# Emission lines in the ISM

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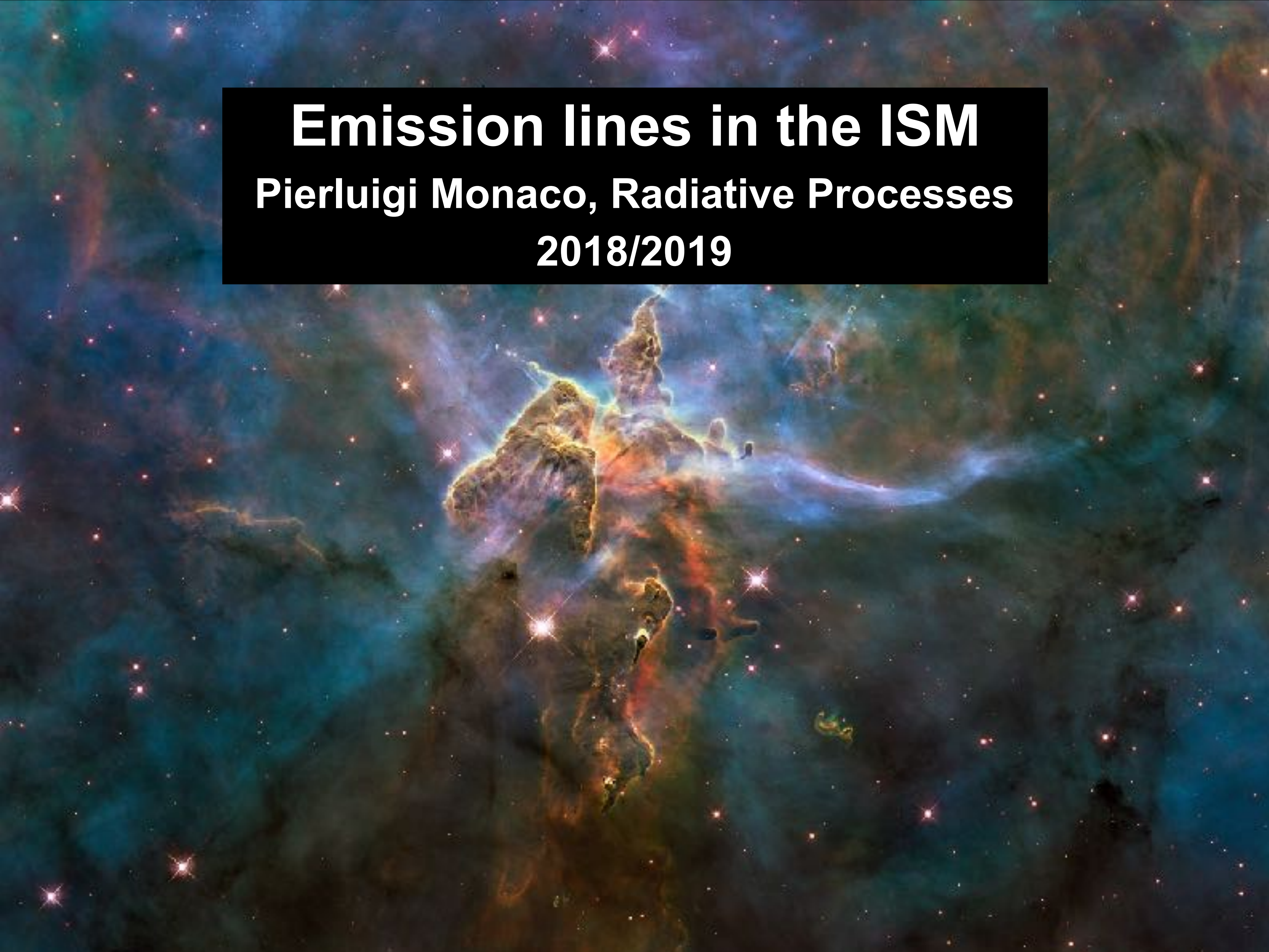
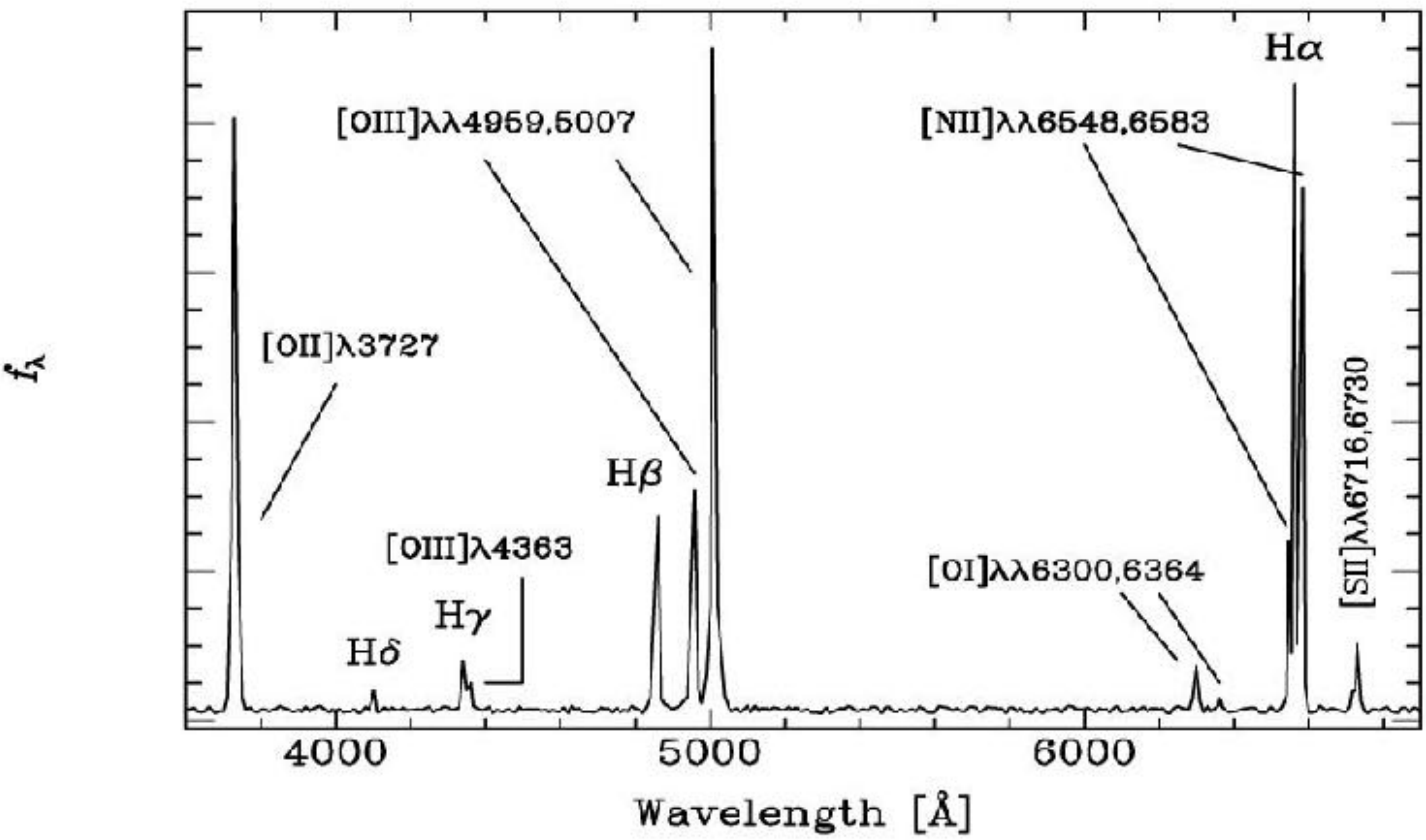


# Spectrum of an HII region: emission lines



# Emission lines (no scatter)

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

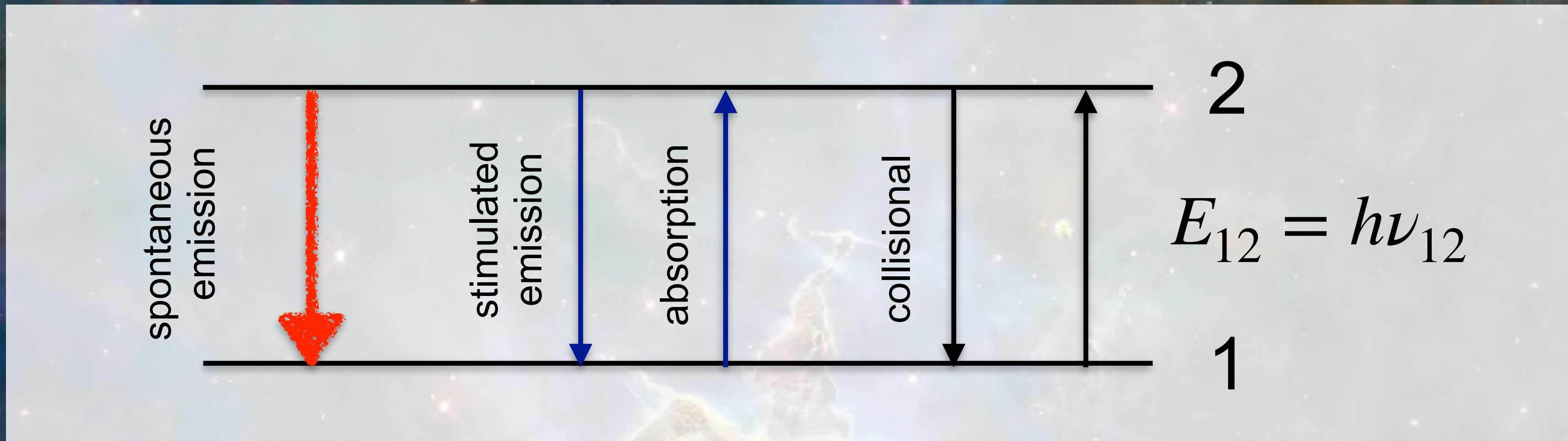
$$\tau \ll 1 \implies I_\nu(\tau_\nu) \simeq I_\nu(0)(1 - \tau_\nu) + \tau_\nu S_\nu =$$

$$= I_\nu(0) + \tau_\nu(S_\nu - I_\nu(0))$$

Emission line when $\tau_\nu \propto \alpha_\nu$ has a peak for an atomic transition and

1. $I_\nu(0) = 0$ - thermal emission from thin cloud
2. $S_\nu > I_\nu(0)$ - thin layer with higher temperature than background radiation

# Radiative transitions with collisions




The probability per unit time of having a transition 2→1 is:

$$P_{21} = A_{21} + B_{21}\bar{J}, \quad \bar{J} = \int J_\nu \phi(\nu) d\nu$$

Spontaneous emission coefficients are of the order:

$A_{21} \sim 10^4 - 10^8 \ s^{-1}$ permitted

$A_{21} \sim 10^{-4} - 1 \ s^{-1}$ forbidden

The rate of transitions stimulated by collisions (with electrons) can be shown to be expressed as $N_e C_{21}$:

$$C_{21} = \left(\frac{2\pi}{kT_e}\right)^{0.5} \frac{\hbar^2}{m^{2/3}} \frac{\Omega(1,2)}{g_2} = \frac{8.63 \times 10^{-6}}{\sqrt{T_e / 1 \ \mathrm{K}}} \frac{\Omega(1,2)}{g_2}$$

$\Omega(1,2) \sim 1$ collision strength

$g_2$ statistical weight of state

The corresponding excitation rate will be:

$$C_{12} = \frac{g_2}{g_1} e^{-E_{12}/kT_e} C_{21}$$

For low $\bar{J}$, the transition rate 2→1 results:

$$\frac{dN_2}{dt} = -N_2(A_{21} + N_e C_{21}) + N_1 N_e C_{12}$$

In equilibrium:

$$\frac{N_2}{N_1} = \frac{N_e C_{12}}{A_{21} + N_e C_{21}} = \frac{N_e/N_c \, e^{-E_{12}/kT_e}}{1 + N_e/N_c}$$

where we defined a **critical density**:

$$N_c = A_{21}/C_{21}$$

# Forbidden lines

Typical values of critical density for forbidden transitions:

$$A_{21} \sim 1\ \mathrm{s}^{-1}, \quad C_{21} \sim 10^{-6}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}, \quad N_c = \frac{A_{21}}{C_{21}} \sim 10^6\ \mathrm{cm}^{-3}$$

At higher densities we have thermodynamical equilibrium:

$$N_e \gg N_c: \qquad \frac{N_2}{N_1} = e^{-E_{12}/kT_e}$$

At lower densities:

$$N_e \ll N_c: \qquad \frac{N_2}{N_1} = \frac{N_e}{N_c} e^{-E_{12}/kT_e} \implies N_2 \ll N_1$$

In this case:

$$N = N_1 + N_2 \sim N_1, \quad N_2 A_{21} = N N_e C_{21} e^{-E_{21}/kT_e}$$

Line emissivity:

$$J_{21} = h\nu_{12} \frac{N_2 A_{21}}{4\pi}, \quad L_{21} = 4\pi J_{21} \times \mathrm{Volume}$$

At lower densities (valid also for optically thick gas):

$$J_{21} \propto N N_e \sim N^2$$

It can be shown that at higher densities $J_{21} \propto N_e$, so a forbidden line becomes very weak with respect to permitted ones.

# Collisional ionization equilibrium

A species X is ionised from $X^{+i}$ to $X^{+i+1}$. by a source of luminosity $L_\nu$ at distance r.

In this case the electron transits form a bound to a free state. Ionization is due to photon absorption, recombination is a kind of collision.

$$\text{Ionization rate}: \quad N(X^{+i}) \int_{\nu_i}^{\infty} \frac{L_\nu}{4\pi r^2 \, h\nu} b_\nu d\nu$$

in cm$^{-3}$ s$^{-1}$, where $\nu_i$ corresponds to the ionization energy and the coefficient $b_\nu$ (cm$^2$) gives the probability of ionization by that photon, taking into accout all the excitation states of the species.

$$\text{Recombination rate}: \quad N_e N(X^{+i+1}) \, \alpha(X^{+i}, T_e)$$

where the recombination coefficient $\alpha(X^{+i}, T_e)$ (cm$^3$ s$^{-1}$) depends on electron temperature.

The production rate of photons that ionize $X^{+i}$ or the Hydrogen atom is (units of s$^{-1}$):

$$Q(X^{+i}) = \int_{\nu_i}^{\infty} \frac{L_\nu}{h\nu} d\nu, \quad Q(H) = \int_{\nu_H}^{\infty} \frac{L_\nu}{h\nu} d\nu$$

We define an average ionization coefficient:

$$\bar{b}(X^{+i}) = \frac{1}{Q(X^{+i})} \int_{\nu_i}^{\infty} b_\nu \frac{L_\nu}{h\nu} d\nu$$

The ratio of the two ionized species can then be written with some math as:

$$\frac{N(X^{+i+1})}{N(X^{+i})} = c\eta \, U \frac{\bar{b}(X^{+i})}{\alpha(X^{+i}, T_e)}$$

where we defined two adimensional parameters:

$$\eta = \frac{Q(X^{+i})}{Q(H)} \qquad U = \frac{Q(H)}{4\pi r^2 \, N_e c}$$

U is the **ionization parameter**, roughly giving the number of ionizing photons per electron, i.e. per Hydrogen atom. Even for U~10$^{-3}$ hydrogen is highly ionized.

# Photo-ionization of a cloud

To study, even in simplified terms, the photoionization of a cloud, one has to consider:

- collisional ionization equilibrium for all the atoms and all the ionization states;
- radiative transfer across the cloud;
- electrons must share the same kinetic temperature as the ions;
- thermal equilibrium of gas must be computed, given by:
  - + radiation losses - emission lines;
  - + energy gain from ionizing photons.

Public code: CLOUDY, https://www.nublado.org/