Cosmology 1

2022/2023
Prof. Pierluigi Monaco

Third intermediate test
Topic: early Universe.
Deadline: June 15, 11:00.

Consider the existence of $\mathcal{N}_\nu = 2$, 3 and 4 families of elementary particles, and thus of neutrino pairs. Compute, for each of these values, the following quantities.

(i) The temperature (in GeV) of neutrino decoupling, obtained by assuming $\tau_w = t(T)$, where τ_w is the time between two consecutive neutrino-electron scatterings and t is the age of the Universe, that depends on the temperature of the thermal soup through the energy density.

(ii) The value of the neutron-to-proton ratios at freezing and the resulting value of the Helium weight ratio Y.

(iii) The values of the equivalence redshift.

Report the equations used and the numbers obtained in max one page.

Solution

The temperature of neutrino decoupling $T_{\nu,\rm dec}$ can be computed by equating the time τ between two consecutive scatterings of a neutrino with an electron to the age of the Universe t:

$$\tau = \frac{1}{\sigma_w n_e c} \qquad t = \sqrt{\frac{3c^2}{32\pi G u}}$$

The dependence on $\mathcal{N}_\nu$ comes through

$$u = \frac{1}{2} g^\star a_r T^4$$

where $g^\star = 5.5 + 7/4\mathcal{N}_\nu$, while the electron density

$$n_e = \frac{15\zeta(3)}{\pi^4}\frac{3}{2}\frac{a_r T^3}{k_b}$$

does not depend on $\mathcal{N}_\nu$.

The resulting ratio of neutron to proton abundances is

$$\frac{n_n}{n_p} = \exp\left(-\frac{1.3\ \mathrm{MeV}}{k_b T_{\nu,\rm dec}}\right)$$

where we have set to 1 the ratio of neutron and proton masses. If all neutrons go to ^4He its mass ratio results

$$Y = \frac{2}{1 + n_p/n_n}$$

Finally, the equivalence redshift can be computed as

$$1 + z_{\rm eq} = \frac{\rho_{c0}\Omega_m}{(1 + 0.224\mathcal{N}_\nu)\rho_{\gamma 0}}$$

where $\rho_{c0} = 3H_0^2/8\pi G$ and $\rho_{\gamma 0}^2 = a_r T_{\gamma 0}^4$ with $T_{\gamma 0} = 2.73$ K. This depends on Ω_m and h, we assume $\Omega_m = 0.315$ and $h = 0.674$.

The results are reported in this table.

$\mathcal{N}_\nu$	$g^\star$	$T_{\nu,\rm dec}$ (keV)	n_n/n_p	Y	$z_{\rm eq}$
2	9.0	875	0.226	0.369	3952
3	10.75	901	0.236	0.382	3418
4	12.5	924	0.245	0.393	3011