Notes on Gunn-Peterson effect

Photons bluer than the Lyman-α frequency can be absorbed by neutral hydrogen along the line of sight

We compute the opacity for photons emitted at frequency $\nu_e > \nu_{L\alpha}$ at redshift z_e.

These photons have frequency ν_0 at the observing time

$$d\tau_\nu = \alpha_\nu \, dl = n_{HI} \sigma_\nu \, dl$$ from Ribicky & Lightman

The frequency ν here depends on redshift:

$$\nu_0 (1+z) = \nu$$

The cross section is:

$$\sigma_\nu = \frac{\pi e^2}{m_e c} f \, g(\nu - \nu_{L\alpha}) \, , \quad f = 0.416 \text{ is the oscillator strength}$$

g is the line profile, $\int_0^\infty g(x)\,dx = 1$

Integrating along the line of sight from the emission redshift of the source to now:

$$\tau_{\nu_0} = \int_0^{z_e} n_{HI}(z) \, \sigma_\nu \, \frac{dl}{dz} \, dz \, .$$

dl is the proper length interval for a redshift interval dz

From eq. 2.49 of Vittorio:

$$\frac{dl}{dz} = \frac{c}{H_0} \frac{1}{(1+z)} \frac{1}{\sqrt{\Omega_0 (1+z)^3 + \Omega_\Lambda}} = \frac{c}{H_0 E(z)} \frac{1}{(1+z)} = \frac{c\,a}{H}$$

or:

$$l = a \int \frac{c\,dt}{a} \Rightarrow dl = c\,dt \qquad \text{(obvious!)}$$

$$dt = \frac{dt}{da} \frac{da}{dz} dz = \frac{1}{\dot a} a^2 dz = \frac{a}{H} dz$$

$$dl = \frac{a c}{H} dz \Rightarrow \frac{dl}{dz} = \frac{c}{H} a = \frac{1}{(1+z)} \frac{c}{H_0} \frac{1}{E(z)}$$

So:

$$\tau_{\nu_0} = \int_0^{z_e} n_{HI}(z) \frac{\pi e^2}{m_e c} f\, g(\nu_0(1+z)-\nu_{L\alpha}) \frac{c}{H_0} \frac{1}{E(z)} \frac{1}{(1+z)} dz$$

We want to integrate over the argument of g, and

$$d(\nu_0(1+z)-\nu_{L\alpha}) = \nu_0 dz$$

so we multiply and divide the integrand by ν_0.

All z-dependent functions vary smoothly, so they can be taken out of the integral, obtaining:

$$\tau_{\nu_0} \cong n_{HI}(z) \frac{\pi e^2}{m_e} f \frac{1}{H_0} \frac{1}{E(z)} \frac{1}{\nu_0(1+z)} \int_0^{z_e} g(\nu_0(1+z)-\nu_{L\alpha}) \nu_0 dz$$

$$\simeq n_{HI}(z) \frac{\pi e^2}{m_e} f \frac{1}{H_0} \frac{1}{E(z)} \frac{1}{\nu_{L\alpha}}$$

where z is such that $\nu_0(1+z) = \nu_{L\alpha}$ $\left(= \frac{c}{\lambda_{L\alpha}} = 2.47 \times 10^{15} s^{-1}\right)$

So:

$$\tau_\nu \simeq n_{HI}(z) \left(4.2 \times 10^{10} h^{-1} cm^3\right) \frac{1}{E(z)}$$

If $n_{HI} \simeq n_b$ (no He and no ionized H)

$$n_{HI}(z) = n_{b0}(1+z)^3 \cong 2.5 \times 10^{-7} cm^{-3} \left(\frac{\Omega_b h^2}{0.022}\right)(1+z)^3$$

$$\Rightarrow \quad \tau_{\nu_0} \simeq 10^4 h^{-1} \left(\frac{\Omega_b h^2}{0.022}\right) \frac{(1+z)^3}{E(z)}$$

at, say, $z=2$: $\tau_{\nu_0}(z=2) \simeq 1.3 \times 10^5$ $(h=0.67)$

This means that the flux bluewards of Lyman α should NOT be observed, unless hydrogen is highly ionized