Structure Formation and Spherical Collapse Model in Dark Energy models

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Outline

1. Standard cosmological model
2. Dark energy
3. Spherical collapse model
4. Hydrodynamics
5. Structure formation

Standard cosmological model

Cosmological parameters

$$\Omega_{\mathrm{m},0},\ \Omega_{\mathrm{q},0},\ h_0,\ w(a)$$

Density fluctuations

$$n \approx 1,\ \sigma_8,\ \mathrm{P(k)}$$

Nearly Gaussian and adiabatic initial density fluctuations

Dark energy I: generalities

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- Affects cosmic history and geometry
- Changes structure formation → number counts and growth factor
- $E^2(a) = \frac{\Omega_{\mathrm{r},0}}{a^4} + \frac{\Omega_{\mathrm{m},0}}{a^3} + \frac{\Omega_{\mathrm{K},0}}{a^2} + \Omega_{\mathrm{q},0} g(a)$
- $g(a) = \exp\left(-3 \int_1^a \frac{1+w(a')}{a'} \,\mathrm{d}a'\right)$

Dark energy II: inhomogeneous case

- $w_{\mathrm{c}}(a) = w_{\mathrm{b}}(a)$
- $\rho = \rho_b(1 + \delta)$

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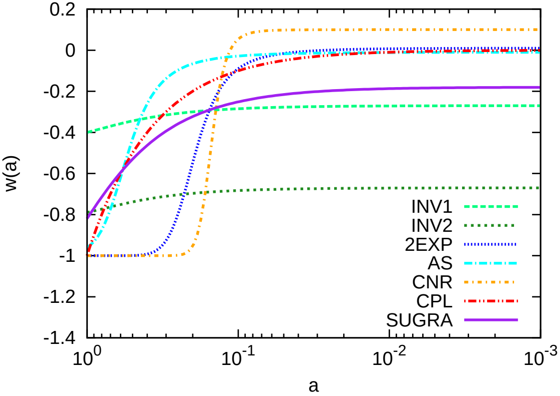
- $\delta P/\delta\rho = w_{\mathrm{eff}} c^2$
- $w_c = w + (w_{\mathrm{eff}} - w)\frac{\delta_{\mathrm{DE}}}{1+\delta_{\mathrm{DE}}}$

Equation of state $w(a)$



(Pace et al. 2010)

Spherical collapse model: generalities

- It follows the evolution of the overdense sphere
- Characterised by 4 parameters: a_{ta}, ζ, δ_{c}, Δ_{V}

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- Two different approaches
  1. Evolution of the radius of the sphere
  2. Evolution of the overdensity (hydrodynamical approach)

Two equations, for the scale factor and for the radius

$$
\begin{aligned}
\dot{x} &= \sqrt{\frac{\omega}{x} + \lambda x^2 g(x) + (1 - \omega - \lambda)} \\
\ddot{y} &= -\frac{\omega\zeta}{2y^2} - \frac{1 + 3w(x)}{2}\lambda g(x) y
\end{aligned}
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ζ and a_{ta} to be determined

$$\begin{aligned} \Delta &= \frac{\zeta x^3}{y^3} \\ \delta_{\mathrm{c}} &= \lim_{x \to 0}\left[\frac{D_+(x_{\mathrm{c}})}{D_+(x)}(\Delta(x) - 1)\right] \end{aligned}$$

Basics of General Relativity

Field's Equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \sum T_{\mu\nu} \Longleftrightarrow R_{\mu\nu} = \frac{8\pi G}{c^4}\left(\sum T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \sum T\right)$$

Energy-momentum tensor ($c = 1$)

$$T^{\mu\nu} = (\rho + P)\tilde{u}^{\mu}\tilde{u}^{\nu} + P g^{\mu\nu}$$

4-velocity

$\tilde{u} = \frac{\mathrm{d}x}{\mathrm{d}\tau}$ with normalization condition $< \tilde{u}^{\mu}, \tilde{u}^{\mu} >= g_{\mu\nu}\tilde{u}^{\mu}\tilde{u}^{\nu} = -1$

Local conservation of the energy-momentum tensor

$$\nabla_{\nu} T^{\mu\nu} = 0$$

Hydrodynamics

Newtonian metric

$$-c^2 \mathrm{d}\tau^2 = \mathrm{d}s^2 = -(1 + 2\Phi/c^2)c^2\mathrm{d}t^2 + (1 - 2\Phi/c^2)\mathrm{d}\vec{x}^2$$

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General Relativity

Newtonian limit

Continuity equation

$$\tilde{u}_\mu \nabla_\nu T^{\mu\nu} = 0$$

$$\frac{\partial\rho}{\partial t} + \nabla_{\vec{r}}\cdot(\rho\vec{v}) + \frac{P}{c^2}\nabla_{\vec{r}}\cdot\vec{v} = 0$$

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General Relativity | Newtonian limit

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Euler's equation

$$(g_{\alpha\mu} + \tilde{u}_\alpha\tilde{u}_\mu)\nabla_\nu T^{\mu\nu} = 0$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla_{\vec{r}})\vec{v} + \nabla_{\vec{r}}\Phi + \frac{c^2\nabla_{\vec{r}}P + \vec{v}\dot{P}}{\rho c^2 + P} = 0$$

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General Relativity	Newtonian limit
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Euler's equation	
$(g_{\alpha\mu} + \tilde{u}_\alpha \tilde{u}_\mu)\nabla_\nu T^{\mu\nu} = 0$	$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla_{\vec{r}})\vec{v} + \nabla_{\vec{r}}\Phi + \frac{c^2\nabla_{\vec{r}}P + \vec{v}\dot{P}}{\rho c^2 + P} = 0$
Poisson's equation	
$R_{00} = \frac{8\pi G}{c^4}\left(\sum_i T^i_{00} - \frac{1}{2}g_{00}\sum_i T^i\right)$	$\nabla^2\Phi = 4\pi G \sum_i \left(\rho_i + \frac{3P_i}{c^2}\right)$

Equations for the overdensity I: assumptions

- Continuity equation: $\dot{\bar{\rho}} + 3H\left(\bar{\rho} + \frac{P}{c^2}\right) = 0$
- Comoving coordinates: $\vec{x} = \vec{r}/a$
- Density: $\rho(\vec{x}, t) = \bar{\rho}(1 + \delta(\vec{x}, t))$
- Equation of state: $P(\vec{x}, t) = w(t)\rho(\vec{x}, t)c^2$ (homogeneous dark energy)
- $\vec{v}(\vec{x}, t) = a[H(a)\vec{x} + \vec{u}(\vec{x}, t)]$

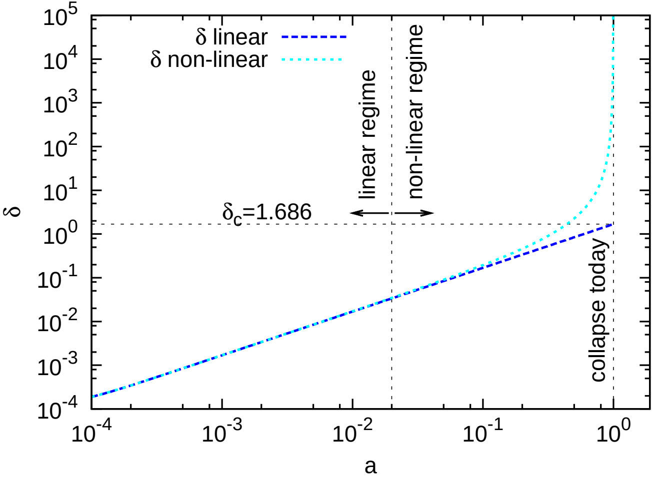
Equations for the overdensity II: complete system

$$\delta_j'' + \left(\frac{3}{a} + \frac{E'}{E} - \frac{w_j'}{1+w_j}\right)\delta_j' - \frac{4+3w_j}{3(1+w_j)}\frac{\delta_j'^2}{1+\delta_j} - \frac{3}{2a^2E^2(a)}(1+w_j)(1+\delta_j)\sum_k \Omega_{\mathrm{k},0}g_k(a)(1+3w_k)\delta_k - \frac{1}{aE^2(a)}(1+w_j)(1+\delta_j)(\sigma^2-\omega^2) = 0$$

$$\delta_j'' + \left(\frac{3}{a} + \frac{E'}{E} - \frac{w_j'}{1+w_j}\right)\delta_j' - \frac{3}{2a^2E^2}(1+w_j)\sum_k \Omega_{\mathrm{k},0}g_k(a)(1+3w_k)\delta_k - \frac{1}{aE^2(a)}(1+w_j)(\sigma^2-\omega^2) = 0$$

Equations for the overdensity III: DM only

$$\begin{aligned} \delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' - \frac{4}{3}\frac{\delta'^2}{1+\delta} - \frac{3}{2a^5E^2(a)}\delta(1+\delta)\Omega_{\mathrm{m},0} &= 0 \\ \delta'' + \left(\frac{3}{a} + \frac{E'}{E}\right)\delta' - \frac{3}{2a^5E^2}\Omega_{\mathrm{m},0}\delta &= 0 \end{aligned}$$


δ linear
δ non-linear
linear regime
non-linear regime
δ_c=1.686
collapse today
δ
a
10^5
10^4
10^3
10^2
10^1
10^0
10^{-1}
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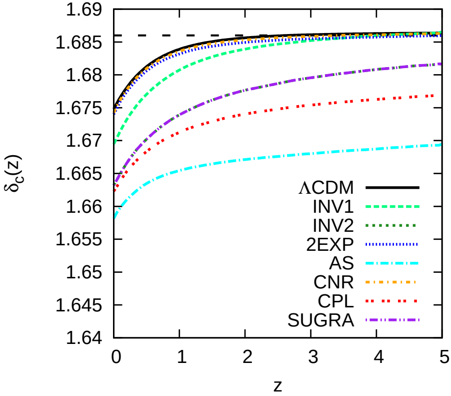
δ_c I: perturbations in DM only



(Pace et al. 2010)

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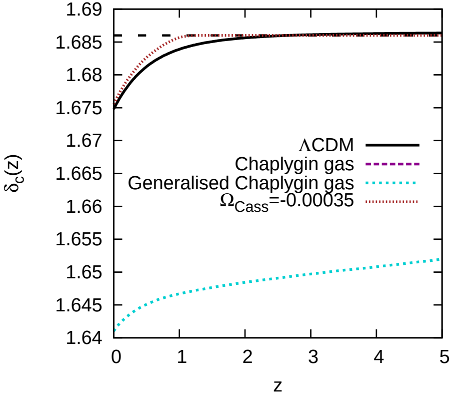

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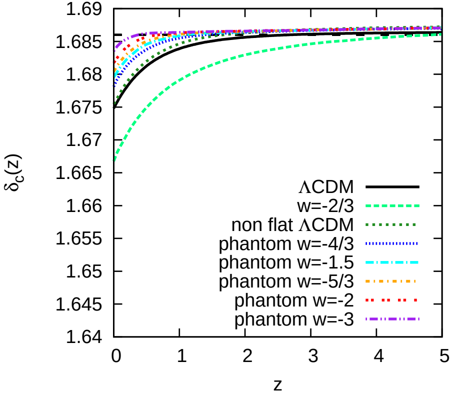

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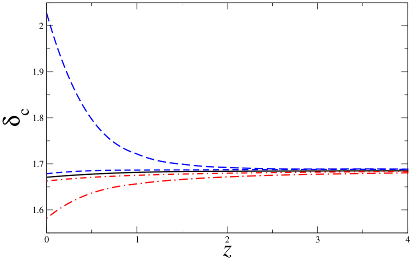
δ_c II: perturbations in DM & DE



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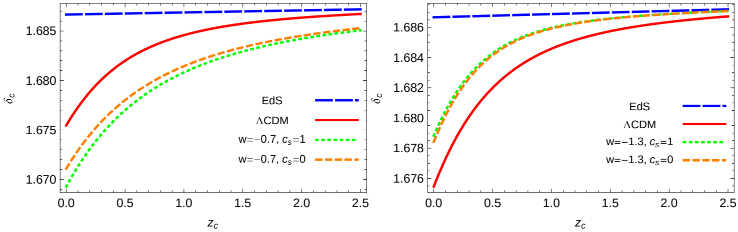

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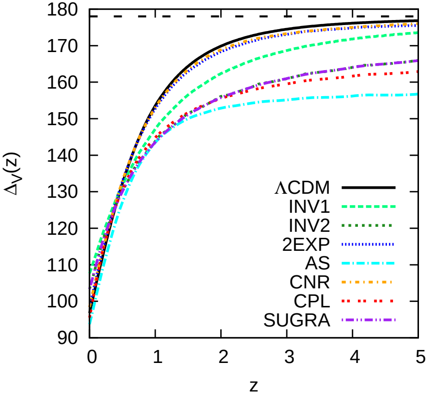
Δ_V



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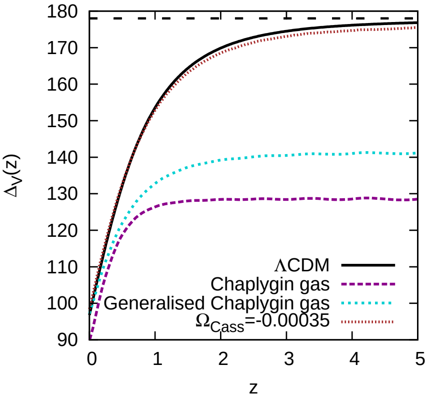

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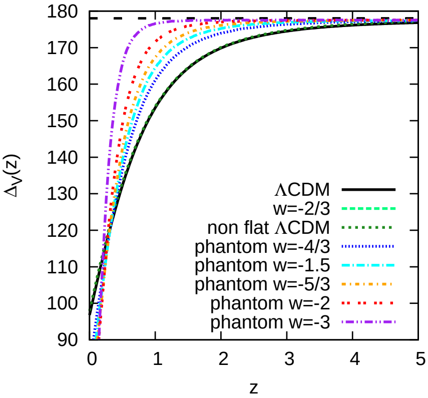
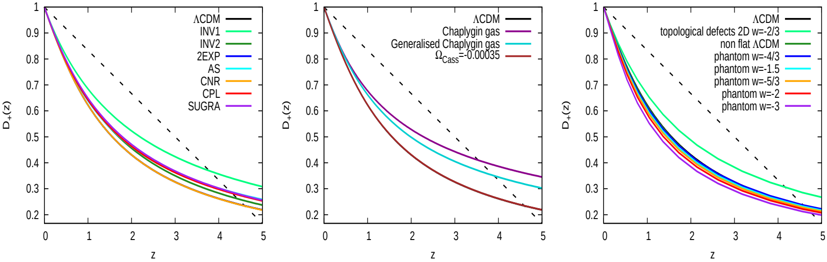


(Pace et al. 2010)

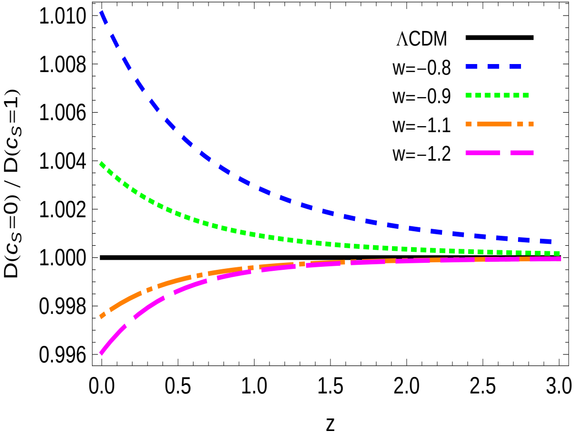
Growth factor

$$
\begin{aligned}
D'' + \left(\frac{3}{a} + \frac{E'}{E}\right)D' - \frac{3}{2a^5E^2}\Omega_{\mathrm{m},0}D &= 0 \\
D_{\mathrm{in}} &= a^n \\
n^2 + \left(2 + \frac{aE'}{E}\right)n - \frac{3}{2a^5E^2}\Omega_{\mathrm{m},0} &= 0
\end{aligned}
$$

Growth factor I: DM perturbations

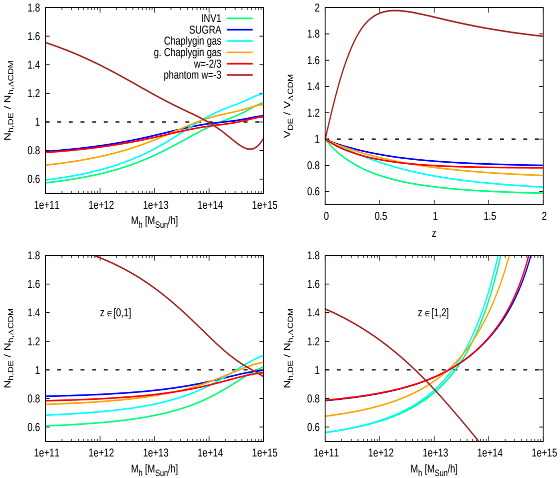


Growth factor II: perturbations in DM & DE



(Creminelli et al. 2010)

Mass function I: perturbations in DM only



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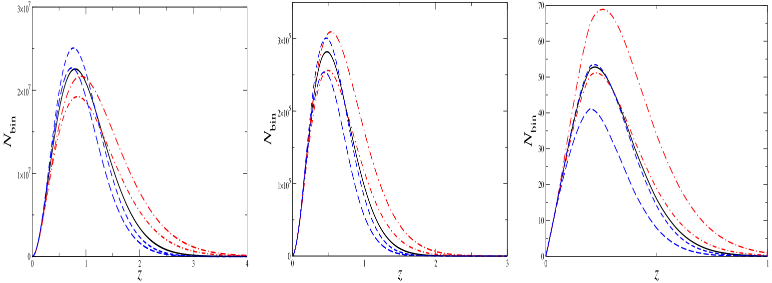
Mass function II: perturbations in DM & DE



$10^{13} < M/(M_\odot/h) < 10^{14}$ $\quad$ $10^{14} < M/(M_\odot/h) < 10^{15}$ $\quad$ $10^{15} < M/(M_\odot/h) < 10^{16}$

(Abramo et. al 2007)

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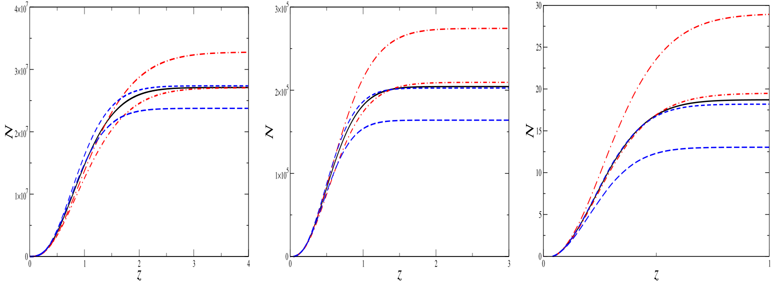
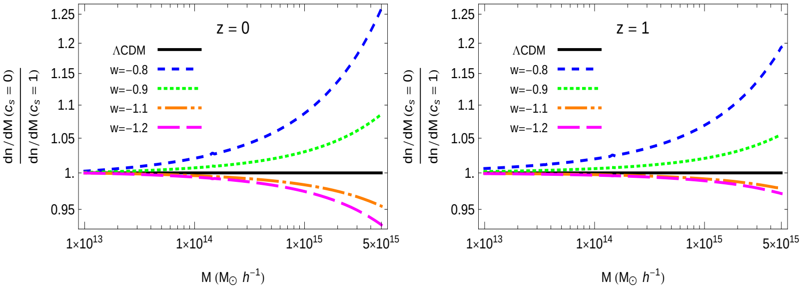


$M/(M_\odot/h) > 10^{13}$ $M/(M_\odot/h) > 10^{14}$ $M/(M_\odot/h) > 10^{15}$

(Abramo et. al 2007)

Mass function III: perturbations in DM & DE



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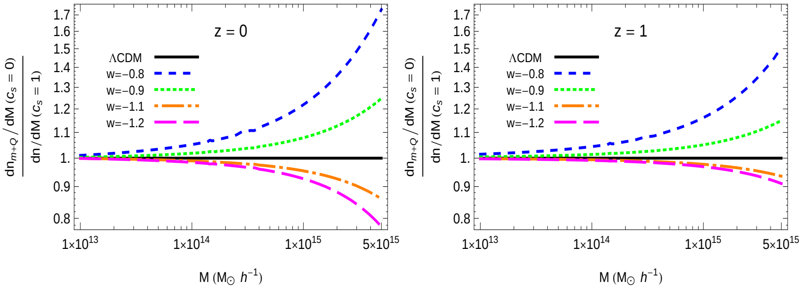

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- Stronger deviations for clumping DE
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- DE mass component affects mass function
- Easy to include ellipticity and rotation
- Possibility to extend the formalism to modified gravity models