Cosmology 1

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Solution of proposed problem, lecture 10
Topic: horizons.

(a) The lecture notes and textbook should be a sufficient guide to draw the past light cone.

(b) It should be clear by the conformal spacetime diagram that $\Delta\eta = \int_{t_1}^{t_2} cdt/a(t)$ is equal to the comoving distance between two observers on the same light cone, i.e. connected by a null path that passes through (t_1, r_1) and (t_2, r_2), $L_{\rm los} = \mathcal{R}_0 \int_{r_1}^{r_2} dr/\sqrt{1-kr^2}$. For an Einstein-de Sitter model, $a(t) = (t/t_0)^{2/3}$ with $t_0 = 2/3H_0 = 13.04$ Gyr, so

$$\Delta\eta = 3ct_0 \left[\left(\frac{t_2}{t_0}\right)^{1/3} - \left(\frac{t_1}{t_0}\right)^{1/3} \right]$$

Redshift $z_{\rm rec} = 1100$ is reached when

$$t_{\rm rec} = t_0(1+z_{\rm rec})^{-3/2} = 2.74 \times 10^{-5} t_0 = 357000 \text{ yr}$$

The conformal time from the Big Bang to recombination is thus

$$\eta_{\rm rec} = 3ct_0(1+z_{\rm rec})^{-1/2} = 362 \text{ Mpc}$$

The comoving distance of the visible horizon it $\eta_0 - \eta_{\rm rec}$; $\eta_0 = 3ct_0 = 12000$ Mpc, so

$$d_{\rm vh} = 11640 \text{ Mpc}$$

This is only 3% smaller than the particle horizon in this model.