Cosmology 1

2023/2024
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Third intermediate test
Topic: Early Universe.
Deadline: June 3, 11:00 am.

Inflation can solve the horizon problem if the comoving Hubble horizon at the beginning of inflation is larger than its value today. The calculation of the number of e-folds performed in class assumed that inflation takes place at an energy of 10^{15} GeV. However, we do not know at what energy inflation took place.

(1) Argue and motivate what is the energy range (in GeV) at which we can reasonably expect inflation to take place.

(2) Recompute the minimal number of e-folds $\mathcal{N}_{\rm min}$ necessary for inflation to solve the horizon problem as a function of energy scale; you can ignore the evolution of $g^\star$ in these order-of-magniture calculations and assume $z_{\rm eq} = 3400$. Comment the result.

(3) Compute the comoving size of the visible Universe (in Mpc), and its corresponding physical size at the beginning and end of inflation (this time in cm), again as a function of the energy scale of inflation, assuming that it lasted for two e-folds more than the minimum: $\mathcal{N}_e = \mathcal{N}_{\rm min} + 2$. For the visible size of the Universe, numerically compute the comoving distance at redshift $z_{\rm rec} = 1100$ assuming a flat ΛCDM model with $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$ and $h = 0.67$ ($H_0 = 100\,h$ km s^{-1} Mpc^{-1}).

(4) Compute the total energy and equivalent mass (in solar masses) contained in a sphere of radius equal to the comoving distance of the visible Universe, at the beginning and at the end of inflation as a function of its energy scale. Report the result in solar masses. This is a bit tricky conceptually: you can compare these numbers to the total mass of the observable Universe, but remember that before equivalence the energy budget is dominated by radiation. Comment what you have obtained, and feel free to speculate.

(5) Argue what happens to entropy along the process (remember that most visible particles are created by the decay of the inflaton at the end of inflation).

Solution

(1) We can set the highest energy value at which inflation can take place at the Planck energy; indeed, the breaking of quantum gravity can trigger inflation. Inflation shuold take place before nucleosynthesis, and the last SSB known to us takes place at the EW breaking; indeed, one considered possibility is that the Higgs field drives inflation. So it is plausibe to set the largest possible energy range from Planck energy, 10^{19} GeV, to EW energy scale, 100 GeV.

(2) In class we learned that the minimal number of e-folds can be computed as:

$$\mathcal{N}_{\rm min} > \ln\left(\frac{\sqrt{a_{\rm eq}}}{a_{\rm f}}\right)$$

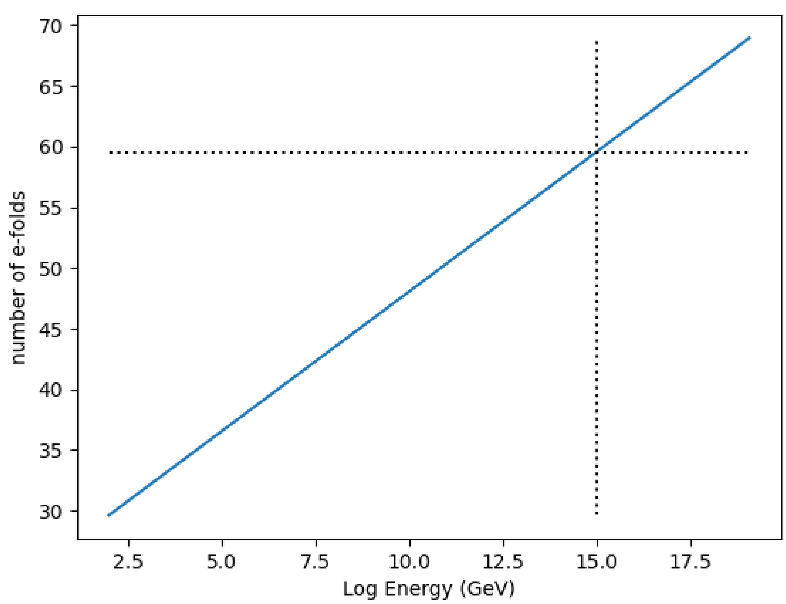
where $a_{\rm eq} = 1/(1+z_{\rm eq})$ is the scale factor at matter-radiation equivalence. The scale factor at the end of inflation can be scaled to the temperature of inflation $T_{\rm infl} = E_{\rm infl}/k_{\rm B}$ as

$$a_{\rm f} = \frac{T_{\rm cmb}}{T_{\rm infl}}$$

where $T_{\rm cmb} = 2.73$ K and we have neglected for simplicity the evolution of the number of degrees of freedom $g^\star$. It then results that

$$\mathcal{N}_{\rm min} \propto \log T_{\rm infl}$$

with inflation at lower energy requiring a lower number of e-folds to solve the horizon problem. This is easy to understand, as the growth of the comoving Hubble horizon from the end of inflation to today is smaller if inflation ends later. We report here a plot of the number of e-folds with respect to the (Log) energy scale, in GeV. The dotted lines report the values for 10^{15} GeV discussed in the class ($\mathcal{N}_{\rm min} = 59.6$).



(3) The comoving distance to the visible horizon can be computed using the machinery developed for the Second Test:

$$d_{\rm vis} = \int_0^{z_{\rm rec}} \frac{c\,dz}{\sqrt{\Omega_m(1+z)^3+\Omega_\Lambda}} = 14100 \text{ Mpc}$$

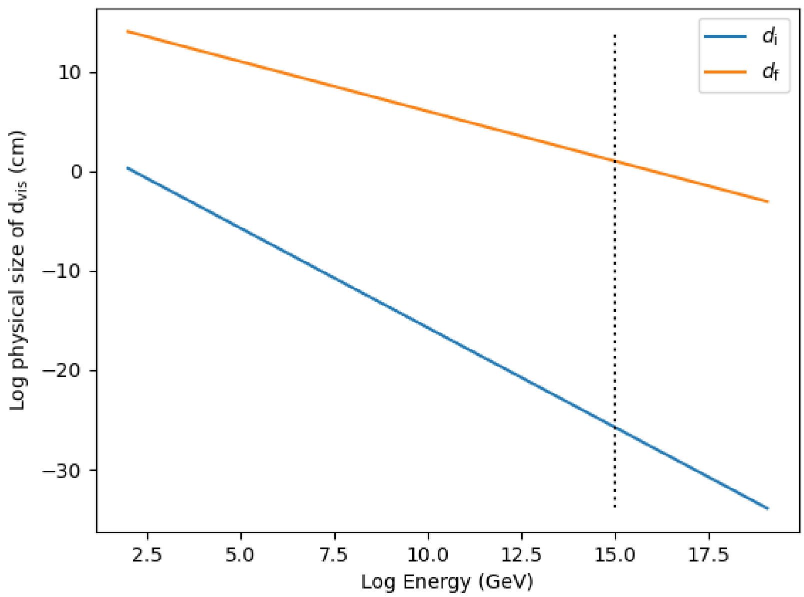
Its physical size at the end of inflation is:

$$d_{\rm f} = a_{\rm f} d_{\rm vis} = \frac{T_{\rm cmb}}{T_{\rm infl}} d_{\rm vis}$$

while at the beginning of inflation:

$$d_{\rm i} = a_{\rm i} d_{\rm vis} = d_{\rm f} / \exp(\mathcal{N}_e)$$

where $\mathcal{N}_e = \mathcal{N}_{\rm min} + 2$. These two distances are reported in the plot below, in cm. For $T_{\rm infl} = 10^{15}$ GeV we have that a 2×10^{-26} cm scale is inflated to 10.4 cm.[1]



(4) The total mass contained today (at $t = t_0$) within the horizon is:

$$M_{\rm h,m} = \rho_c \Omega_m \frac{4\pi}{3} d_{\rm vis}^3 = 0.45 \times 10^{24} \text{ M}_\odot$$

This mass is constant, as long as the particles (dark matter and baryons) exist. However, the total energy budget is dominated by dark energy, that has an equivalent mass of $M_{\rm h,\Lambda} = 1.01 \times 10^{24}$ M$_\odot$, for a total of 1.46×10^{24} M$_\odot$. CMB photons contribute little equivalent mass, $M_{\rm h,r} = 8.12 \times 10^{19}$ M$_\odot$, but when we go to higher redshift this contribution grows like $(1+z)$, while dark energy dies like $(1+z)^{-3}$. So the equivalent mass at the end of inflation can be recovered as:

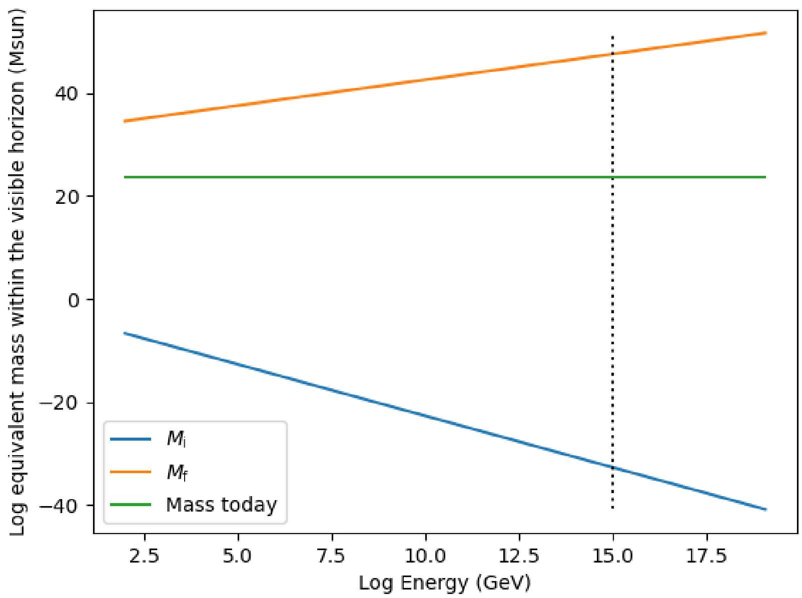
$$M_{\rm f} = M_{\rm h,r}/a_{\rm f}$$

[1] This is a bit larger than what reported in the notes due to different approximations; here $a_{\rm f} = 2.4 \times 10^{-28}$ and the visible horizon is 14.1 Gpc, larger than $ct_0 = 4.2$ Gpc.

During inflation the energy density is constant, so (as for dark energy) the initial equivalent mass within the horizon is obtained by rescaling it by the cube of the ratio of scale factors:

$$M_\mathrm{i} = M_\mathrm{f} \left(\frac{a_\mathrm{i}}{a_\mathrm{f}} \right)^3 = M_\mathrm{f} \exp(-3\mathcal{N}_e)$$

The figure below reports these masses, together with the total mass (given by matter) of the visible Universe as a reference.



Clearly the total energy in a comoving volume is not a constant during the Universe evolution, growing if $w < 0$ and decreasing if $w > 0$. If inflation took place at 10^{15} GeV, the visual horizon started from ~ 5 g and was inflated to 4×10^{47} $\mathrm{M}_\odot$. Less drastic growth is obtained at lower energies, but the initial mass always keeps below the solar mass. This is a possible hint for speculating about the initial conditions that gave rise to inflation, where what we need is a macroscopic but small amount of matter compressed to huge density and at huge energy, and fastly expanding.

(5) Approximating $g^\star$ to a constant, for a radiative universe the entropy in a comoving volume is proportional to the number of particles in it. Inflation should be driven by a phase transition, and while it happens the transition is far from equilibrium; also, if $p = -u$, then $\sigma = (u+p)/T$ goes to zero, and this shows that the entropy formulation found in Lecture 15, based on a radiative universe, does not apply during inflation. Associating a temperature to the expanding vacuum is not trivial, one may associate it to entropy thus predicting a drastic decrease of temperature. However, reheating following the end of inflation leads to the creation of all particles (as in the final stage of a phase transition happening out of equilibrium). We can assume that reheating marks the return to a radiative universe in thermodynamical equilibrium.

To deepen the subject, let's discuss about the number of particles in the comoving observable Universe and assume that it does not vary during inflation (all pre-inflation particles are stable). One can compute the number of particles at the beginning of inflation within the horizon ($g^\star = 2$):

$$N_{\rm i} = n_{\rm i} \frac{4\pi}{3} (a_i d_{\rm vis})^3 = \frac{40\zeta(3)}{\pi^3} \frac{a_{\rm r} T_{\rm infl}^3}{k_{\rm B}} (a_i d_{\rm vis})^3$$

For $T_{\rm infl} = 10^{15}$ GeV this results as large as $\sim 10^8$ particles, but it should be compared to its value at the end of inflation, that is boosted by a factor $\exp(3(\mathcal{N}_e + 2)) \sim 10^{80}$, and to the number of photons within the visible horizon. If $g^\star$ does not change, so that $T \propto a^{-3}$, it is easy to see that the dependences of the scale factor in $(a d_{\rm vis})^3$ and in T^3 cancel, so the number of relativistic particles is a constant from t_f to t_0 and is:

$$N_\gamma = \frac{40\zeta(3)}{\pi^3} \frac{a_{\rm r} T_{\rm cmb}^3}{k_{\rm B}} d_{\rm vis}^3 \sim 10^{89}$$

Even when comparing this number with the number of baryons within the horizon, $N_{\rm b} = \eta N_\gamma \sim 10^{80}$, the number of pre-inflation particles is tiny, testifying that entropy is created by reheating (and that any monopole problem is solved).

The figure below reports the number of pre-inflation particles within the horizon as a function of energy scale of inflation. The higher the energy the more drastic is the increase of entropy at the end of inflation. While a Planck-scale inflation makes it unlikely to have even a single pre-inflation particle within the horizon, an EW-scale inflation makes $\sim 10^{47}$ particles survive the extreme dilution, that is however limited to "only" ~ 30 e-folds. This is yet smaller by 33 orders of magnitude than the baryon fraction.