Cosmology 1

2022/2023
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Second intermediate test
Topic: FRW models.
Deadline: May 25, 11:00.

The results of the Planck satellite favour a flat ΛCDM model with $h = 0.674$ and $\Omega_m = 0.315$. Curvature is constrained to be $\Omega_k = -0.011 \pm 0.013$, so a closed or an open model would fit the data as well. In this exercise we will investigate the observational consequences of curvature.

It is useful to know that:

$$\int_t^{t_0} \frac{cdt}{a(t)} = \int_0^z \frac{cdz}{H(z)}$$

where t is the cosmic time at redshift z. This makes the numerical computation of the comoving distance easy. We will neglect radiation in the following computations. Moreover, we will assume that recombination takes place at $z_{\rm rec} = 1100$, and that the comoving size of the sound horizon is $r_d = 148$ Mpc. These two numbers depend on the model so they should be computed self-consistently, but for simplicity we will keep them constant.

(i) Assume that the error on Ω_k is Gaussian. Then compute and report, for a set of values of Ω_k that sample the distribution of its true value, (a) the size of the Hubble horizon c/H_0, (b) the size of the visible horizon, (c) the size of the particle horizon, (d) the size of the curvature radius $\mathcal{R}_0$.

(ii) Compute and report, for the various models defined above, the angle subtended by the sound horizon, and infer the precision with which this angle has been measured in order to provide the constrain on Ω_k reported above.

(iii) A friend that is studying engineering asks you about this test; you decide to produce some plots that are as clear as possible. Find the clearest way to visualize the effect of curvature on measurable quantities, and the relation among the horizon scales and the curvature radius.

Solution

(i) The size of the Hubble horizon is simply $c/H_0 = 4447$ Mpc, and is the same for all curvature parameter values. The size of the visible and particle horizons must be computed by numerically evaluating the integral reported above, with integration limits from 0 to $z_{\rm rec}$ for the visible horizon and from 0 to infinity for the particle horizon. The curvature radius is computed as:

$$\mathcal{R}_0 = \frac{c}{H_0\sqrt{|\Omega_k|}}$$

The table below gives the values for these quantities for Ω_k values that sample a normal distribution from -3σ to 3σ, in steps of σ. The Ω_Λ parameter is tuned to preserve the sum of all Ω's to be unity.

	Ω_k	Ω_Λ	c/H_0 Mpc	$d_{\rm vis}$ Mpc	$d_{\rm ph}$ Mpc	$\mathcal{R}_0$ Mpc
-3σ	-0.050	0.735	4448	14107	14585	19892
-2σ	-0.037	0.722	4448	14061	14539	23124
-1σ	-0.024	0.709	4448	14016	14494	28711
	-0.011	0.696	4448	13972	14449	42410
1σ	0.002	0.683	4448	13928	14405	99459
2σ	0.015	0.670	4448	13884	14362	36317
3σ	0.028	0.657	4448	13842	14319	26582

One can notice a few facts: the visible and particle horizons are ~ 3 times the Hubble horizon, in an Einstein-de Sitter universe the particle horizon would be twice the Hubble horizon; this is mostly an effect of the lower matter density. The particle horizon (without inflation of course) is just marginally larger than the visible horizon, by only ~ 480 Mpc with little dependence on cosmological parameters. The curvature radius results larger than the particle horizon in all cases, but with the lowest value of $\Omega_k = -0.05$, $\mathcal{R}_0$ is only marginally larger than $d_{\rm ph}$.

(ii) The sound horizon r_d is given as a comoving quantity, its physical size at $z = 1100$ is $r_d/(1 + z_{\rm rec}) = 0.134$ Mpc. The angular diameter distance $d_D(z)$ must be computed from the comoving distance $d_c(z)$ taking into account the curvature of the universe:

$$d_D(z) = \frac{1}{1+z}\mathcal{R}_0 \sin\left(\frac{d_c(z)}{\mathcal{R}_0}\right)$$

if the universe is closed, otherwise for an open universe sin must be substituted with sinh. The resulting angles are given in the table below.

	Ω_k	Ω_Λ	angle (deg)
-3	-0.050	0.735	0.6546
-2	-0.037	0.722	0.6419
-1	-0.024	0.709	0.6297
0	-0.011	0.696	0.6180
1	0.002	0.683	0.6069
2	0.015	0.670	0.5961
3	0.028	0.657	0.5858

A circle of radius r_d (and diameter twice as large) would then subtend an angle of roughly one degree. This implies that measures of CMB fluctuations must have a resolution that is well below 1 deg, to properly resolve this feature. If we want to use it to constrain Ω_k, we must resolve the difference in angles corresponding to 1σ, that is ~ 0.01 deg angles, amounting to less than an arcmin.

(iii) I give no answer to this question, every group will find the best way to visualize their geometrical intuition. The reason why I use the concept of "engineer friend" is because I want students to focus on their understanding, and a good way to do it is to explain it to a person that has a good general background but no specific knowledge in cosmology.