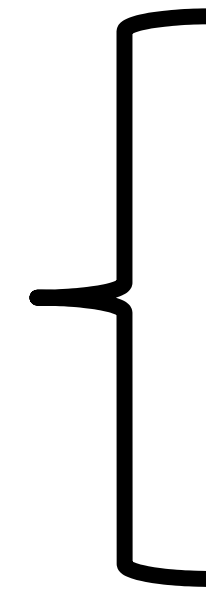


# **A radio astronomy primer**

**Maurilio Pannella - [mpannella@units.it](mailto:mpannella@units.it) - May 2021**

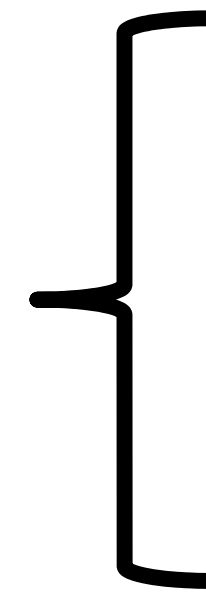
# Agenda and Outline of this primer

6 May



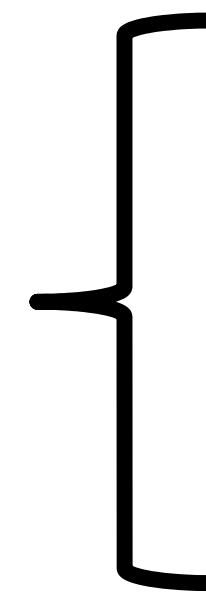
- What is radio astronomy
- Historical background
- Physical processes

25 May



- Collecting radio signals
- A brief introduction to interferometry

27 May



- Radio astronomy and galaxy evolution
- The present and future of radio astronomy

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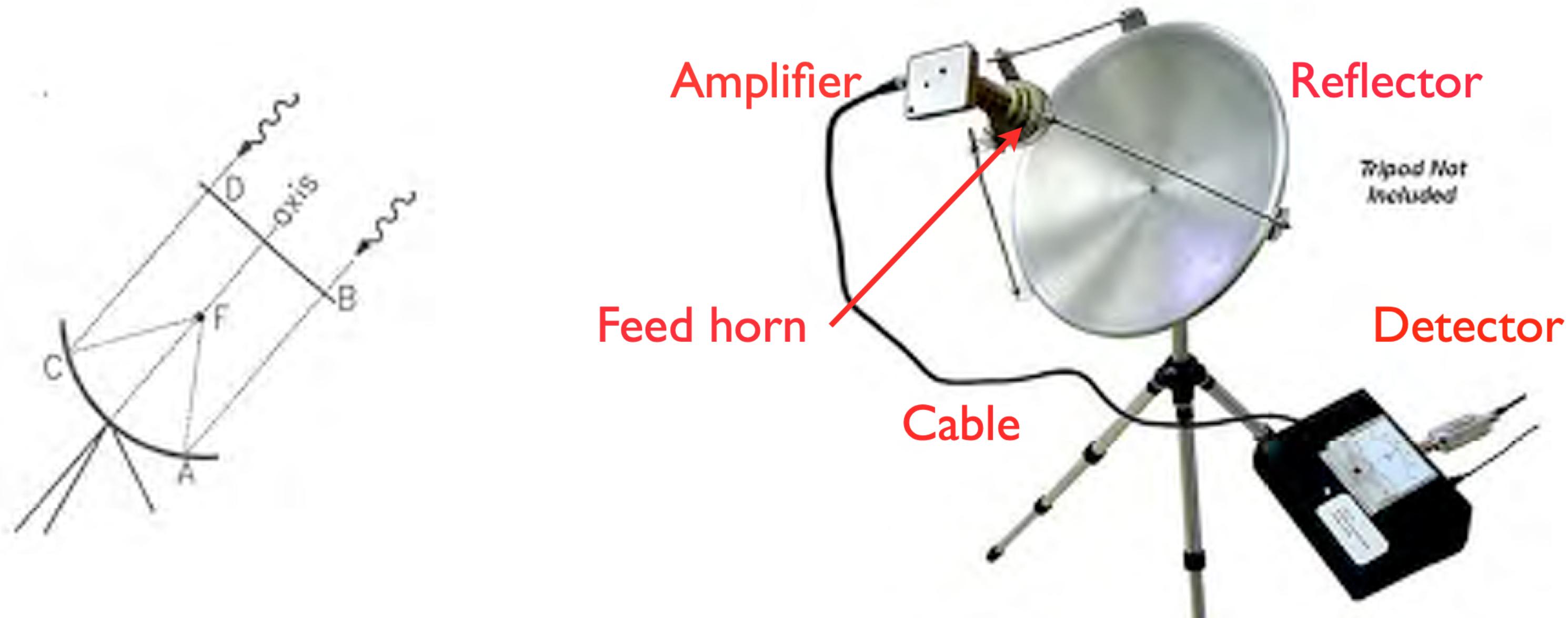
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# Collecting radio signals

Radio photons are too wimpy to do very much - we cannot usually detect individual photons

- e.g. optical photons of 600 nanometre  $\Rightarrow$  2 eV or 20000 Kelvin ( $h\nu/kT$ )
  - e.g. radio photons of 1 metre  $\Rightarrow$  0.000001 eV or 0.012 Kelvin
- ➔ Photon counting in the radio is not usually an option, we must think classically in terms of measuring the source electric field etc.

i.e. measure the voltage oscillations induced in a conductor (antenna) by the incoming EM-wave. Example:





# Collecting radio signals

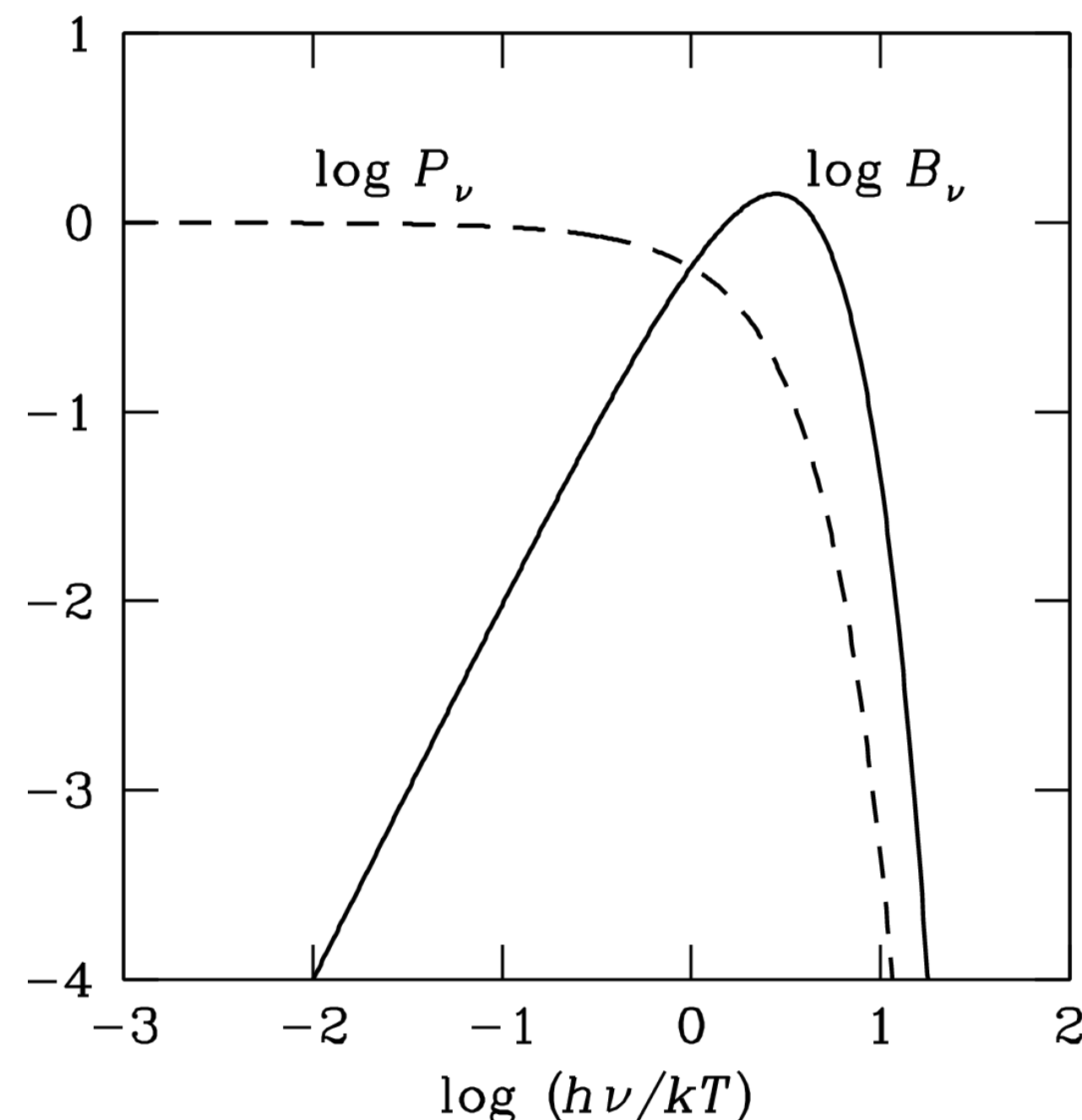
Nyquist theorem and noise Temperature

A resistor (even without current) at a certain temperature  $T$  produces noise power:  $P_\nu d\nu = kT d\nu$  (when  $h\nu \ll kT$ )

# Collecting radio signals

## Nyquist theorem and noise Temperature

A resistor (even without current) at a certain temperature  $T$  produces noise power:  $P_\nu d\nu = kT d\nu$  (when  $h\nu \ll kT$ )



$$P_\nu = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

At low frequencies  $\nu \ll kT/h$ , the specific intensity  $B_\nu$  of blackbody radiation in three dimensions (solid curve) is proportional to  $\nu^2$ . Its one-dimensional analog, the spectral power density of noise generated by a resistor (dashed curve), is proportional to  $\nu^0$ . Quantization causes the sharp exponential cutoffs of both curves at high frequencies.

# Collecting radio signals

Nyquist theorem and noise Temperature

A resistor (even without current) at a certain temperature  $T$  produces noise power:  $P_\nu d\nu = kT d\nu$  (when  $h\nu \ll kT$ )

If we could connect the resistor to the telescope (without any loss) then we'd be able to measure the telescope temperature

A radio telescope is about measuring noises/temperatures

# Collecting radio signals

EM power in bandwidth  $\delta\nu$  from solid angle  $\delta\Omega$  intercepted by surface  $\delta A$  is:

$$\delta W = I_\nu \delta\Omega \delta A \delta\nu$$

Defines surface brightness  $I_\nu$  ( $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ ; aka specific intensity)

Flux density  $S_\nu$  ( $\text{W m}^{-2} \text{Hz}^{-1}$ ) – integrate brightness over solid angle of source

$$S_\nu = \int_{\Omega_s} I_\nu d\Omega$$

Convenient unit – the **Jansky**  $\rightarrow 1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{Hz}^{-1} = 10^{-23} \text{ erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$

Note:  $S_\nu = L_\nu / 4\pi d^2$  ie. distance dependent

$\Omega \propto 1/d^2 \Rightarrow I_\nu \propto S_\nu / \Omega$  ie. distance independent

# Collecting radio signals

Recall : 
$$\delta W = I_\nu \delta\Omega \delta A \delta\nu$$

Telescope of effective area  $A_e$  receives power  $P_{rec}$  per unit frequency from an unpolarized source but is only sensitive to one mode of polarization:

$$P_{rec} = \frac{1}{2} I_\nu A_e \delta\Omega$$

Telescope is sensitive to radiation from more than one direction with *relative* sensitivity given by the normalized antenna pattern  $P_N(\theta, \varphi)$ :

$$P_{rec} = \frac{1}{2} A_e \int_{4\pi} I_\nu(\theta, \varphi) P_N(\theta, \varphi) d\Omega$$

# Collecting radio signals

In general surface brightness is position dependent, ie.  $I_\nu = I_\nu(\theta, \phi)$

$$I_\nu(\theta, \phi) = \frac{2k\nu^2 T(\theta, \phi)}{c^2}$$

(if  $I_\nu$  described by a blackbody in the Rayleigh-Jeans limit;  $h\nu/kT \ll 1$ )

Back to flux:

$$S_\nu = \int_{\Omega_s} I_\nu(\theta, \phi) d\Omega = \frac{2k\nu^2}{c^2} \int T(\theta, \phi) d\Omega$$

In general, a radio telescope maps the *temperature distribution of the sky*

# Collecting radio signals

$$S_\nu = \int_{\Omega_s} I_\nu(\theta, \varphi) d\Omega = \frac{2k\nu^2}{c^2} \int T(\theta, \varphi) d\Omega$$

$$S_\nu = \int_{\Omega_s} I_\nu d\Omega = \frac{2k\nu^2}{c^2} \int T_B d\Omega$$

$$P_{rec} = \frac{A_e}{2} \int_{4\pi} I_\nu(\theta, \varphi) P_N(\theta, \varphi) d\Omega$$

$$\therefore T_A = \frac{A_e}{2k} \int_{4\pi} I_\nu(\theta, \varphi) P_N(\theta, \varphi) d\Omega$$

*Antenna temperature* is what is observed by the radio telescope.

A “convolution” of sky brightness with the beam pattern

It is an inversion problem to determine the source temperature distribution.

$$S_\nu = \frac{2k}{A_{eff}} T_A$$



# Collecting radio signals

An antenna is a passive device that converts electromagnetic radiation in space into electrical currents in conductors or vice versa, depending on whether it is being used for receiving or for transmitting, respectively.

Radio telescopes are receiving antennas, and radar telescopes are also transmitting antennas. It is often easier to calculate the properties of transmitting antennas and to measure the properties of receiving antennas.

The reciprocity theorem states that most characteristics of a transmitting antenna (e.g., its radiation pattern) are unchanged when that antenna is used for receiving, so any analysis of a transmitting antenna can be applied to a receiving antenna used in radio astronomy, and any measurement of a receiving antenna can be applied to that antenna when used for transmitting.

The two cases are equivalent because of time reversibility:

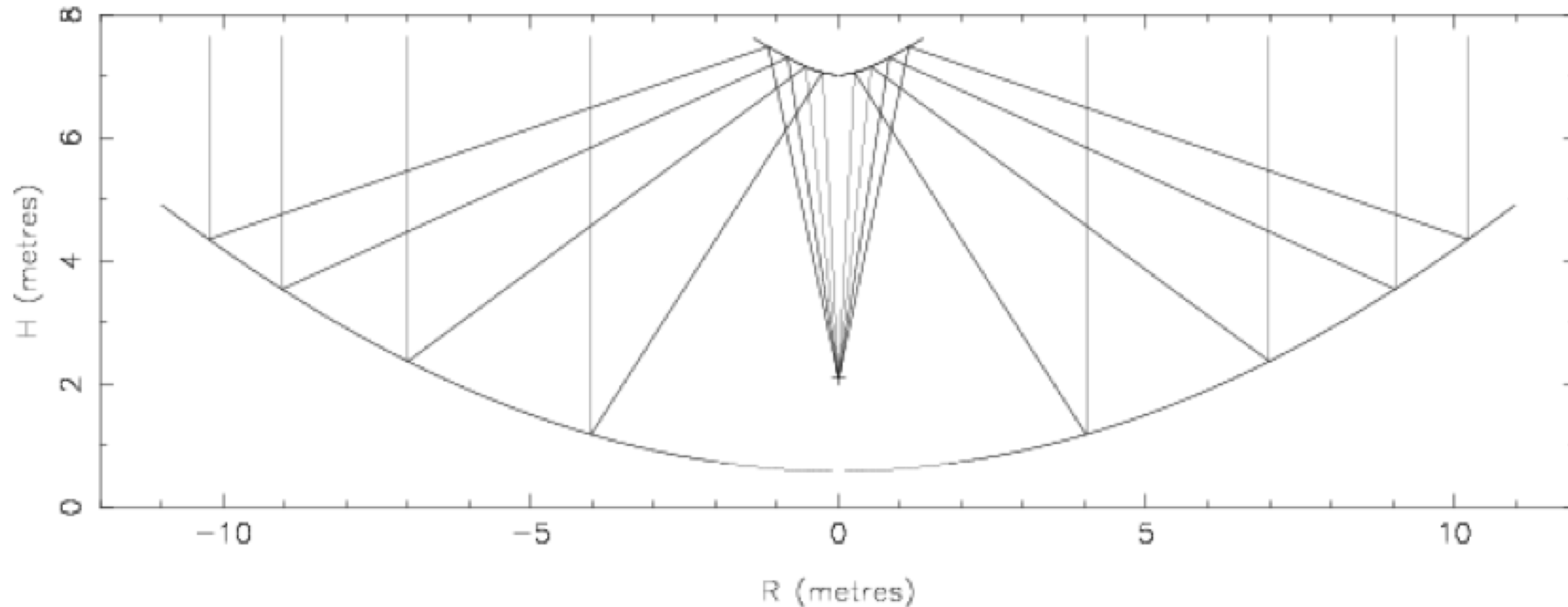
the solutions of Maxwell's equations are valid when time is reversed!



# Collecting radio signals

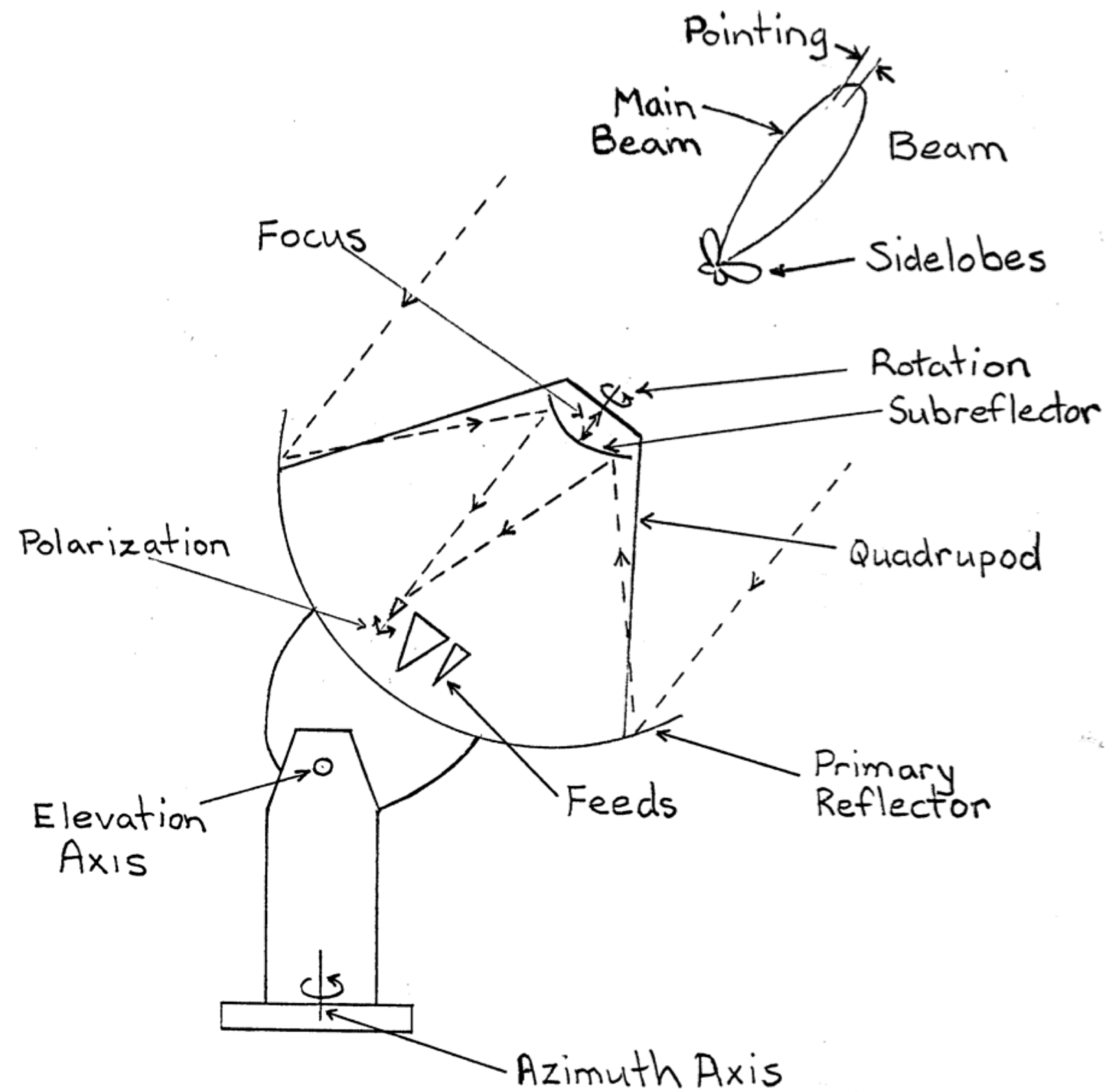
The *antenna* collects the E-field over the aperture at the focus

The *feed horn* at the focus adds the fields together, guides signal to the *front end*





# Collecting radio signals

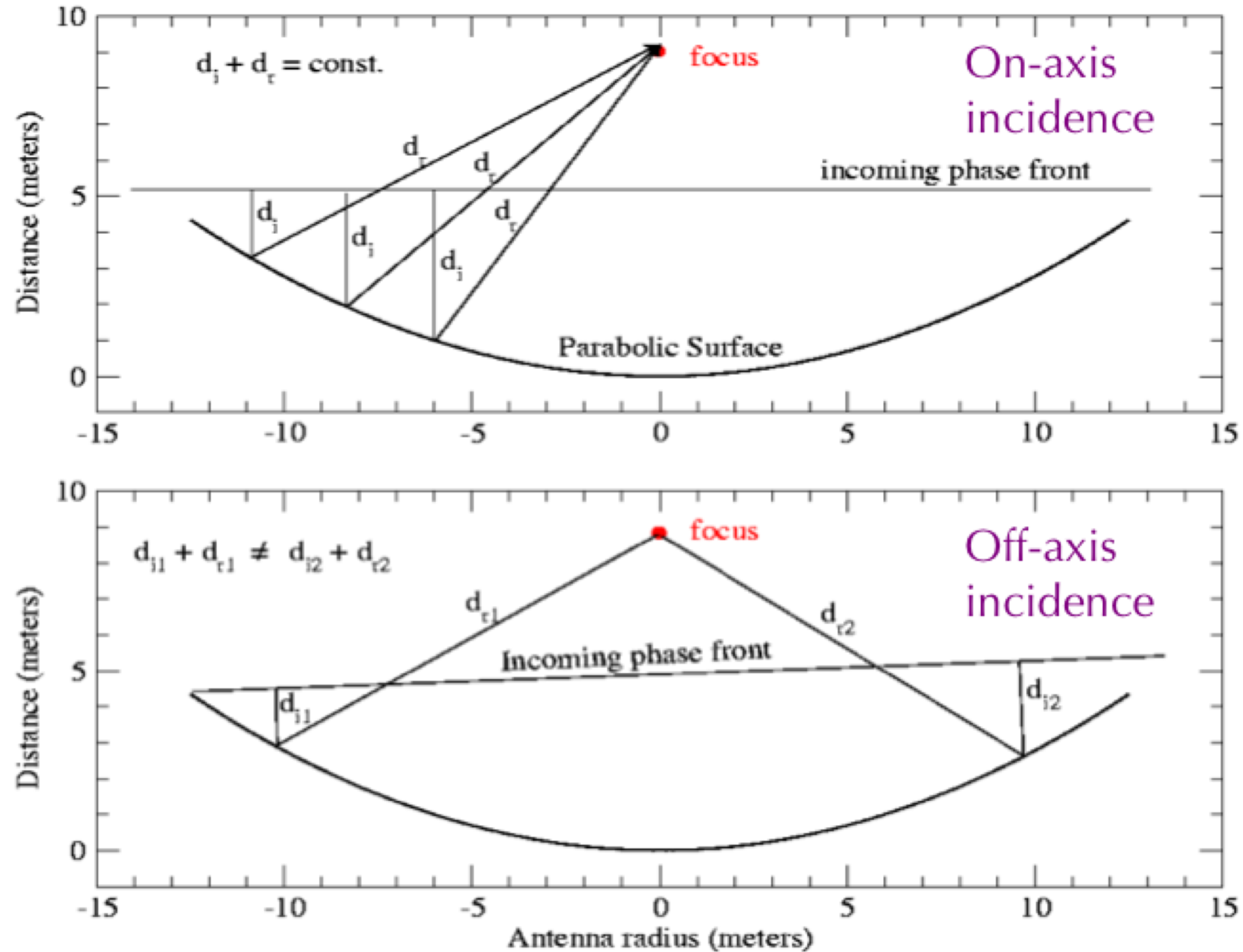


The Parkes 64-m telescope, Australia



# Collecting radio signals

- Antenna response is a coherent phase summation of the E-field at the focus
- First null occurs at the angle where one extra wavelength of path is added across the full aperture width, i.e.,  $\theta \sim \lambda/D$



# Collecting radio signals

The feed can illuminate the aperture antenna with a sine wave of fixed frequency  $\nu = \omega / (2\pi)$  and electric field strength  $g(x)$  that varies across the aperture.

The illumination induces currents in the reflector. The currents will vary with both position and time:

$$I \propto g(x) \exp(-i\omega t)$$

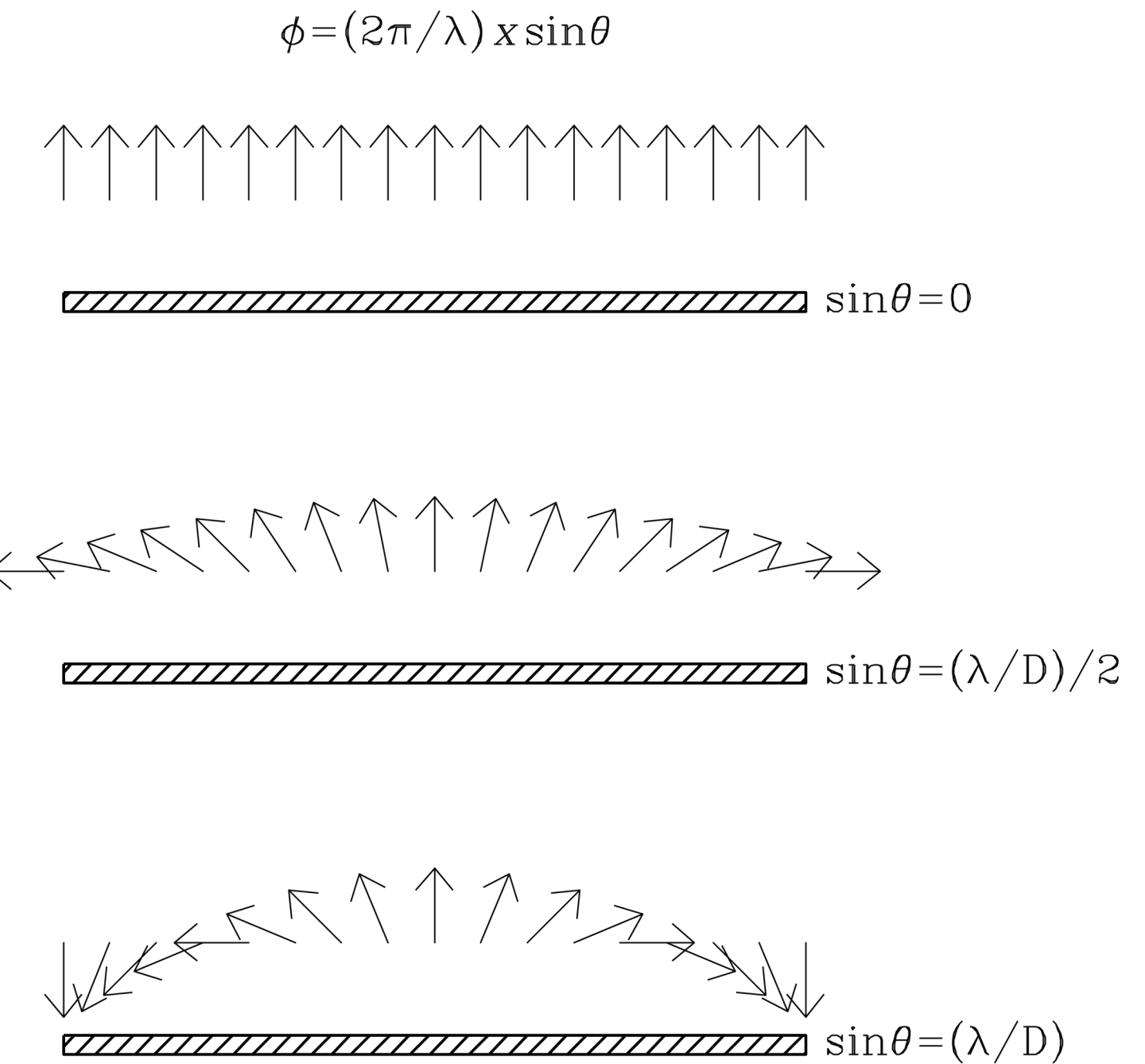
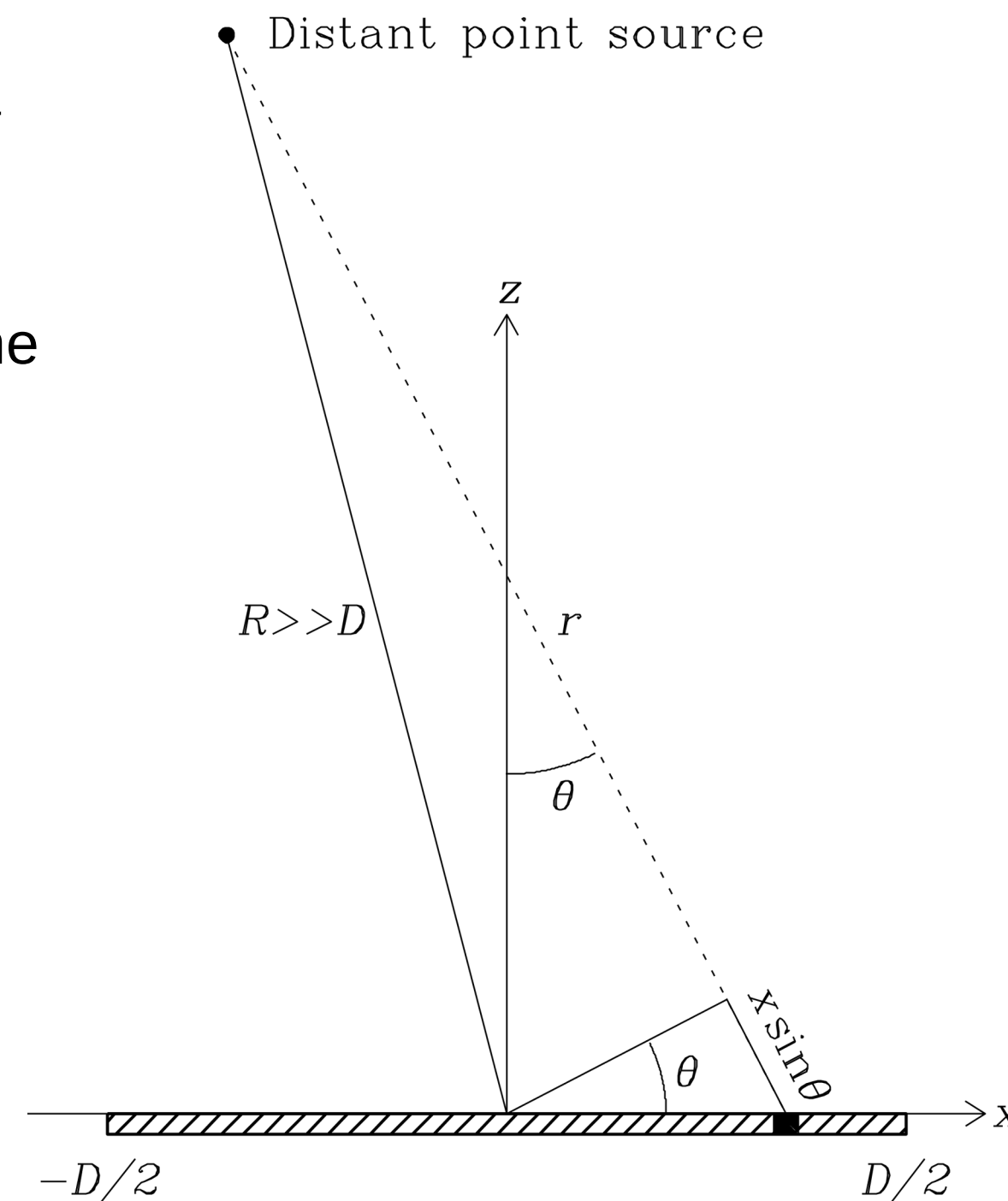
Huygens's principle asserts that the aperture can be treated as a collection of small elements which act individually as small antennas.

The field from each element extending from  $x$  to  $x+dx$  is:

$$df \propto g(x) \exp(-i2\pi r(x)/\lambda) r(x) dx,$$

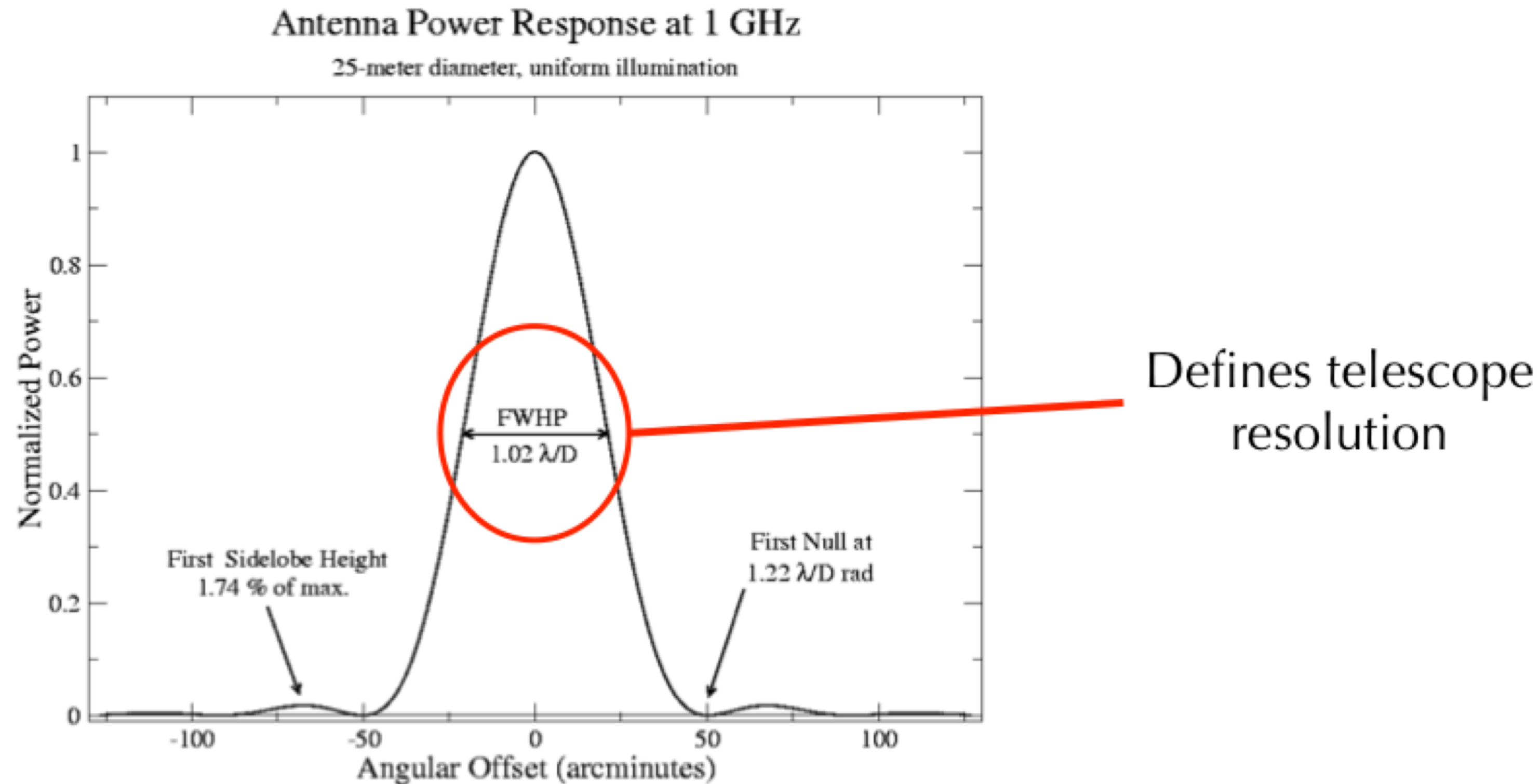
$$f(l) = \int_{\text{aperture}} g(u) e^{-i2\pi l u} du.$$

The electric-field pattern  $f(l)$  of an aperture antenna is the Fourier transform of the electric field distribution  $g(u)$  illuminating that aperture



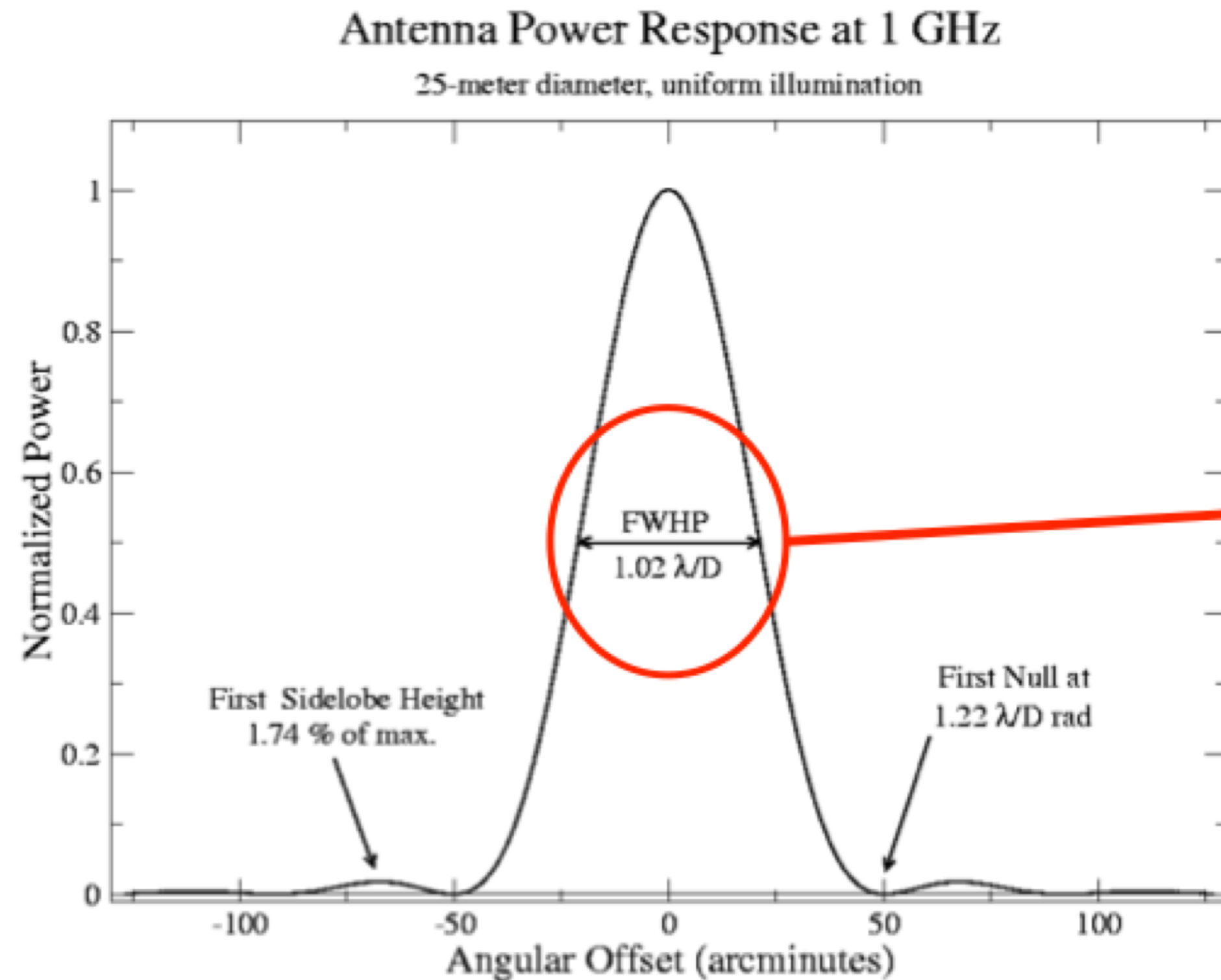
with  $l \equiv \sin \theta$  (far field approximation) and  $u \equiv x/\lambda$

# Collecting radio signals



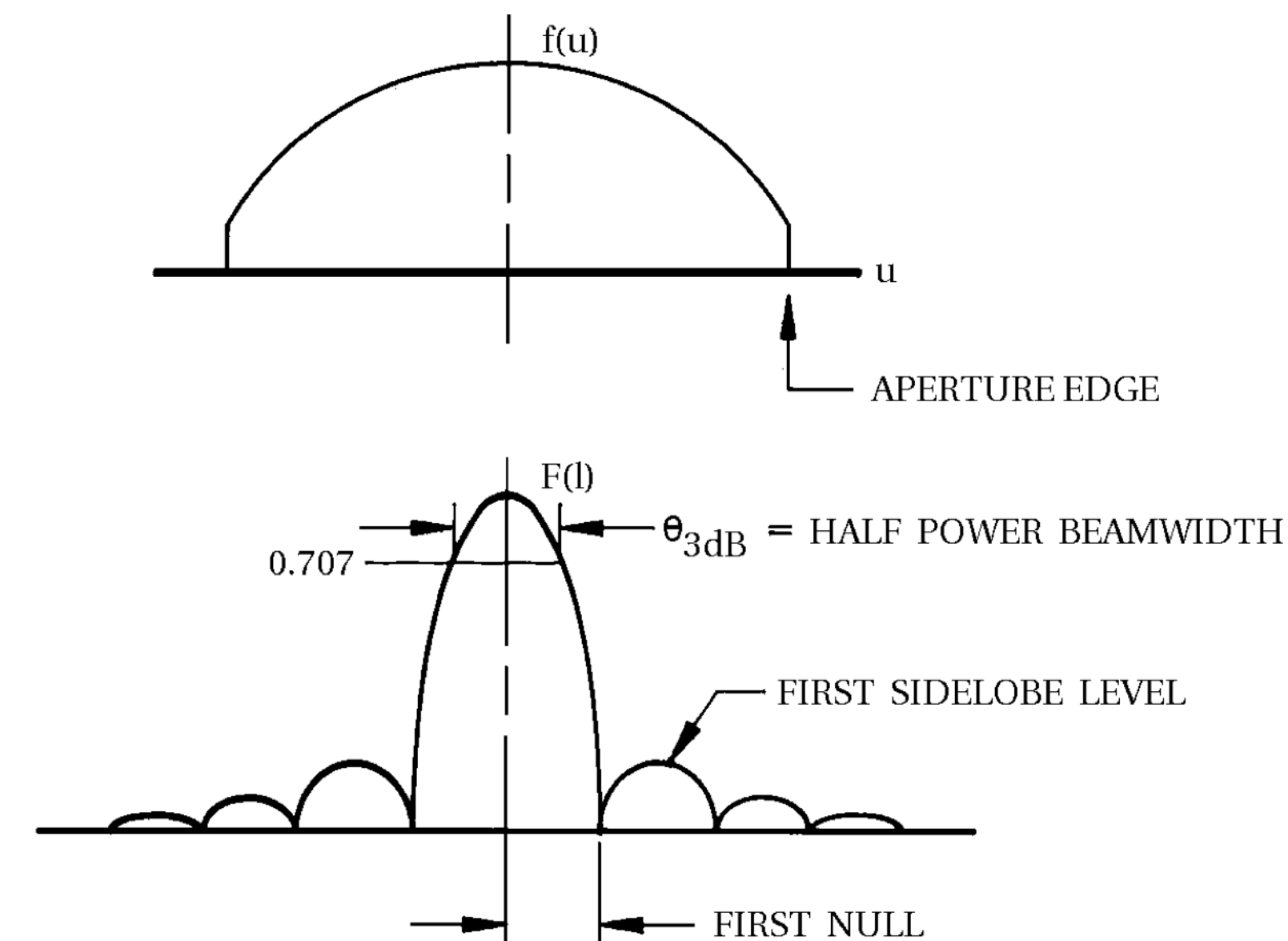
- The voltage response pattern is the FT of the aperture distribution
- The power response pattern,  $P(\theta) \propto V^2(\theta)$ , is the FT of the autocorrelation function of the aperture

# Collecting radio signals



Defines telescope resolution

- The voltage response pattern is the FT of the aperture distribution
- The power response pattern,  $P(\theta) \propto V^2(\theta)$ , is the FT of the autocorrelation function of the aperture
- for a uniform circle,  $V(\theta)$  is  $J_1(x)/x$  and  $P(\theta)$  is the Airy pattern,  $(J_1(x)/x)^2$





# Collecting radio signals

$$P(\theta, \varphi, \nu) = A(\theta, \varphi, \nu) I(\theta, \varphi, \nu) \Delta \nu \Delta \Omega$$

effective collecting area  $A(\nu, \theta, \varphi)$  [m<sup>2</sup>]

on-axis response

$$A_0 = \eta A$$

$\eta$  = aperture efficiency

Normalized pattern  
(primary beam)

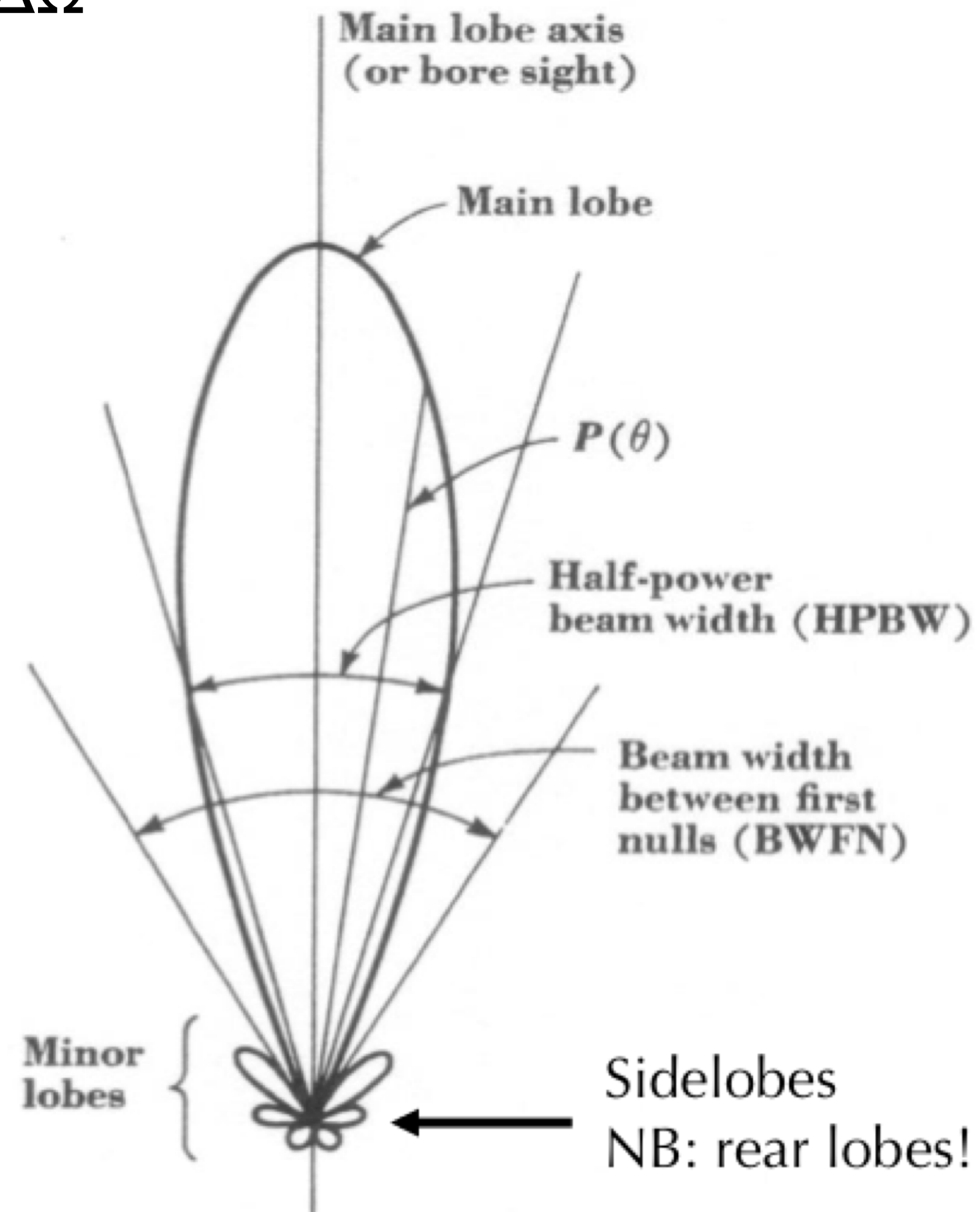
$$\mathbf{A}(\nu, \theta, \varphi) = A(\nu, \theta, \varphi) / A_0$$

Beam solid angle

$$\Omega_A = \iint \mathbf{A}(\nu, \theta, \varphi) d\Omega \text{ all sky}$$

$$\mathbf{A}_0 \Omega_A = \lambda^2$$

$\lambda$  = wavelength,  $\nu$  = frequency



# Collecting radio signals

$$P(\theta, \varphi, \nu) = A(\theta, \varphi, \nu) I(\theta, \varphi, \nu) \Delta \nu \Delta \Omega$$

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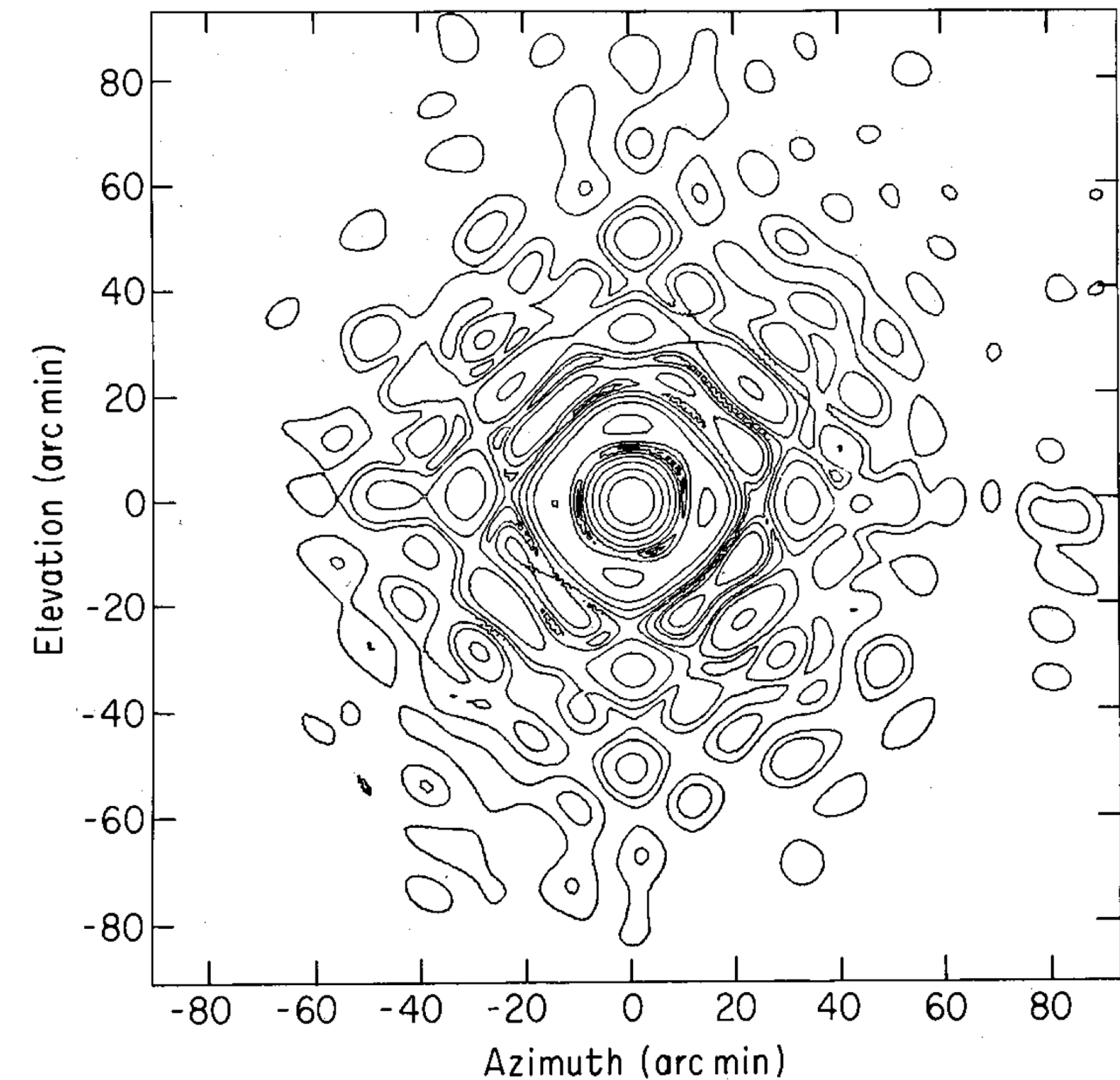
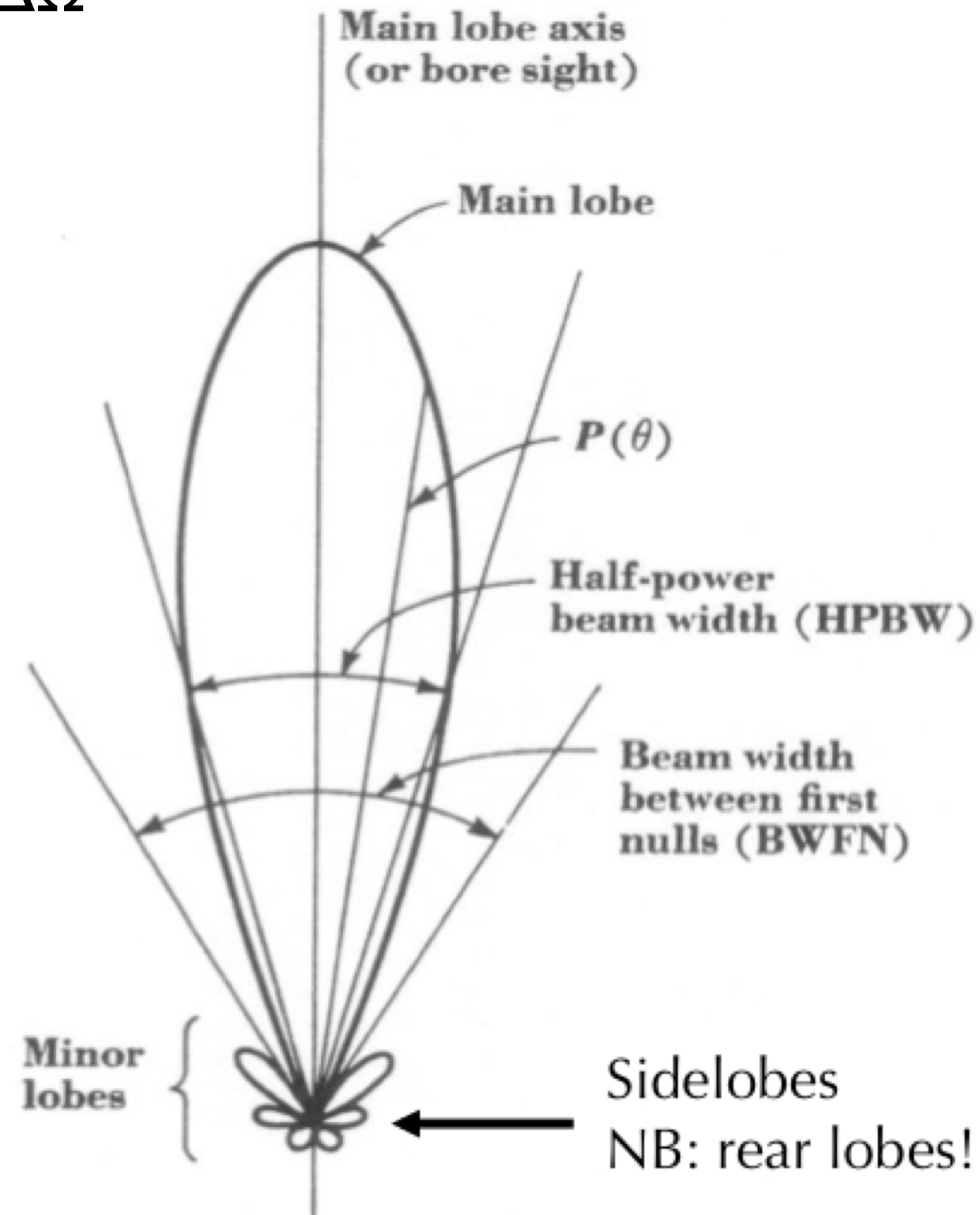
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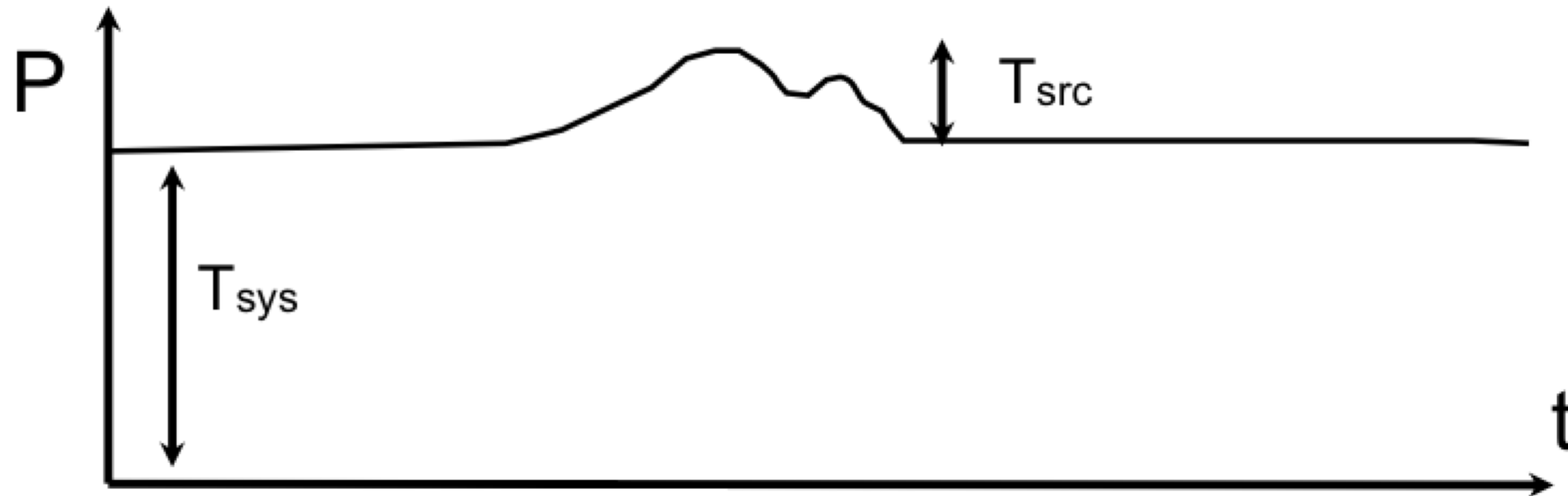
$\lambda$  = wavelength,  $\nu$  = frequency





# Collecting radio signals

Imaging of the sky with a single-dish can be achieved by letting the source drift across the telescope beam and measuring the power received as a function of time. This provides a 1-D cut across the source intensity. Usually, the area of interest is measured at least twice, in orthogonal directions (sometimes referred to as “basket weaving”).



# Collecting radio signals

## Aperture Efficiency

$$A_0 = \eta A, \eta = \eta_{sf} \times \eta_{bl} \times \eta_s \times \eta_t \times \eta_{misc}$$

$\eta_{sf}$  = reflector surface efficiency

$\eta_{bl}$  = blockage efficiency

$\eta_s$  = feed spillover efficiency

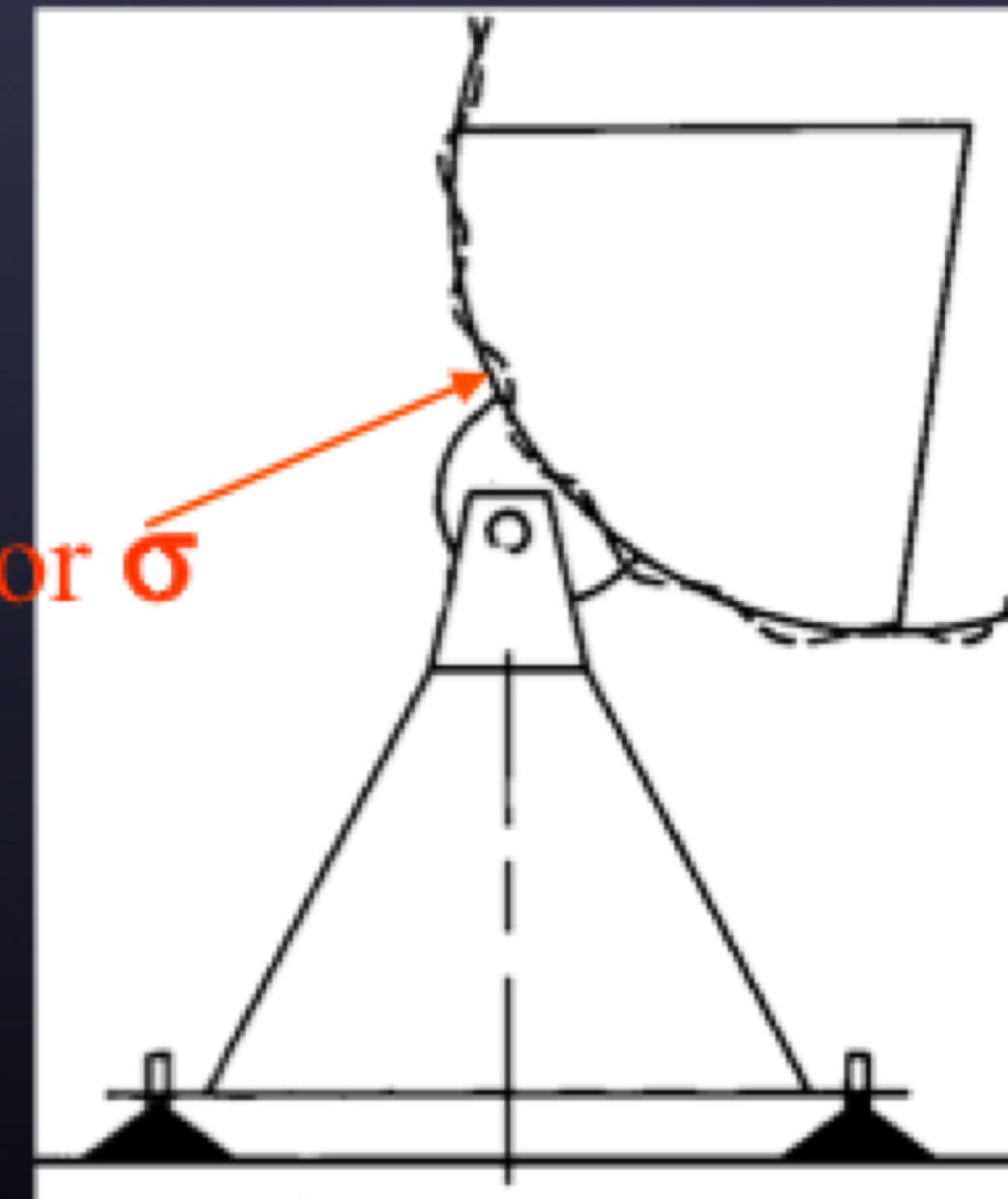
$\eta_t$  = feed illumination efficiency

$\eta_{misc}$  = diffraction, phase, match, loss

$$\eta_{sf} = \exp(-(4\pi\sigma/\lambda)^2)$$

e.g.,  $\sigma = \lambda/16$ ,  $\eta_{sf} = 0.5$

rms error  $\sigma$



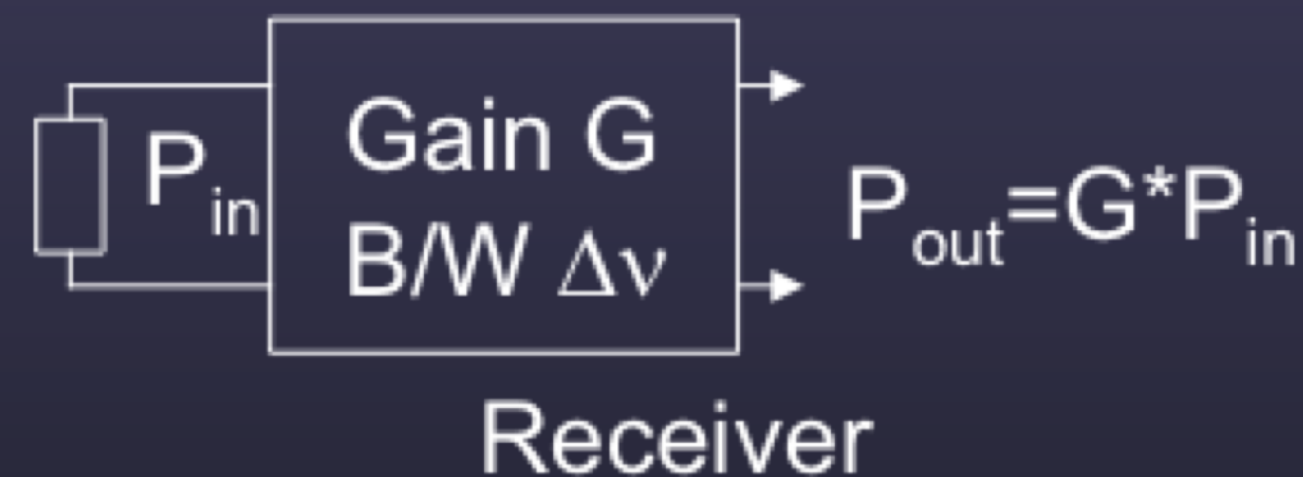
# Collecting radio signals

Reference received power to the equivalent temperature of a matched load at the input to the receiver

Rayleigh-Jeans approximation to Planck radiation law for a blackbody

$$P_{in} = k_B T \Delta\nu \quad (W)$$

Matched load  
@ temp  $T$  ( $^{\circ}K$ )



$$k_B = \text{Boltzmann's constant } (1.38 \cdot 10^{-23} \text{ J}/^{\circ}K)$$

When observing a radio source,  $T_{total} = T_A + T_{sys}$

- $T_{sys}$  = system noise when not looking at a discrete radio source
- $T_A$  = source antenna temperature



# Collecting radio signals - calibrating a telescope

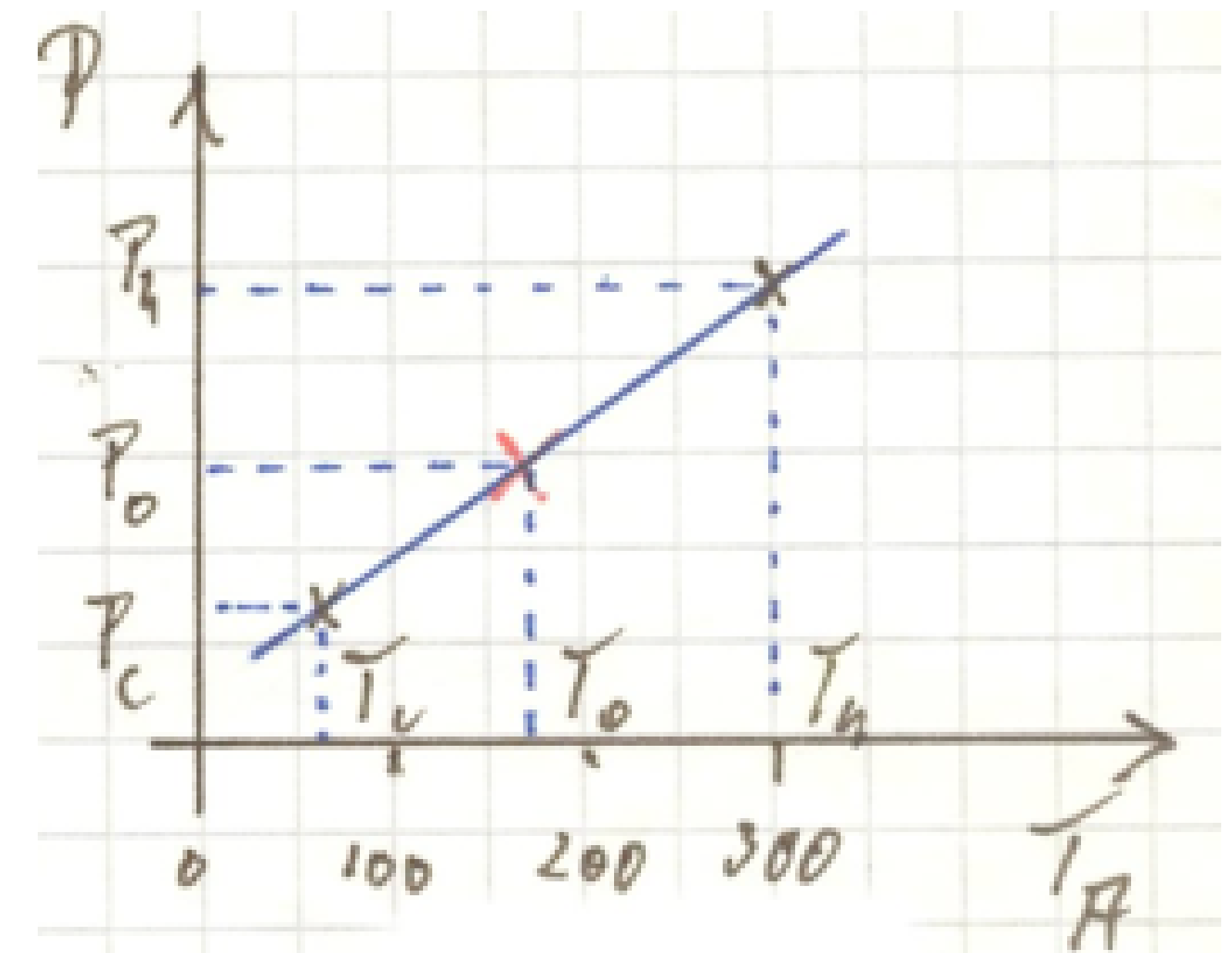
$$S_\nu = \frac{2k}{A_{eff}} T_A$$

relate the voltages measured at the receiver system to the antenna temperature

hot = absorbing material (300 K)  
cold = soaked in liquid nitrogen (77 K)

problem is that we do not know  $A_{eff}$  in general  
for a horn antenna  $A_{eff}$  can be calculated analytically  
now we can relate source flux density with antenna temperature

receiver system needs to be linear



# Collecting radio signals - calibrating a telescope

known flux density of the source can be used to calibrate other telescope

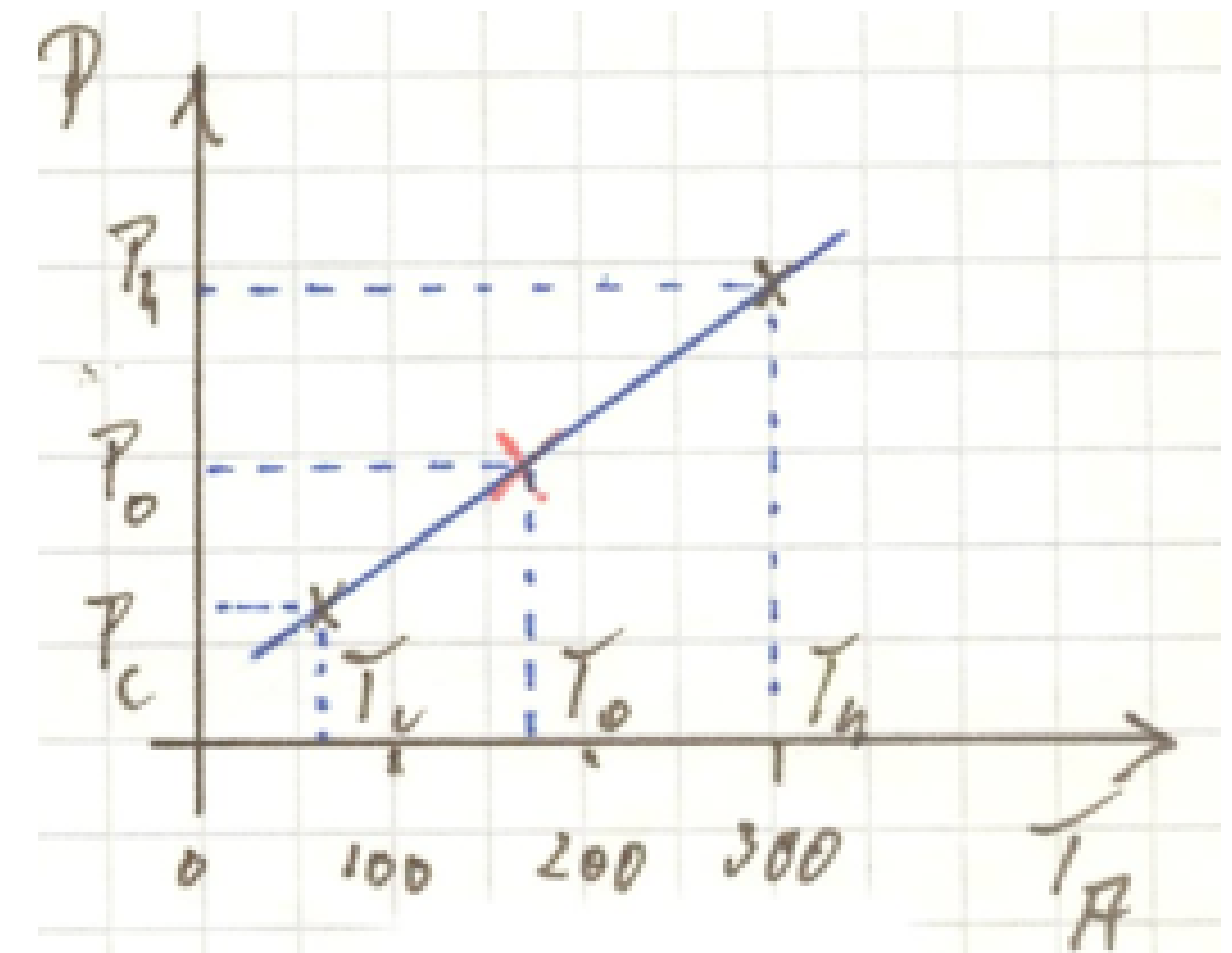
$$S_0 = \cancel{S_\nu} = \frac{2k}{A_{eff}} T_A$$

hot = absorbing material (300 K)  
cold = soaked in liquid nitrogen (77 K)

antenna temperature for another telescope

$$A_{eff} = 2k \cdot \frac{T_{A0}}{S_0} \quad \eta_A = \frac{A_{eff}}{A_{geo}} = \frac{8k T_{A0}}{\pi D^2 S_0}$$

receiver system needs to be linear



# Collecting radio signals - calibrating a telescope

With the known parameters of a telescope we can simply bootstrap the flux densities of sources to be measured.

All we need is a calibration source not too far away from the target source:

$$S_{\text{target}} = S_{\text{cal}} \frac{U_{\text{cal}}}{U_{\text{target}}}$$

Target flux density      Calibrator flux density      Calibrator voltage      Target voltage

# Collecting radio signals

Unfortunately, the telescope system itself contributes noise to the the signal detected by the telescope, i.e.,

$$P_{out} = P_A + P_{sys} \rightarrow T_{out} = T_A + T_{sys} \quad (\text{with } T_A \ll T_{sys})$$

The *system temperature*,  $T_{sys}$ , represents noise added by the system:

$$T_{sys} = T_{bg} + T_{sky} + T_{spill} + T_{loss} + T_{cal} + T_{rx}$$

$T_{bg}$  = microwave and galactic background (3K, except below 1GHz)

$T_{sky}$  = atmospheric emission (increases with frequency--dominant in mm)

$T_{spill}$  = ground radiation (via sidelobes) (telescope design)

$T_{loss}$  = losses in the feed and signal transmission system (design)

$T_{cal}$  = injected calibrator signal (usually small)

$T_{rx}$  = receiver system (often dominates at cm — a design challenge)

Note that  $T_{bg}$ ,  $T_{sky}$ , and  $T_{spill}$  vary with sky position and  $T_{sky}$  is time variable

# Collecting radio signals

When Penzias & Wilson made their measurements, they found:

$$T_{\text{atm}} = 2.3 \pm 0.3 \text{ K},$$

$$T_{\text{loss}} = 0.9 \pm 0.4 \text{ K},$$

$$T_{\text{spill}} < 0.1 \text{ K}.$$

And they expected  $T_{\text{sky}} \sim 0$ .

So looking straight up, they expected to measure  $T_A$ ,

$$T_A = 2.3 + 0.9 + 0.1 + 0 = 3.2 \text{ K}.$$

What they found was  $T_A = 6.7$  Kelvin!

The excess was the CMB and Galactic emission.



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Bell lab advert (right) - 1963 - 3 years before the CMB was detected - and featuring the Penzias & Wilsons horn antenna.

**FIRST PHONE CALL VIA MAN-MADE SATELLITE!**

Think of watching a royal wedding in Europe by live TV, or telephoning to Singapore or Calcutta - by way of man-made satellite? A mere dozen or so years ago, this idea is now a giant step closer to reality.

Bell Telephone Laboratories recently took the step by successfully bouncing a phone call between its Holmdel, N. J., test site and the Jet Propulsion Laboratory of the National Aeronautics and Space Administration (NASA) in California. The reflector was a 100-foot sphere of aluminum plastic orbiting the earth 1000 miles up.

**Dramatic application of telephone science**

Sponsored by NASA, this dramatic experiment - known as "Project Echo" - culled heavily on telephone science for its fulfillment...

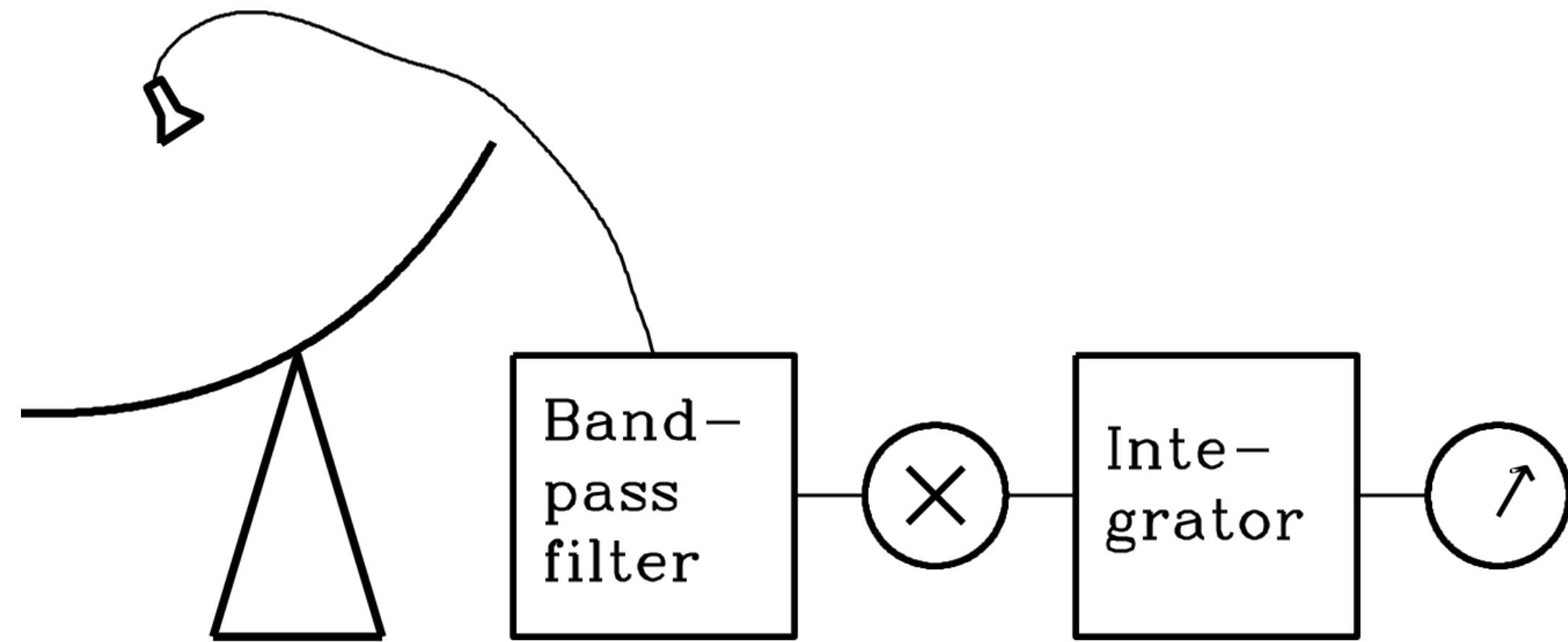
- The Delta rocket which carried the satellite into space was steered into a precise orbit by the Bell Laboratories Command Guidance System. This is the same system which recently guided the remarkable Titan II weather satellite into its unpowered circular orbit.
- To pick up the signals, a special horn-reflector antenna was used. Previously prepared by Bell Laboratories for microwave radio relay, it is virtually immune to common radio "noise" interferences. The amplifier - also a laboratory development - was a traveling wave "noise" with very low noise susceptibility. The signals were still further protected from noise by a special FM switching technique invented at Bell Laboratories.

**BELL TELEPHONE LABORATORIES**  
MADE POSSIBLE BY SUBSCRIPTIONAL SERVICE AND EQUIPMENT

# Collecting radio signals

Q: How can you detect  $T_A$  (signal) in the presence of  $T_{sys}$  (noise)?

# Collecting radio signals

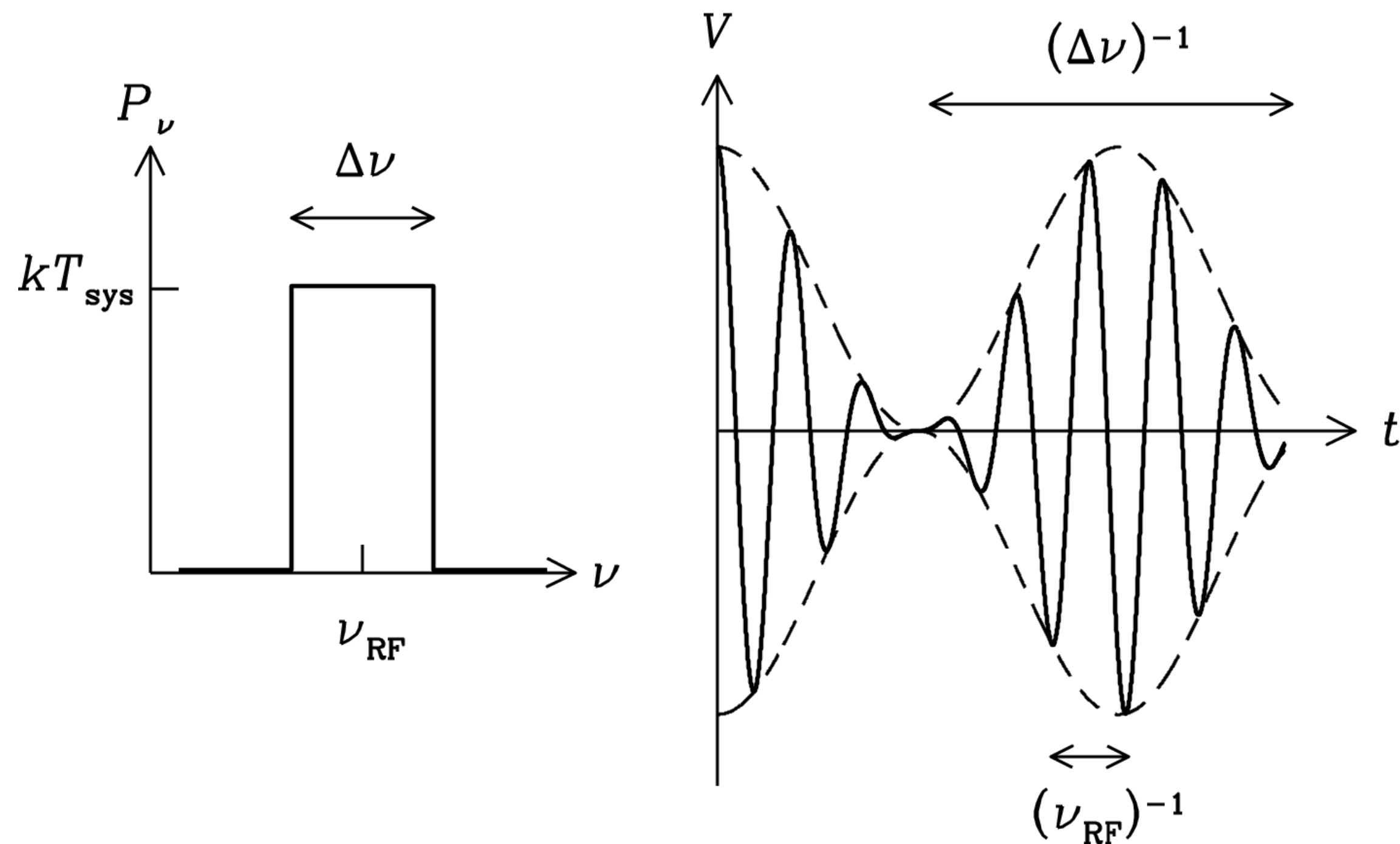
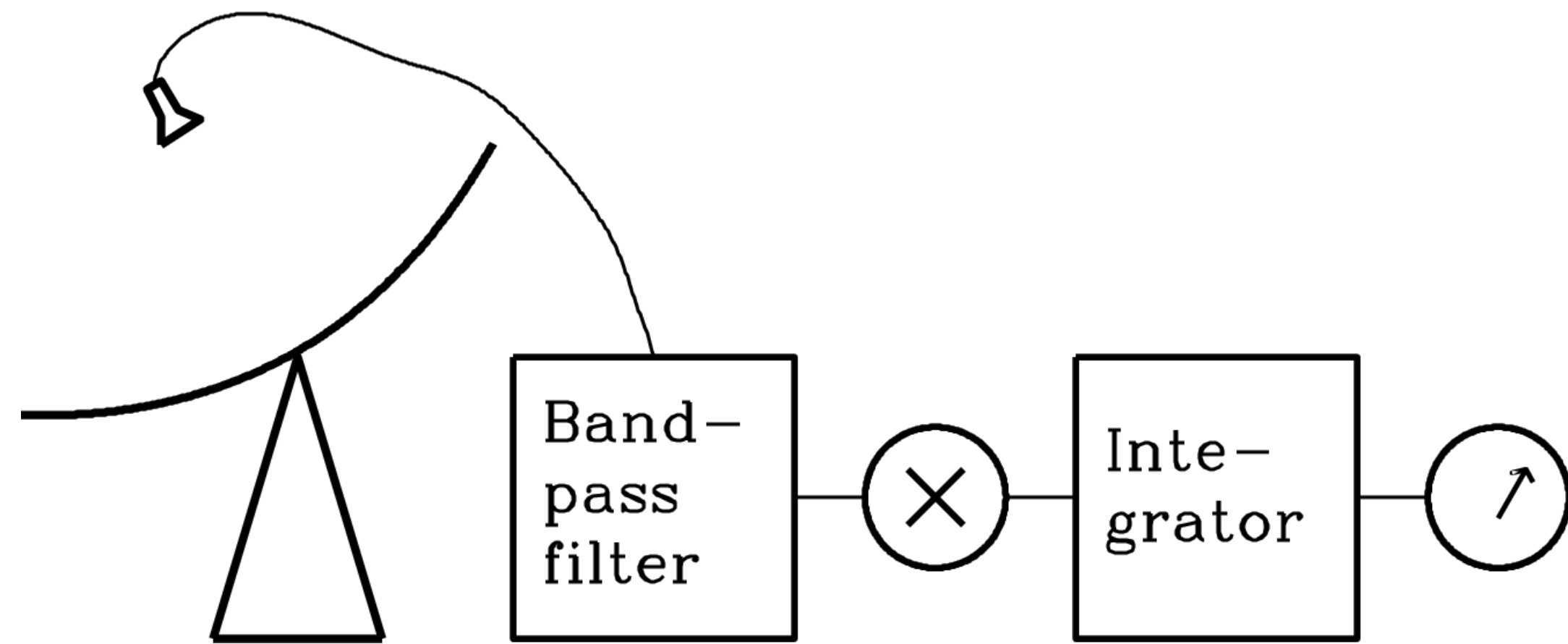


The simplest possible radiometer:

- filters the broadband noise coming from the telescope
- multiplies the filtered voltage by itself (square-law detection)
- smooths the detected voltage, and measures the smoothed voltage.

The function of the detector is to convert the noise voltage, which has zero mean, to noise power, which is proportional to the square of voltage.

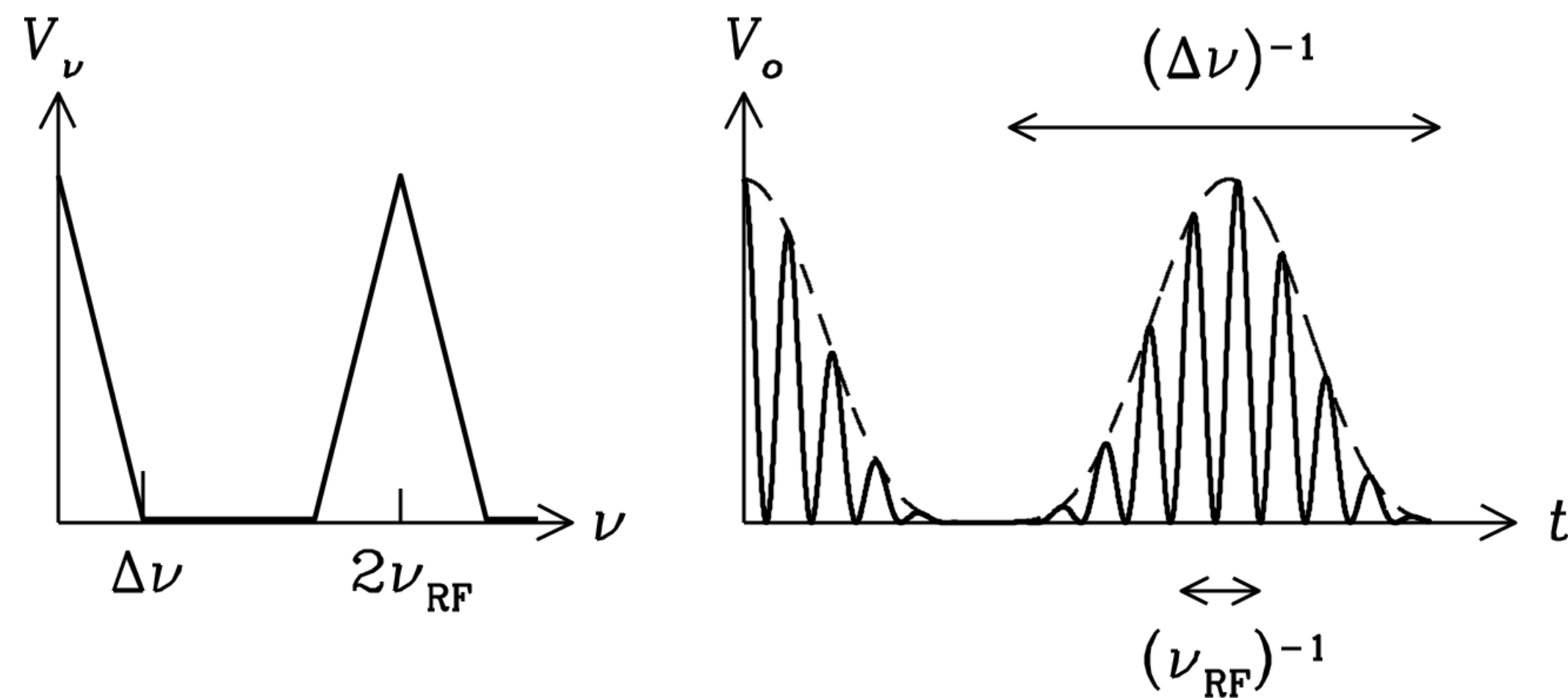
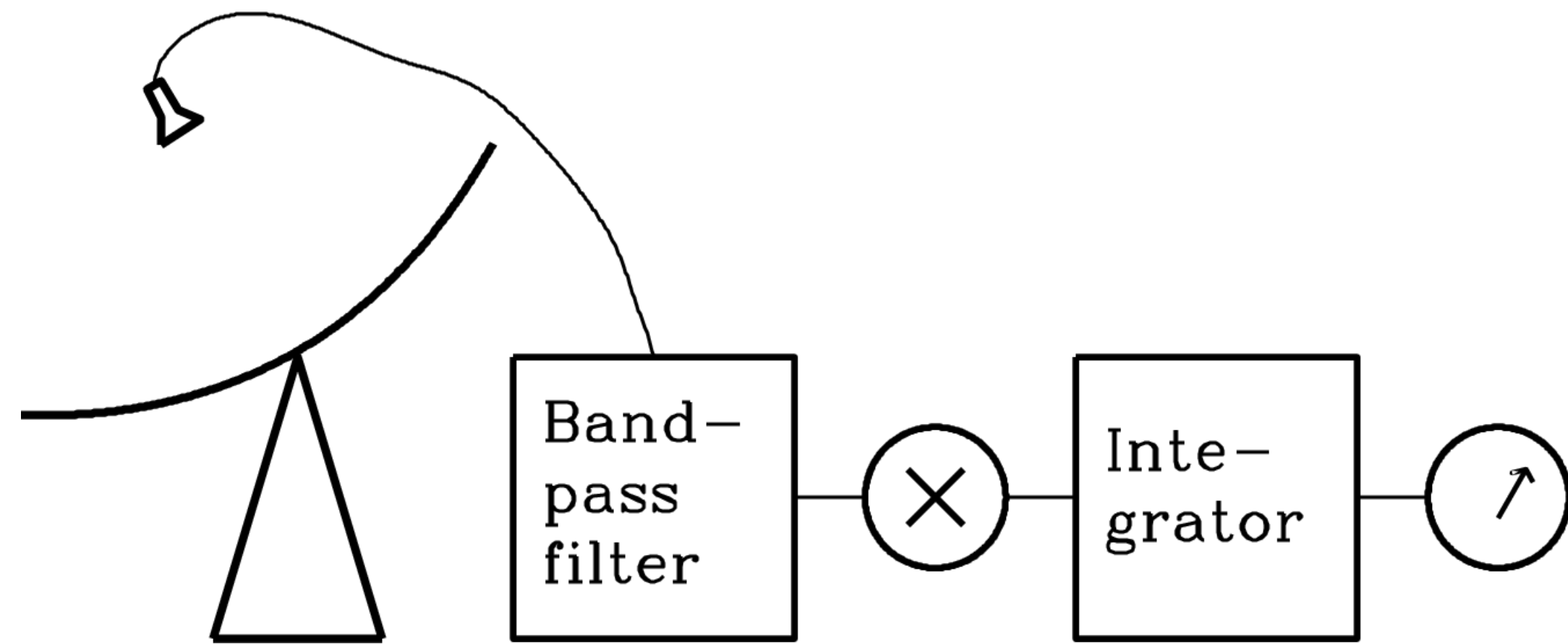
# Collecting radio signals



The voltage output  $V(t)$  of the filter with center frequency  $\nu_{\text{RF}}$  and bandwidth  $\Delta\nu < \nu_{\text{RF}}$  is a sinusoid with frequency  $\nu_{\text{RF}}$  whose envelope (dashed curves) fluctuates on timescales  $(\Delta\nu)^{-1} > (\nu_{\text{RF}})^{-1}$



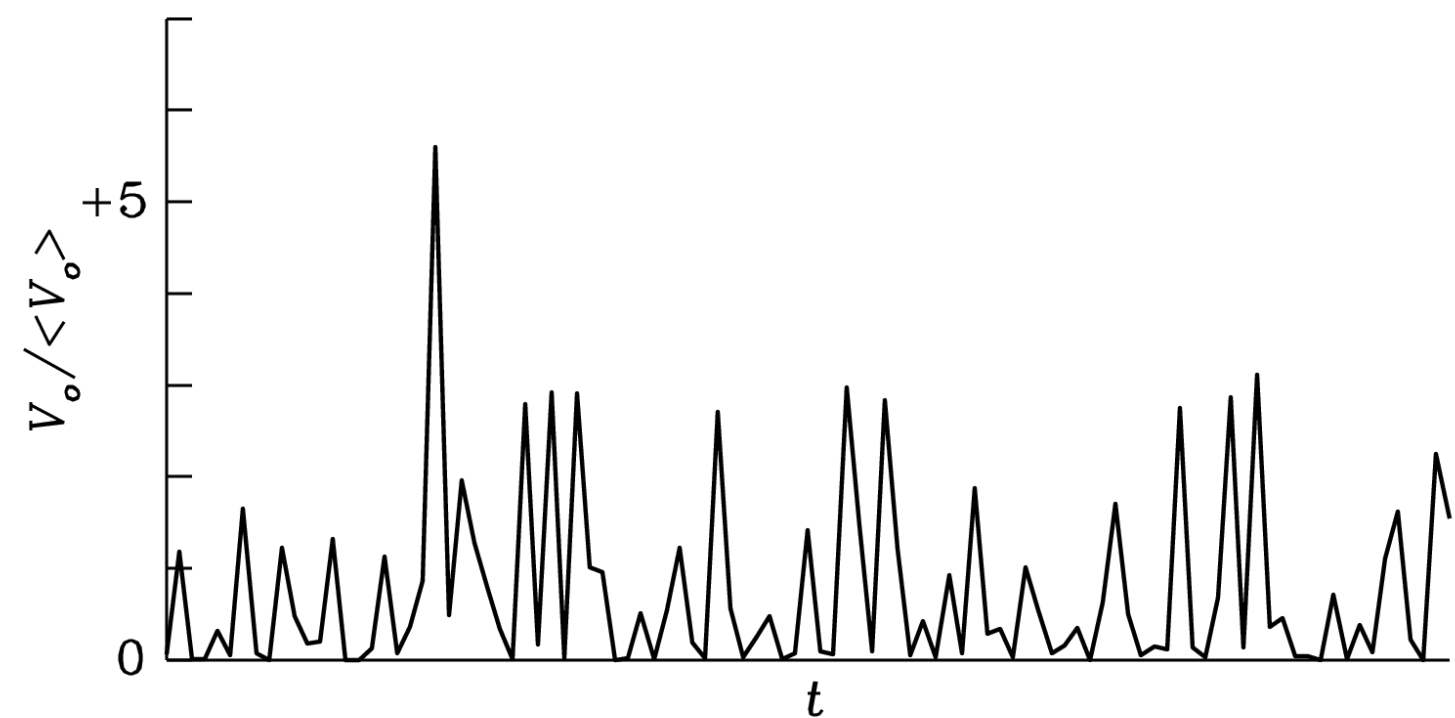
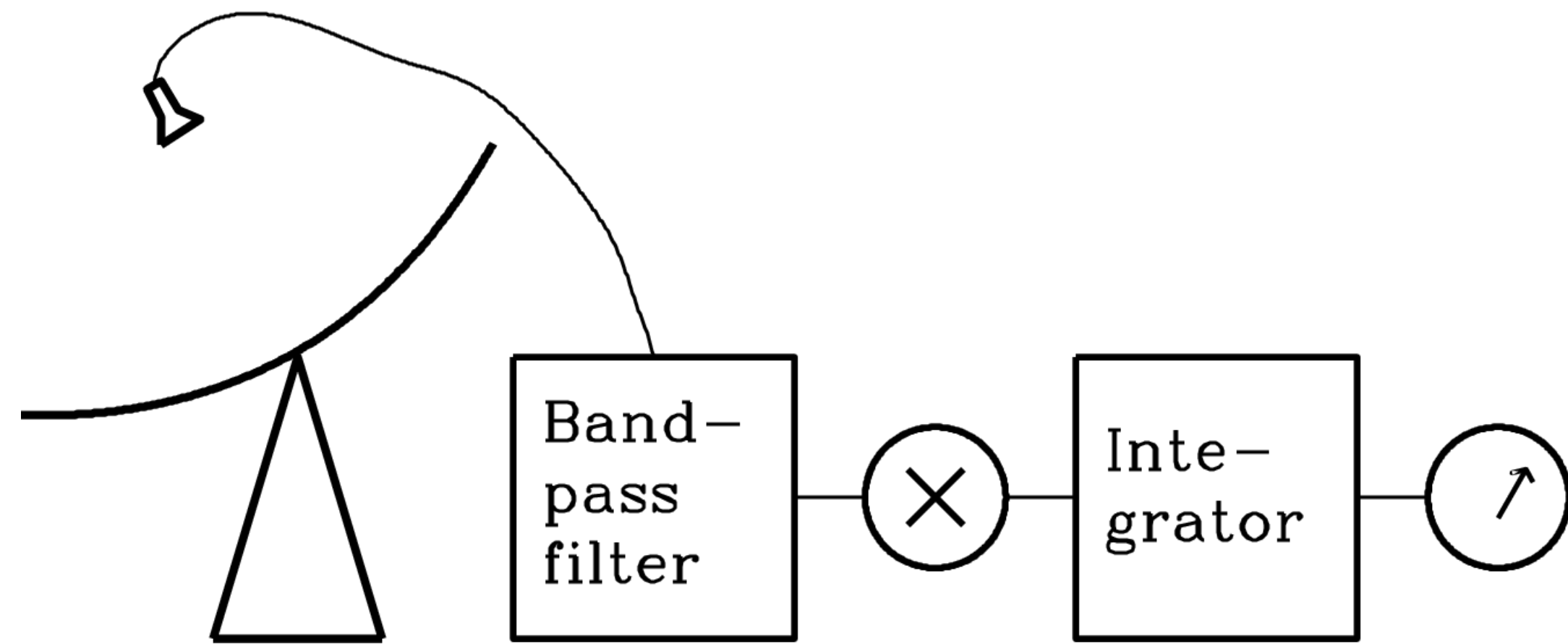
# Collecting radio signals



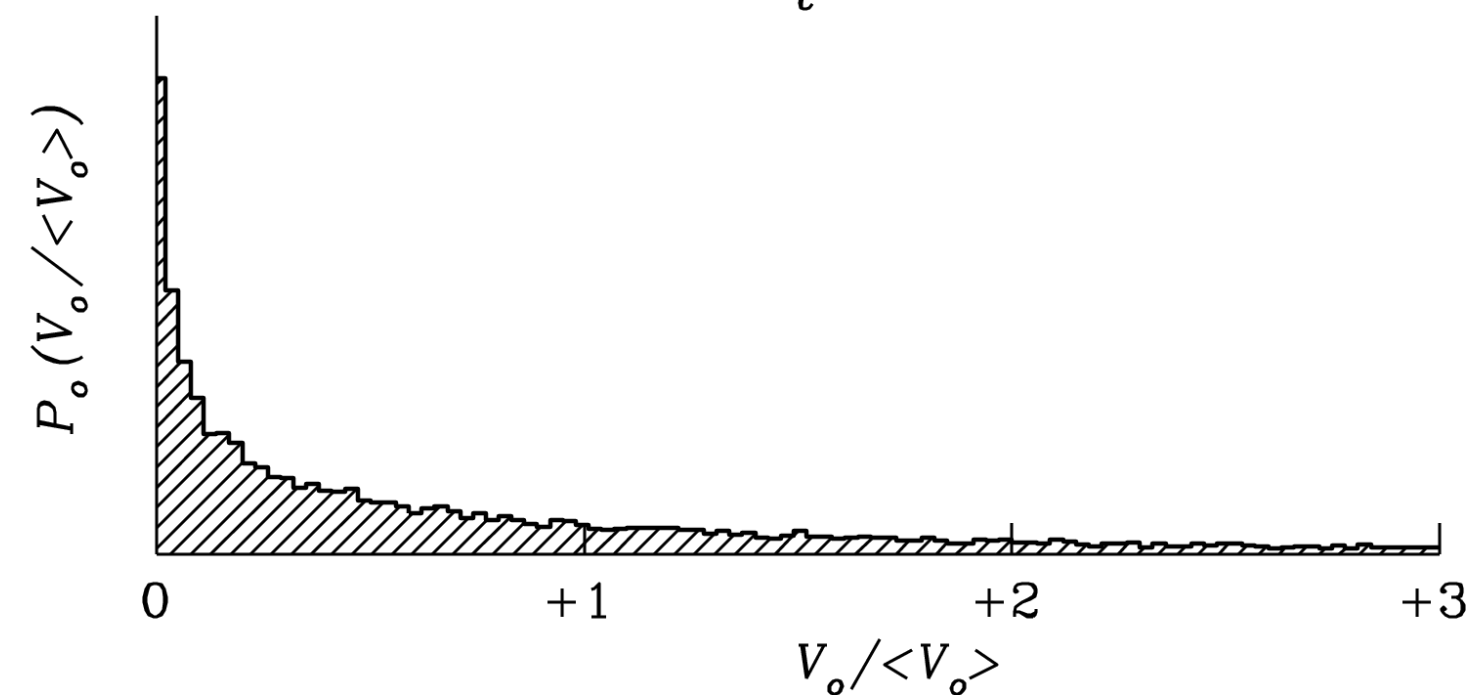
The output voltage  $V_o$  of a square-law detector is proportional to the square of the input voltage. It is always positive, so its mean (DC, or zero-frequency component) is positive and proportional to the input power. The high frequency ( $\nu \approx 2\nu_{RF}$ ) fluctuations add no information about the source and are filtered out in the next stage.

For a narrowband input voltage  $V_i \approx \cos(2\pi\nu_{RF}t)$  at frequency  $\nu_{RF}$ , the detector output voltage would be  $V_o \propto \cos^2(2\pi\nu_{RF}t) = [1 + \cos(4\pi\nu_{RF}t)]/2$ , a function whose mean value is proportional to the average power of the input signal.

# Collecting radio signals

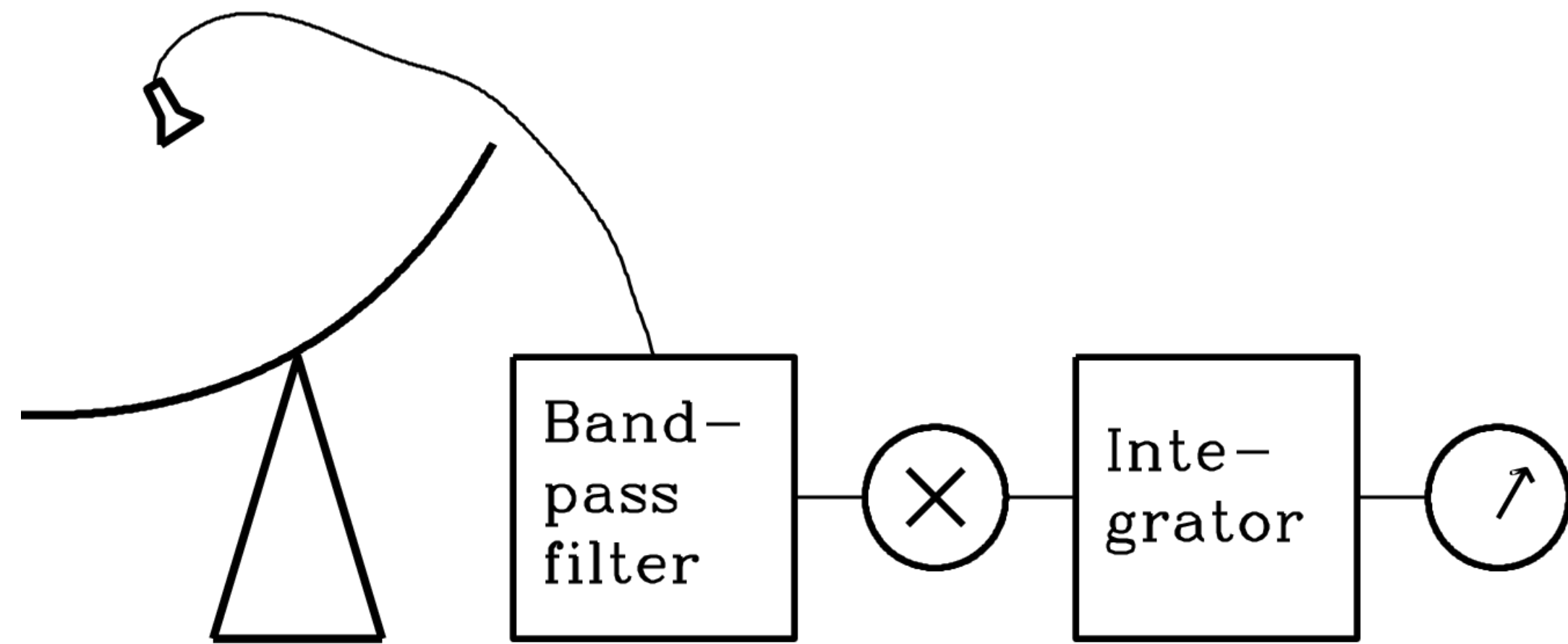


The upper plot shows the output voltage  $V_o$  of a square-law detector whose input is Gaussian noise.



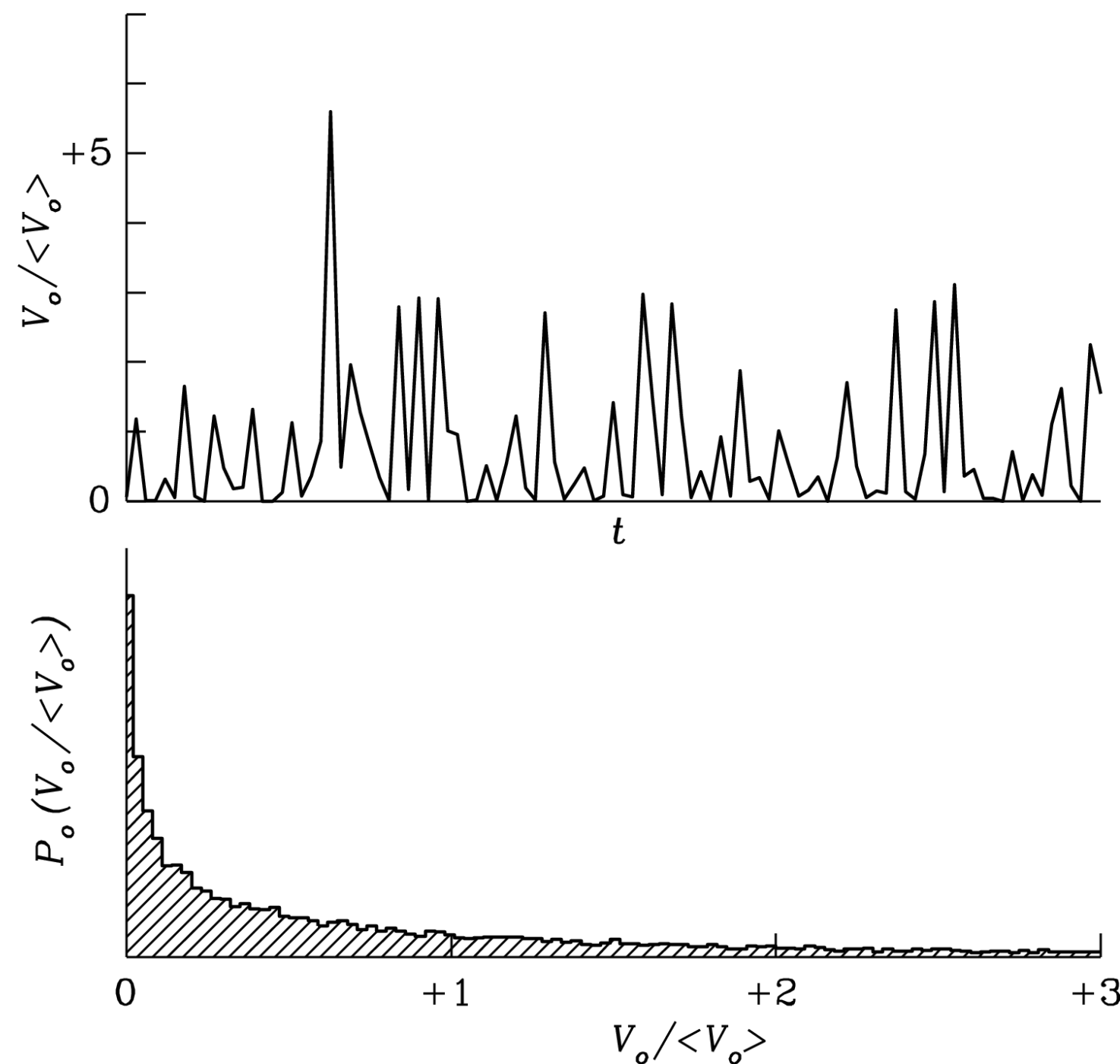
The output voltage histogram is peaked sharply near zero and has a long positive tail. The mean detected voltage  $\langle V_o \rangle$  equals the mean squared input voltage, and the rms of the detected voltage distribution is  $2^{1/2} \langle V_o \rangle$

# Collecting radio signals



The rapidly varying component at frequencies near  $2\nu_{\text{RF}}$  and its envelope vary on timescales that are normally much shorter than the timescales on which the average signal power  $\Delta T$  varies.

The unwanted rapid variations can be suppressed by taking the arithmetic mean of the detected envelope over some timescale  $\tau \gg (\Delta\nu)^{-1}$  by integrating or averaging the detector output.



The upper plot shows the output voltage  $V_o$  of a square-law detector whose input is Gaussian noise.

The output voltage histogram is peaked sharply near zero and has a long positive tail. The mean detected voltage  $\langle V_o \rangle$  equals the mean squared input voltage, and the rms of the detected voltage distribution is  $2^{1/2} \langle V_o \rangle$

# Collecting radio signals

Q: How can you detect  $T_A$  (signal) in the presence of  $T_{sys}$  (noise)?

A: The signal is correlated from one sample to the next but the noise is not

For bandwidth  $\Delta\nu$ , samples taken less than  $\Delta\tau = 1/\Delta\nu$  are not independent  
(Nyquist sampling theorem!)

Time  $\tau$  contains  $N = \tau/\Delta\tau = \tau \Delta\nu$  independent samples

For Gaussian noise, total error for  $N$  samples is  $1/\sqrt{N}$  that of single sample

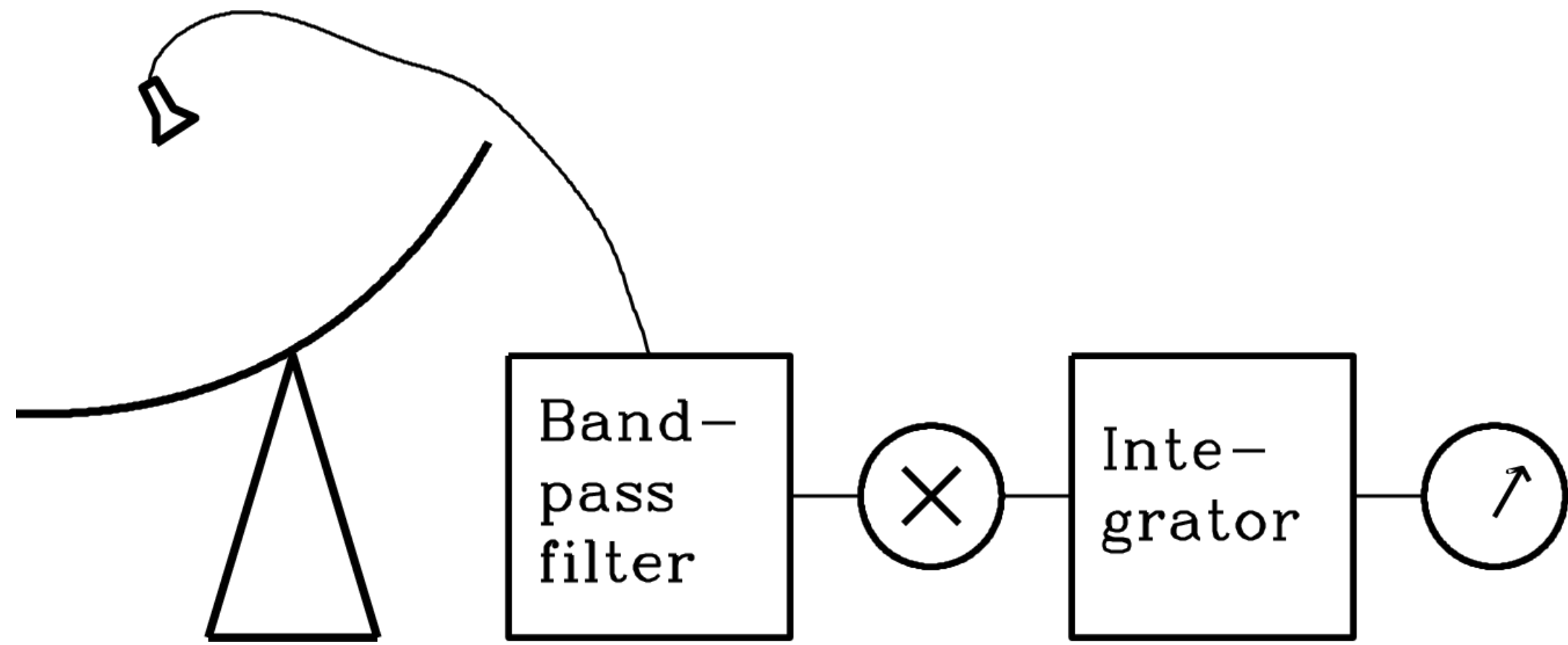
$$\therefore \frac{\Delta T_A}{T_{sys}} = \frac{1}{\sqrt{\tau \Delta\nu}}$$

Radiometer equation

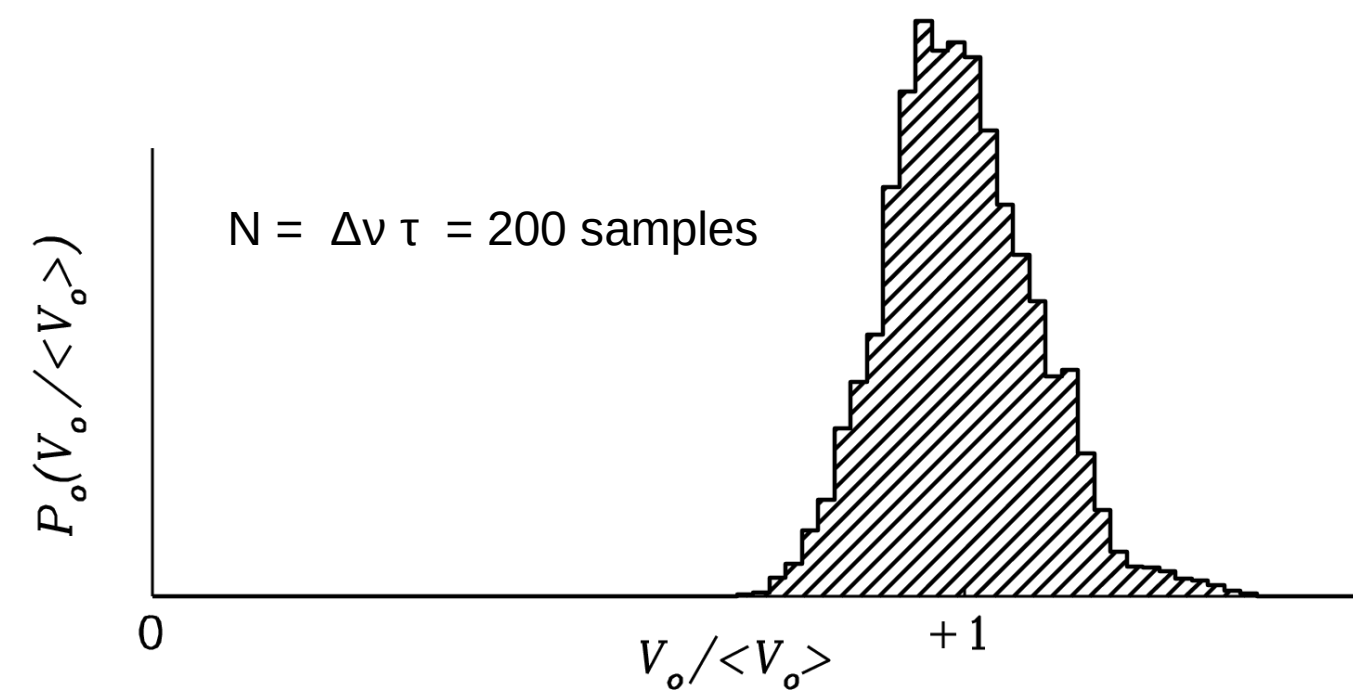
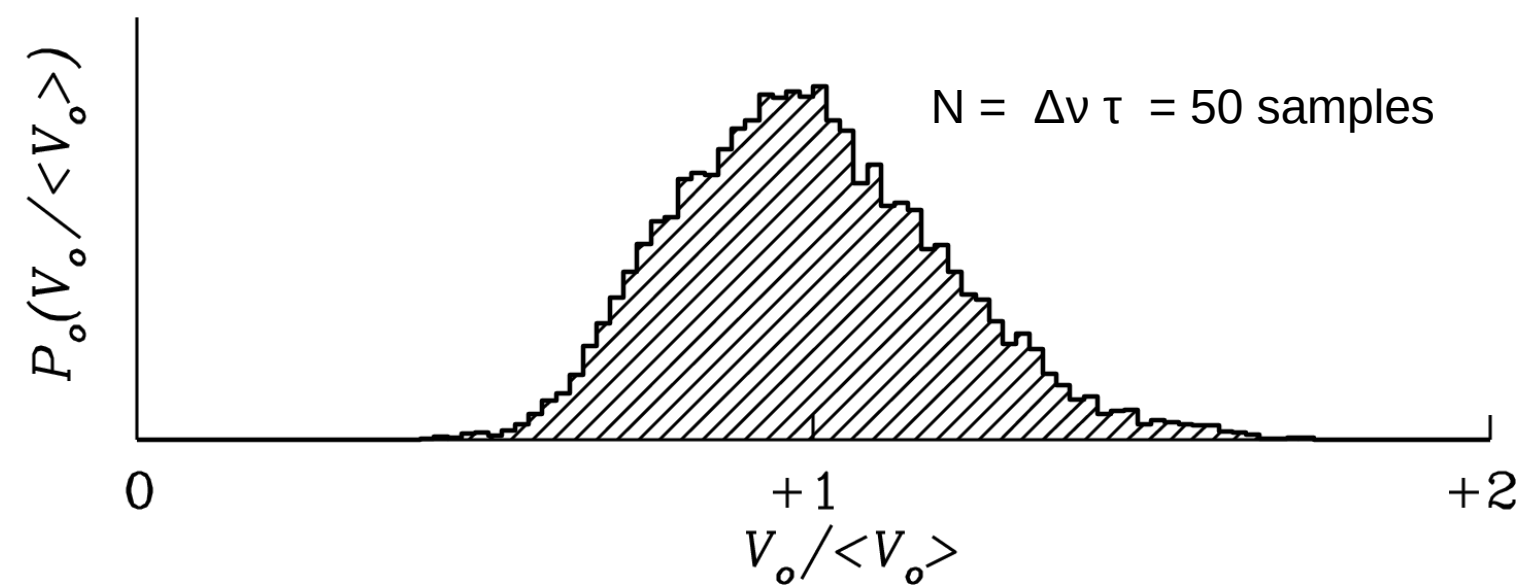
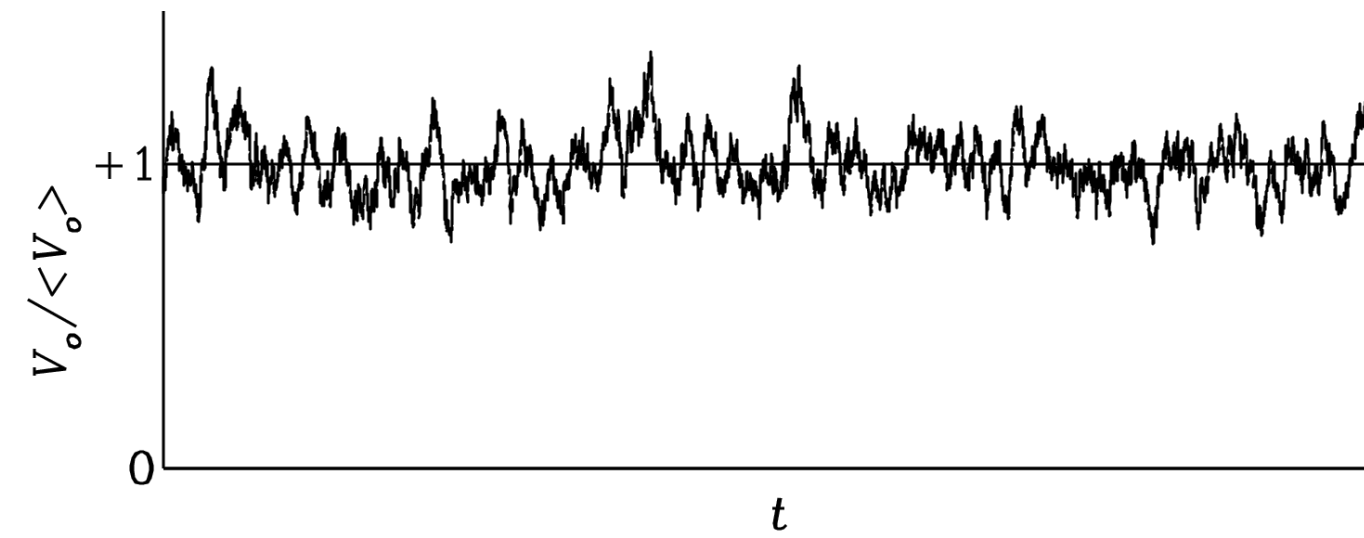
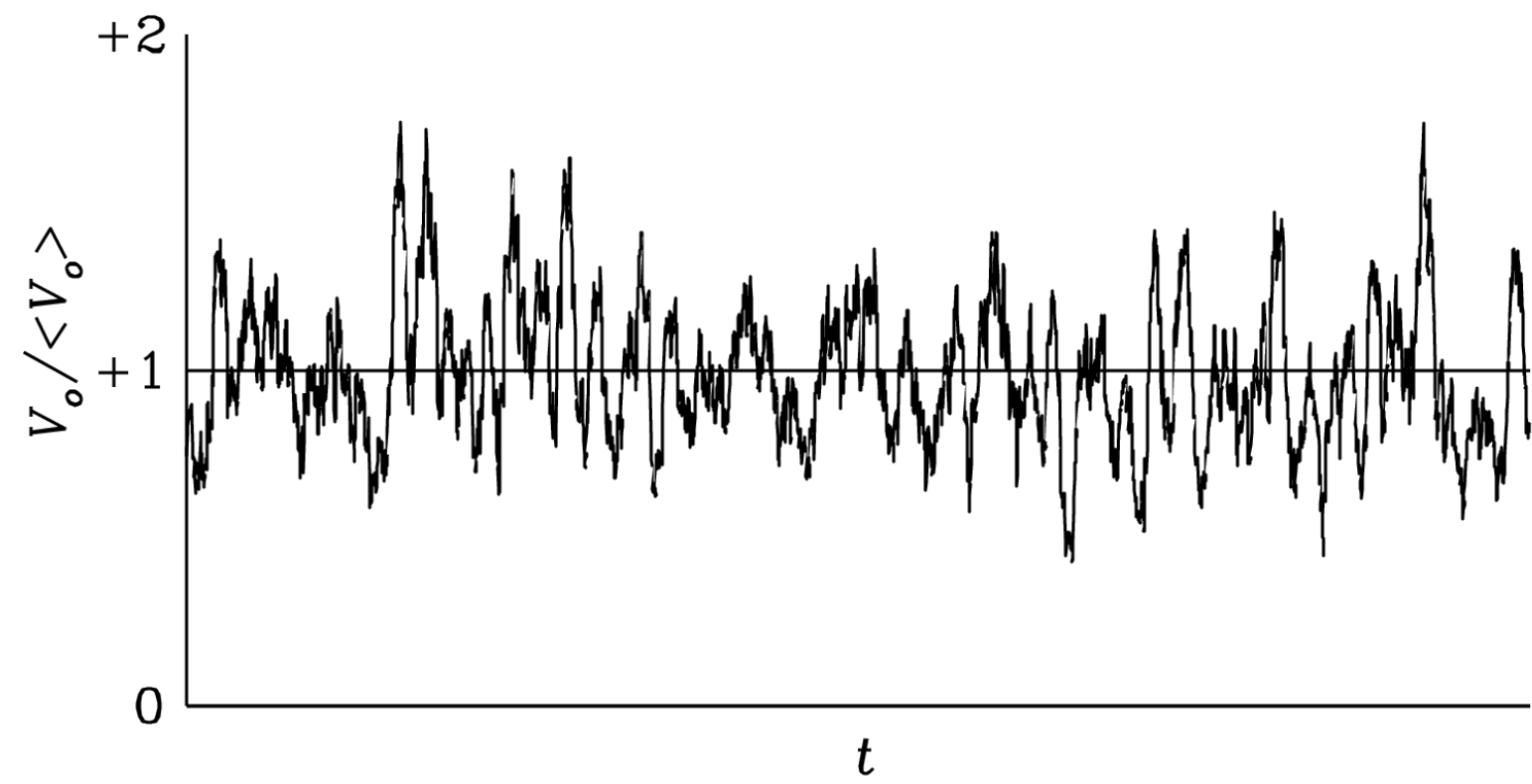
$$SNR = \frac{T_A}{\Delta T_A} = \frac{T_A}{T_{sys}} \sqrt{\tau \Delta\nu}$$



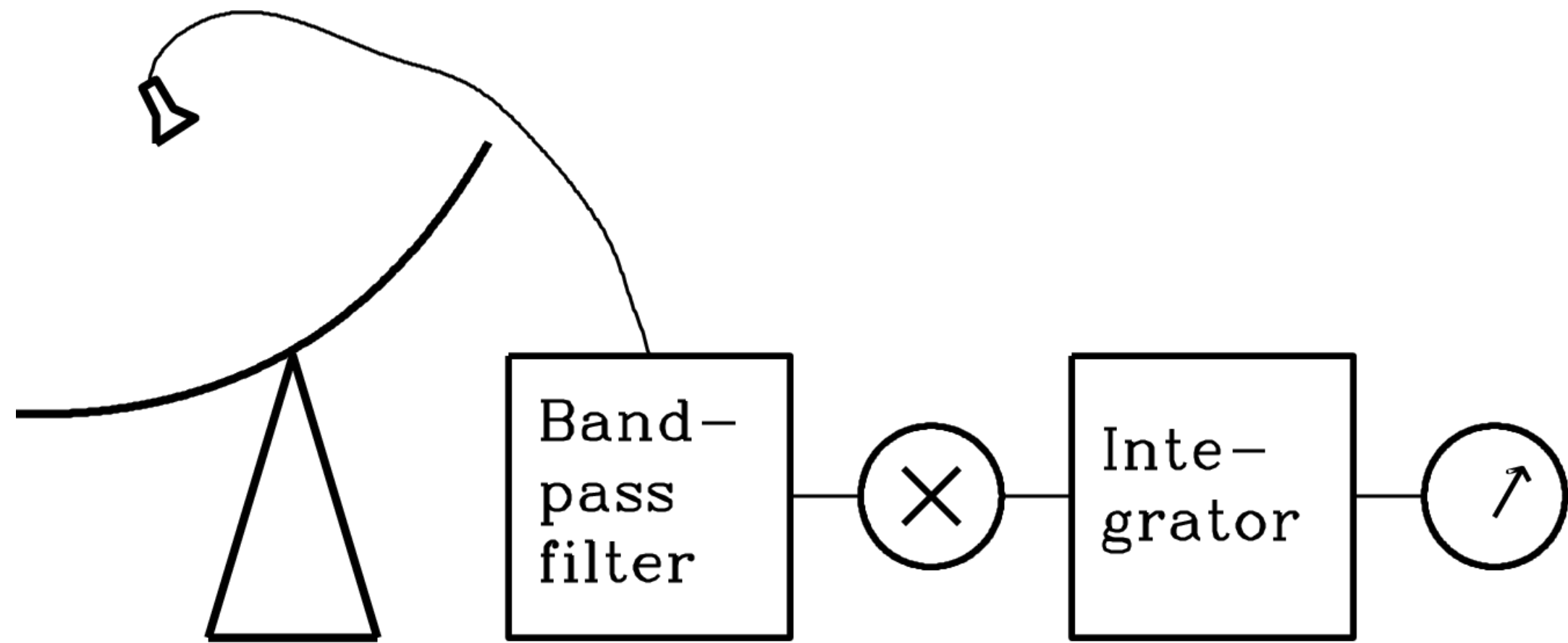
# Collecting radio signals



$$\sigma_T \approx \frac{T_s}{\sqrt{\Delta\nu\tau}}$$

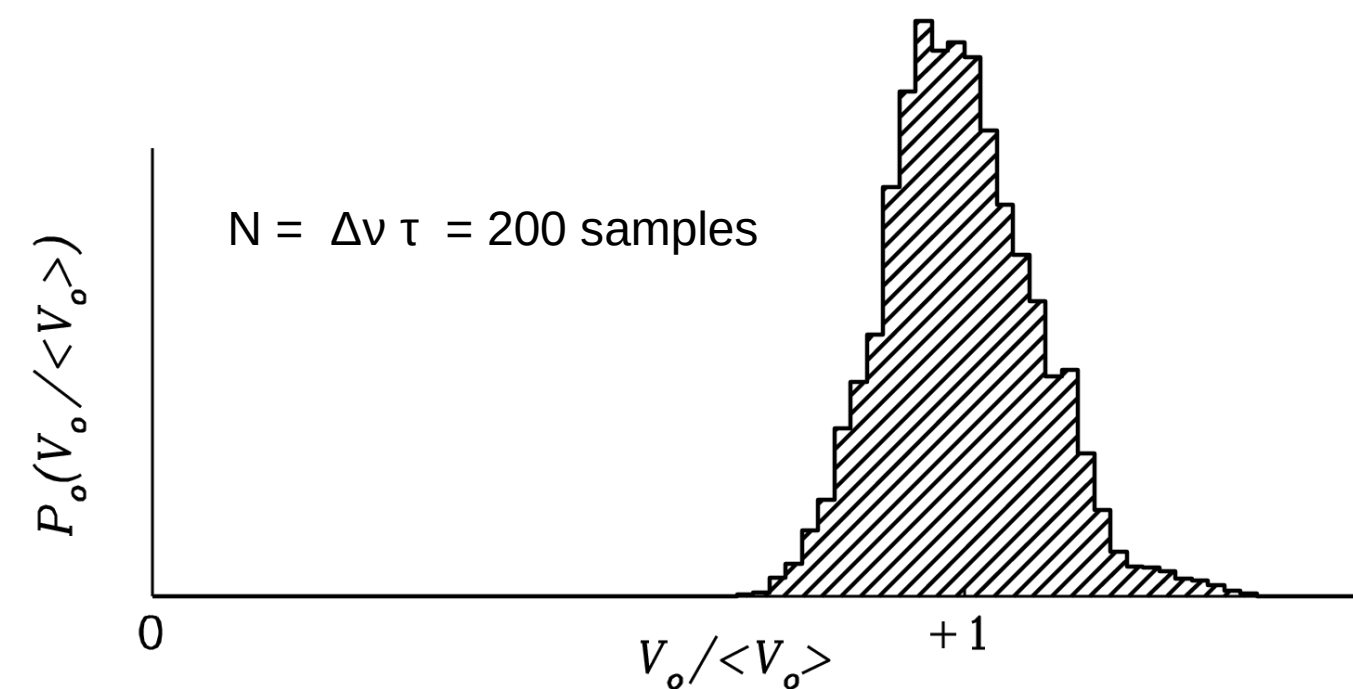
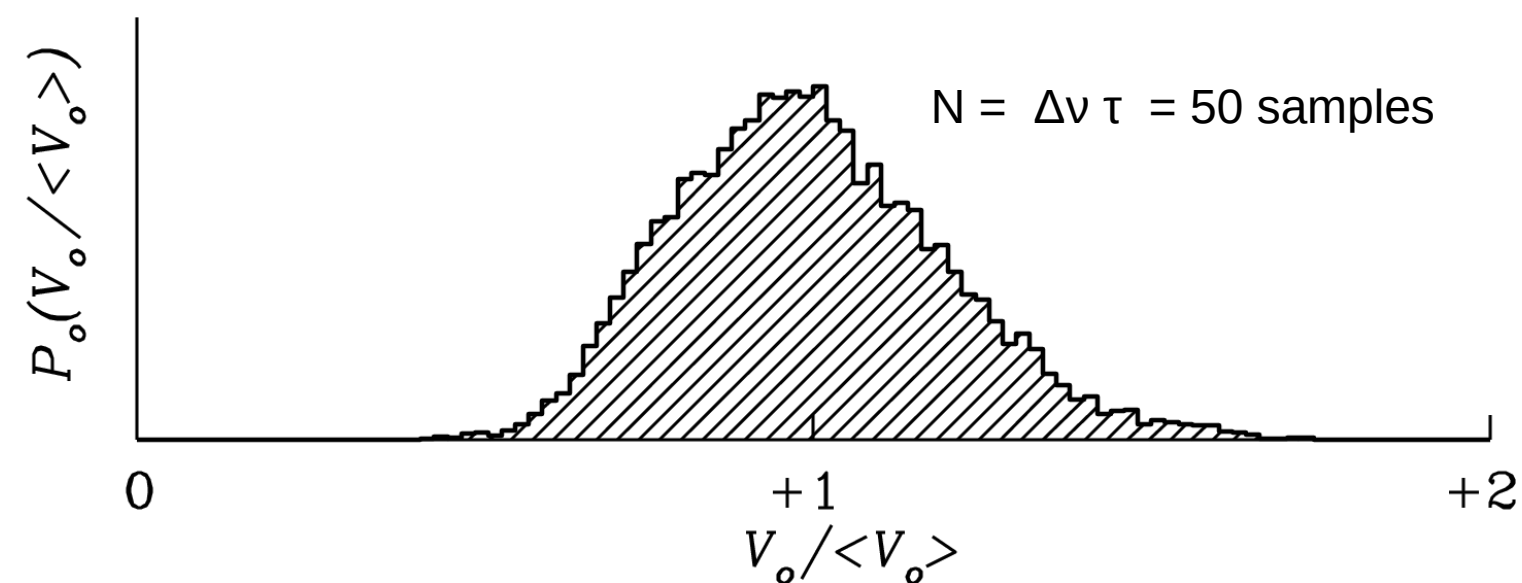
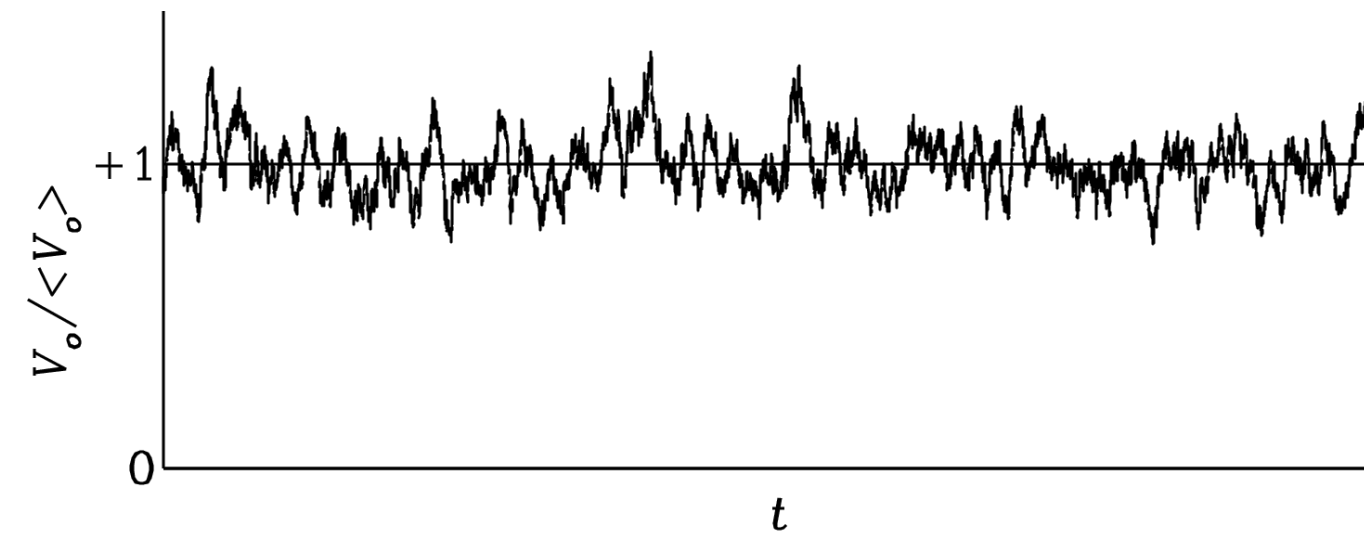
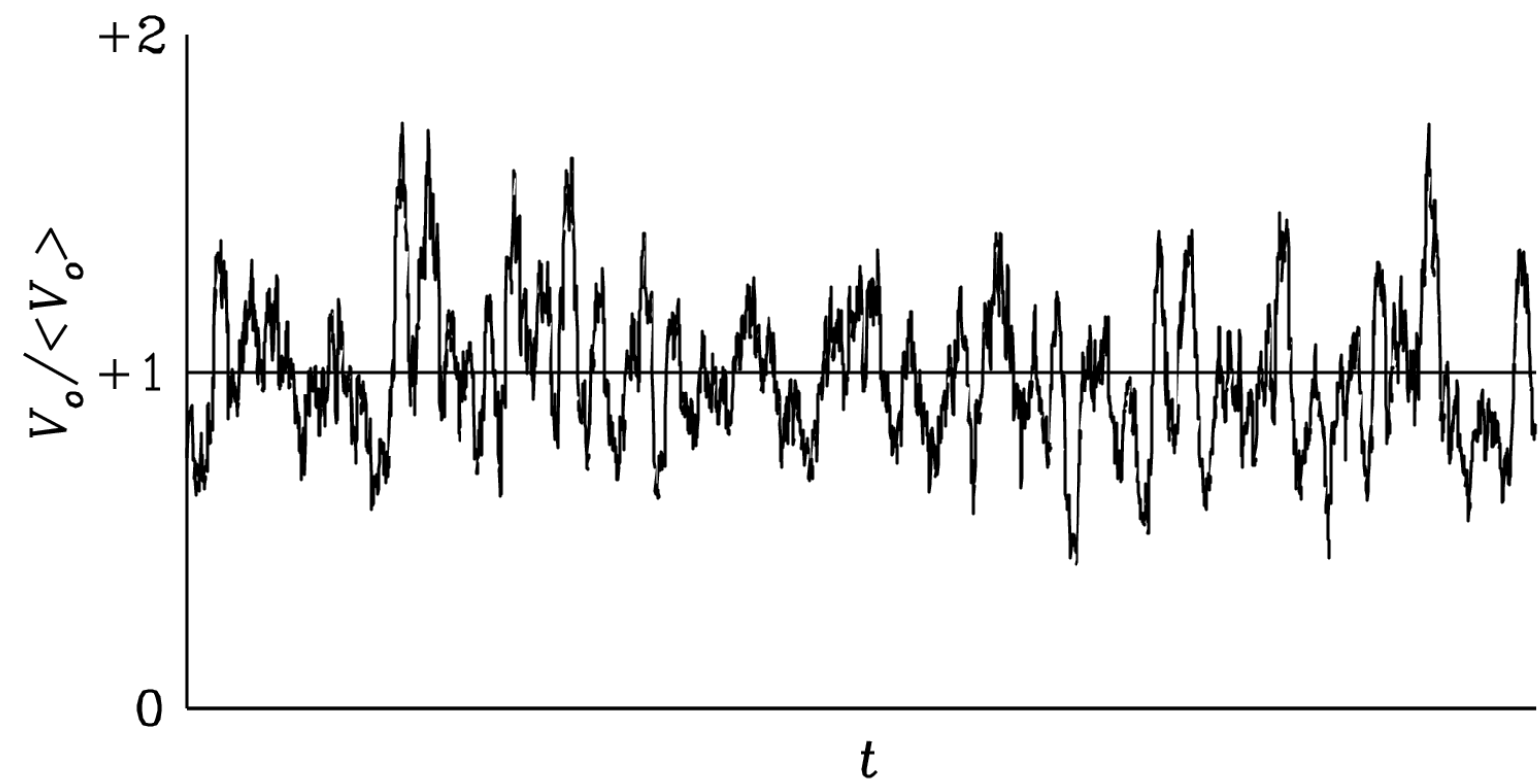


# Collecting radio signals



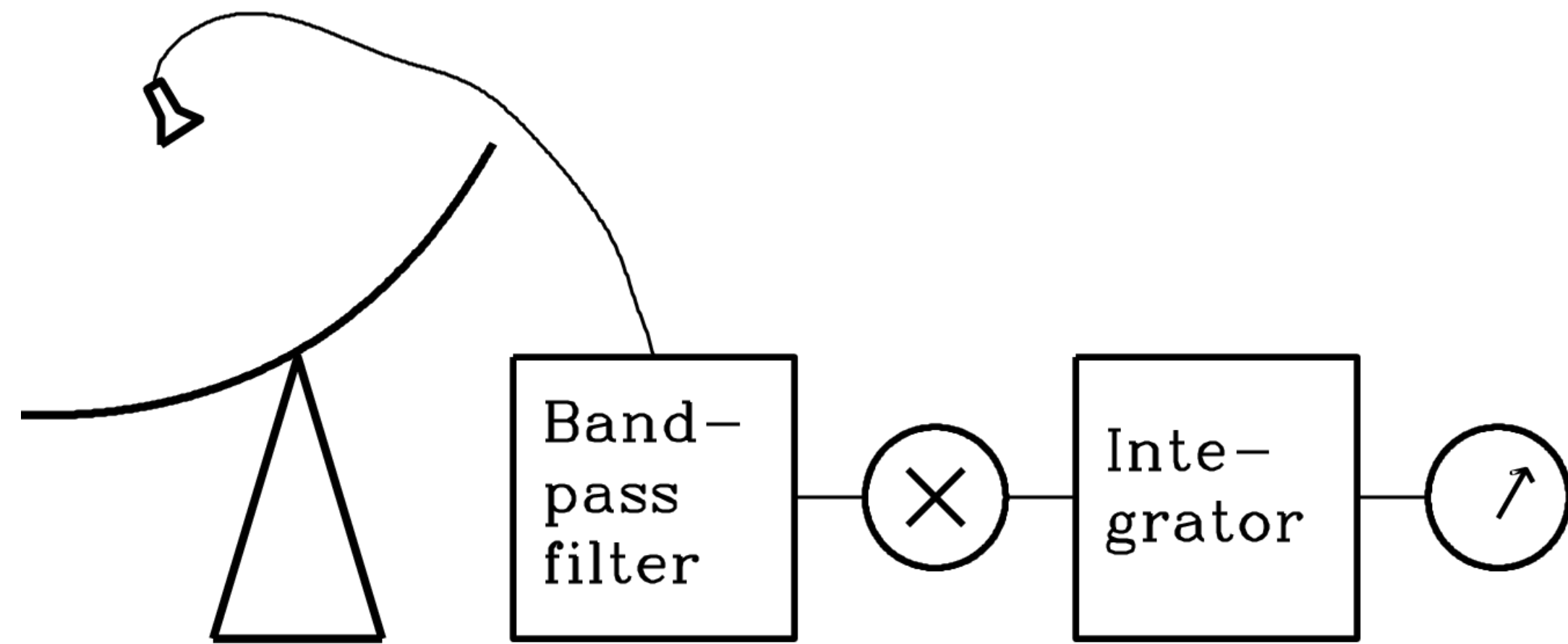
$$\sigma_T \approx \frac{T_s}{\sqrt{\Delta\nu\tau}}$$

$$\Delta\nu\tau > 10^8$$



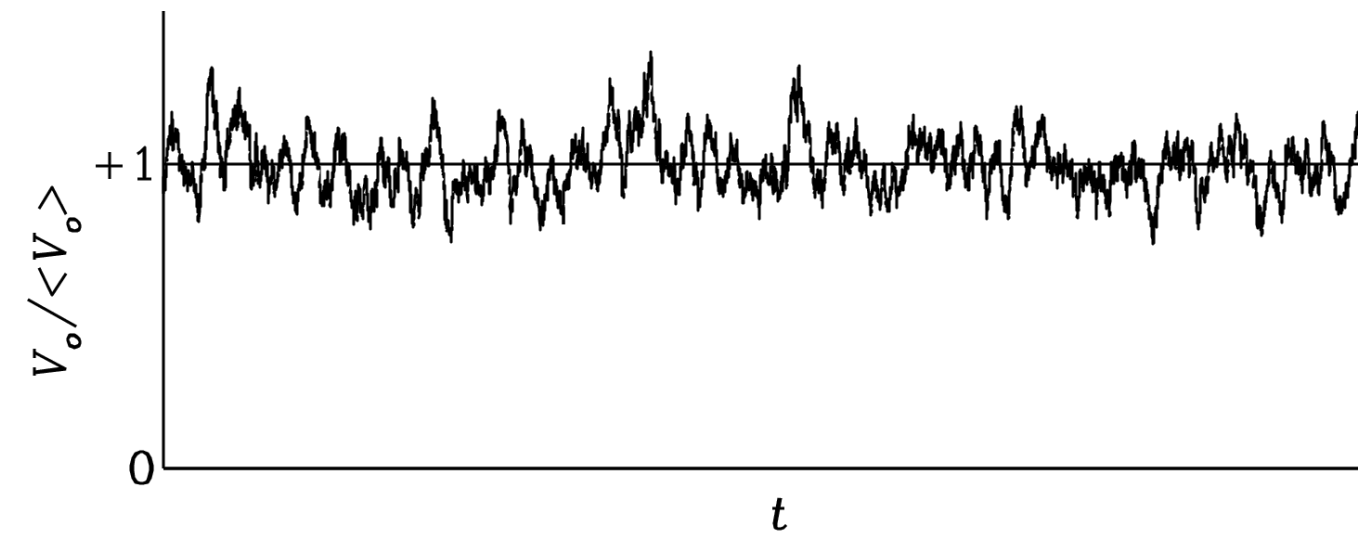
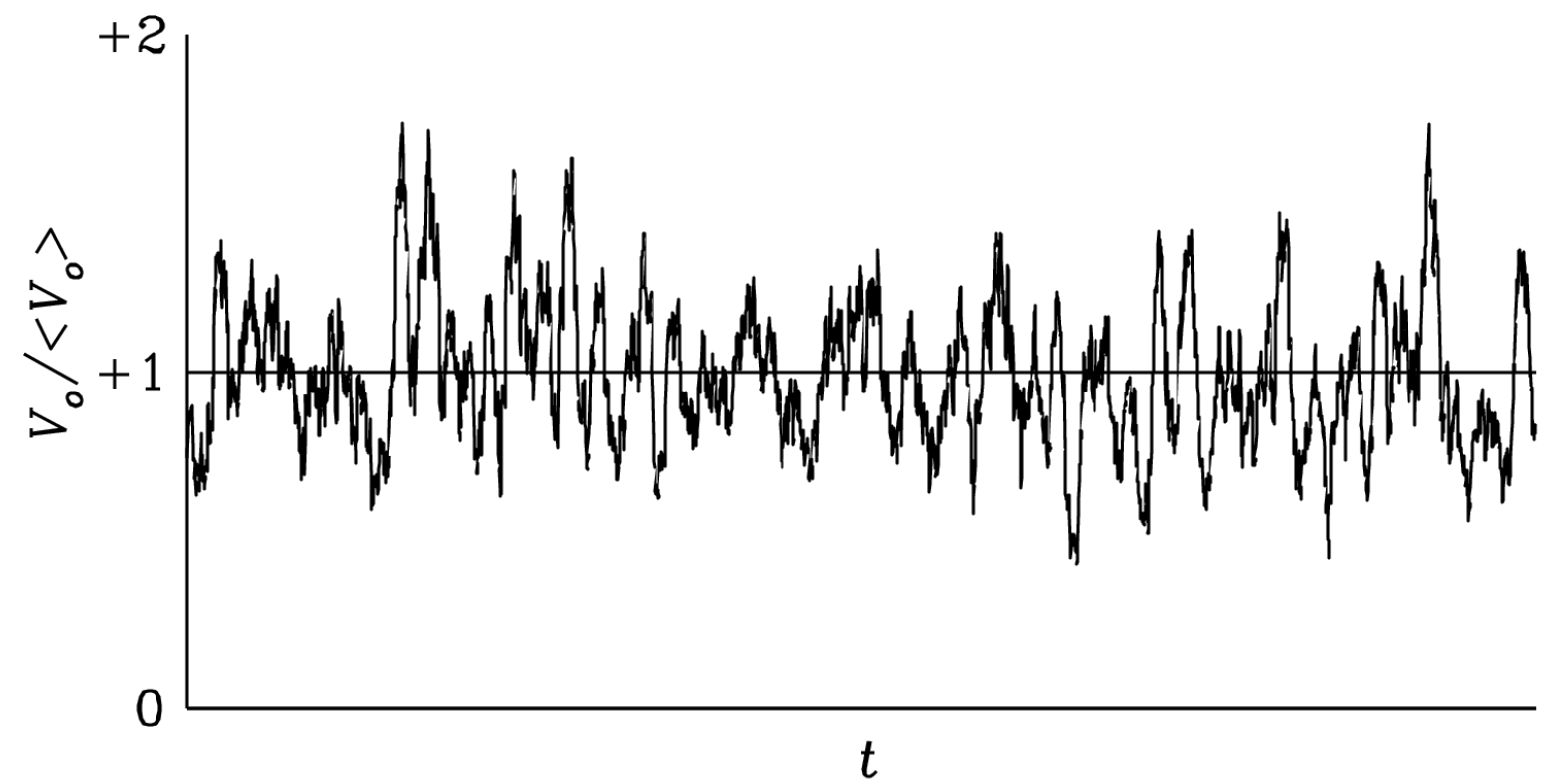
$$\Delta T \sim 5 \times 10^{-4} T_s$$

# Collecting radio signals

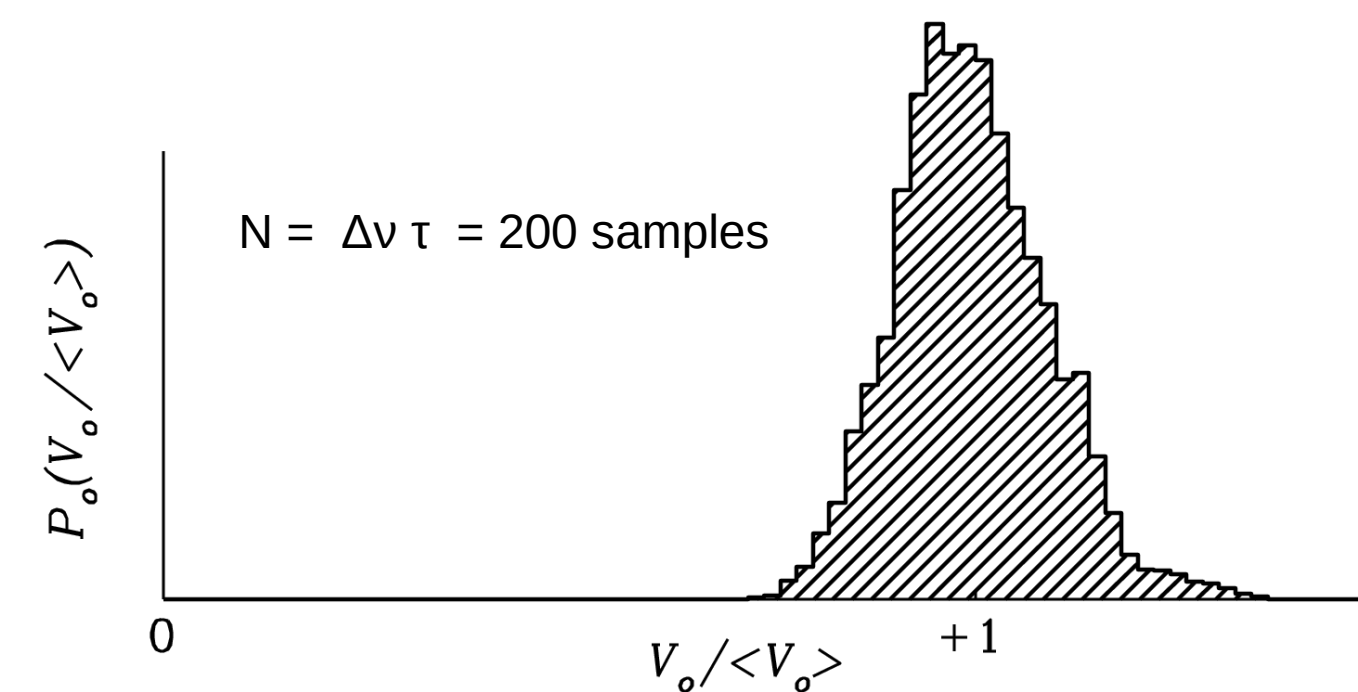
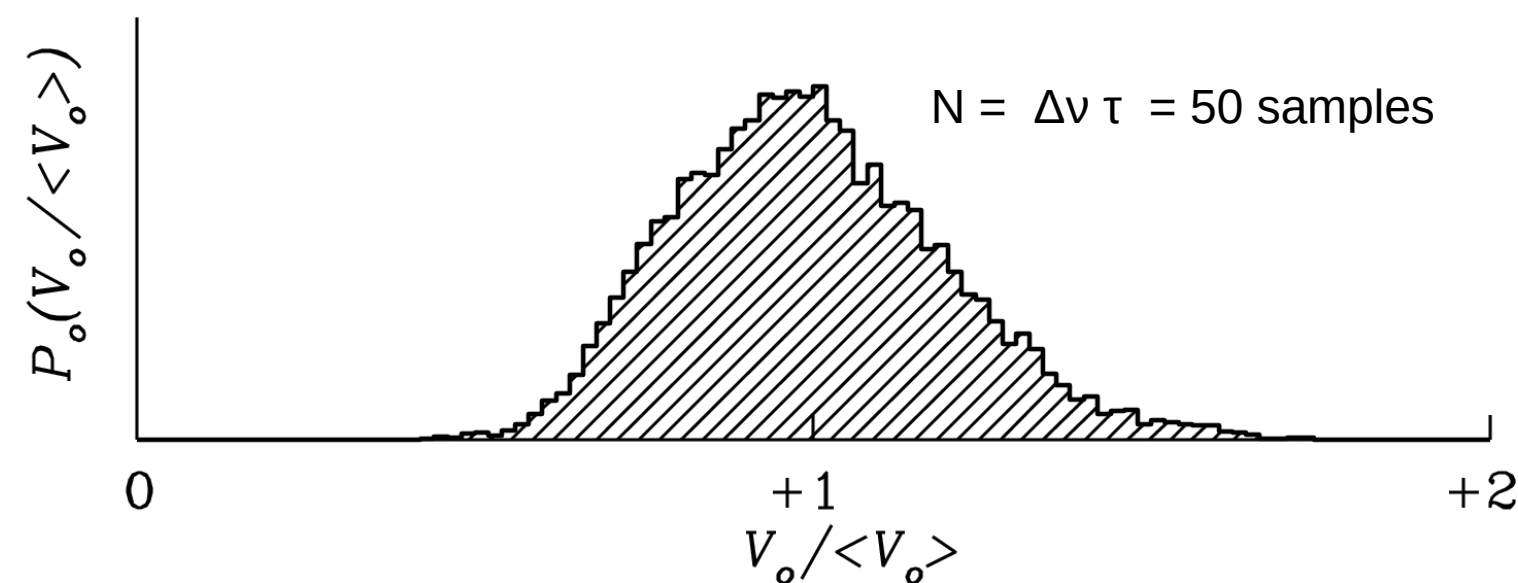


$$\sigma_T \approx \frac{T_s}{\sqrt{\Delta\nu\tau}}$$

$$\Delta\nu\tau > 10^8$$



$$\Delta T \sim 5 \times 10^{-4} T_s$$



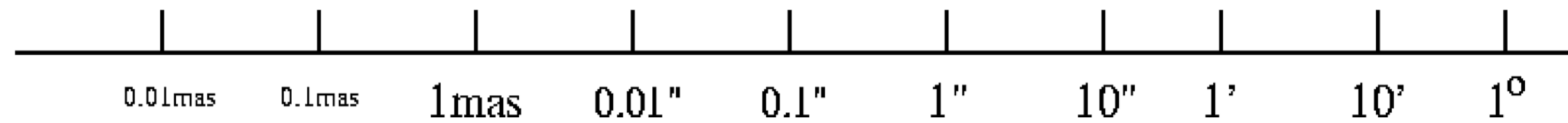
The weakest detectable signals  $T_A (= \Delta T)$  only have to be a few times the output rms  $\sigma_T$  given by the radiometer equation and not several times the total system noise  $T_s$

# Interferometry for the faint of heart

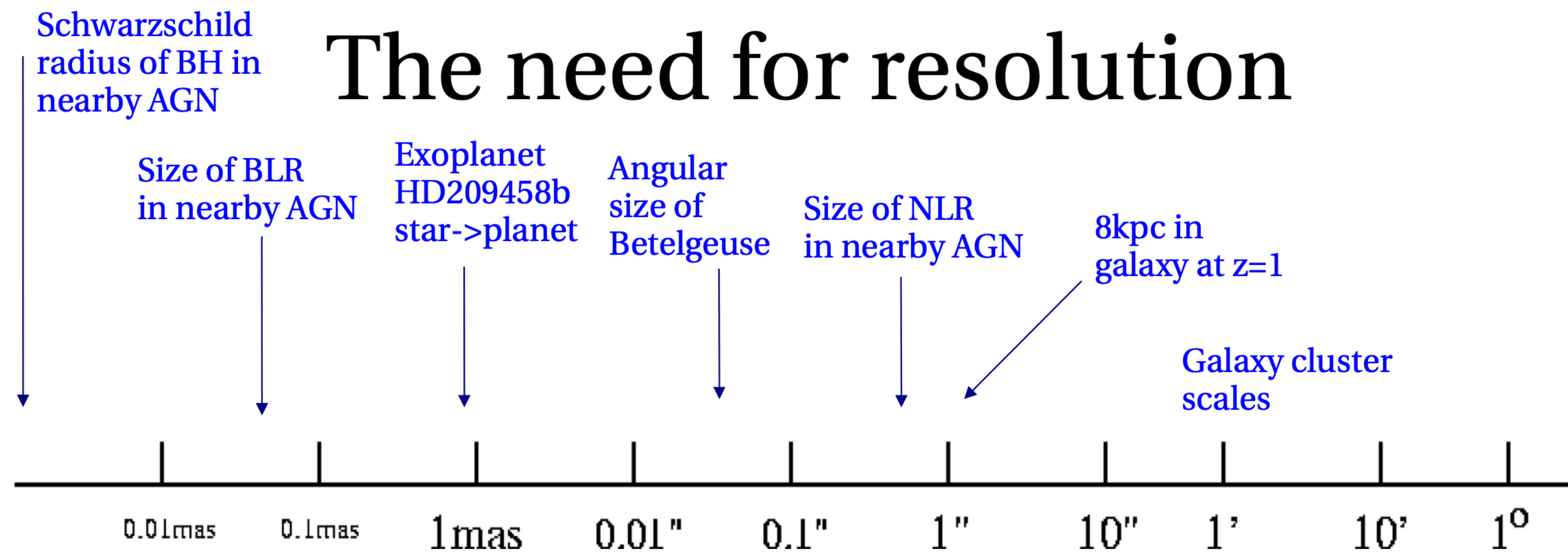
- The simple device just described defines a ‘total power radio telescope’.
- Conceptually simple – power in, power out.
- But the angular resolution of a single antenna is limited by diffraction to:  $\theta \sim \lambda/D$  (radians)
- In ‘practical’ units:  $\theta_{arcsec} \approx 2\lambda_{cm}/D_{km}$
- For arcsecond resolution, we need km-scale antennas, which are obviously impractical.
- We seek a method to ‘synthesize’ a large aperture by combining signals collected by separated small apertures.

# Interferometry for the faint of heart

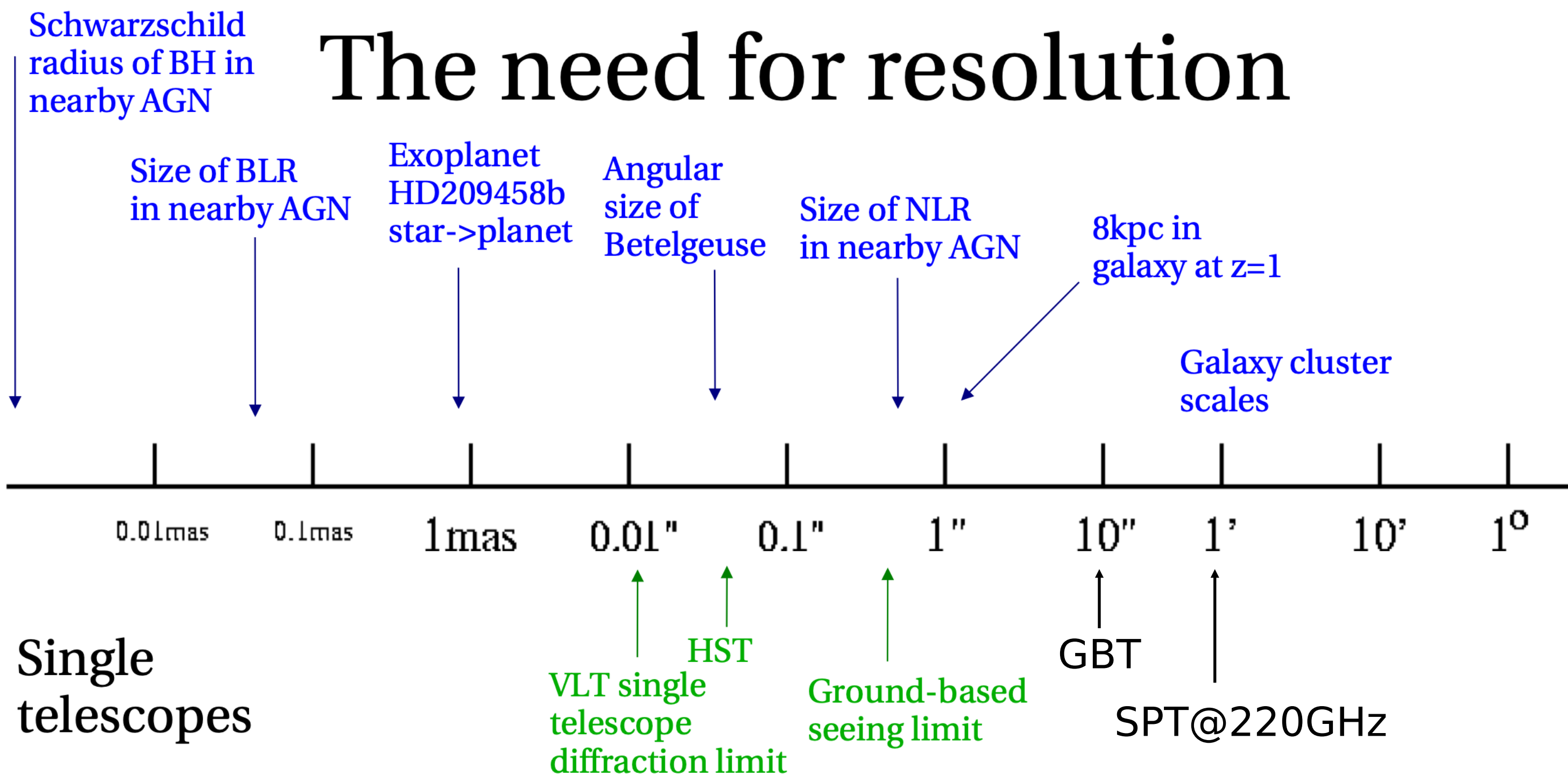
The need for resolution



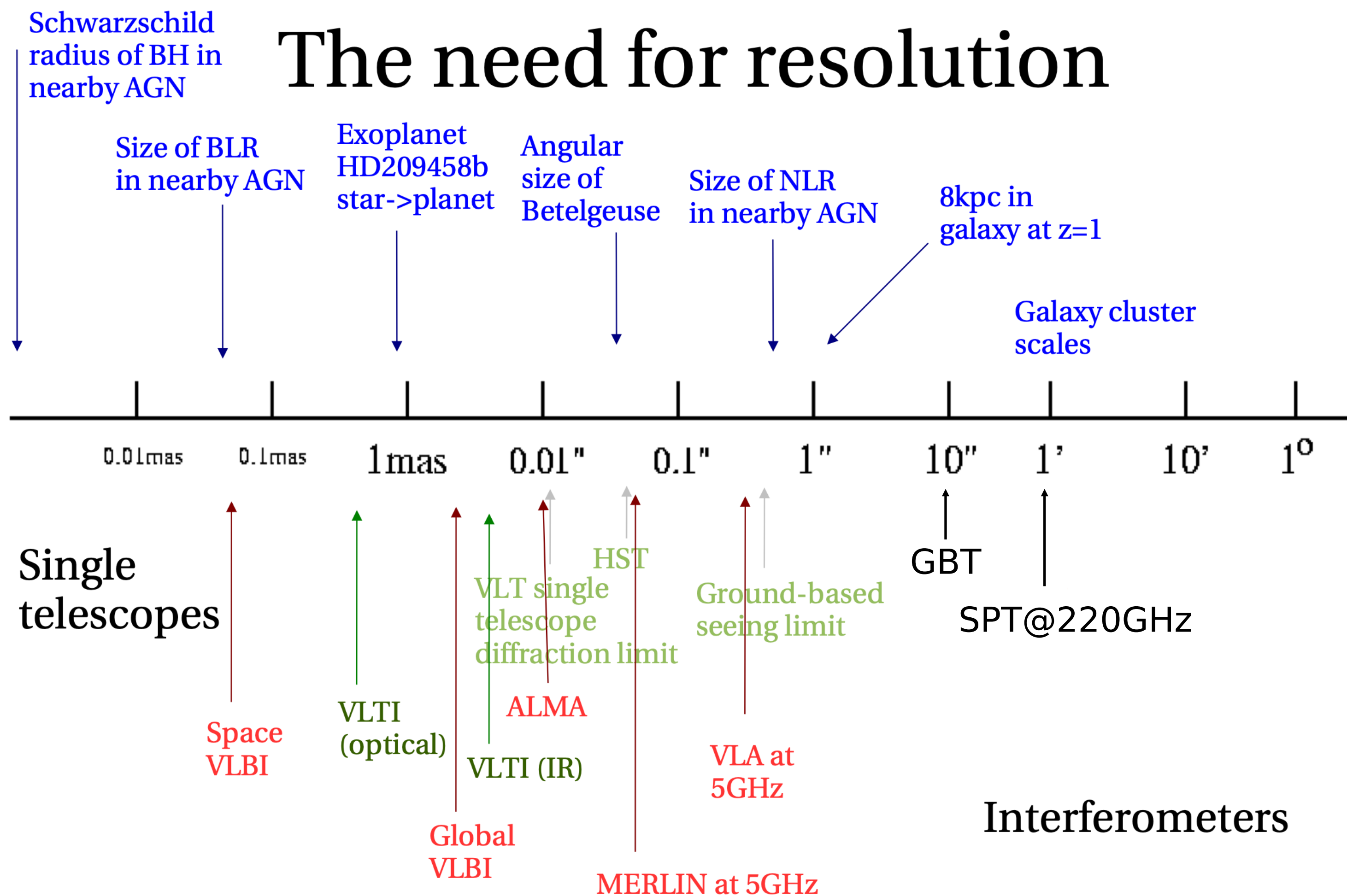
# Interferometry for the faint of heart



# Interferometry for the faint of heart



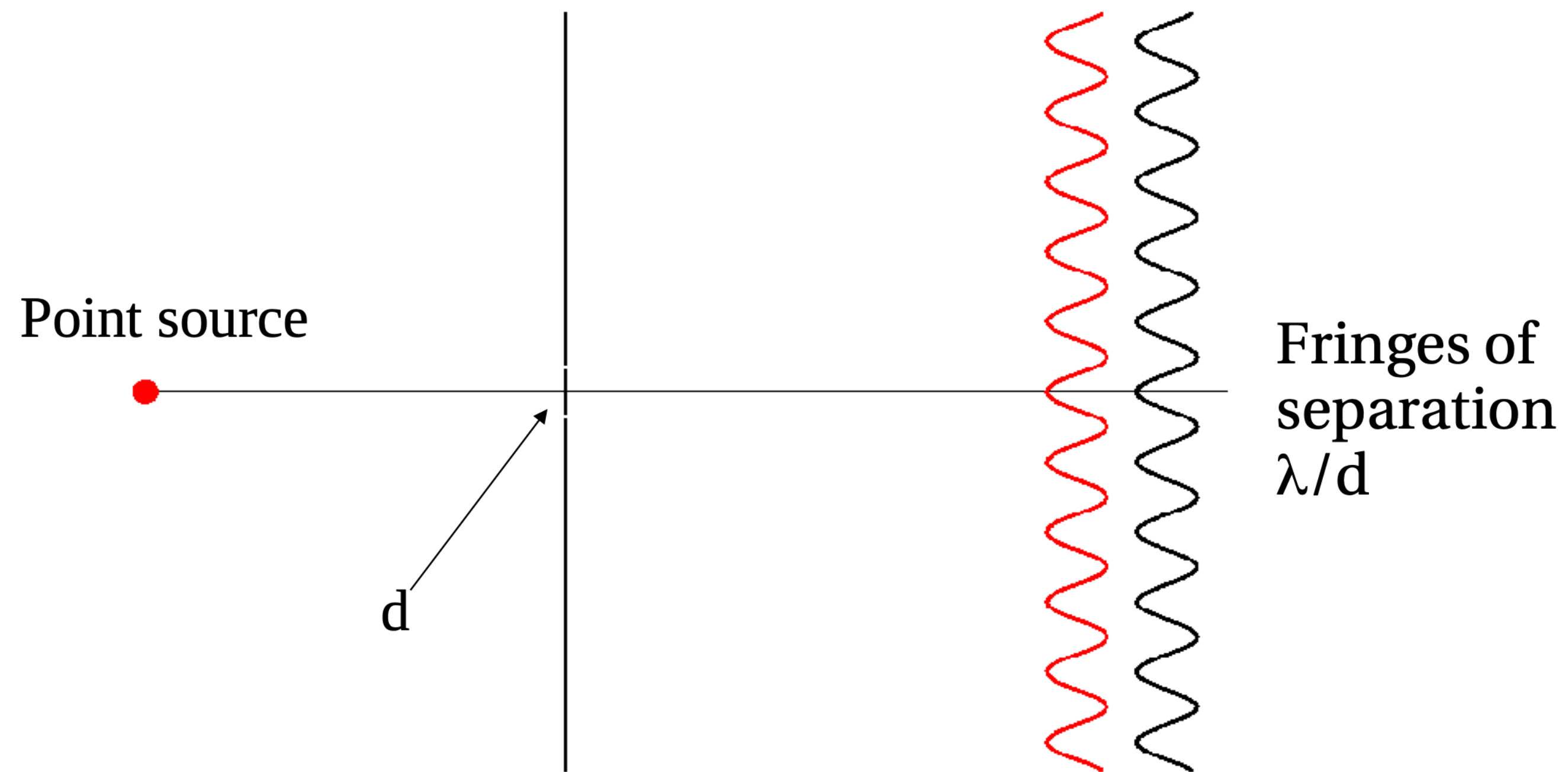
# Interferometry for the faint of heart





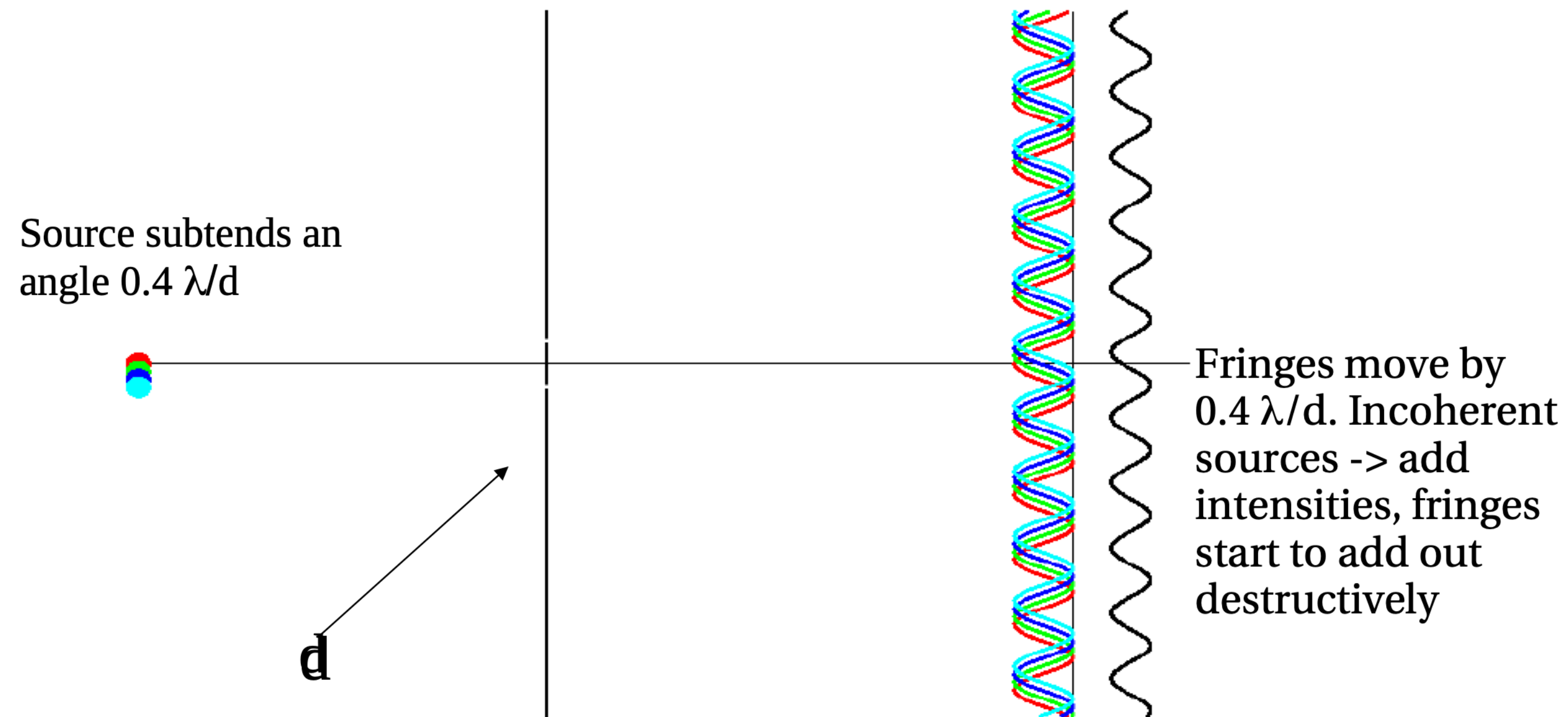
# Interferometry for the faint of heart

## Young's slits revisited



# Interferometry for the faint of heart

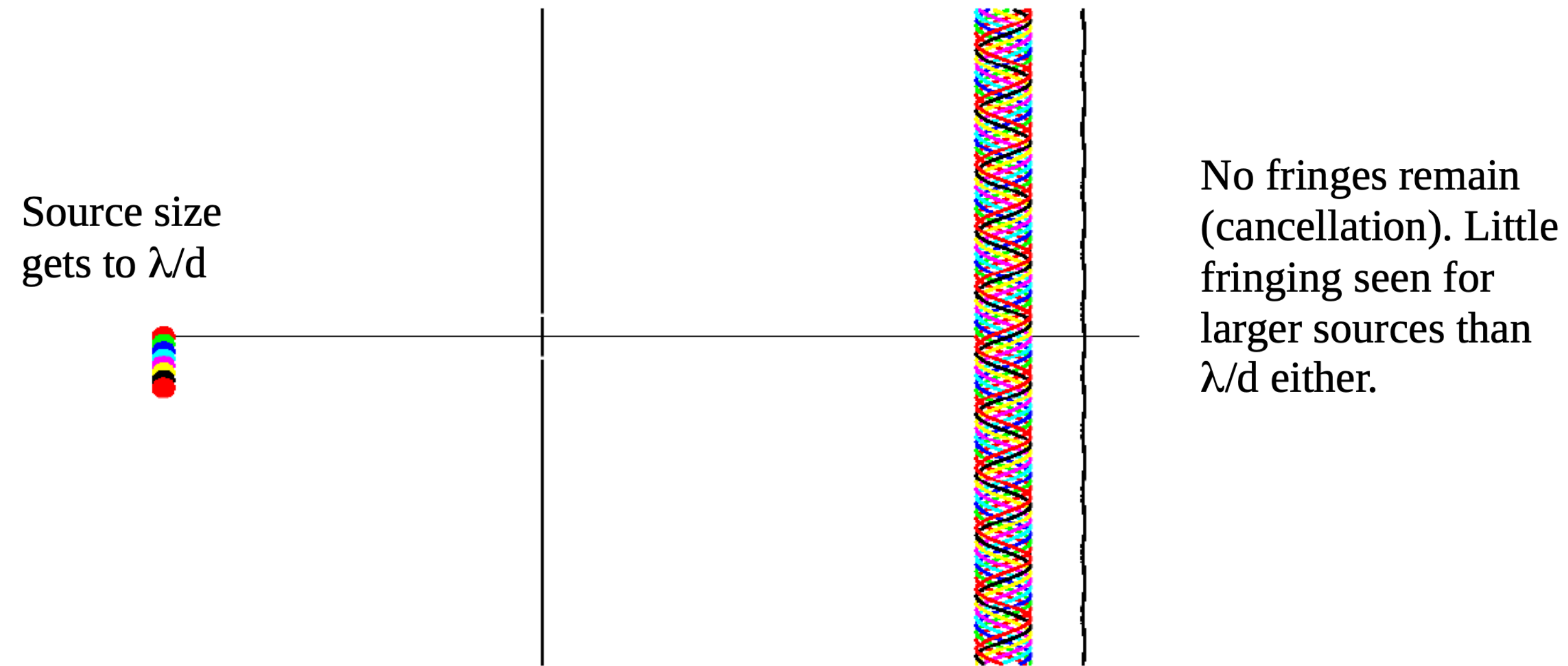
## Larger source



Define |fringe visibility| as  $(I_{\max} - I_{\min}) / (I_{\max} + I_{\min})$

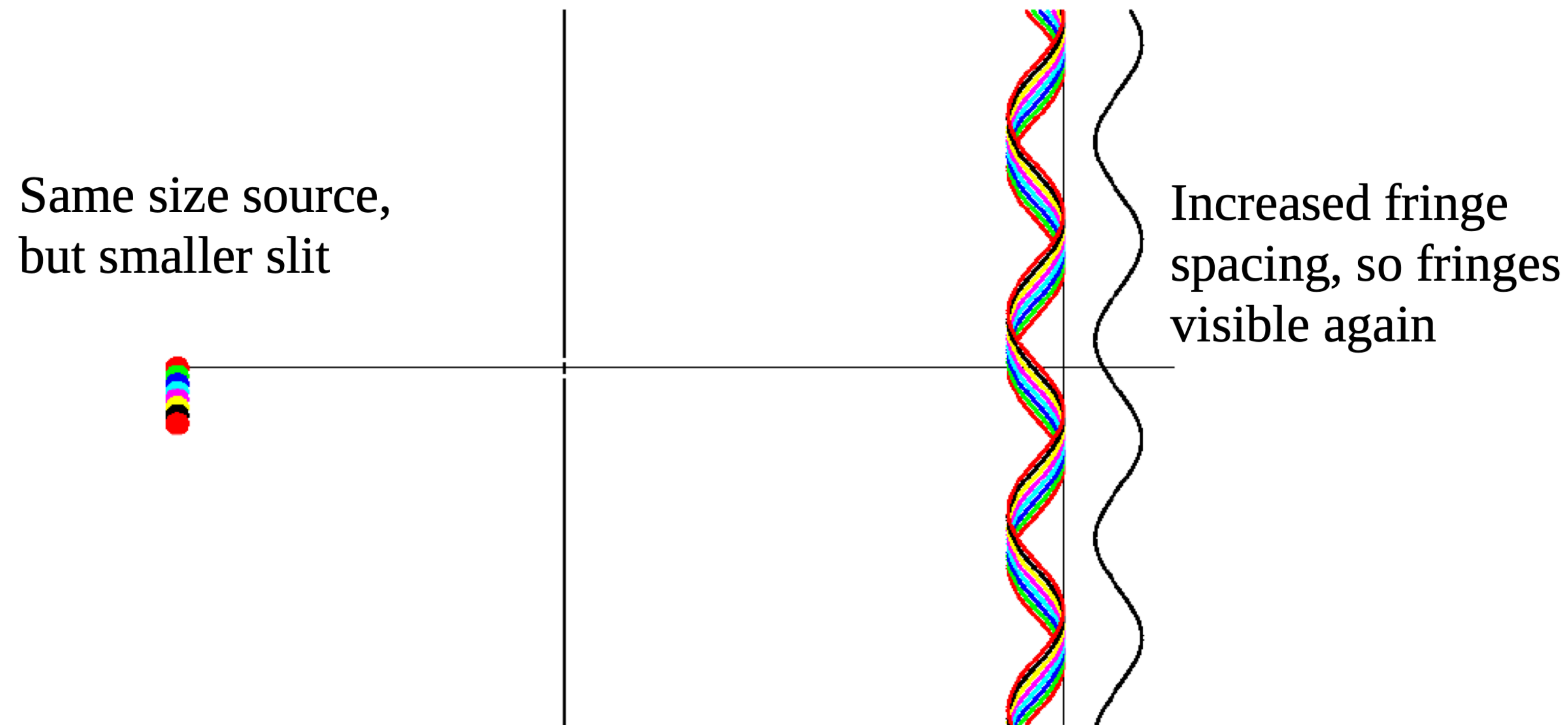
# Interferometry for the faint of heart

## Still larger source



# Interferometry for the faint of heart

## Effect of slit size



# Interferometry for the faint of heart

## Young's slits: summary

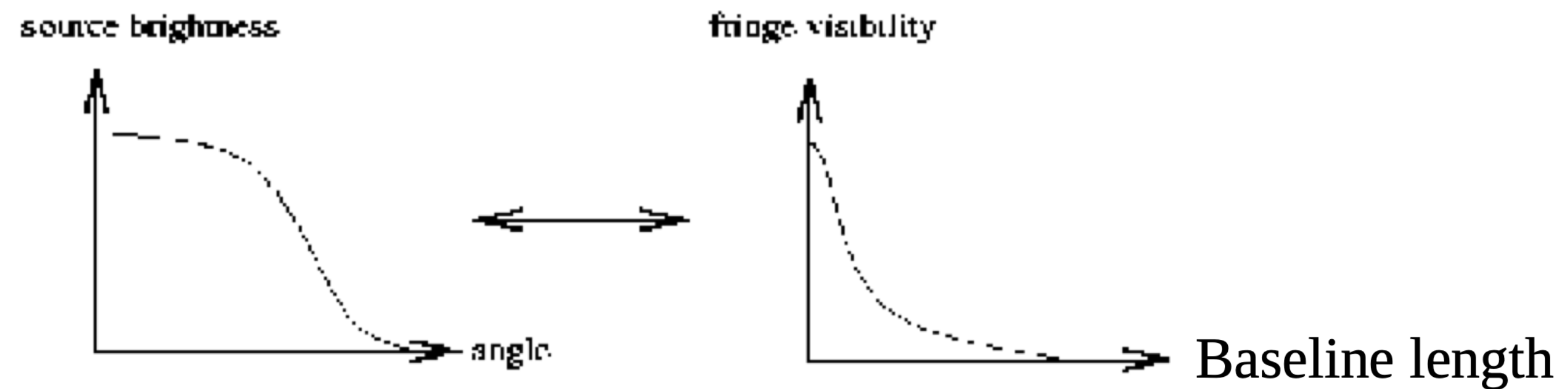
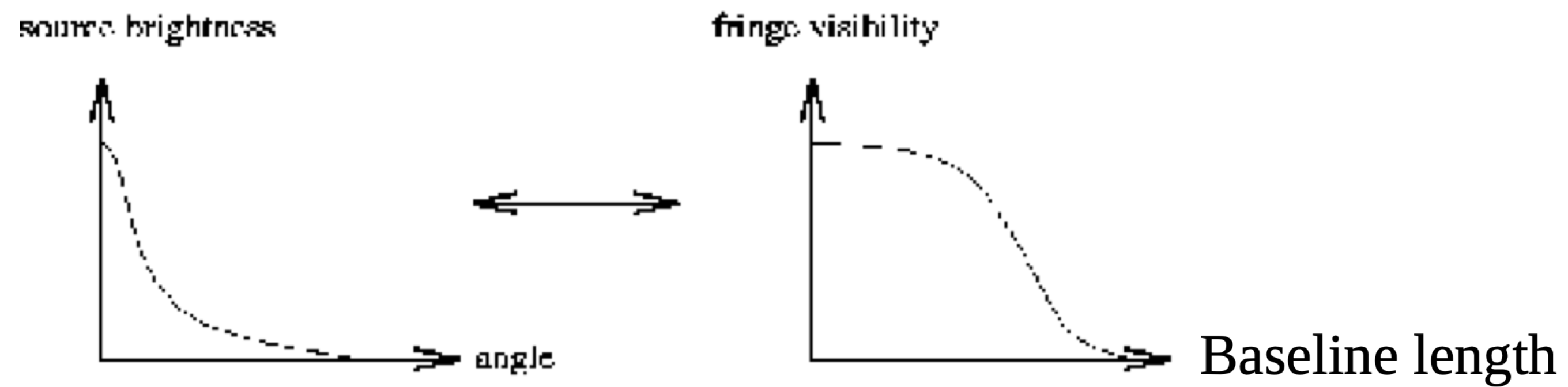
### Visibility of interference fringes

- Decreases with increasing source size
- Goes to zero when source size goes to  $\lambda/d$
- For given source size, increases for decreasing separation
- For given source size and separation, increases with  $\lambda$



# Interferometry for the faint of heart

## Summary in pictures



# Interferometry for the faint of heart

## It's a Fourier transform!

The fringe visibility of an interferometer gives information about the Fourier transform of the sky brightness distribution.

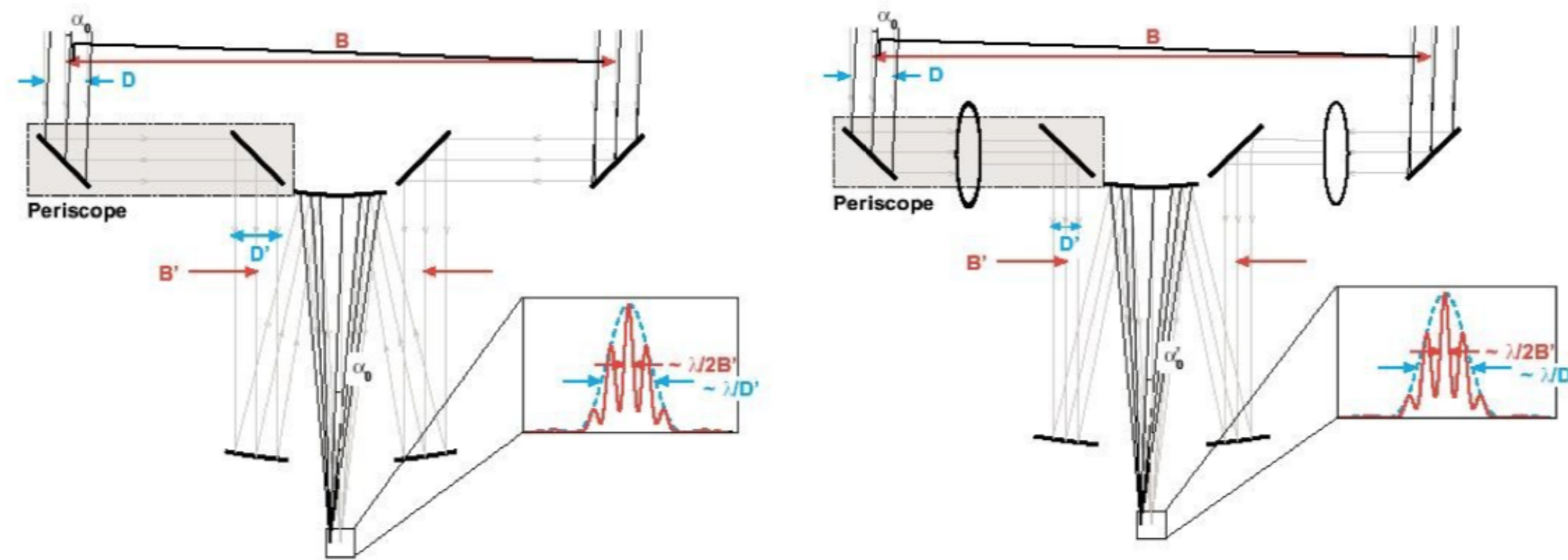
Long baselines record information about the small-scale structure of the source but are **INSENSITIVE** to large-scale structure (fringes wash out)

Short baselines record information about large-scale structure of the source but are **INSENSITIVE** to small-scale structure (resolution limit)

# Interferometry for the faint of heart

Non-photon-limited: electronic, relatively straightforward  
can clone and combine signals  
“correlation” (multiplication+delay)  
can even record signals and combine later

Photon-limited case: use classical Michelson/Fizeau arrangements  
delay lines for manipulation  
cannot clone photons



# Interferometry for the faint of heart

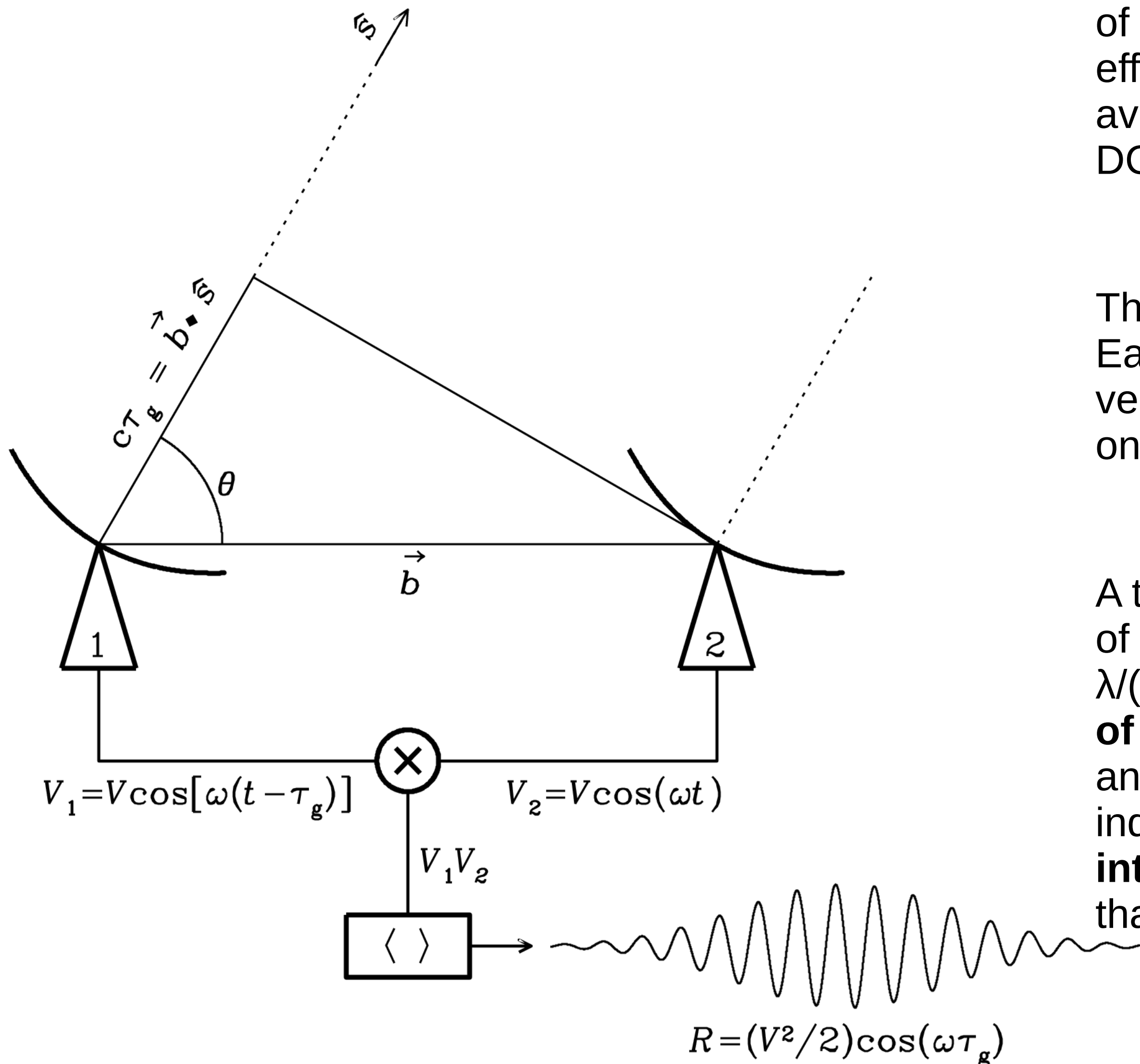
- Replacing a single large telescope by a collection of small telescopes filling the large one
- Signals received by telescopes are combined by pairs
- Each antenna pair (**baseline**) measures a Fourier component of the source brightness distribution (**visibility**)
- Given sufficient number of measurements the source brightness distribution can be obtained by Fourier inversion

# Interferometry for the faint of heart

The correlator output amplitude  $V^2/2$  is proportional to the flux density  $S$  of the point source multiplied by  $(A_1A_2)^{1/2}$ , where  $A_1$  and  $A_2$  are the effective collecting areas of the two antennas. Notice that the time-averaged response  $R$  of a multiplying interferometer is zero: there is no DC output.

The correlator output voltage  $R=(V^2/2)\cos(\omega\tau_g)$  varies sinusoidally as the Earth's rotation changes the source direction relative to the baseline vector. These sinusoids are called fringes, and the fringe phase depends on  $\theta$  and is an exquisite measure of the source position.

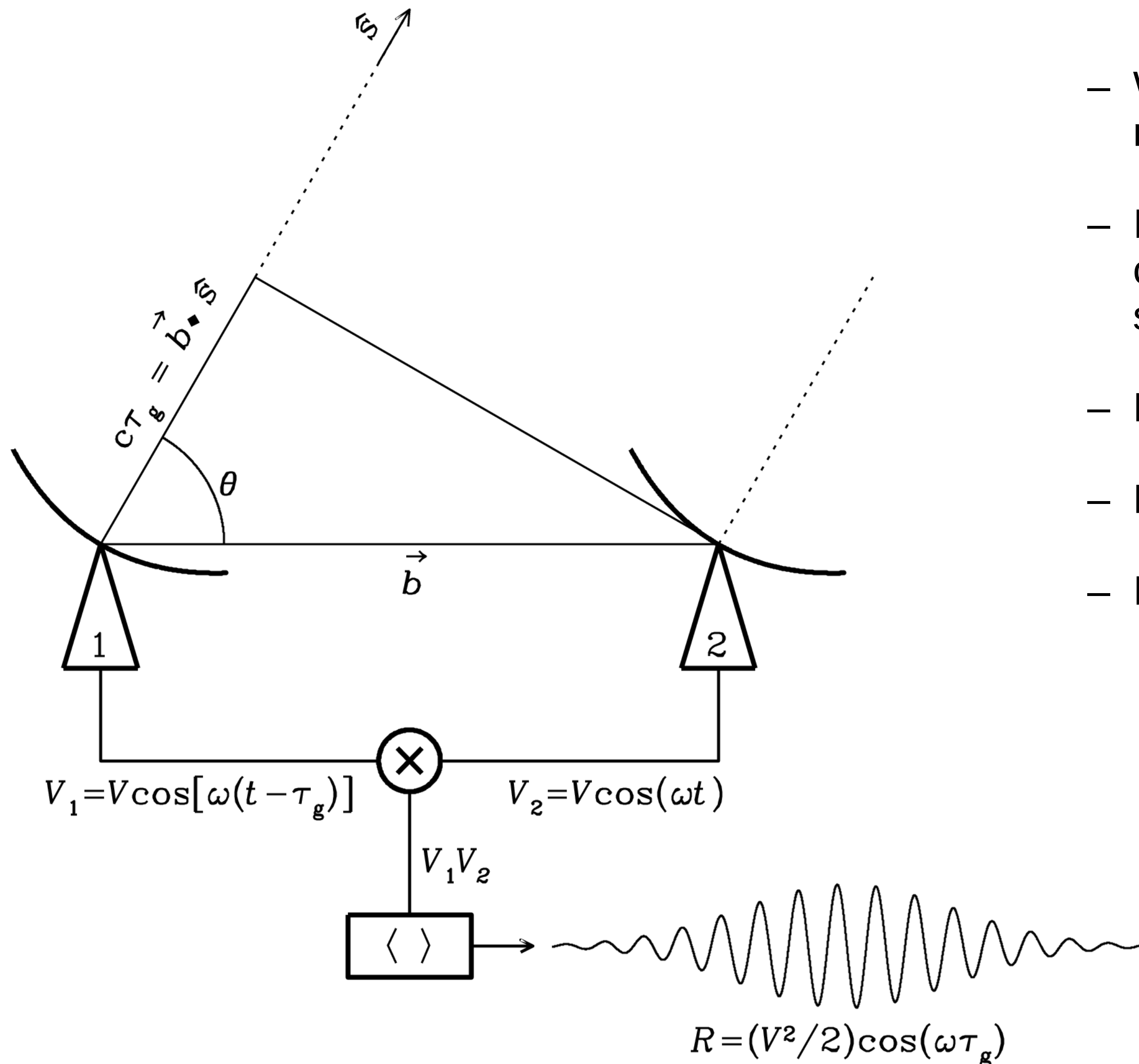
A two-element interferometer is sensitive to only one Fourier component of the sky brightness distribution: the component with angular period  $\lambda/(b\sin\theta)$ . **The response  $R$  is that sinusoid multiplied by the product of the voltage patterns of the individual antennas.** Normally the two antennas are identical, so this product is the power pattern of the individual antennas and is called **the primary beam of the interferometer**. The primary beam is usually a Gaussian much wider than a fringe period.





# Interferometry for the faint of heart

- We want to increase the number of antennae to measure possibly many physical scales
- In order to describe slightly extended sources we need a complex correlator to be able to treat at the same time the symmetric and anti-symmetric parts (cos-even plus sin-odd parts)
- Finite bandwidths and averaging times (to increase sensitivity)
- Earth-Rotation Aperture Synthesis
- Interferometers in Three Dimensions



# Interferometry for the faint of heart

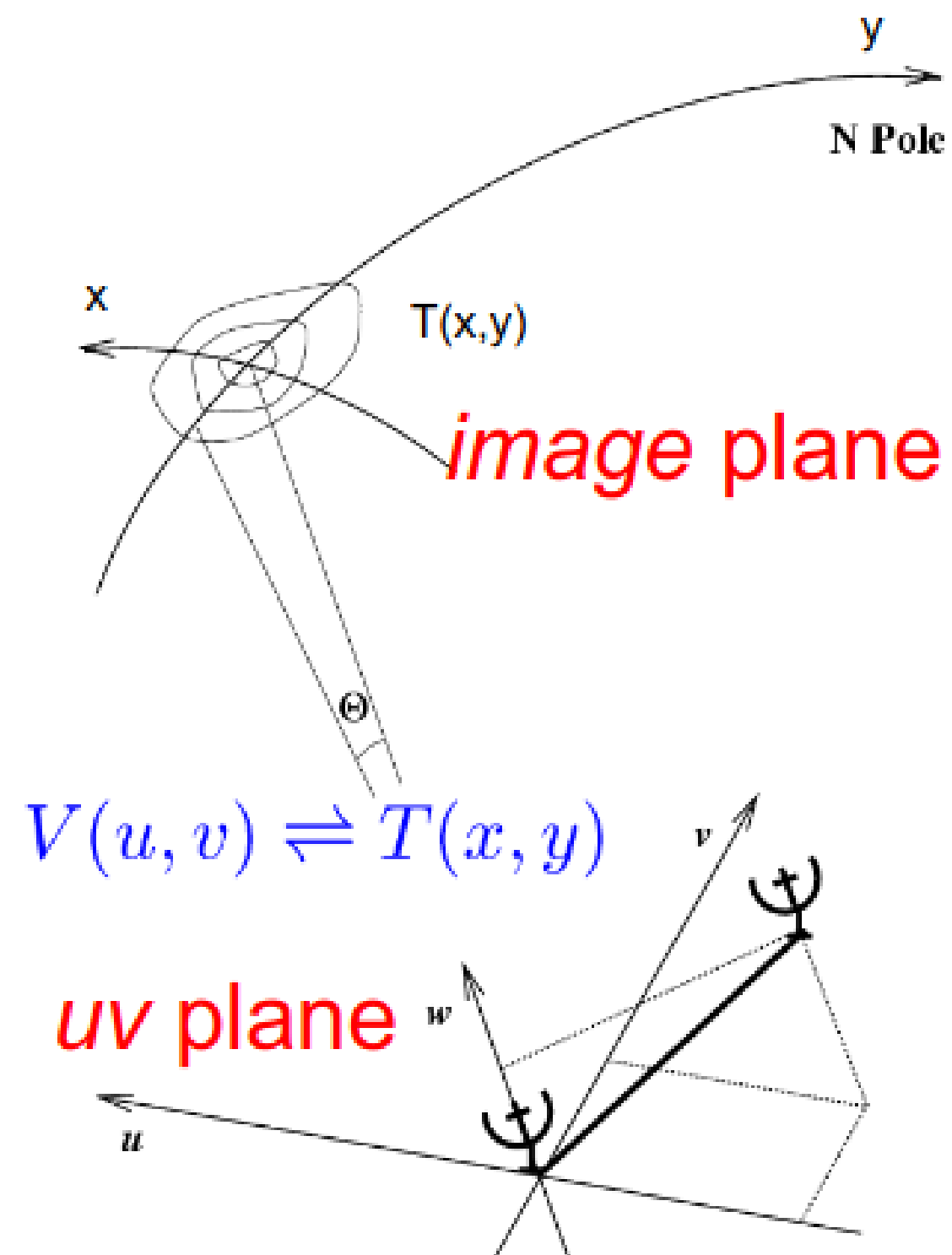
## Visibility and Sky Brightness

For small fields of view: the complex visibility,  $V(u,v)$ , is the 2D Fourier transform of the brightness on the sky,  $T(x,y)$

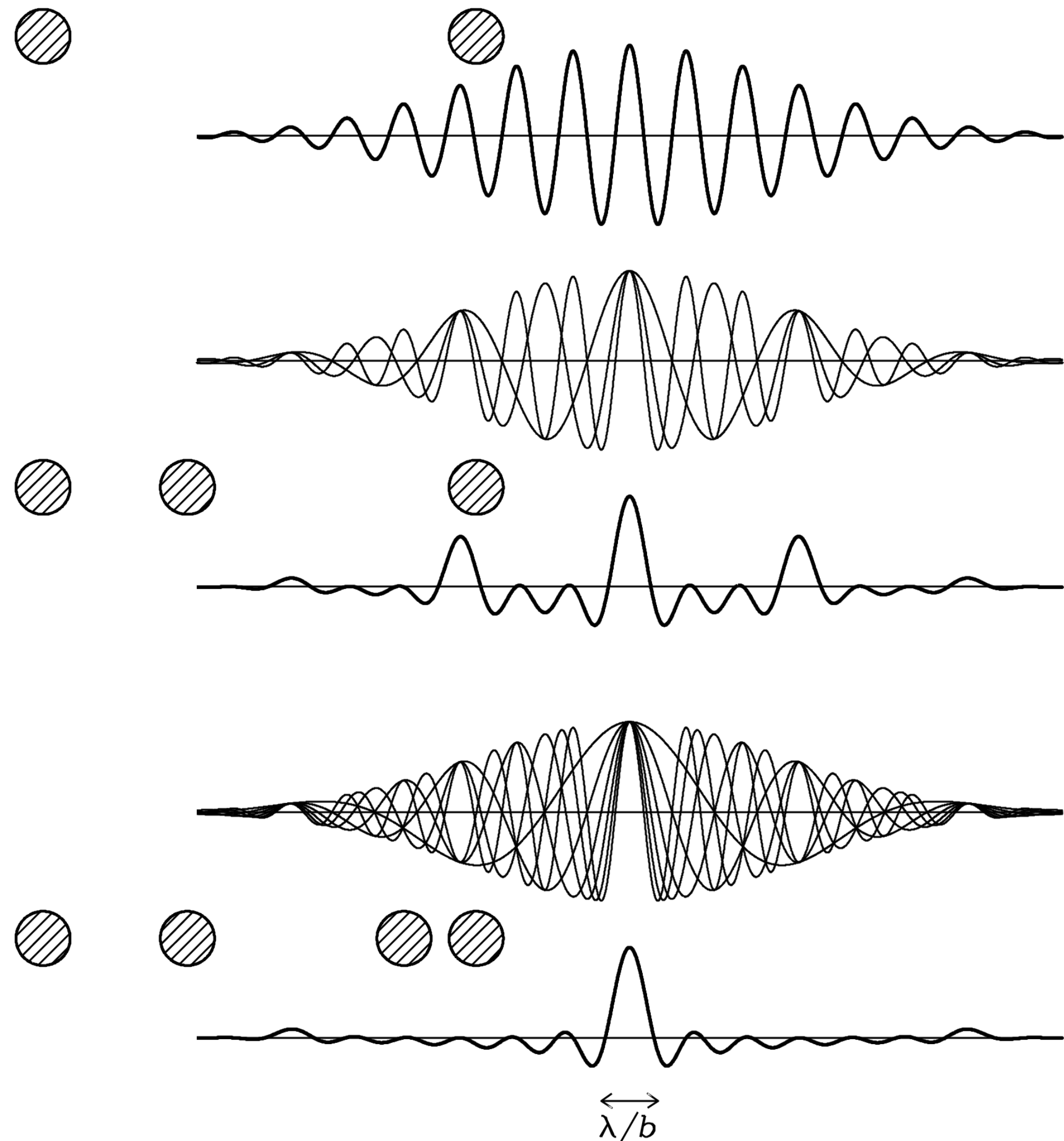
$$V(u, v) = \iint T(x, y) e^{2\pi i (ux + vy)} dx dy$$

$$T(x, y) = \iint V(u, v) e^{-2\pi i (ux + vy)} du dv$$

- $u, v$  (wavelengths) are spatial frequencies in E-W and N-S directions, i.e. the baseline lengths
- $x, y$  (rad) are angles in tangent plane relative to a ref position in E-W and N-S directions



# Interferometry for the faint of heart



The instantaneous point-source responses of interferometers with overall projected length  $b$  and two, three, or four antennas distributed as shown are indicated by the thick curves.

The synthesized main beam of the four-element interferometer is nearly Gaussian with angular resolution  $\theta \approx \lambda/b$ , but the sidelobes are still significant and there is a broad negative “bowl” caused by the lack of spacings shorter than the diameter of an individual antenna.

Thus the **synthesized beam is sometimes called the dirty beam**. The instantaneous dirty beam of the multielement interferometer is the arithmetic mean of the individual responses of its component two-element interferometers.

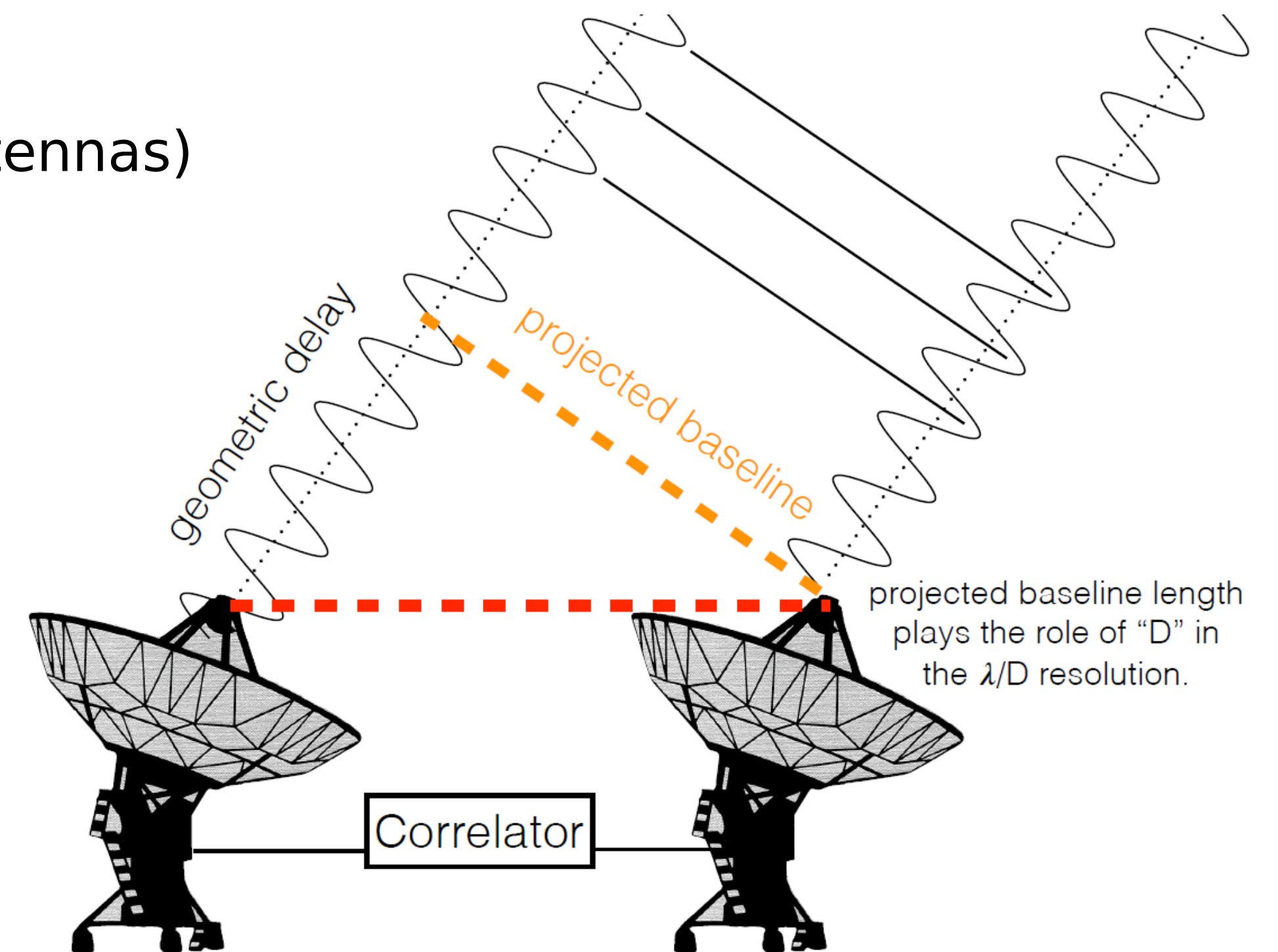
The individual responses of the three two-element interferometers comprising the three-element interferometer and of the six two-element interferometers comprising the four-element interferometer are plotted as thin curves.

# Interferometry for the faint of heart

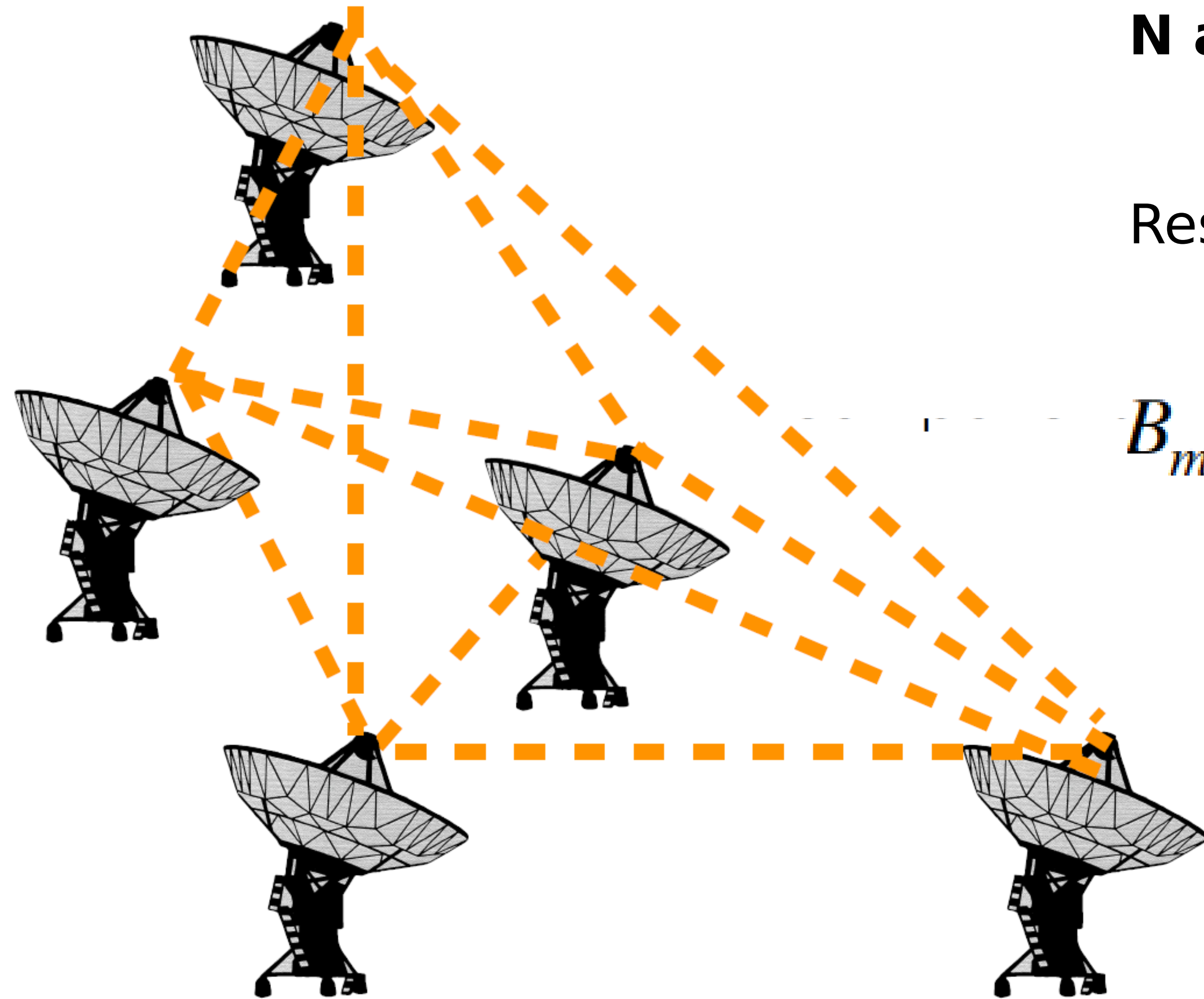
'Synthesize' a large aperture by combining signals collected by separated antennas

It measures the interference pattern produced by the two apertures, which is related to the source brightness

**2 antennas:** projected baseline length  $B$  (distance between antennas) plays the role of  $D$  in the resolution, i.e.  $\Delta\theta \sim \lambda/B$



# Interferometry for the faint of heart



**N antennas** :  $N(N-1)/2$  baselines

Resolution:  $\lambda/B_{max}$

$B_{max}$  = maximum projected baseline



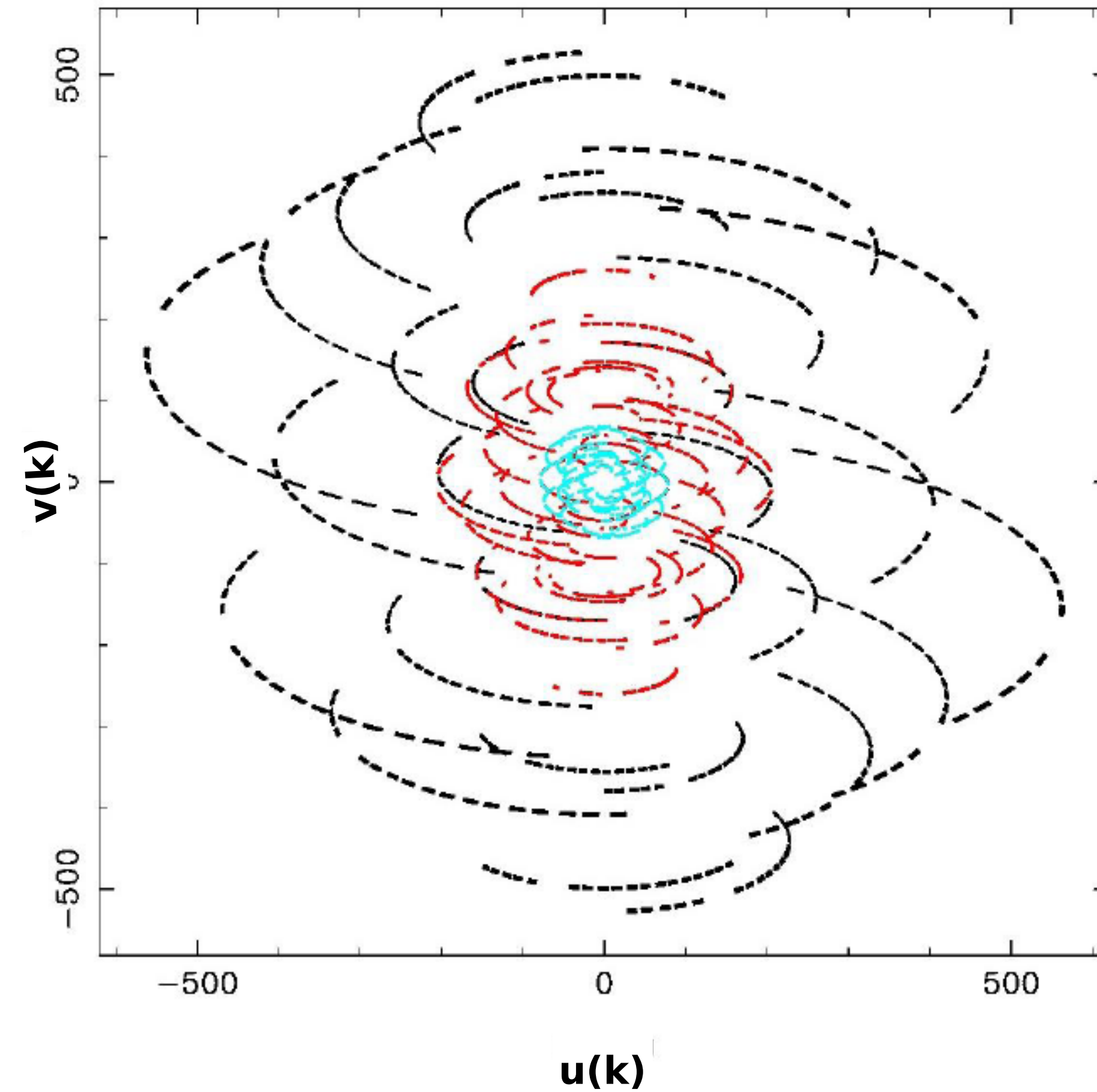
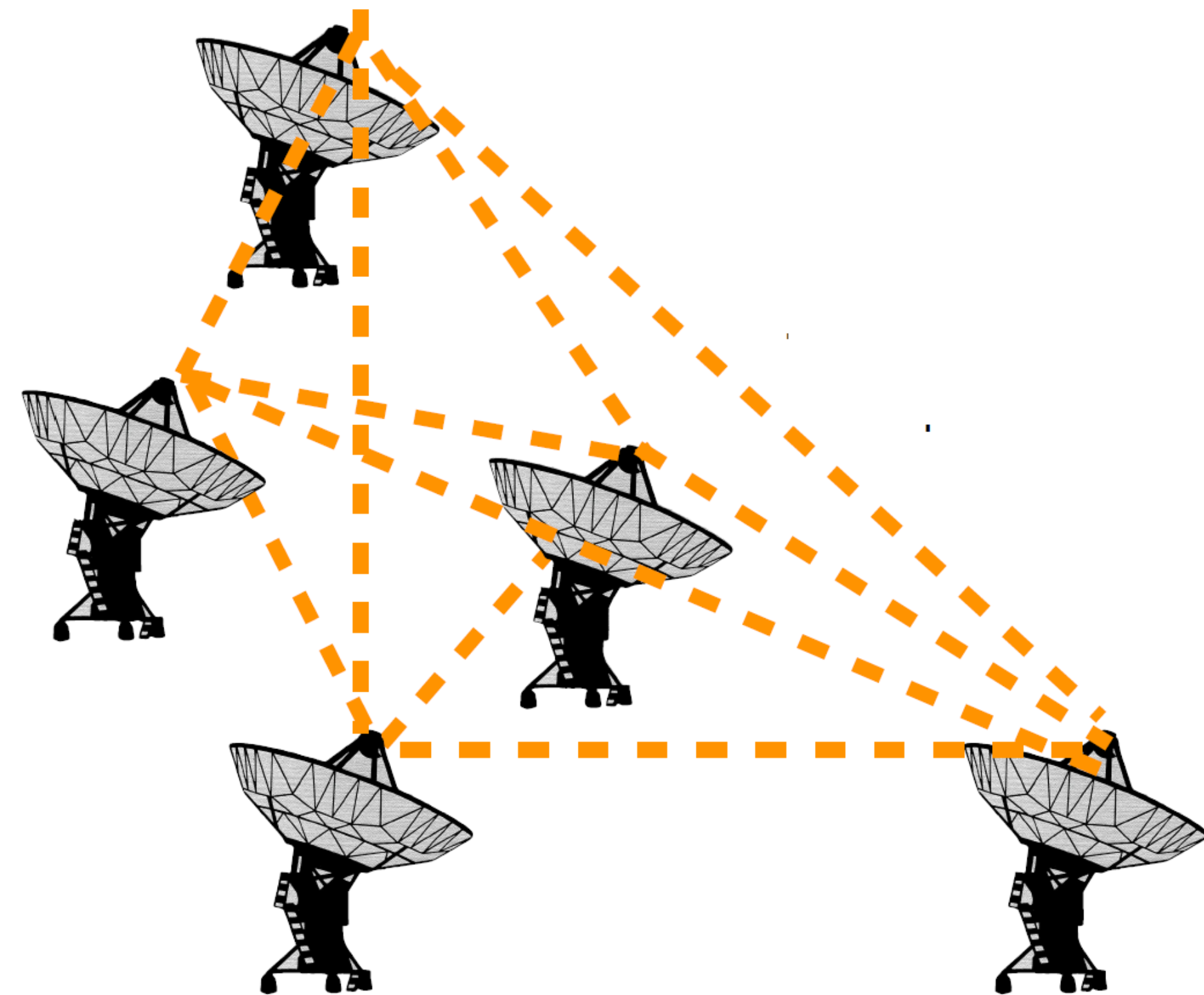
# Interferometry for the faint of heart

An array with  $N$  antennas

will have  $N(N-1)/2$  baselines

and each baseline will measure a visibility  
for each frequency channel and  
for each integration time

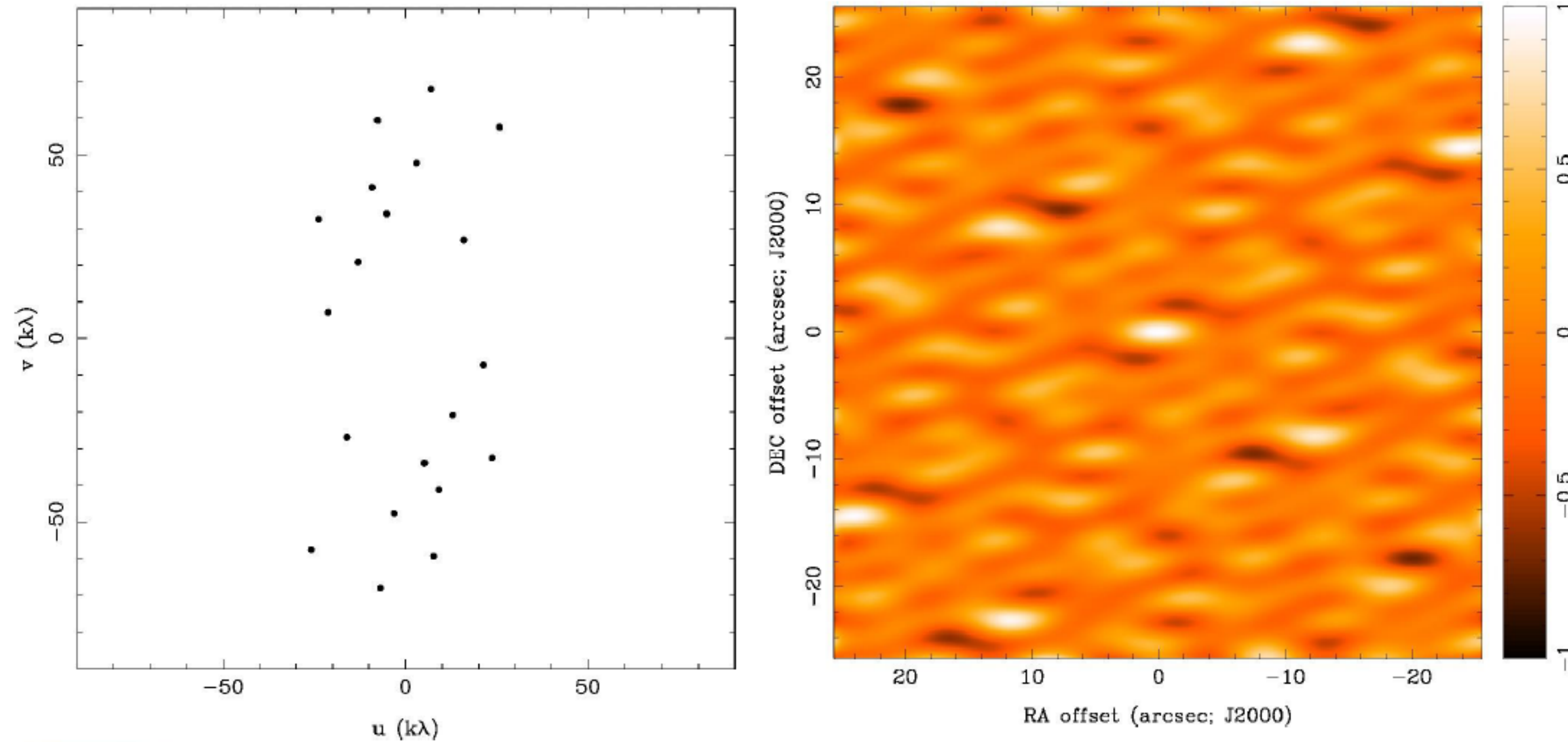
(Earth rotation helps covering the  $uv$  plane)



# Interferometry for the faint of heart

The better we sample the uv plane, the better we can recover the true brightness of your target

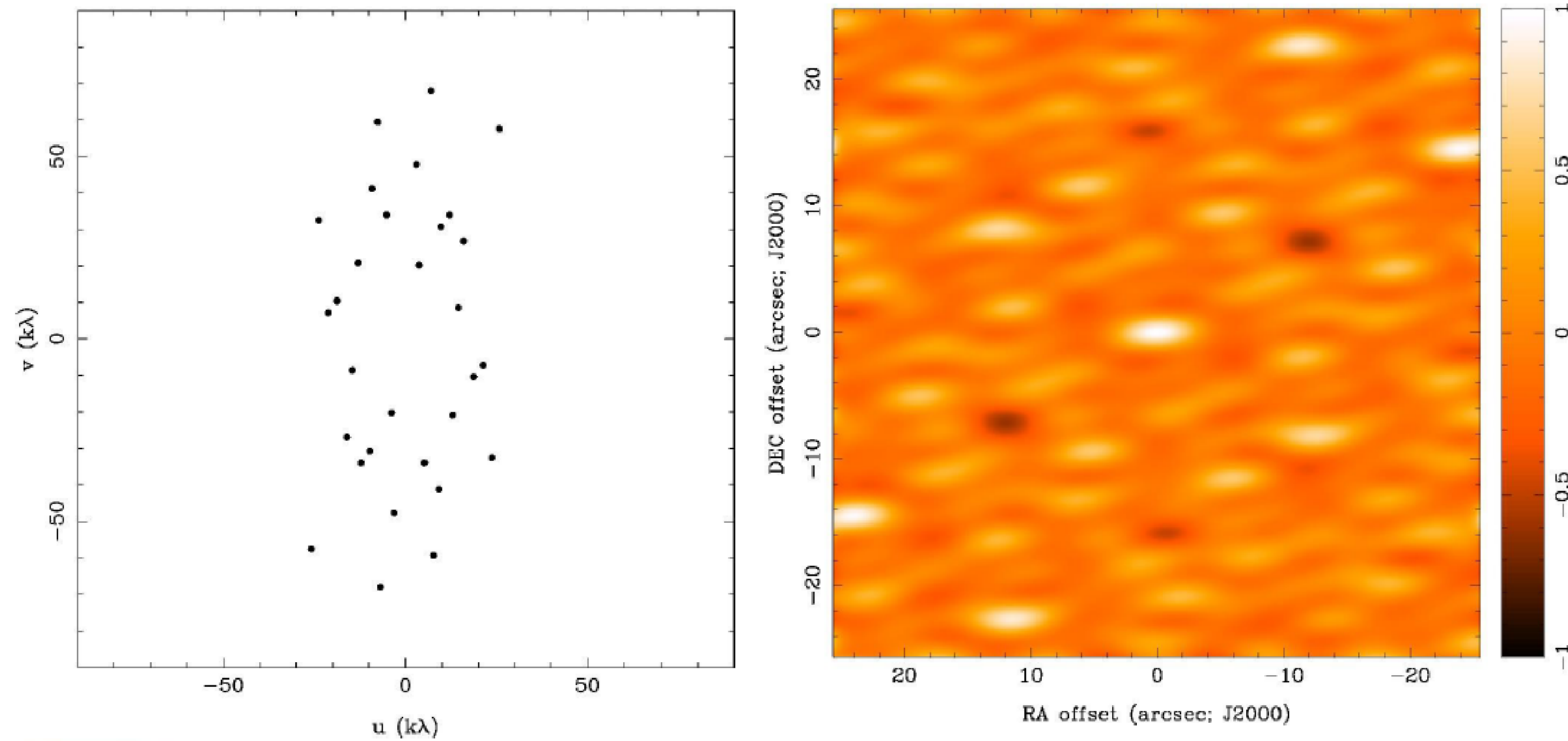
5 Antennas, 1 min observing



# Interferometry for the faint of heart

The better we sample the uv plane, the better we can recover the true brightness of your target

6 Antennas, 1 min observing

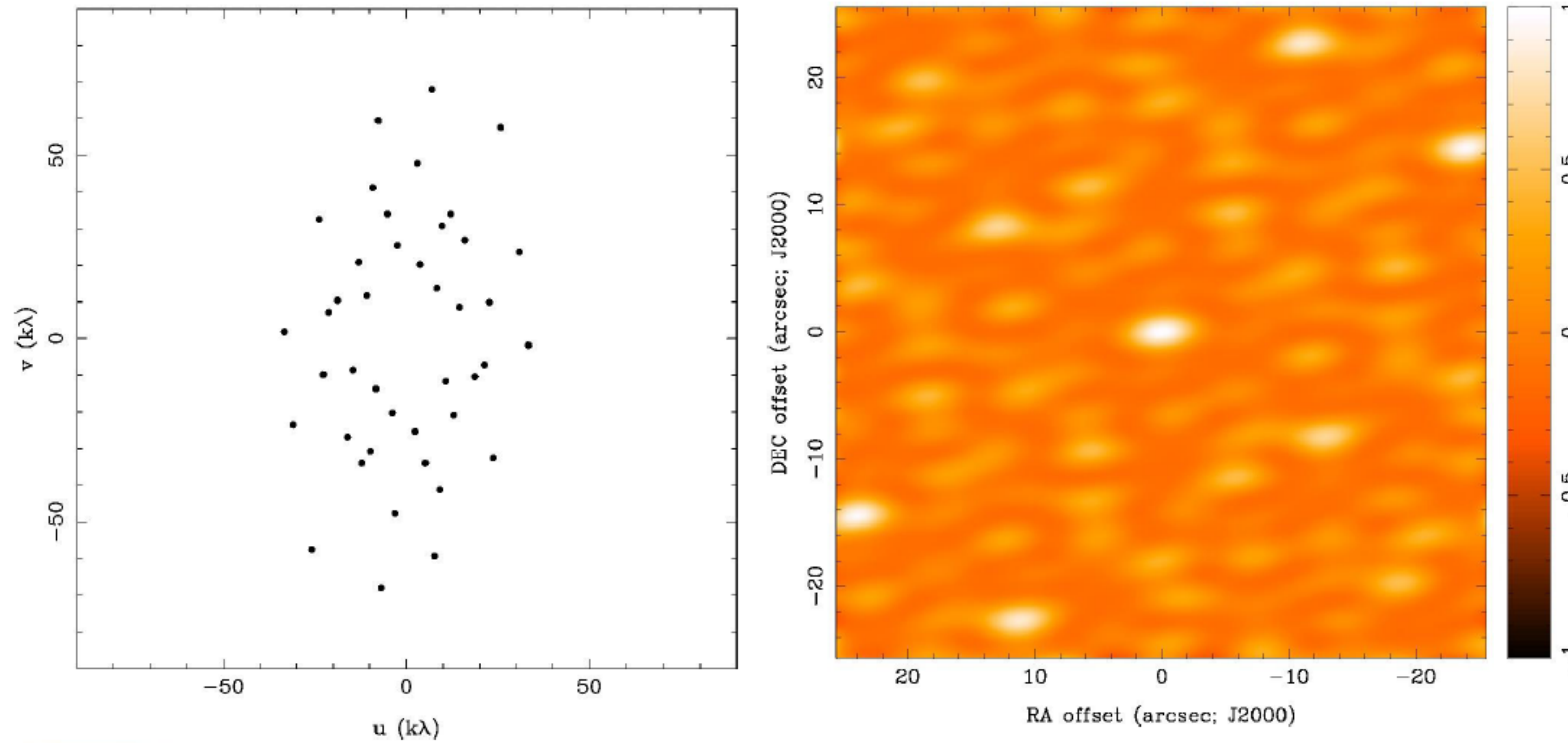




# Interferometry for the faint of heart

The better we sample the uv plane, the better we can recover the true brightness of your target

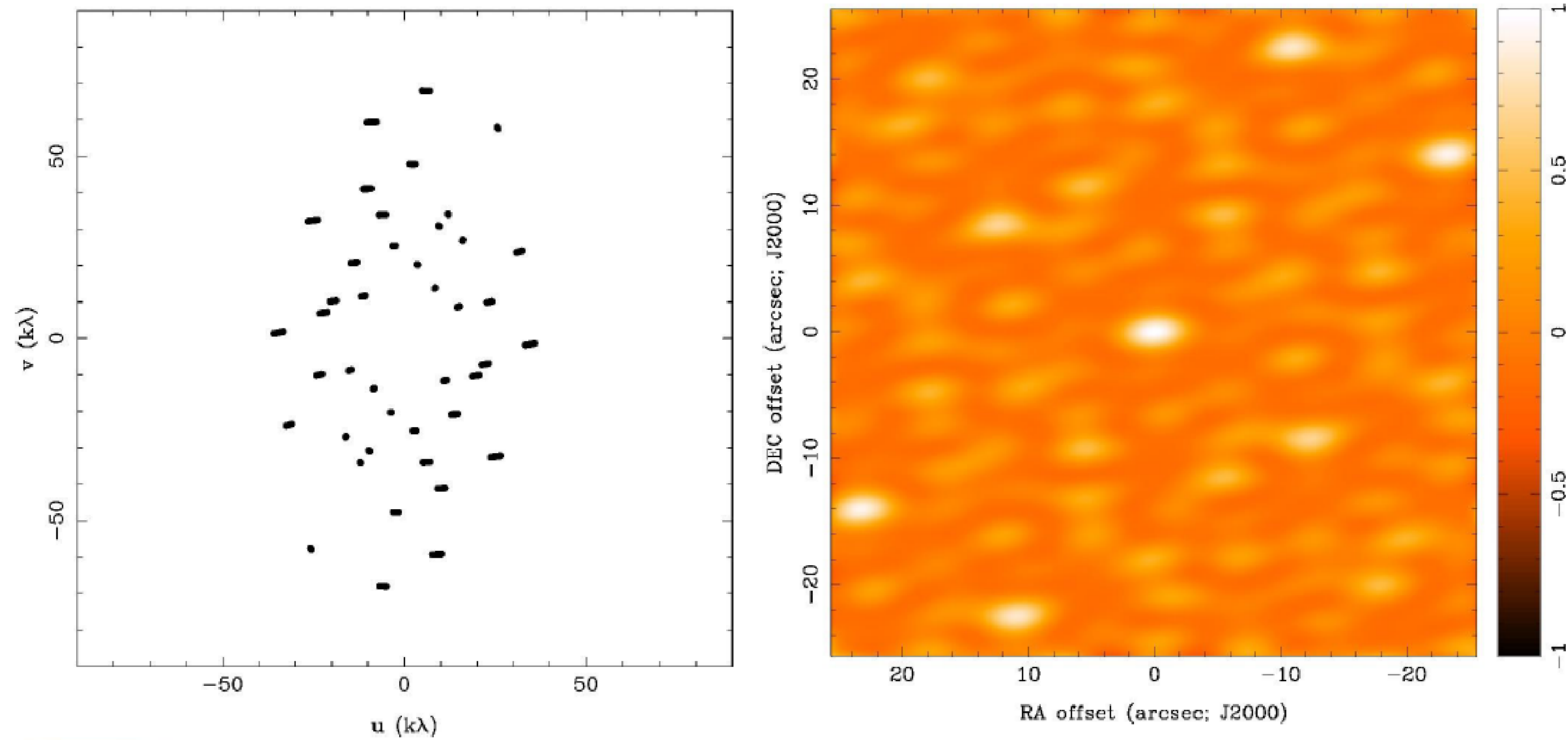
7 Antennas, 1 min observing



# Interferometry for the faint of heart

The better we sample the uv plane, the better we can recover the true brightness of your target

7 Antennas, 10 min observing

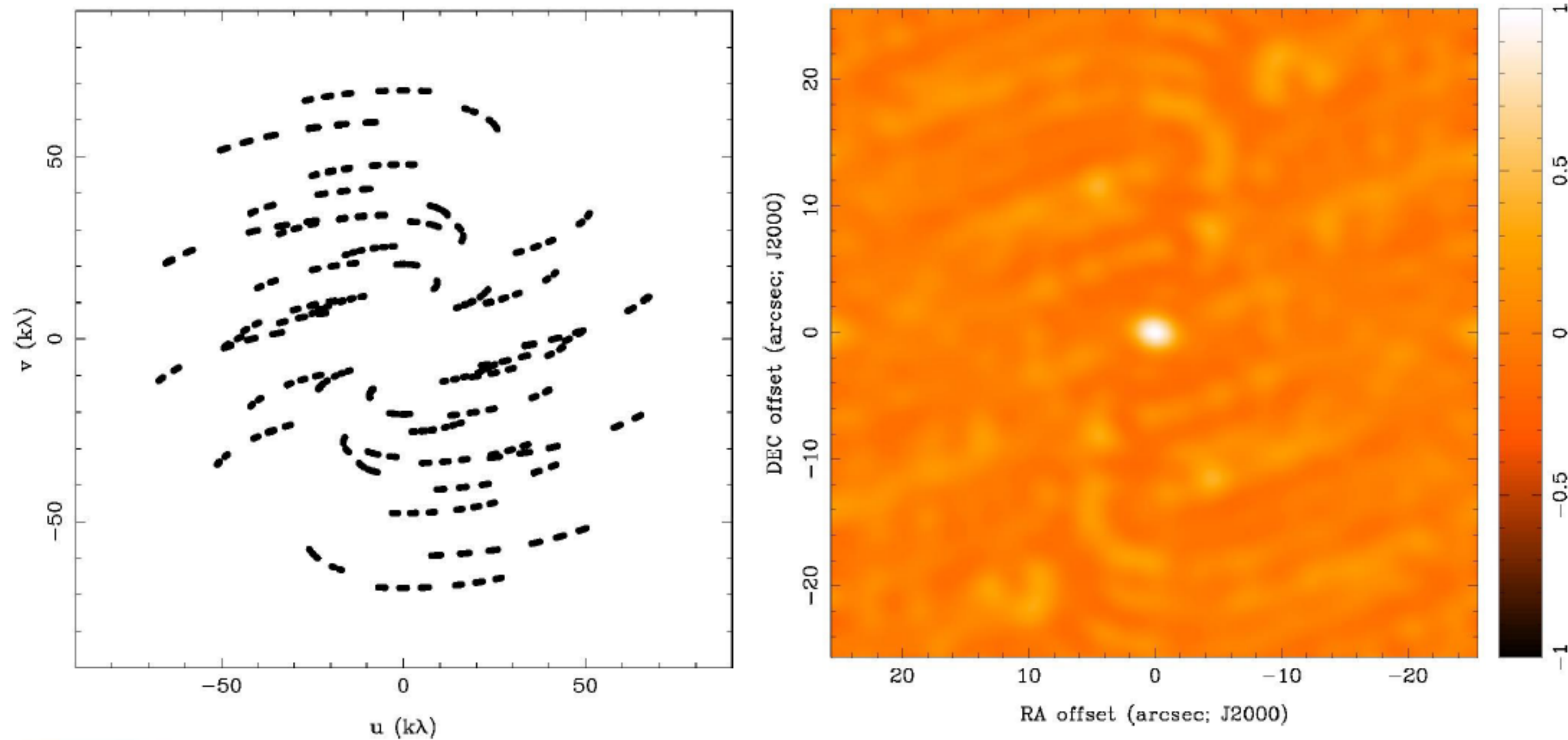




# Interferometry for the faint of heart

The better we sample the uv plane, the better we can recover the true brightness of your target

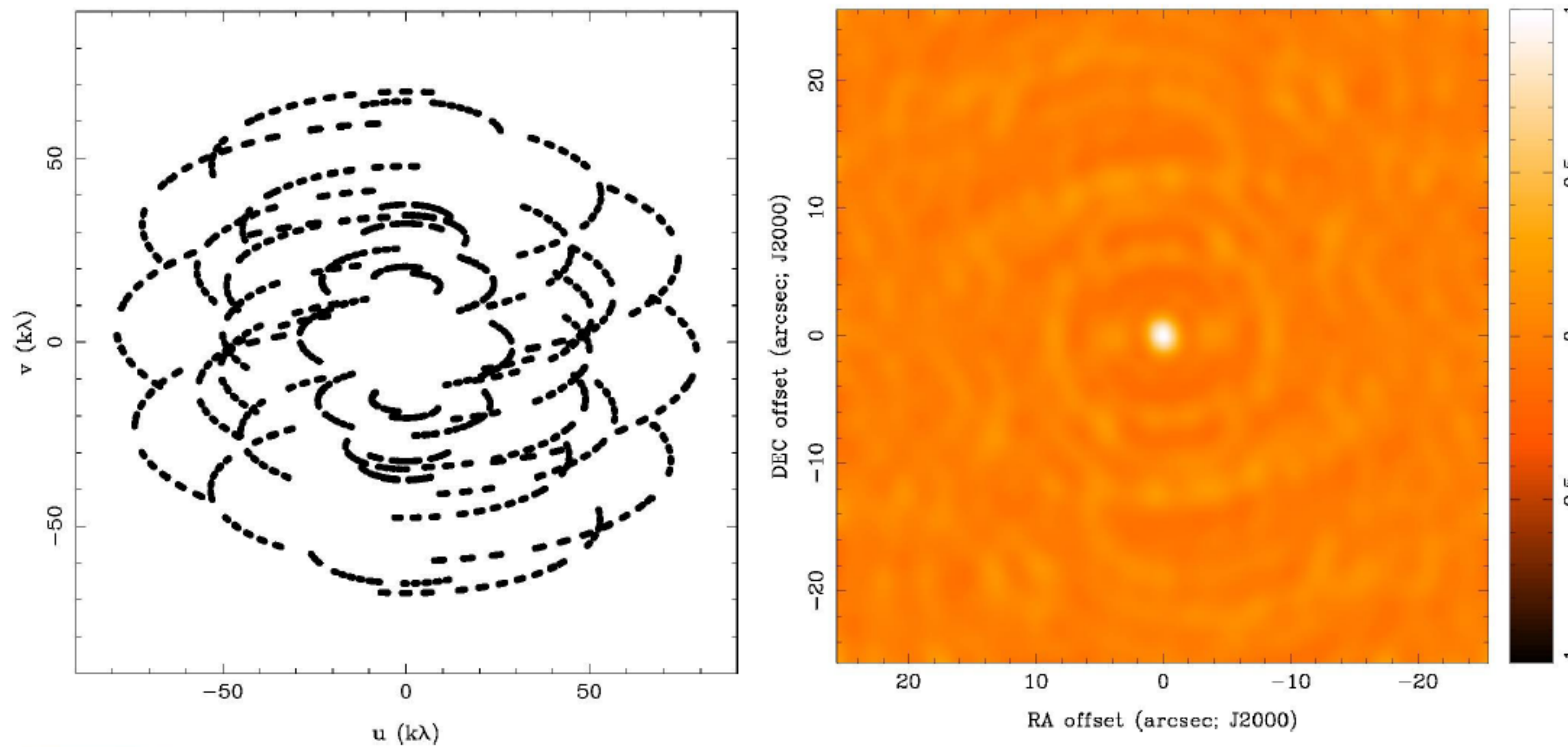
7 Antennas, 3 hours observing



# Interferometry for the faint of heart

The better we sample the uv plane, the better we can recover the true brightness of your target

7 Antennas, 8 hours observing



# Interferometry for the faint of heart

## RELEVANT QUANTITIES IN INTERFEROMETRY

$$\mathbf{FOV} \approx \frac{\lambda}{D}$$

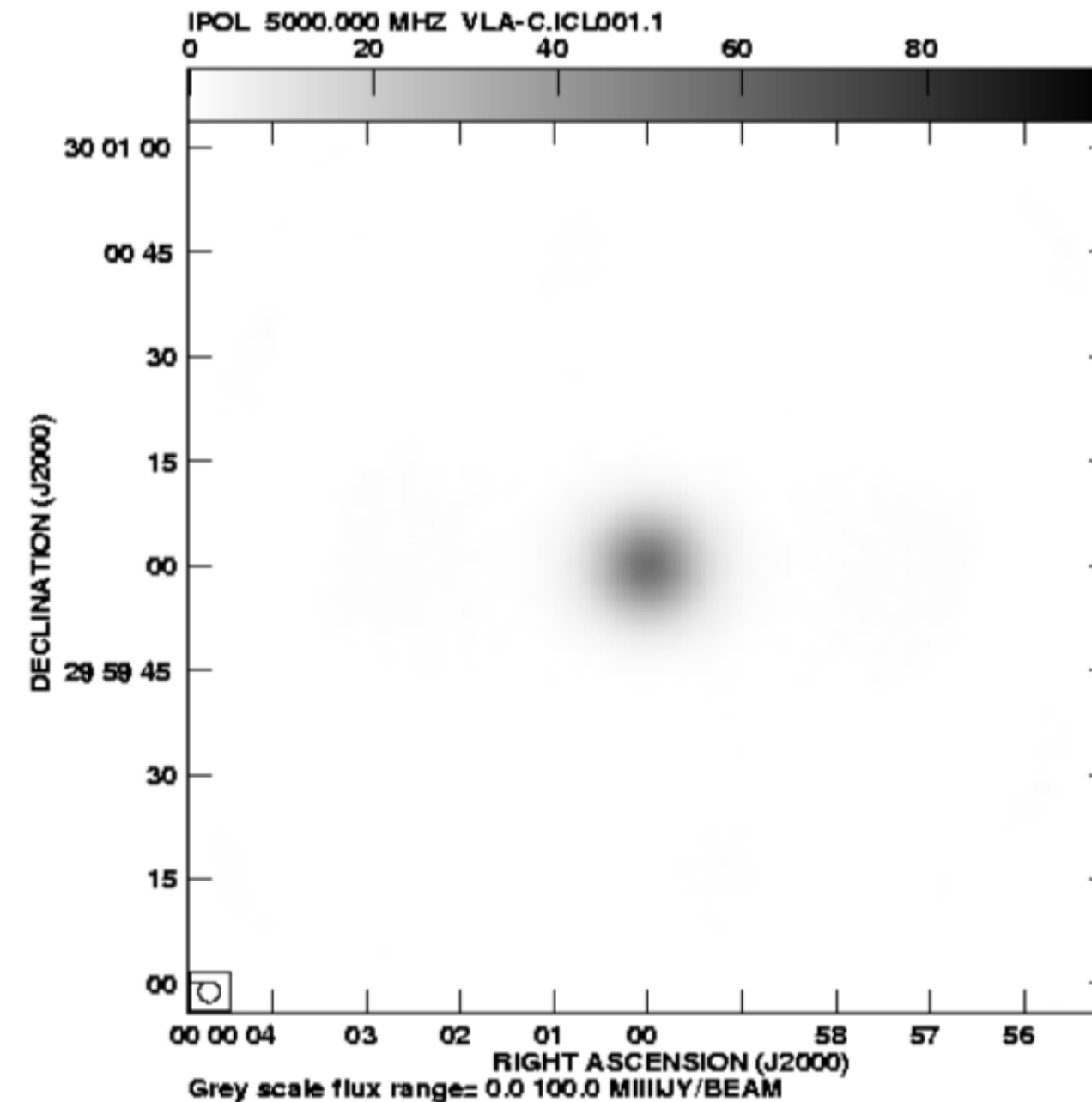
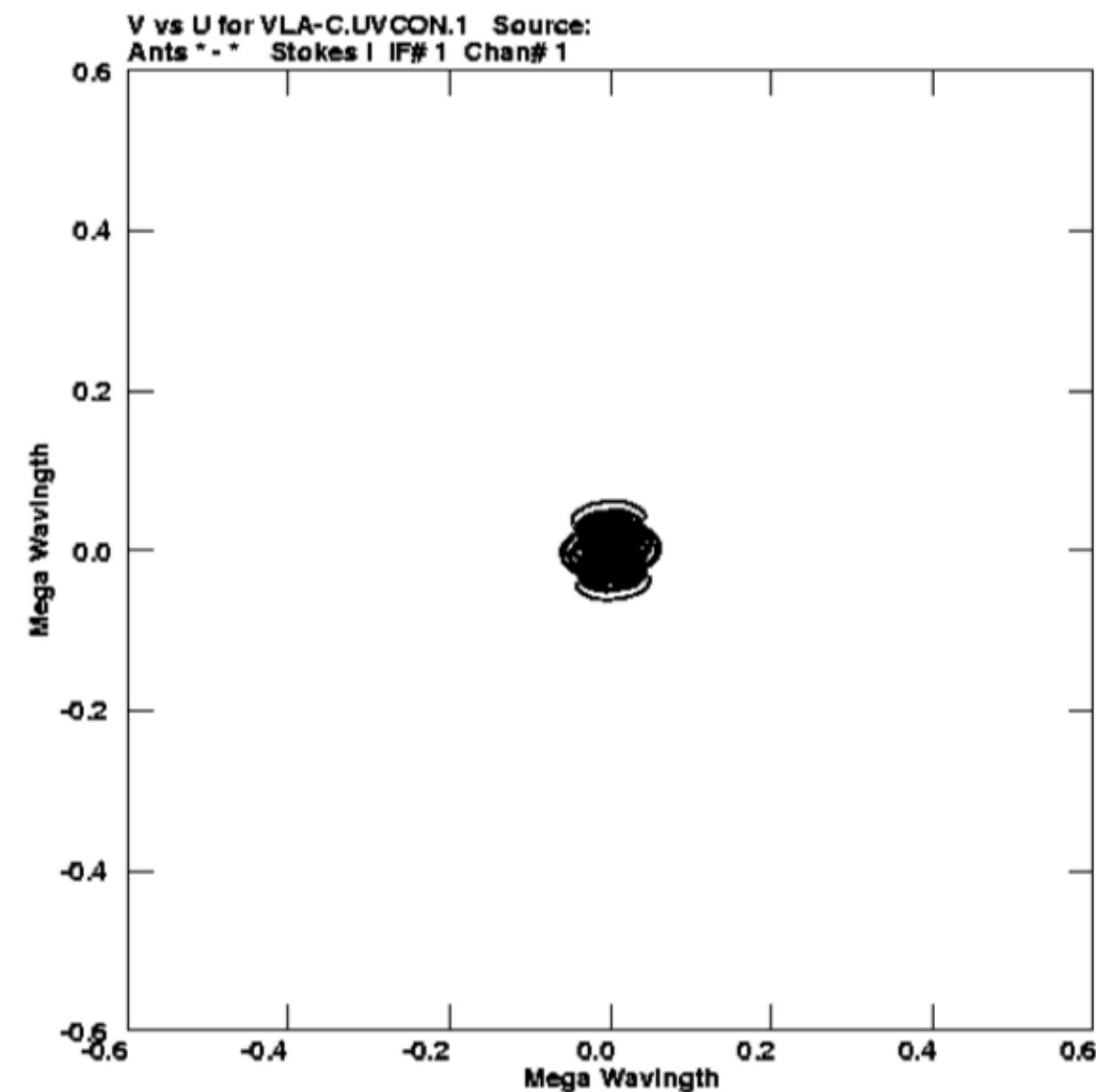
$$\mathbf{Resolution} \approx \frac{\lambda}{B_{max}}$$

$$\mathbf{Maximum\ scale} \approx \frac{\lambda}{B_{min}}$$

$$\mathbf{Sensitivity} = \frac{T_{sys}}{A_{eff} \sqrt{N(N-1) \Delta \nu \tau}}$$

# Interferometry for the faint of heart

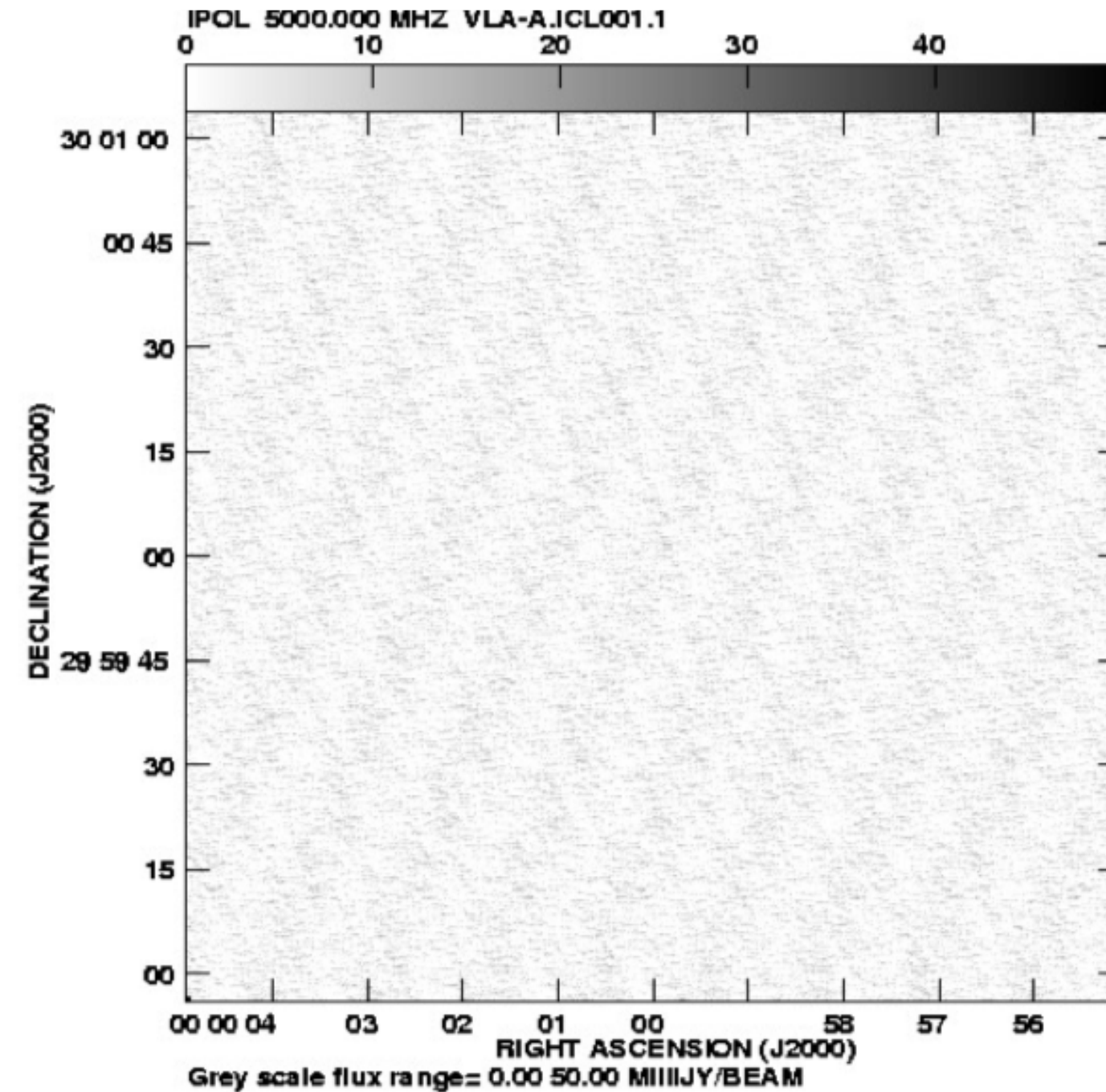
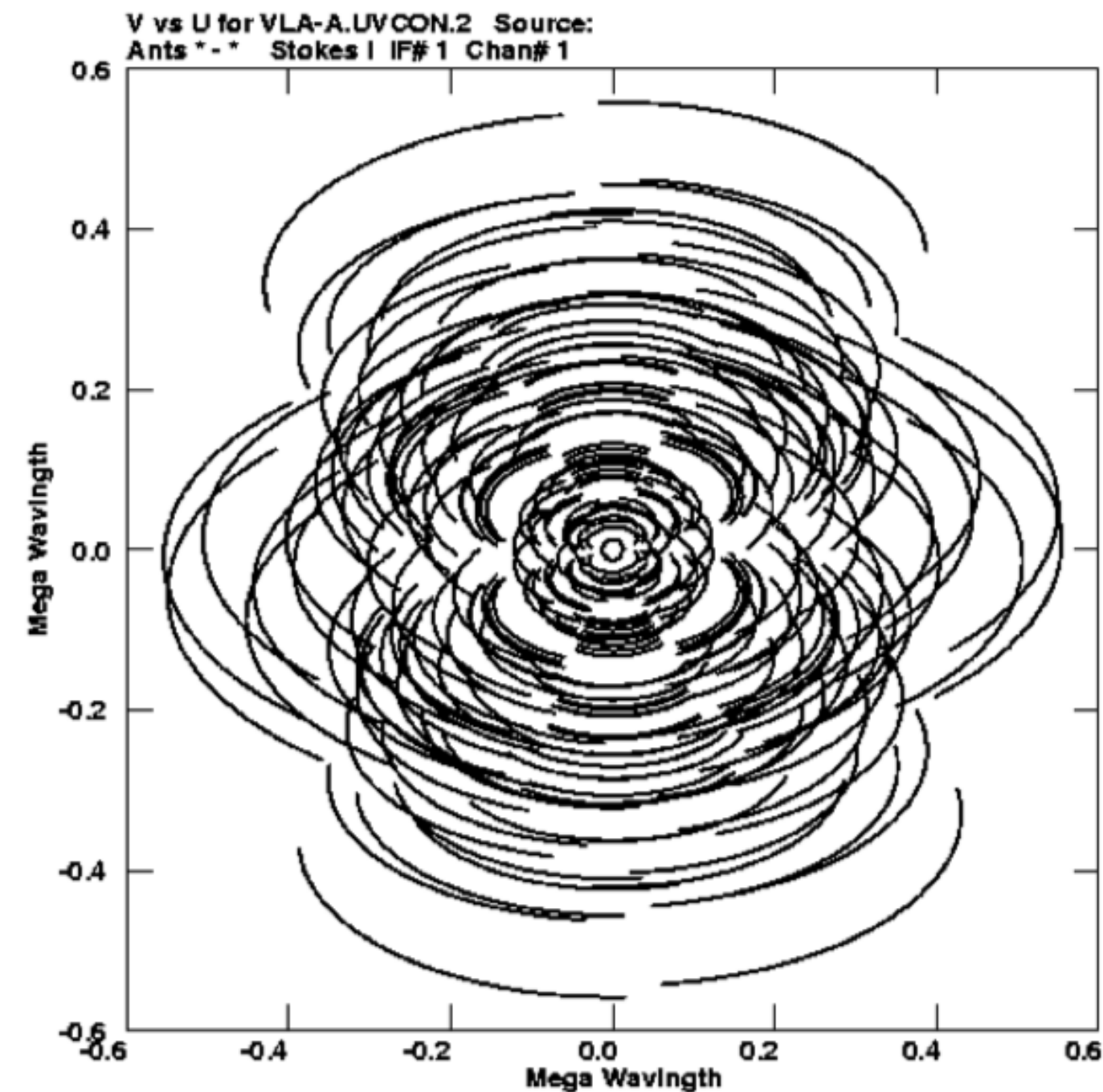
## FT imaging is not like direct imaging!



This is how a 12" uniform source in the sky is mapped by a uv coverage producing a 3" resolution imaging

# Interferometry for the faint of heart

## FT imaging is not like direct imaging!

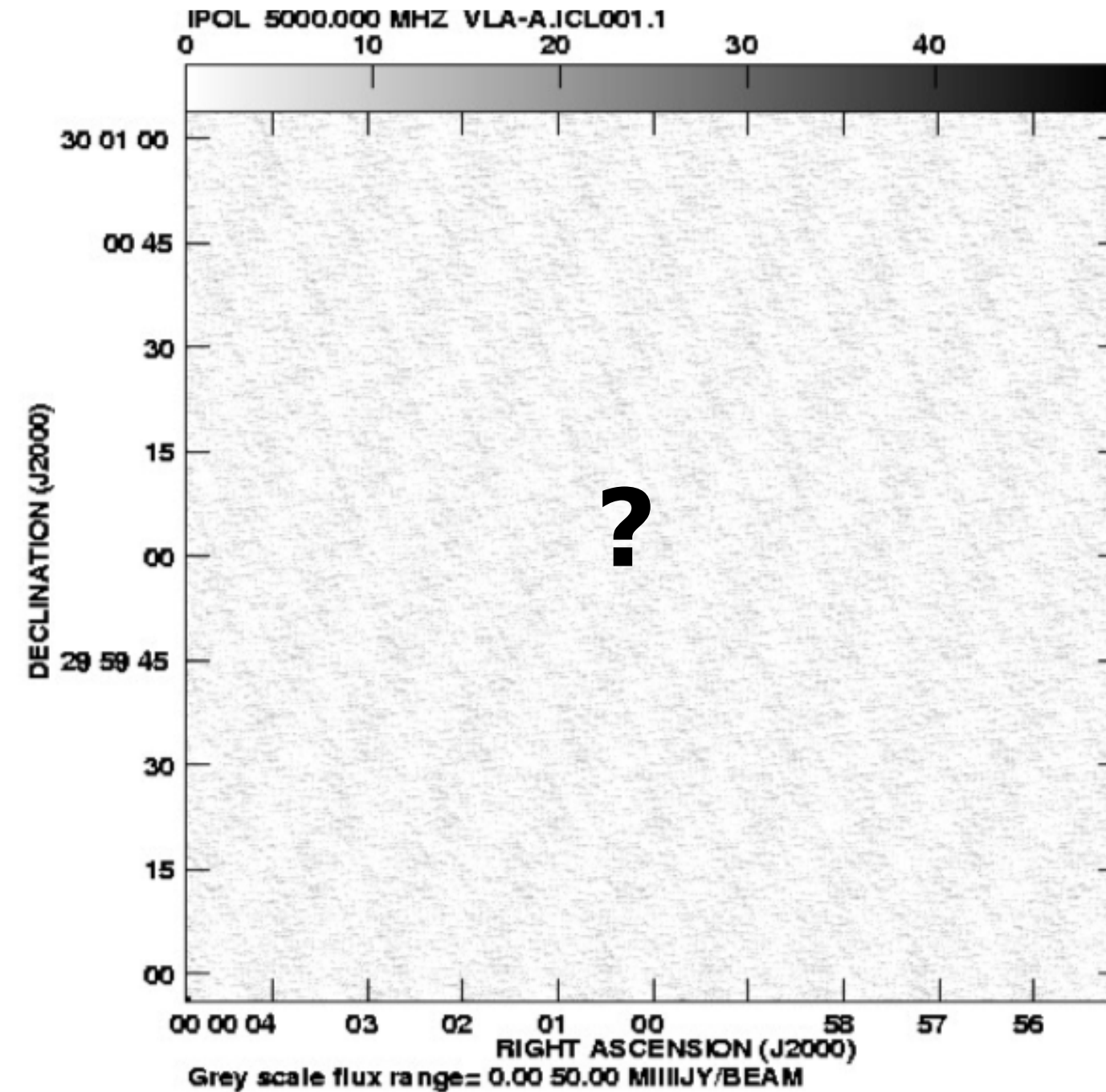
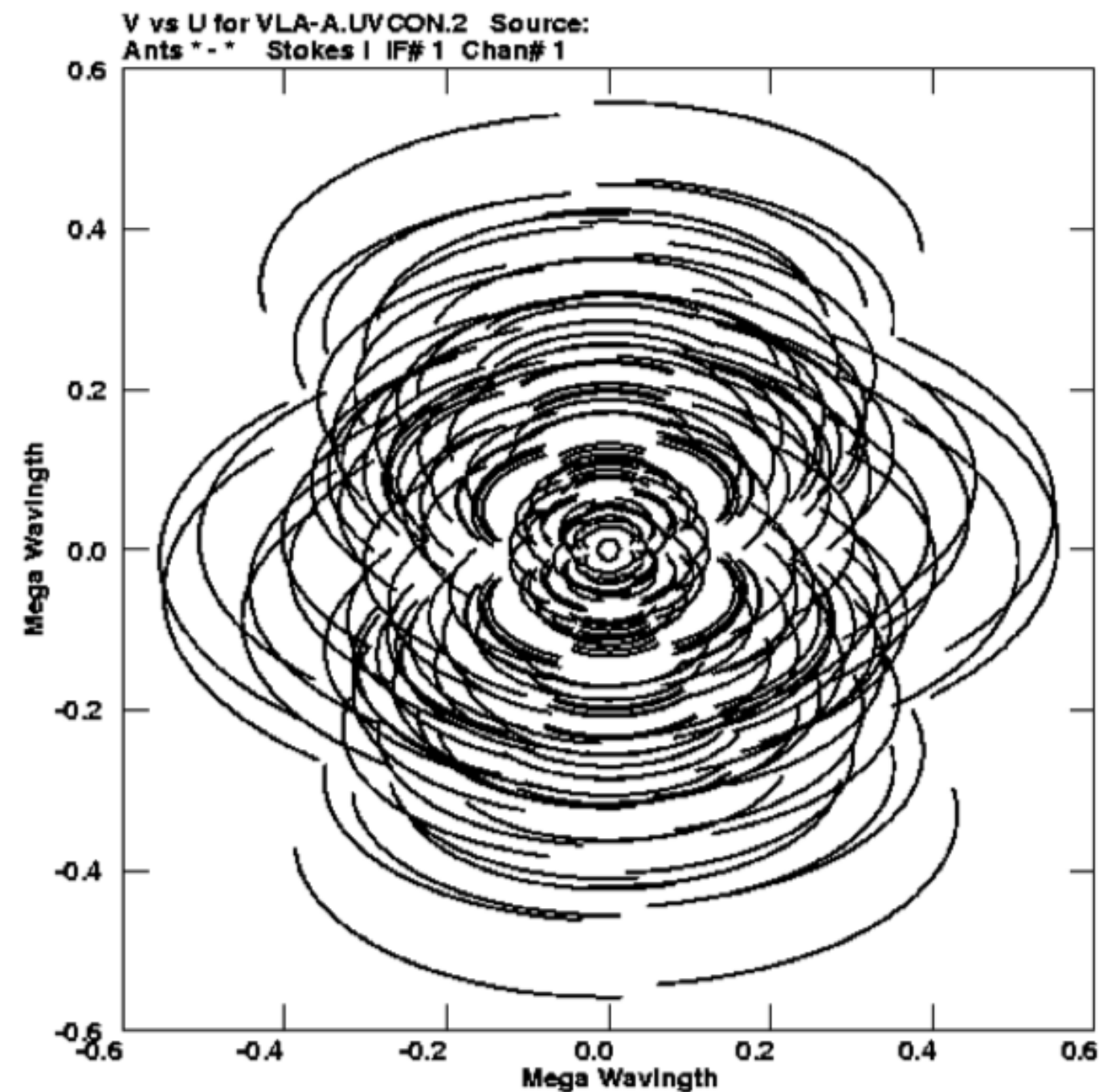


This is how the same 12" source in the sky is mapped by a uv coverage producing a 0.3" resolution imaging



# Interferometry for the faint of heart

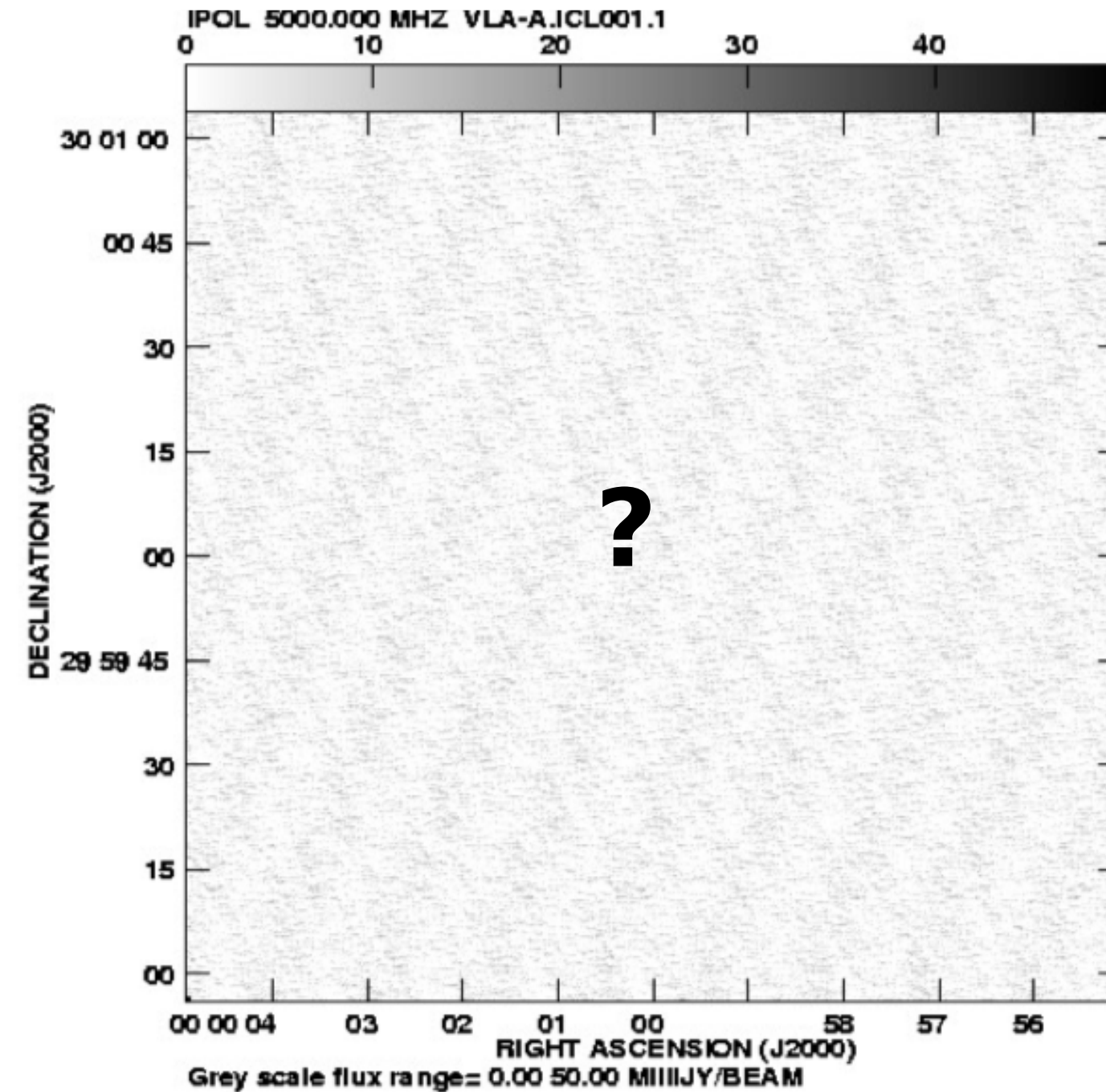
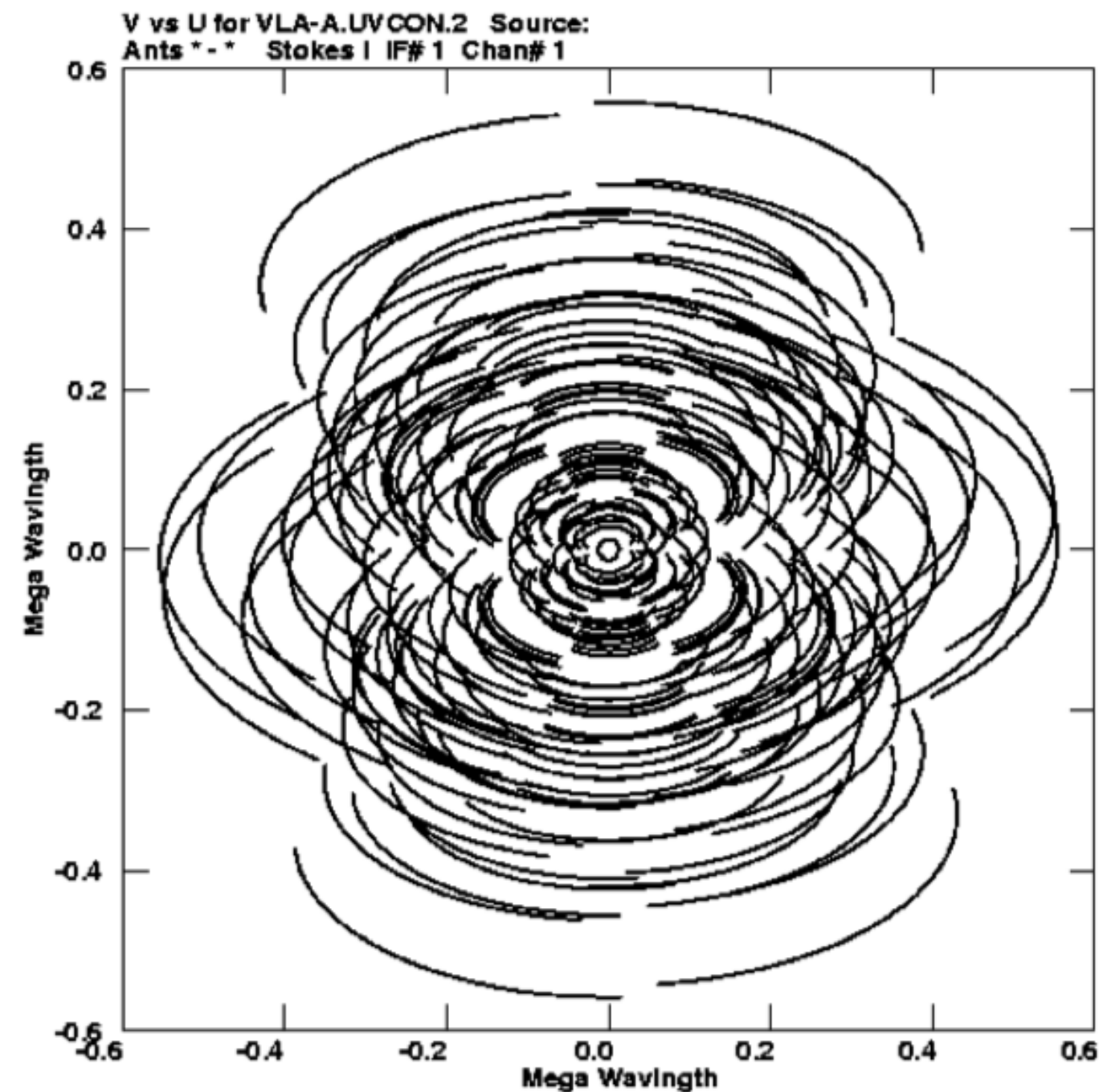
## FT imaging is not like direct imaging!



This is how the same 12" source in the sky is mapped by a uv coverage producing a 0.3" resolution imaging

# Interferometry for the faint of heart

## FT imaging is not like direct imaging!



**Smoothing cannot help  
FT imaging if the relevant  
fourier components are  
not sampled at all**

**The source is lost  
irretrievably!**

**We'd have wasted  
telescope time :(((**

This is how the same 12" source in the sky is mapped by a uv coverage producing a 0.3" resolution imaging

# Interferometry for the faint of heart

## RELEVANT QUANTITIES IN INTERFEROMETRY

$$\mathbf{FOV} \approx \frac{\lambda}{D}$$

$$\mathbf{Resolution} \approx \frac{\lambda}{B_{max}}$$

$$\mathbf{Maximum\ scale} \approx \frac{\lambda}{B_{min}}$$

$$\mathbf{Sensitivity} = \frac{T_{sys}}{A_{eff} \sqrt{N(N-1) \Delta \nu \tau}}$$

Synthesis array (Fourier Transform imaging) is 'blind' to structures on angular scales both smaller and larger than the range of fringe spacings given by the antenna distribution, i.e., the array configuration.