Cosmology 1

2024/2025
Prof. Pierluigi Monaco

Third intermediate test
Topic: Early Universe.
Please deliver your test by June 11th.
Strict deadline: June 13th, 11:00.

The comoving Hubble horizon at matter-radiation equality leaves an observable imprint on the matter power spectrum. This happens because fluctuations on smaller scales enter the horizon during the radiation-dominated era and cannot grow till equality, while larger scales enter the horizon in the matter-dominated era and suffer no damping. At the same time, this comoving scale depends on the number of relativistic species present in the early Universe, in particular on the number of neutrinos and on any "dark radiation" that may be present in the dark sector.

We will again assume a flat ΛCDM model with $\Omega_m = 0.319$, and $H_0 = 67$ km/s/Mpc, and assume that flatness is achieved by having $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$ (though the Ω_Λ term is negligible at equality). We will call $\mathcal{N}_\nu$ the number of neutrino families.

(1) Compute the size of the comoving Hubble horizon $d_{\rm eq}$ at equality as a function of $\mathcal{N}_\nu$, and quantify it for $\mathcal{N}_\nu = 2$, 3 and 4, in Mpc and h^{-1} Mpc.

(2) Recast this in terms of the wavenumber $k_{\rm eq} = 1/d_{\rm eq}$, in h Mpc^{-1}. How accurate should a measurement of $k_{\rm eq}$ be to enable us to measure the number of neutrino families?

Let's now assume $\mathcal{N}_\nu = 3$. Neutrinos have a small mass, so they belong to the dark matter sector and at low redshift they contribute to the matter budget.

(3) Suppose that one of the neutrinos is much more massive than the others, what should its mass be to explain dark matter?

(4) Compute Ω_ν today if the mass of the most massive neutrino is $m_\nu = 0.1$ eV.

(5) At what time does this neutrino become non-relativistic?

Solution

(1) To compute the comoving Hubble horizon at equality we need the second Friedmann equation,

$$H^2 = H_0^2 \left(\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda \right)$$

where the photon density is

$$\Omega_\gamma = \frac{a_r T_{\gamma 0}^4}{c^2 \rho_{c0}} = 5.56 \times 10^{-5}$$

(ρ_{c0} is the critical density today) and

$$\Omega_r = \Omega_\gamma (1 + 0.227 \mathcal{N}_\nu) = 9.34 \times 10^{-5}$$

where 0.227 is the value of $7/8 \times (4/11)^{4/3}$. At such high redshifts the Ω_Λ term in the Friedmann equation is negligible, so we can modulate it to keep the universe flat while varying the number of neutrinos, with no significant effect on the results. The matter-radiation equality redshift is:

$$z_{\rm eq} = \frac{\Omega_m}{\Omega_r} - 1$$

Neglecting Ω_Λ, and exploiting the fact that the two other terms in the Friedmann equation are equal at $z_{\rm eq}$, it is easy to obtain that:

$$d_{\rm eq} = \frac{c}{\Omega_m H_0} \sqrt{\frac{\Omega_r}{2}}$$

Its values are reported in the table below, in Mpc and h^{-1} Mpc.

$\mathcal{N}_\nu$	$d_{\rm eq}$		$k_{\rm eq}$	
2	89.2 Mpc	59.7 h^{-1} Mpc	0.0112 Mpc^{-1}	0.0167 h Mpc^{-1}
3	95.9 Mpc	64.2 h^{-1} Mpc	0.0104 Mpc^{-1}	0.0156 h Mpc^{-1}
4	102.1 Mpc	68.4 h^{-1} Mpc	0.0098 Mpc^{-1}	0.0146 h Mpc^{-1}

(2) We have that:

$$k_{\rm eq} = \frac{\Omega_m H_0}{c} \sqrt{\frac{2}{\Omega_r}}$$

Its values are reported above, in Mpc^{-1} and h Mpc^{-1}. The relative variation of $k_{\rm eq}$ from 2 to 4 neutrinos is $\sim 14\%$ so this quantity should be measured to a few percent precision level to have a good measure.

(3) The density parameter of dark matter is:

$$\Omega_{\rm dm} = \Omega_m - \Omega_b$$

If the heaviest neutrino species has a mass m_ν and the others are negligible, then its mass density is:

$$\rho_\nu = m_\nu n_{\nu 1} = m_\nu \frac{3\zeta(3)}{2\pi^2} \left(\frac{k_B T_\nu}{\hbar c} \right)^3$$

where $n_{\nu 1}$ is the number density of a single neutrino species and

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

that is 1.94 K today. We equate the mass density ρ_ν, computed at $t = t_0$, to the dark matter density, $\rho_{\rm dm,0} = \rho_{c0}\Omega_{\rm dm}$, obtaining:

$$m_\nu = \frac{\rho_{\rm dm,0}}{n_{\nu 1}(t_0)}$$

This results in a mass of $m_\nu = 11.4$ eV.

(4) It is easy to obtain Ω_ν for the smaller mass by using the expression for $\rho_\nu(t_0)$ for $m_\nu = 0.1$ eV. The result is $\Omega_\nu = 2.38 \times 10^{-3}$, much larger than Ω_γ but still a minor contribution to Ω_m.

(5) The redshift when neutrinos become non-relativistic can be computed as the time at which

$$T_\nu(z) = T_{\nu 0}(1 + z_{\rm nr}) = \frac{m_\nu c^2}{k_B}$$

We obtain

$$z_{\rm nr} = 594$$