Cosmology 1

2021/2022
Prof. Pierluigi Monaco

Second intermediate test
Topic: FRW models.
Deadline: May 26, 11:00.

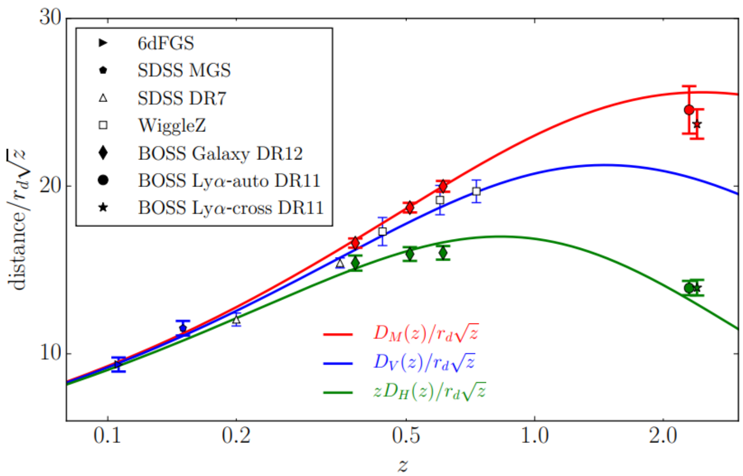


Figure 14. The "Hubble diagram" from the world collection of spectroscopic BAO detections. Blue, red, and green points show BAO measurements of D_V/r_d, D_M/r_d, and D_H/r_d, respectively, from the sources indicated in the legend. These can be compared to the correspondingly coloured lines, which represents predictions of the fiducial Planck ΛCDM model (with $\Omega_m = 0.3156$, $h = 0.6727$). The scaling by $\sqrt{z}$ is arbitrary, chosen to compress the dynamic range sufficiently to make error bars visible on the plot. For visual clarity, the Lyα cross-correlation points have been shifted slightly in redshift; auto-correlation points are plotted at the correct effective redshift. Measurements shown by open points are not incorporated in our cosmological parameter analysis because they are not independent of the BOSS measurements.

Source: `https://arxiv.org/pdf/1607.03155.pdf`.

The Baryonic Oscillations Spectroscopic Survey (BOSS) collaboration has performed detailed measurements of the Baryonic Acoustic Oscillation (BAO) scale at two redshifts, thus determining with accuracy three distances that are directly related to the cosmological parameters of an FRW model. These distances are:

(i)
$$D_M(z) := (1+z)D_A(z)$$

is the comoving angular diameter distance (here $D_A(z)$, the angular diameter distance, is what is called $d_D(z)$ in my notes),

(ii)
$$D_V(z) := \left[\frac{czD_M^2(z)}{H(z)}\right]^{1/3}$$

where $H(z)$ is the usual Hubble parameter, is a suitable combination of D_M and H,

(iii)

$$D_H(z) := \frac{c}{H(z)}$$

is the Hubble horizon.

The figure shown above reports these distances divided, for convenience, by $r_d\sqrt{z}$, where $r_d = 147.78$ Mpc is the size of the sound horizon, measured from the position of the acoustic peaks in the angular power spectrum of CMB temperature fluctuations; the horizon distance D_H is also multiplied by z.

The measurements of these three distances are the following:

	$z = 0.32$	$z = 0.57$
D_V	1270 ± 14 Mpc	2033 ± 21 Mpc
D_M	1294 ± 21 Mpc	2179 ± 35 Mpc
H	78.4 ± 2.3 km s^{-1} Mpc^{-1}	96.6 ± 2.4 km s^{-1} Mpc^{-1}

The test consists in using these data to obtain constrains on cosmological parameters.

(1) Using a flat ΛCDM cosmology with $\Omega_m = 0.3156$ and $h = 0.6727$, reproduce the BOSS figure given above, reporting the model predictions and the measurements in the table. Notice that the figure reports other measurements.

(2) Compute a χ^2 for the comparison of measurements and data, assuming that the observational errors are uncorrelated, and check that it is acceptable. Then try to estimate errorbars on the two parameters Ω_m and h, obtained from comparing the model to these data. How do they compare with Planck errorbars on the same parameters?

(3) Relax the flatness hypothesis, can you get a measure of Ω_k?

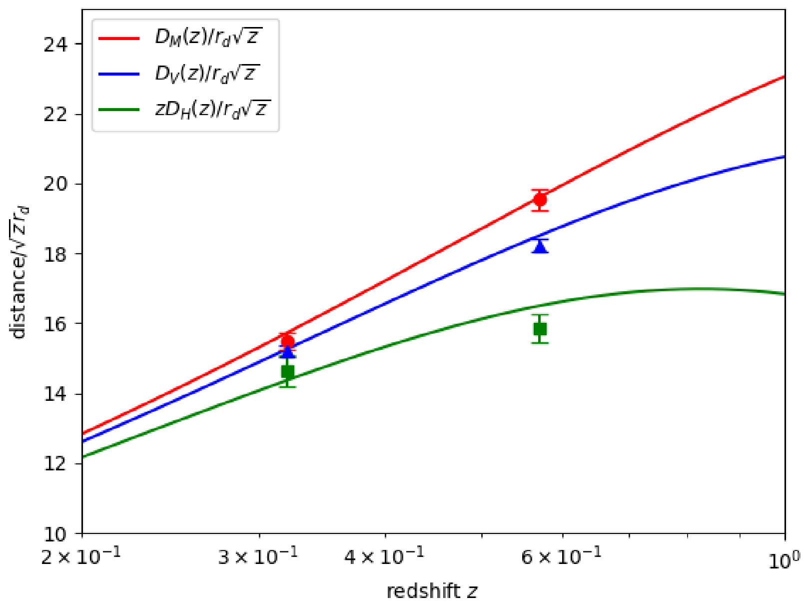
The degree of sophistication of the statistical analysis will depend on the past experience of participants, sophistication will be appreciated but is not required. A very crude approach may be valid in this context, provided that a discussion of its limitations is given.

Solution

Reproducing the BOSS plot in figure 1 is a matter of computing the comoving angular diameter distance, $D_M(z)$ and the Hubble parameter $H(z)$, implementing the equations discussed during the course. To simplify the solution, it is very important to notice that

$$\int \frac{cdt}{a(t)} = \int \frac{cdz}{H(z)}$$

It is also important to notice that the figure is logscale in redshift. For a flat universe, the comoving angular diamenter distance coincides with the comoving distance, while this is not true in presence of curvature. The figure below shows the results I have obtained, more focused on the data points to make errorbars visible.



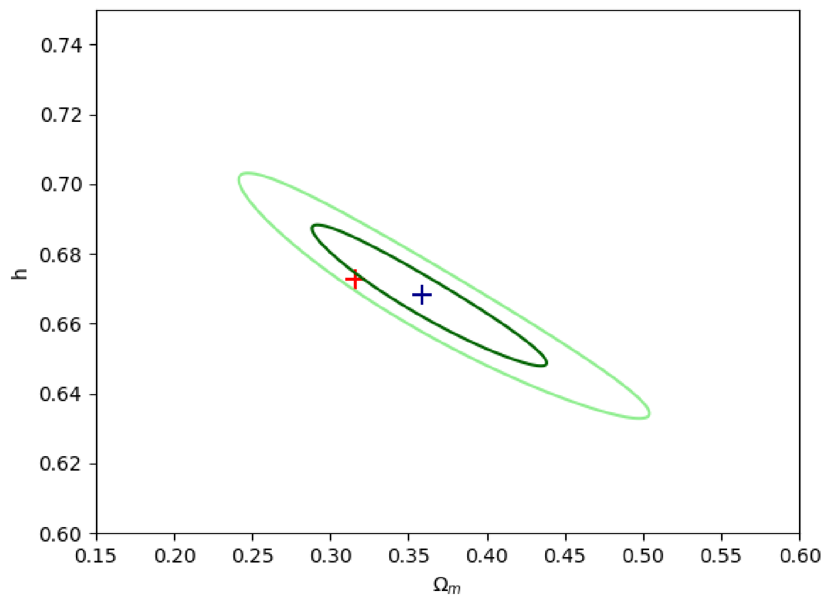
I compute the χ^2 statistics as usual: let's call **d** the data vector, given by the six measurements, **e** the errors on the measurements, and $\mathbf{t}(\Omega_m, \Omega_k, h)$ the theory predictions for the six measurements, that depend on model parameters:

$$\chi^2 = \Sigma_i \left(\frac{d_i - t_i}{e_i}\right)^2$$

I assume here that the measurements are independent, through we know that they are not, but taking account of their covariance is beyond our interests. Using values $\Omega_m = 0.3156$, $h = 0.6727$ and $\Omega_k = 0$, I obtain $\chi^2 = 6.088$, with an associated probability of 29.7% for 6 degrees of freedom.

This result is not trivial at all: we are comparing the predictions of a model, whose parameters are determined using Planck's observations of CMB temperature fluctuations, with results obtained using measurements of galaxy clustering at redshift $z \sim 0.5$. Their consistence is a remarkable confirmation of the validity of the ΛCDM cosmological model. Notice that these data measure the Hubble constant through the baryonic oscillations, and use no SNIa data; we know that the Hubble constant from SN data is in tension with that from CMB data, while this determination is based on a theoretical measure of the sound horizon r_d; the consistency of the two datasets is due to the consistency of the BAO feature across redshift.

Assuming a flat universe, and making the simplifying assumption that the sound horizon r_d does not depend on cosmological parameters (this is not correct, but its variation is beyond our interests here), it's easy to compute the best fit and the confidence intervals by minimising the χ^2. I compute the levels corresponding to 68% and 95% from the integral of the χ^2 distribution with 4 degrees of freedom. Using this procedure, I obtained these confidence intervals:

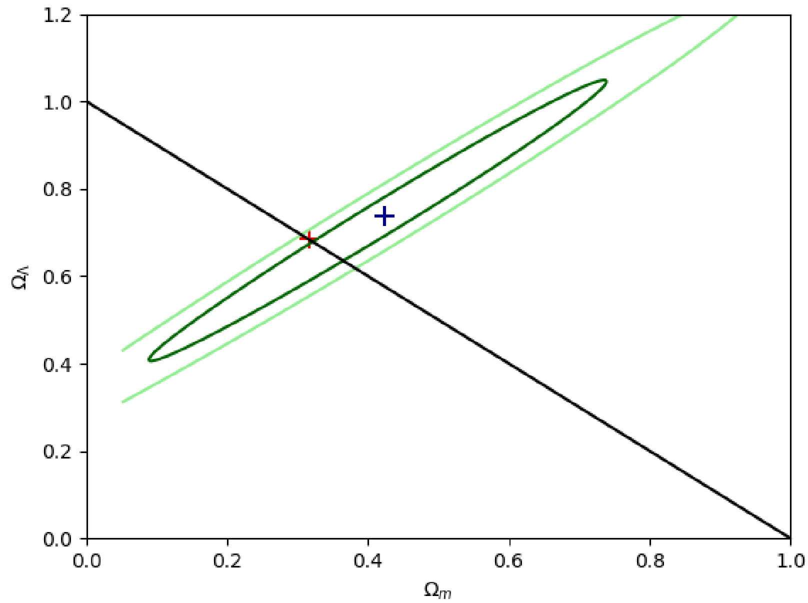


There is a significant level of degeneracy between parameters, a very common feature in this kind of analysis. The resulting measures with 1-σ errorbars are

$$\Omega_m = 0.359 \pm 0.075 \quad \text{and} \quad h = 0.668 \pm 0.020$$

The relative errors are respectively $\sim 21\%$ and 3%, much larger than Planck's values of 2.3% and 0.8%; it's clear that Planck is much more powerful in constraining parameters. However, the power of these measurements comes from the fact that they are obtained at a completely different redshift, and they can be improved by observing larger cosmological volumes. Also, one can construct a combination of Ω_m and h, following the degeneracy, that would be constrained with a smaller error.

We can relax the condition of flat universe, and consider non-flat cosmologies. The computation of comoving angular diameter distance is more complex in this case, one has first to compute $\mathcal{R}_0 = c/H\sqrt{|\Omega_k|}$, then $r = d_c/\mathcal{R}_0$ and $L_{\rm los} = \mathcal{R}_0\chi(r)$, where $\chi(r) = \sin^{-1}(r)$ or $\sinh^{-1}(r)$ according to the sign of Ω_k. I have kept h fixed, and varied Ω_m and Ω_k to obtain confidence intervals, with this result:

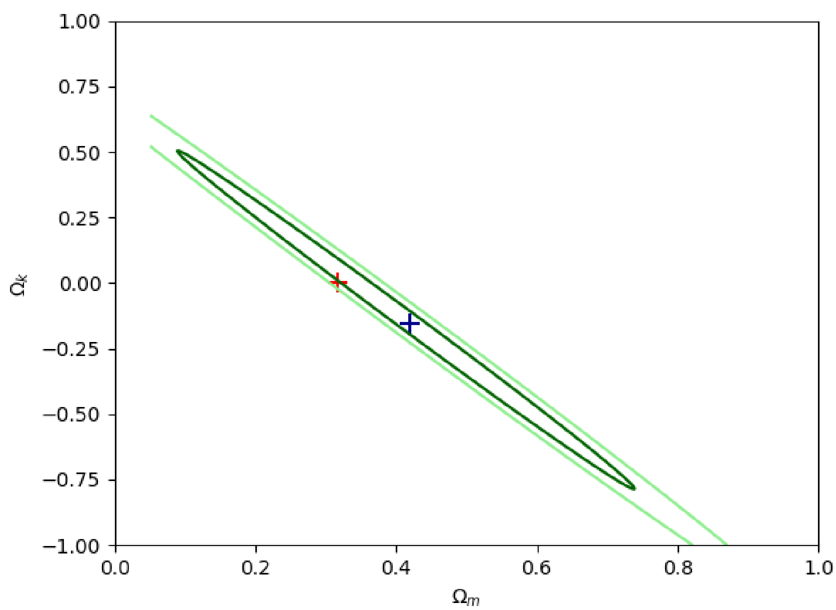


Here the black line marks the flat universes. The result is disappointing, the confidence intervals are highly degenerate along a direction nearly perpendicular to the flat universe line, so the errorbars are much larger:

$$\Omega_m = 0.42 \pm 0.32 \quad \text{and} \quad \Omega_\Lambda = 0.74 \pm 0.32$$

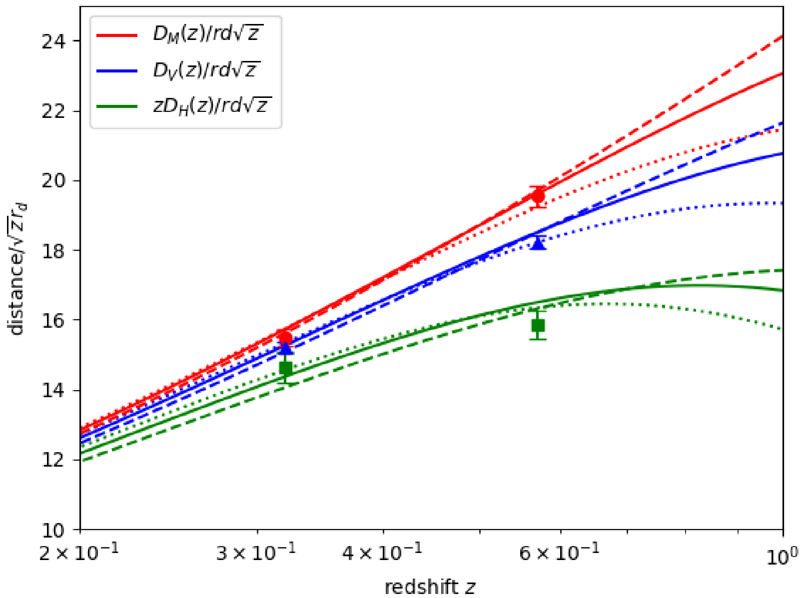
The error on Ω_k results very large, if this is used as a parameter in place of Ω_Λ:

$$\Omega_m = 0.42 \pm 0.32 \quad \text{and} \quad \Omega_k = -0.15 \pm 0.65$$



Things do not improve when all three parameters are left free to vary; in this case the error on h does not grow, but the errors on the Ω's remain large.

These data simply cannot constrain curvature. To better illustrate this degeneracy, I plot here predictions for three models, the Planck one, a closed model with $\Omega_m = 0.67$ and $\Omega_\Lambda = 1.0$, so that $\Omega_k = -0.67$ (dotted lines), and an open model with $\Omega_m = 0.05$ and $\Omega_\Lambda = 0.4$, so that $\Omega_k = 0.55$ (dashed lines). These models are within the 1-σ confidence intervals, and they are pretty similar at the redshifts of the data, and diverge at higher redshift.



So it's clear that to constrain Ω_k we have to go to higher redshift. Again, we have kept r_d constant, so these conclusions are to be taken as purely indicative.