Cosmology 1

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Prof. Pierluigi Monaco

Solution of proposed problem, lecture 20
Topic: statistical mechanics.

After the annihilation of mesons, the particles present in the universe are:

	mass	contribution to $g^\star$	contribution to $\tilde{g}$
muon pairs: $\mu^+\mu^-$	106 MeV	$2 \times 2 \times 7/8 = 3.5$	$2 \times 2 \times 3/4 = 3$
neutrinos: $\nu_e\bar{\nu}_e$, $\nu_\mu\bar{\nu}_\mu$, $\nu_\tau\bar{\nu}_\tau$	negligible	$6 \times 1 \times 7/8 = 5.25$	$6 \times 1 \times 3/4 = 4.5$
electron pairs: e^+e^-	511 keV	$2 \times 2 \times 7/8 = 3.5$	$2 \times 2 \times 3/4 = 3$
photons: γ	0	2	2

The values of $g^\star$ and $\tilde{g}$ are thus, in the four intervals defined in the text:

from end of hadron era to muon annihilation	$g^\star = 14.25$	$\tilde{g} = 12.5$
from muon annihilation to neutrino decoupling	$g^\star = 10.75$	$\tilde{g} = 9.5$
from neutrino decoupling to electron annihilation	$g^\star = 5.5$	$\tilde{g} = 5$
from electron annihilation to now	$g^\star = 2$	$\tilde{g} = 2$

Muons are bosons and are of three kinds, so they contribute 3 to $g^\star$ (and to $\tilde{g}$). This allows to compute the cosmic time at muon annihilation. Using the equation given in the proposed problem, the resulting numbers are (excluding the present age, for which the formula of radiative universe is clearly not applicable):

end of hadron era	$t = 3.5 \times 10^{-5}$ s
muon annihilation	$t = 6.4 \times 10^{-5}$ s
neutrino decoupling	$t = 0.9$ s
electron annihilation	$t = 4$ s