Cosmology 1

2025/2026
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Second intermediate test
Topic: FLRW models.
Deadline: 15 April, 9:00 am.

While examining the large-scale structure mapped by a spectroscopic survey, an astronomer identifies a population of (identical) bubble-like structures in the galaxy distribution. To characterize their typical shape, she aligns the objects on their centers and reconstructs the average profile in the plane defined by the directions parallel and perpendicular to the line of sight. Repeating the analysis under different cosmological assumptions to convert observed angles and redshifts into distances, she finds that the apparent shape of the profile changes. Assuming that the bubbles are intrinsically spherical, she therefore decides to use this geometric distortion to constrain cosmological parameters. Take this task under her supervision.

This is commonly called the Alcock–Paczyński test. Throughout the exercise, neglect radiation and peculiar velocities, and work with comoving distances unless otherwise stated.

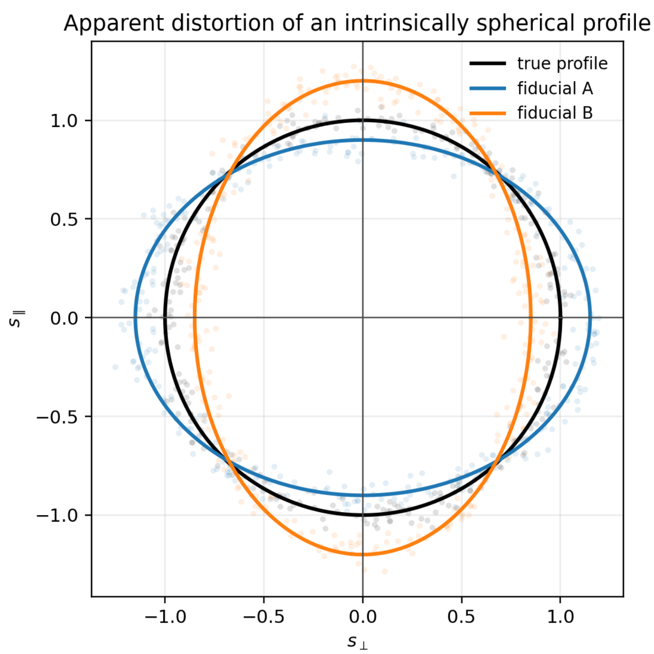

Figure 1: Illustration of how an intrinsically spherical profile can appear distorted when the data are converted into distances with different fiducial cosmologies. The lighter points represent the galaxies tracing the mean profile.

(1) Consider one bubble-like object traced by galaxies, centered at redshift z. It is characterized by an angular extent $\Delta\theta$ transverse to the line of sight (on the sky) and by a redshift extent Δz along the line of sight. Assuming that the object is sufficiently small that the background cosmology does not vary appreciably across its extent, and working in the small-angle approximation, derive the relation between the comoving separations perpendicular and parallel to the line of sight inferred assuming a fiducial cosmology, $(s_\perp, s_\parallel)$, and the corresponding separations in the true cosmology, $(r_\perp, r_\parallel)$. Hence show how this intrinsically spherical object appears distorted when an incorrect cosmology is assumed.

(2) Taking a sample of several bubble-like objects, the astronomer divides them into three redshift bins and measures, in each bin, the mean angular extent $\Delta\theta$ and the mean redshift extent Δz, obtaining:

z	$\Delta\theta$ (rad)	Δz
0.9	0.03091 ± 0.00066	0.03561 ± 0.00077
1.2	0.02492 ± 0.00053	0.04190 ± 0.00092
1.5	0.02139 ± 0.00046	0.04894 ± 0.00108

Assuming that the objects are, on average, spherically symmetric in the true cosmology, (i) derive the Alcock–Paczyński condition relating $\Delta\theta$ and Δz; (ii) assuming a flat ΛCDM cosmology, which of the two parameters H_0 and Ω_m can be constrained by the test? (iii) using standard error propagation, work out constraints for the two parameters.

(3) An independent analysis of the CMB leads to a measurement of the intrinsic comoving size of the bubble-like structures:

$$r_b = 100 \text{ Mpc}.$$

Using the same measurements of $\Delta\theta$ and Δz and assuming again a flat ΛCDM cosmology: (i) determine $H(z)$ and $d_A(z)$ in each redshift bin, together with their uncertainties; (ii) determine the region of the (Ω_m, H_0) parameter space compatible with the data.

(4) So far, spatial flatness has been assumed. Discuss in detail what changes if spatial curvature is allowed. In particular, write the expression of the angular diameter distance in a curved FLRW universe and explain how the Alcock–Paczyński test is modified. If you are not tired of the exercise, fix H_0 to the value measured by the Planck satellite ($h = 0.674$) and determine the the region of the (Ω_m, Ω_k) parameter space compatible with the data.