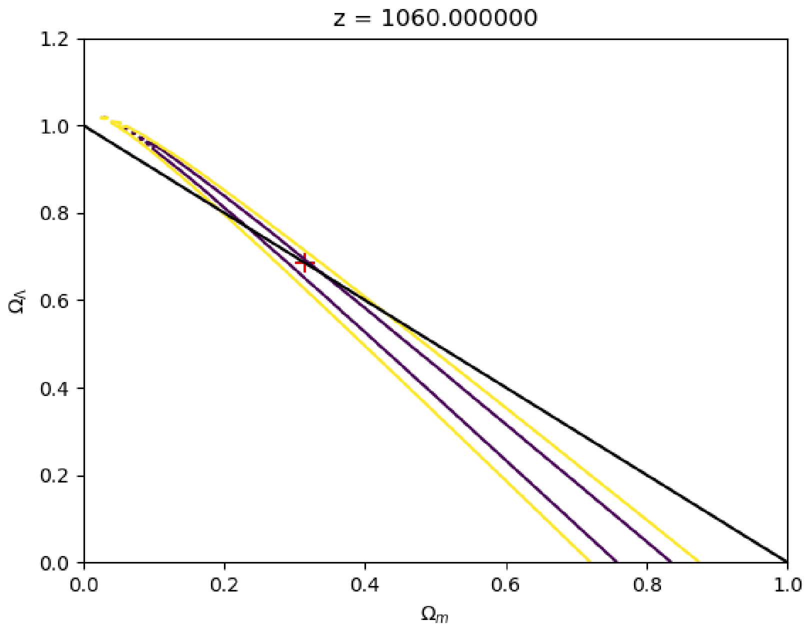
The comoving value of 166 Mpc is larger than the 147 Mpc value quoted in the Planck paper by a factor 1.130. The reason why our estimation is approximated is explained above; if we keep this approximation we bias the parameter values obtained by comparing our inaccuate theory with the observation. A better (quick and dirty) solution can be obtained by multiplying our estimate of the sound horizon, **for all parameter values**, by the inverse of 1.130, so as to obtain 147 Mpc for the Planck cosmology. Of course, following the approach of the Vittorio textbook would be a much better option, but it would not allow us to solve this problem in time.

(3) The simplest way to sample a 2D parameter space is to compute the theoretical expectation on a grid of parameter values. I used a grid with $\Omega_m \in [0,1]$ and $\Omega_\Lambda \in [0,1.2]$, with 100 values per side for a total of 10000 evaluations; I avoided negative values of Λ that are of little interest to us. I computed the χ^2-like quantity $(\theta_{\rm predicted} - \theta_{\rm observed})^2/\sigma^2_{\rm observed}$, where $\theta_{\rm predicted} = d_{\rm sh,p}(\Omega_m, \Omega_\Lambda)/d_D(z_{\rm rec})$ and plotted the contours where this quantity has values 1 and $2^2 = 4$. The computation takes less than a minute on my laptop. The result is given below, the blue and yellow curves refer to $1-\sigma$ and $2-\sigma$. The black line gives the values relative to a flat universe ($\Omega_m - \Omega_\Lambda = 1$, $\Omega_K = 0$), while the red cross gives the true cosmology, that lies well inside the allowed region.



Two things are evident from this plot. First, there is a degeneracy between the two parameters, meaning that several different combinations of Ω_m and Ω_Λ give the same prediction. It is clear that this observation alone cannot constrain the parameters, but if we combined it with another observation that has a different degeneracy, like the Hubble diagram of type Ia SNe, we could combine the two to have a very good constraint. The other evident thing is that the Ω_m parameter is constrained better than Ω_Λ.

(4) Computing the parameter values compatible with the extension of the first acoustic peak in the CMB temperature fluctuations requires the same computation as above, one has only to change the redshift and the angle value. The result is however different:



Clearly the degeneracy is almost aligned with the lines of constant Ω_K, so one can argue that this observation is able to constrain the curvature parameter Ω_K much tighter than the others. This is correct even if Ω_K is not a free parameter here but depends on the others; one could decide for example to use it as a free parameter in place of Ω_Λ. As a matter of fact, the position of the first acoustic peak gives the clearest evidence that we live in a flat universe.