Cosmology 1

2025/2026
Prof. Pierluigi Monaco

Third intermediate test
Topic: early Universe.
Deadline: 22 May, 9:00 am.

First problem: inflation

Inflation dramatically dilutes the particles present before its beginning. Assuming that inflation took place at an energy $T = 10^{15}$ GeV, and making some reasonable assumptions on $g_\star$, it is possible to compute the number of relic particles, present at the beginning of inflation, that we may have within our Hubble horizon c/H_0. We assume here that $H_0 = 67$ km s^{-1} Mpc^{-1}. As a note, it's clear that these relic particles will still exist only if they are stable and are not reprocessed by the other particles; in the following calculations we ignore their interactions after inflation.

(1) Compute the number of relic particles within the horizon as a function of the number of e-folds $\mathcal{N}_e$.

(2) Are there configurations where this number is not much smaller than one? How does this result depend on the energy scale of inflation?

(3) Discuss your result by comparing the abundance of relic particles with that of other particle species present today.

Second problem: Big Bang nucleosynthesis

A very recent measurement of primordial Helium abundance (arXiv:2601.22239) gives:

$$Y_p = 0.2458 \pm 0.0013$$

Compute the implied number of neutrino families N_ν (treated as a real number) by following this path (see the Bonometto textbook):

(1) Assuming that the temperature of neutrino decouping is $T_{\rm dec,3} = 900$ keV when $N_\nu = 3$, work out the dependence of this energy on N_ν.

(2) Assume now that BBN ends at $T_{\rm bbn} = 44$ keV and work out the cosmic time at this temperature, as a function of N_ν.

(3) Finally assume that after decoupling the neutron-to-proton ratio decays exponentially due to beta decay (with $\tau = 878.3$ s) until $t_{\rm bbn}$, and that at this time all neutrons combine to form ^{4}He.

(4) Compare the measurement to this prediction and obtain an estimation of N_ν.

As a note, $T_{\rm bbn} = 44$ keV has been fine-tuned to obtain, within this simplified approach, a prediction that is very similar to the one reported in the quoted paper, in normalization and in its dependence on N_ν.