Cosmology 1

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Solution of proposed problem, lecture 21
Topic: Big Bang nucleosynthesis.

At 100 keV only photons are present in the primordial soup, so

$$\rho = \frac{a_r T^4}{c^2} \sim 15 \text{ g cm}^{-3}$$

this allows to compute the age at that time (neglecting neutrinos):

$$t = \sqrt{\frac{3}{32\pi G\rho}} \sim 170 \text{ s}$$

The thermal velocity (on one direction) of baryons at that temperature is:

$$v = \sqrt{\frac{k_B T}{m_p}} \sim 3 \times 10^8 \text{ cm s}^{-1}$$

so the timescale for deuterium production is:

$$t_d = \frac{1}{n\sigma v} \sim 2 \times 10^{20} \frac{1}{n}$$

Equating $t = t_d$ we obtain $n \sim 2 \times 10^{18}$ cm^{-3}. Then:

$$T_0 = T \left(\frac{n_0}{n}\right)^{1/3} \sim 4.3 \text{ K}$$

From this temperature one can estimate the number of photons per baryon, and thus have a confirmation that nucleosynthesis must wait for photons with $h\nu > 2.2$ MeV to be rare. It is also possible, by solving the above equations analytically, to demonstrate that $T_0 \propto T^{1/2}$, so if one started from $T = 2.2$ MeV one would have obtained a higher T_0 and a lower η; one can then decrease T until the probability of having a photon with $h\nu > 2.2$ MeV becomes $\sim \eta$.