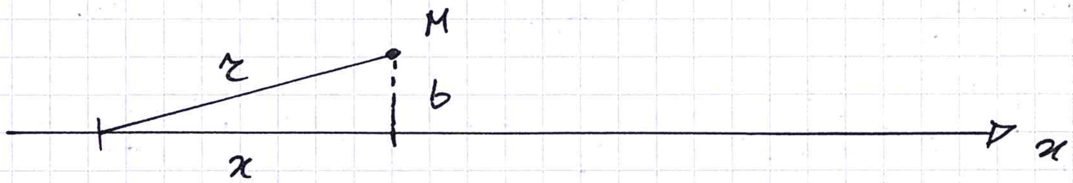
Notes on EHT

→ Photon trajectory in Newtonian dynamics:



$$x = ct \,, \quad r^2 = b^2 + x^2 = b^2 + c^2t^2 \Rightarrow 2r\frac{dr}{dt} = 2c^2 t$$

$$\Rightarrow \left(\frac{dr}{dt}\right)^2 = c^2\left(1 - \frac{b^2}{r^2}\right) \,, \quad V_{eff}(r) = \frac{c^2b^2}{r^2} = \frac{h^2}{r^2}$$

the straight path of the photon implies a centrifugal barrier

→ Impact parameter for a particle around a black hole:

at infinity $L = pb \Rightarrow b = \frac{L}{\sqrt{E^2 - m^2}}$, for a photon $b = \frac{L}{E}$

→ Capture radius of a black hole:

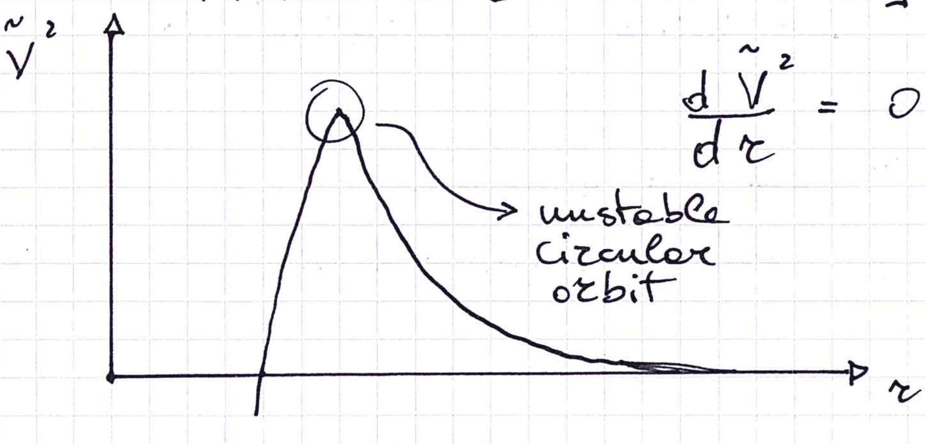
For a photon $(m = 0)$:

$$\left(\frac{dr}{d\lambda}\right)^2 = \tilde{E}^2 - \left(1 - \frac{2GM}{r}\right)\frac{\tilde{L}^2}{r^2} = \tilde{E}^2\left[1 - \left(1 - \frac{2GM}{r}\right)\frac{\tilde{L}^2}{\tilde{E}^2}\frac{1}{r^2}\right]$$

$$\tilde{E} \to E_\infty \,, \quad \frac{\tilde{L}}{\tilde{E}} = b$$

$$\left(\frac{dr}{d\lambda}\right)^2 = E_\infty\left[1 - \left(1 - \frac{2GM}{r}\right)\frac{b^2}{r^2}\right] \,, \quad \frac{\tilde{V}^2}{\tilde{E}^2} = \left(1 - \frac{2GM}{r}\right)\frac{b^2}{r^2}$$

$$\frac{d\tilde{V}^2}{dr} = 0 \Rightarrow r = 3GM$$



→ Photon capture radius:

The smallest radius for the photon orbit can be computed by setting

$$\frac{dr}{d\varphi} = \frac{dr}{d\lambda}\frac{d\lambda}{d\varphi} = 0 \quad , \quad \frac{d\varphi}{d\lambda} = \frac{\tilde{L}}{r^2} \qquad \left(\tilde{L} = r^2 \frac{d\varphi}{d\lambda} \text{ is conserved}\right)$$

$$\left(\frac{dr}{d\varphi}\right)^2 = \frac{r^4}{b^2}\left[1 - \frac{b^2}{r^2}\left(1 - \frac{2GM}{r}\right)\right] = 0 \qquad \text{for } b^2(r_{min} - 2GM) = r_{min}^3$$

This means that the smallest r for the orbit is the solution of this cubic equation.
But if $r_{min} = 3GM$ the photon falls onto the black hole.

This happens if $\quad b^2 < \dfrac{(3GM)^3}{(3GM - 2GM)} = 27\,(GM)^2$

This defines the photon capture radius:

$$b_{max} = \sqrt{27}\, R_G \qquad (R_G = GM\,,\ \text{gravit. radius})$$

→ The shadow of the black hole is larger on the sky, due to extreme lensing

$$\vartheta_G \equiv \frac{GM}{c^2}\frac{1}{D} \quad , \quad d = \alpha\,\vartheta_G \quad , \quad \alpha \simeq 10$$

$$\alpha \simeq 2\sqrt{27} \quad \text{but geometry is more complex}$$

M87 is at $D = 16.8 \pm 0.8$ Mpc, the BH mass of M87* is $M = 6.5 \pm 0.7 \times 10^9\, M_\odot$, so:

$$R_G = \frac{GM}{c^2} = 8.85 \times 10^{14}\ \text{cm} = 2.86 \times 10^{-4}\ \text{pc}$$

$$\vartheta_G = \frac{R_G}{D} = 1.85 \times 10^{-11}\ \text{rad} = 3.81\ \mu\text{as}$$

$$d \simeq 42\ \mu\text{as} \quad \text{and} \quad \frac{2\pi R_G}{c} \simeq 1.8 \times 10^5\ \text{s} \simeq 51\ \text{h}$$

→ For Sgr A*: $\quad D = 8$ kpc $\quad , \quad M = 4 \times 10^6\, M_\odot$.

$$R_G = 5.9 \times 10^{12}\ \text{cm}$$

$$\vartheta_G = 2.39 \times 10^{-11}\ \text{rad} = 4.92\ \mu\text{as}$$

$$d \simeq 50\ \mu\text{as} \quad , \quad \text{but} \quad \frac{2\pi R_G}{c} = 124\ \text{s}$$

Notes on the BH mass function

+ Sphere of influence: $\frac{G M_{BH}}{r_{BH}} = \sigma_*^2$

$$\Rightarrow r_{BH} = \frac{G M_{BH}}{\sigma_*^2} = 10.7\,\mathrm{pc}\left(\frac{M_{BH}}{10^8 M_\odot}\right)\left(\frac{\sigma_*}{200\,\mathrm{km\,s^{-1}}}\right)^{-2}$$

$$\vartheta_{BH} = 0.11''\left(\frac{M_{BH}}{10^8 M_\odot}\right)\left(\frac{\sigma_*}{200\,\mathrm{km\,s^{-1}}}\right)^{-2}\left(\frac{D}{20\,\mathrm{Mpc}}\right)^{-1}$$

+ Bulge / spheroid luminosity function

$\phi(L)\,dL$: LF of galaxies, Mpc^{-3} ($\phi : \mathrm{Mpc}^{-3} L_\odot^{-1}$)

$$\frac{B}{T} = \frac{L_{BUL}}{L}, \quad L_{BUL} = \frac{B}{T} L$$

$\rightarrow \phi(L_{BUL})\,dL_{BUL}$

BUL :
- → ALL GALAXY (Elliptical)
- → "CLASSICAL" BULGE (S0, Sa)
- → "PSEUDO-BULGES" (late types)

+ From L_{BUL} (M_{BUL}) and σ_e to M_{BH}

$$\phi(M_{BH}) = \int_0^\infty P(M_{BH}|L_{BUL})\,\phi(L_{BUL})\,dL_{BUL}$$

$$\phi(M_{BH}) = \int_0^\infty P(M_{BH}|\sigma_e)\,\phi(\sigma_e)\,d\sigma_e$$

(consistent? if there are no biases ...)

+ BH mass density

$$\rho_{BH} = \int_0^\infty M_{BH}\,\phi(M_{BH})\,dM_{BH} \simeq 3\text{-}5 \times 10^5\,M_\odot\,\mathrm{Mpc}^{-3}$$

+ From AGN shining to BH masses

$$\begin{cases} L_Q = \eta \dot{M}_{acc} c^2 , \\ \dot{M}_{BH} = (1-\eta)\dot{M}_{acc} \end{cases} \Rightarrow \dot{M}_{BH} = \frac{1-\eta}{\eta c^2} L_Q$$

Quasar LF: $\phi_Q(L_Q; z)\, dL_Q$ (Mpc^{-3})

Accretion rate on BHs @ z : $\dot{M}_{BH}(L)\phi_Q(L_Q; z)\, dL_Q$
$(M_\odot\, yr^{-1}\, Mpc^{-3})$

Accreted mass per unit redshift: $\dot{M}_{BH}(L)\phi_Q(L_Q; z)\left|\frac{dt}{dz}\right| dL_Q dz$
$(M_\odot\, Mpc^{-3})$

$$\rho_{BH} = \int_0^\infty dz \int_0^\infty dL_Q \left(\frac{1-\eta}{\eta c^2} L\right) \phi_Q(L_Q; z) \left|\frac{dt}{dz}\right| \quad (M_\odot\, Mpc^{-3})$$

+ Soltan argument:

$$u = \int_0^\infty dz \int_0^\infty dL_Q L_Q \phi_Q(L_Q; z) \left|\frac{dt}{dz}\right|$$

luminosity density of quasars $(erg\, Mpc^{-3})$

$$\Rightarrow \rho_{BH} = \frac{1-\eta}{\eta c^2} u$$

+ AGN LF: $\phi(L, z)$ in some band
+ correction for obscured systems
+ bolometric correction
$\Rightarrow$ bolometric LF

+ From AGN LF to BH MF:

$$\dot{M}_{acc} = \lambda \dot{M}_{EDD} = \frac{\lambda M_{BH}}{t_{EDD}}$$

so $\phi_Q(L_Q | z)$ $\Rightarrow$ $\phi(M_{BH})$
quasar LF local BH MF