Cosmology 1

2023/2024
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Second intermediate test
Topic: FLRW models.
Deadline: April 29, 11:00.

Olber's paradox is based on the fact that in a static universe filled with stars the sky surface brightness should be of the same order of the surface brightness of an average star, so the night sky should not be dark.

We will compute, under very simplistic assumptions, the surface brightness of the sky due to all the visible galaxies, called the extragalactic background light (EBL), in a cosmological model; we will assume here as a baseline the flat ΛCDM model with $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$ and $h = 0.67$ ($H_0 = 100\,h$ km s^{-1} Mpc^{-1}).

We will assume that all visible matter is in galaxies. We define the galaxy luminosity function $\Phi(L)$, as the number of galaxies in a comoving volume dV with luminosity in an interval dL around L, divided by $dV dL$, with units (comoving) Mpc^{-3} L$_\odot^{-1}$. This is assumed to have the form:

$$\Phi(L)dL = \Phi_* \left(\frac{L}{L_\star}\right)^\alpha \exp\left(-\frac{L}{L_\star}\right)\frac{dL}{L_\star}$$

with $\Phi_\star = 5 \times 10^{-3}$ Mpc^{-3}, $\alpha = -1.2$ and $L_\star = 5 \times 10^{10}$ L$_\odot$, where L$_\odot$ = 3.84×10^{26} W (or 3.84×10^{33} erg s^{-1}) is the solar luminosity. We assume for simplicity that all galaxies appear at $z_{\rm start} = 5$ and that their luminosity function does not evolve in redshift after their birth. Also, we assume that galaxies have a minimum luminosity of 10^6 L$_\odot$.

(1) The first step is to compute $dN/dz\,d\Omega\,df$, the number of galaxies per unit solid angle Ω, at redshift in an interval dz around z, with observed flux in an interval df around f. To this aim it is useful to express the integral needed to compute the comoving distance as an integral in z.

(2) The second step is to integrate $dN/dz\,d\Omega\,df$ in redshift and flux, so as to obtain the total number of potentially visible galaxies per unit solid angle (please use the square degree instead of the steradian). However, this number in itself is not going to be very useful; a typical observation resolves objects down to a flux limit f_0, a more useful quantification is the number of visible galaxies as a function of the flux limit. Hint: for the values of f_0 refer to the observed flux of an $L_\star$ galaxy at some intermediate redshift, say $z = 2$, and then vary it around this value.

(3) The third step is to integrate the flux of all galaxies, in redshift and flux, to obtain the surface brightness of the EBL. Compute this quantity (in W m$^-$2 sq.deg.$^{-1}$, or erg s^{-1} cm^{-2} sq.deg.$^{-1}$) and compare it to the Sun surface brightness. It is again interesting to divide this integral into galaxies above and below the limiting flux: to what flux should we push our observations to resolve 90% of the EBL?

(4) For those that are not too tired after the first three points, try to see what happens when other cosmological models (maybe non-flat ones) are used: can the EBL be used to infer cosmological parameters?

Comment the results in view of the Olbers paradox: is this quantity much lower than the sun surface brightness? why does it not diverse? is the value of $z_{\rm start}$ essential?

Solution

(1) It is easy to see that:

$$d_c(z) = \int_{t(z)}^{t_0} \frac{cdt}{a(t)} = \int_0^z \frac{cdz}{H(z)}$$

so that $d(d_c)/dz = c/H(z)$, where (neglecting radiation)

$$H(z) = H_0 E(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda + (1 - \Omega_m - \Omega_\Lambda)(1+z)^2}$$

The comoving volume contained in an interval dz and subtending a solid angle $d\Omega$ is then:

$$dV = \frac{c}{H_0} \frac{d_c^2(z)}{E(z)} \, dz \, d\Omega$$

The relation between f and L is $L = f \, 4\pi d_L(z)^2$, so $df/f = dL/L$. I first use $d\ln L$ as a differential (NB: here $\ln L$ is an abbreviation of $\ln(L/L_\odot)$); it is easy to obtain that, if $dN/dV\,dL = \Phi(L)$, then $dN/dV\,d\ln L = \tilde{\Phi}(\ln L) = L\Phi(L)$, so:

$$\frac{dN}{dz\,d\Omega\,d\ln L} = \frac{dN}{dV\,d\ln L} \frac{dV}{dz\,d\Omega} = \frac{c}{H_0} \frac{d_c^2(z)}{E(z)} L(f)\Phi(L(f))$$

where only in this equation we make it explicit that L is a function of f. Then, given that $df = f\,d\ln f = f\,d\ln L$, we have $d\ln L/df = 1/f$, so:

$$\frac{dN}{dz\,d\Omega\,df} = \frac{dN}{dz\,d\Omega\,d\ln L} \frac{d\ln L}{df} = \frac{4\pi c}{H_0} \frac{d_L^2 d_c^2}{E} \Phi(4\pi d_L^2 f)$$

In the implementation, it is more convenient to work in $\ln L$.

(2) The galaxy luminosity function in this exercise does not depend on redshift, the total number density of galaxies (per Mpc^3) is:

$$\bar{n} = \int_{L_{\min}}^{\infty} \Phi(L) dL = \int_{\ln L_{\min}}^{\infty} L\Phi(L)\, d\ln L = 0.189 \ \mathrm{Mpc}^{-3}\,.$$

This integral must be evaluated numerically; it is convenient to perform it in $\log L$, achieveing a much more stable numerical result.

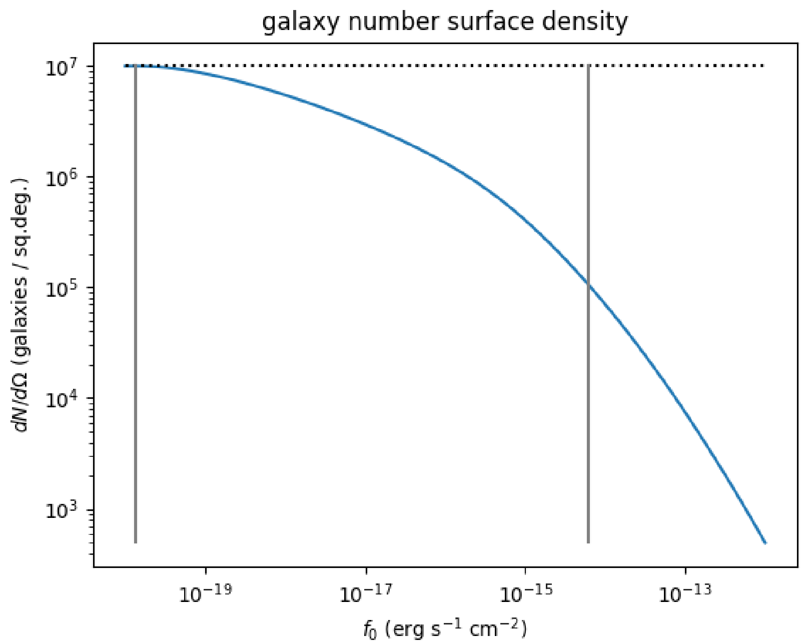
The surface density (number per sq. deg.) of galaxies that are in principle visible can then be computed by multiplying this constant density by the comoving volume subtended by a sq. deg. to $z_{\rm start}$:

$$\frac{dN_{\rm tot}}{d\Omega} = \bar{n} \times \int_0^{z_{\rm start}} \frac{c}{H_0} \frac{d_c^2}{E} dz = 9.93 \times 10^6 \ \text{galaxies sq.deg.}^{-1}\,.$$

To compute the number surface density of galaxies above a flux f_0 one cannot separate any more the double integral in z and L, as the lower limit in L (or $\ln L$) depends on redshift. Let's call $L_0 = 4\pi d_L^2 f_0$ the luminosity of the galaxy with flux f_0 at z, then the integral will be integrating from the (log of the) max of L_0 and $L_{\rm min}$:

$$\frac{dN(f_0)}{d\Omega} = \int_0^{z_{\rm start}} dz\, \frac{c}{H_0}\frac{d_c^2}{E} \int_{\ln\max(L_0(z),L_{\rm min})}^{\infty} L\Phi(L)\, d\ln L$$

Let's call $f_\star(z)$ the flux with which a galaxy of luminosity $L_\star$ is observed at redshift z, we have that $f_{\rm ref} := f_\star(z=2) = 6.17 \times 10^{-15}$ erg s^{-1} cm^{-2}. Moreover, let's call $f_{\rm min} = 1.38 \times 10^{-20}$ erg s^{-1} cm^{-2} the flux of the fainters visible galaxy of luminosity $L_{\rm min}$ at $z_{\rm start}$. We show below the galaxy surface density as a function of f_0, marking the position of $f_{\rm ref}$ and $f_{\rm min}$.



The galaxy surface density grows continually with decreasing f_0, until it saturates at $f_{\rm min}$; at $f_{\rm ref}$ we only see $\sim 1\%$ of the galaxies.

(3) Let's call $dF/dz\, d\Omega\, d\ln L$ the flux received from all galaxies in a comoving volume in $dz\, d\Omega$ from galaxies that have luminosity L:

$$\frac{dF}{dz\, d\Omega\, d\ln L} = f\frac{dN}{dz\, d\Omega\, d\ln L} = \frac{c}{H_0}\frac{d_c^2}{E}\frac{L^2}{4\pi d_L^2}\Phi(L)$$

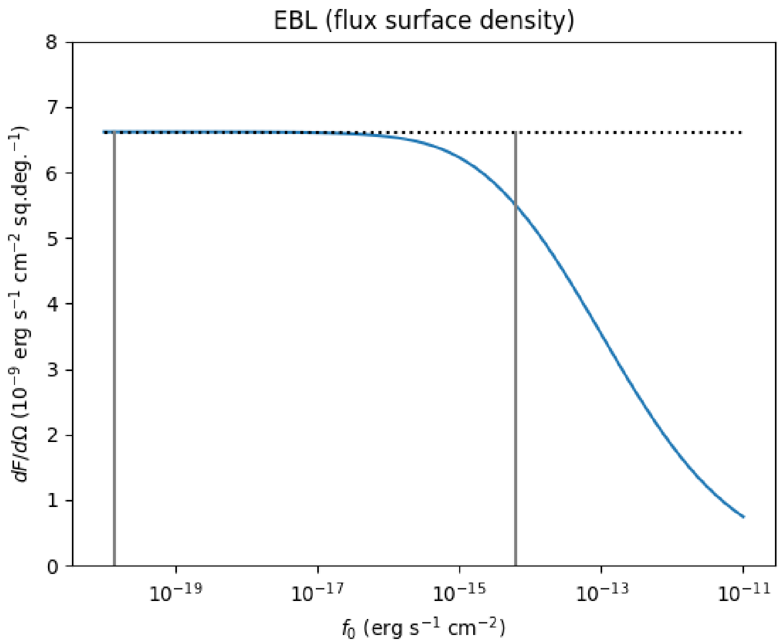
This EBL is computed by integrating this term in $\ln L$ and z; if a limiting flux is used, the integral in $\ln L$ again evaluated from the max of L_0 and $L_{\rm min}$:

$$B_{\rm EBL} := \frac{dF}{d\Omega} = \int_0^{z_{\rm start}} dz\, \frac{c}{4\pi H_0}\frac{d_c^2}{E d_L^2} \int_{\ln\max(L_0(z),L_{\rm min})}^{\infty} L^2\Phi(L)\, d\ln L$$

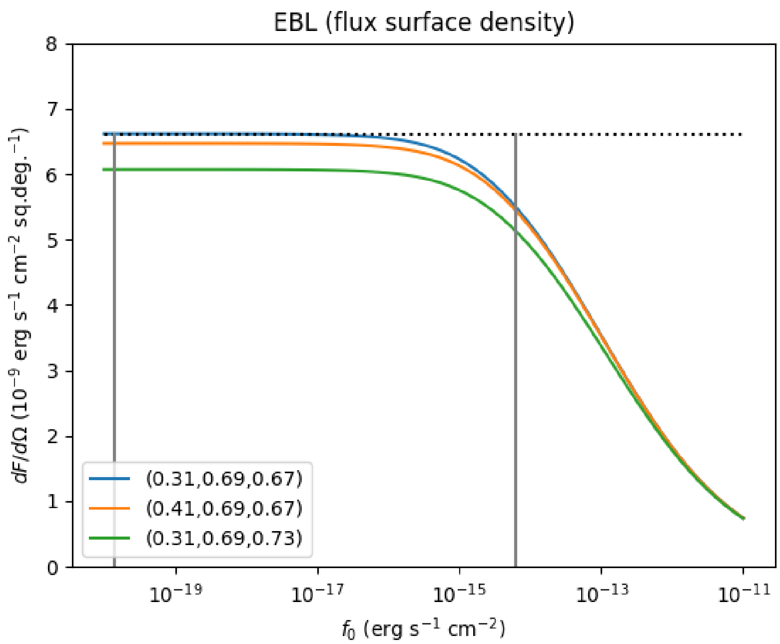
For $f_0 \to 0$ the value results:

$$B_{\rm EBL} = 6.61 \times 10^{-9} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sq.deg.}^{-1}$$
$$= 6.61 \times 10^{-12} \text{ W m}^{-2} \text{ sq.deg.}^{-1}$$

The figure below shows the flux from "resolved" sources versus the EBL (the horizontal dotted line). Again, the values of $f_{\rm ref}$ and $f_{\rm min}$ are reported as vertical lines. This time the integral converges much faster, it is at 89% of the final value at $f_{\rm ref}$ and goes to 95% at $0.13 f_{\rm ref}$, to 99% at $0.0155 f_{\rm ref} \simeq 7000 f_{\rm min}$.



(4) Apart from final numbers, none of the conclusions given above depends on cosmological parameters. If the functions $H(z)$, $d_c(z)$ and $d_L(z)$ are adapted to work with a generic (non-flat) cosmology, the computation of the integrals can be redone to obtain solutions for different cosmologies. As an example I report the computation of the EBL for three cosmologies, the baseline one used above, a flat cosmology with $h = 0.73$ and a closed cosmology with $\Omega_m = 0.41$. The figure below reports these results; numbers in parenthesis are $(\Omega_m, \Omega_\Lambda, h)$. However, this cannot be exploited to constrain cosmological parameters because its determination is dominated by the uncertainty on the galaxy luminosity function.



The bolometric flux from the Sun is $f_\odot = L_\odot/4\pi(\mathrm{AU})^2 = 1.36 \times 10^6$ erg s^{-1} cm^{-2}, where AU $= 1.50 \times 10^{13}$ erg s^{-1} is the Astronomical Unit. This is distributed over a circle of angular radius $R_\odot/AU = 0.265$ deg (after transforming radians to degrees), so the area of the sun is $\Omega_\odot = 0.222$ sq.deg. Then for the Sun, $B_\odot = f_\odot/\Omega_\odot = 6.11 \times 10^6$ erg s^{-1} cm^{-2} sq.deg.$^{-1}$. On the other hand, the radiation emitted by a black body of temperature T per unit (physical) area is $\sigma_{\mathrm{SB}}T^4$, where $\sigma_{\mathrm{SB}} = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-1} is the Stefan-Boltzmann constant. Then $L_\odot = 4\pi R_\odot^2 \sigma_{\mathrm{SB}} T_\odot^4$, where $T_\odot = 5775$ K is the effective temperature of the sun. It is easy to demonstrate that $B_\odot = \sigma_{\mathrm{SB}} T_\odot^4/\pi$, depening only on the sun effective temperature.

We see that $B_\odot$ is 15 orders of magnitudes higher than the B_{EBL} (in this computation). Clearly the presence of a horizon is key to avoiding the divergenge of the integral. The starting redshift z_{start} has little role here, it is easy to compute that increasing z_{start} leads to a $\sim 1.5\%$ increase at most, and galaxies at $z < 2$ give more than 90% of the EBL, while the volume from $z = 2$ to infinity is still large, so cosmological redshift, with the implied decline of flux due to the luminosity distance, is the strongest contributor to the covergence of the integral.