Galactic flat rotational curves and Pioneers' anomalous acceleration

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Based on

Possible relation between galactic flat rotational curves and the Pioneers' anomalous acceleration, New Astron. **12** *(2006) 142-145*

and on articles by Milgrom, Sanders, Bekenstein, McGaugh, Anderson et al., and others

Sections

- Flat rotational curves, MOND and dark matter.
- The Pioneers' anomaly.
- A possible relation? A simple model.

Flat rotational curves

Spiral galaxies present almost flat peripheral rotational curves. The velocity is measured using the emmission lines (21 cm) from atomic hydrogen and removing the average redshift. The remaining, point dependent redshift is due to the Doppler effect, that is to the relative velocity of the particle with respect to the center of the galaxy.

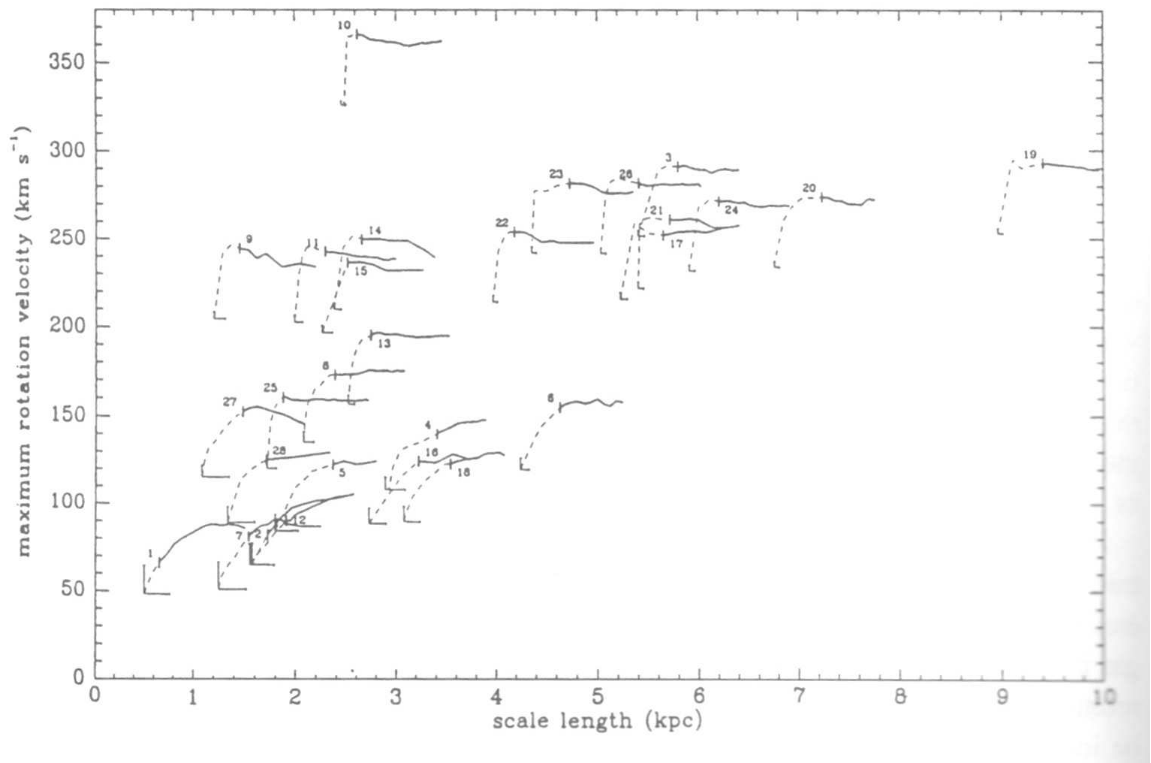


Figure taken from Casertano and Van Gorkom (1991).

Flat rotational curves II

- The hydrogen emission line is ideally suited for measurements of the rotational curves, indeed hydrogen extends well beyond the optical disk radius.
- With only few exceptions (NGC 2683 and NGC 3521) there is no evidence of a decreasing profile outside the optical disk.
- The hydrogen mass outside the optical disk is negligible with respect to the visible baryonic mass inside the optical disk. Thus $M_{galaxy}(r) \to M = cnst$. Thus Newton's law would predict for the peripheral velocity of hydrogen atoms

$$m_H a = G \frac{m_H M}{r^2}$$

$$\frac{V^2(r)}{r} = G \frac{M}{r^2} \Rightarrow V(r) = \sqrt{\frac{GM}{r}}$$

$$\text{thus } V(r) \to 0 \text{ as } r \to +\infty$$

Flat rotational curves III

In general it is difficult to find $M(r)$ because it depends on the assumed mass-to-light ratio.

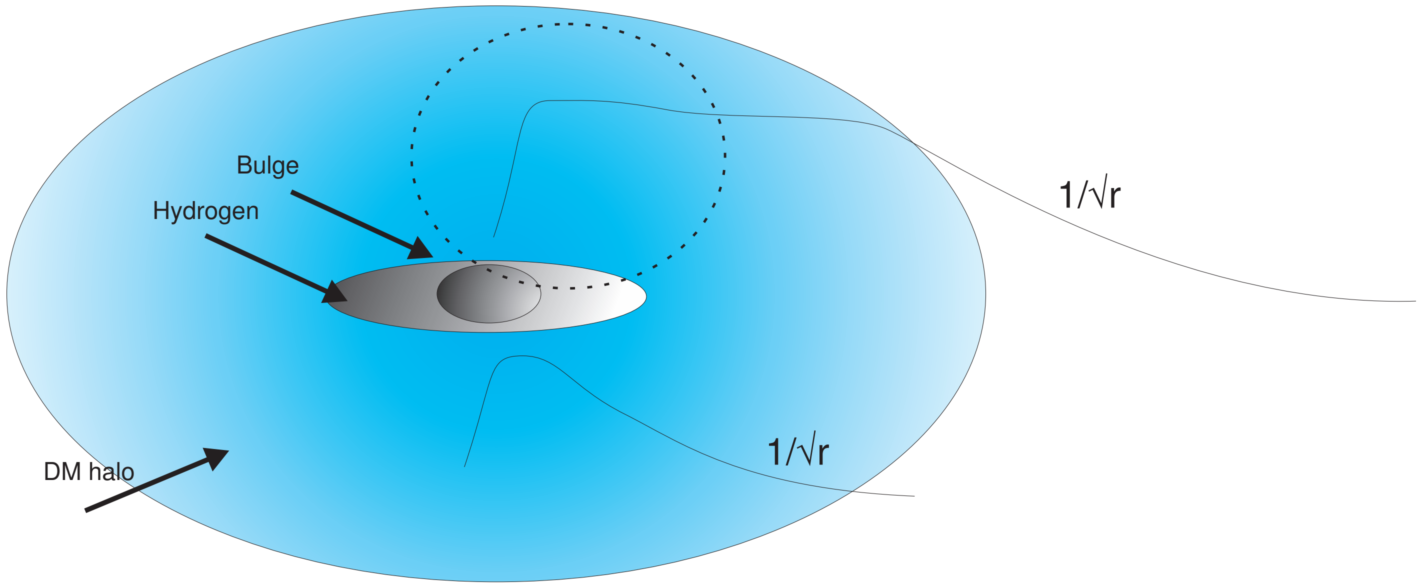
Nevertheless, if most of the mass is inside the optical radius then the assumption $M(r) \rightarrow const.$ still holds.

The problem of the previous inconsistency is solved along two strategies

(a) Dark Matter. The visible mass is not the whole mass and hence the function $M(r)$ may differ considerably from the one deduced from the visible matter. Still $M(r) \rightarrow cnst.$ but this happens at a length scale which is not probed by the visible mass.

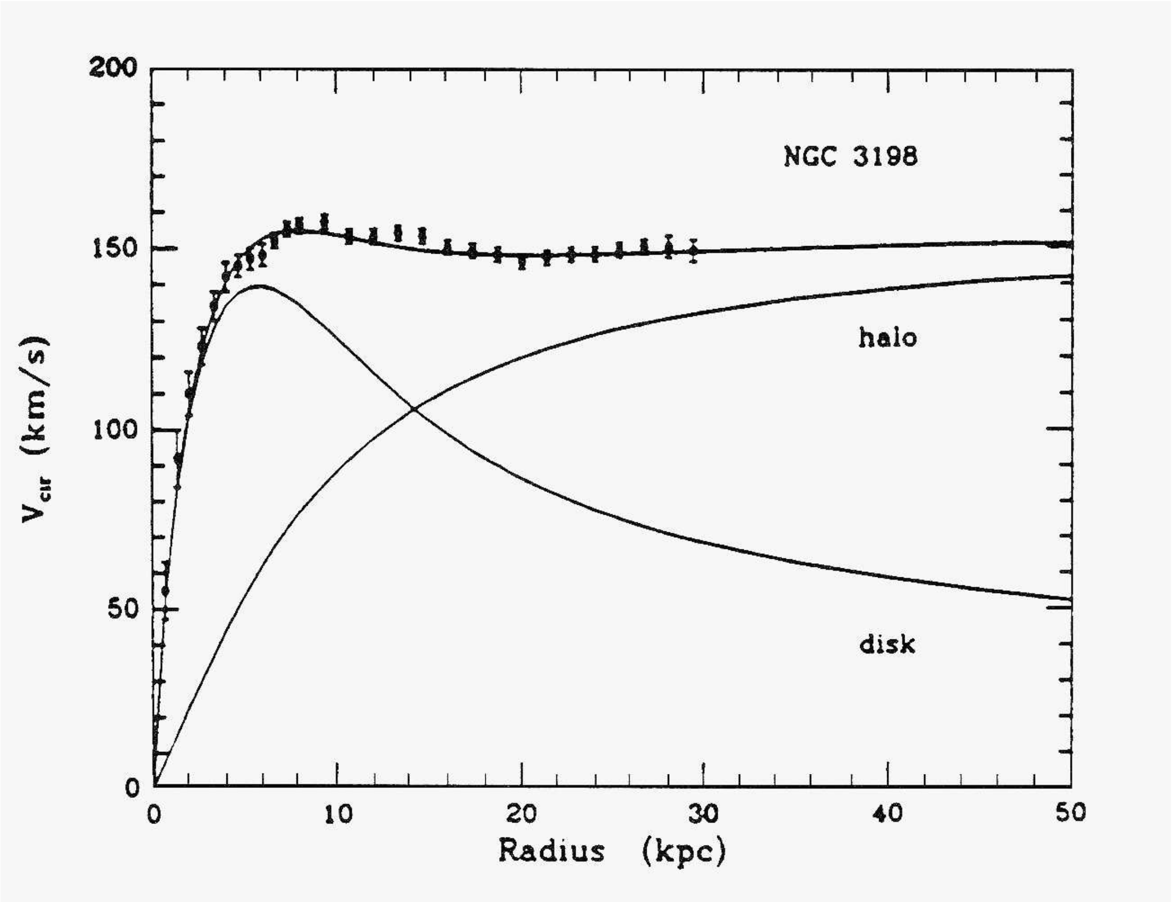
(b) The Newton's force law does not hold at the galactic scale.

Dark matter



- Since $V^2 \sim GM(r)/r$, it must be $M(r) \sim r$ at least inside the observed region. In particular the dark matter is predominant over the visible matter in the outer part and negligible in the inner part of the optical disk. The model is supported by the fact that an isothermal sphere has this distribution.
- Unfortunately, the numerical simulations do not give $\rho(r) \propto 1/r^2$ but rather the Navarro, Frenk and White profile $1/r^3$.

Dark matter II



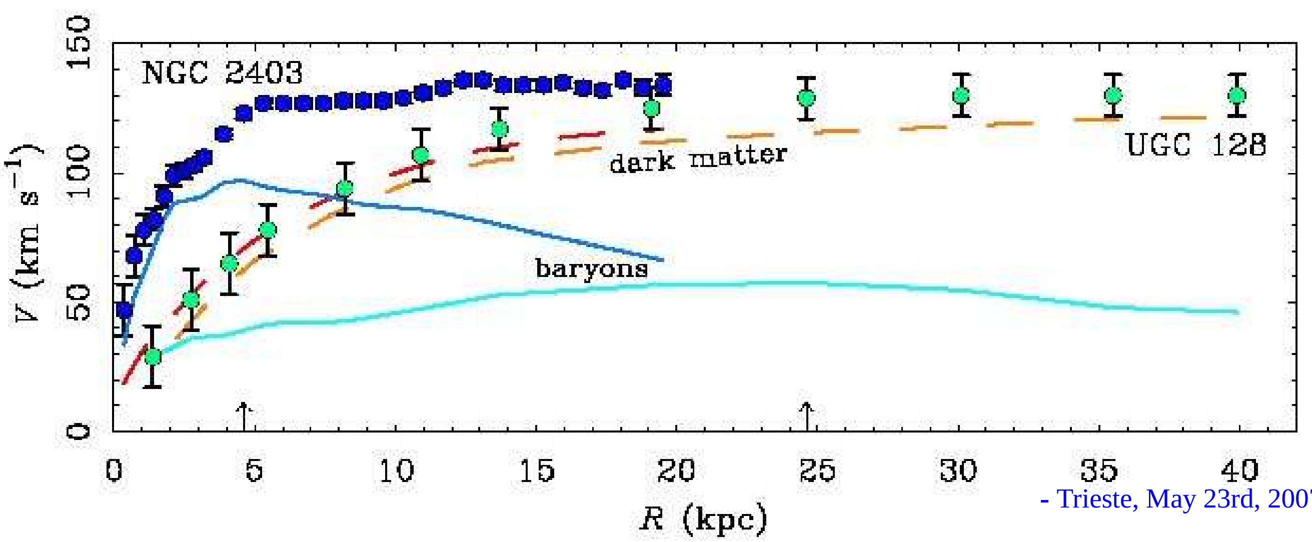
- If the dynamics of the dark matter halo and the baryonic matter are decupled how can they cospire to give a nearly flat profile? The inner part of the velocity profile depends only on the stellar disk while the outer part only on the dark matter halo (recall $M(r) \sim r$ for the DM halo). There is a problem of fine tuning (Conspiracy).

Tully-Fisher law

Spiral galaxies obey the Tully-Fisher law

$$L \propto V^p$$

where $p \in [2.5, 5]$, with the smallest scatter in the near-infrared where $p \sim 4$. This relation has been improved showing that L, the luminosity, can be replaced with the baryonic mass (giving the Baryonic Tully-Fisher relation). The puzzling fact is that V, the peripheral, maximum, or other similar characteristic velocity, is independent of the distribution of the mass, and in particular of the scale of the galaxy.



Conspiracy: a closer look at a study by S. McGaugh.

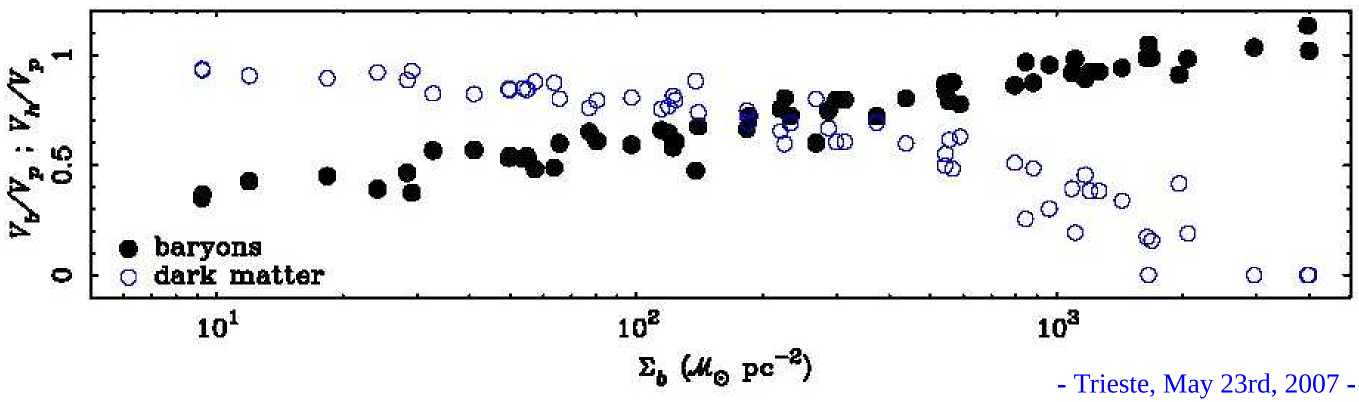
Let V_p be the observable peak in the velocity and R_p the corresponding radius. Since the Tully-Fisher law holds it is observed that V_p is independent of R_p. However, if the Newton force law holds, given a set of galaxies with about the same (baryonic) mass

$$V_p^2 \propto \frac{M_b}{R_p} \Rightarrow \frac{d \ln V_p}{d \ln R_p} = -\frac{1}{2}$$

instead of 0. One may argue that most of the mass is dark but this is not so, defined

$$V_b^2 \propto \frac{M_b}{R_p}; \quad V_{DM}^2 \propto \frac{M_{DM}}{R_p}$$

and $V_p^2 = V_b^2 + V_{DM}^2$ so that V_p is almost independent of R_p. It is found that V_b/V_p is correlated with the baryonic surface density. The more the baryons the less the dark matter needed.



Modify Newton’s force law

First idea, to change the distance dependence

$$\frac{GM}{r^2} \to \frac{GM}{r^2}[1 + f(r/r_0)]$$

with $f(x) \sim x$ for $x \gg 1$ and $f(x) \sim 0$ for $x \ll 1$.
Usual argument: it does not work because for large r

$$\frac{V^2}{r} = \frac{GM}{rr_0} \Rightarrow M \propto V^2$$

wrong exponent in Tully-Fisher! But note the tacit assumption that r_0 is a universal constant independent of M.

Moreover, large galaxy clusters show moderate mass discrepancy while small low surface brightness (LSB) galaxies exhibit large discrepancies.

A figure from Sanders (2002)

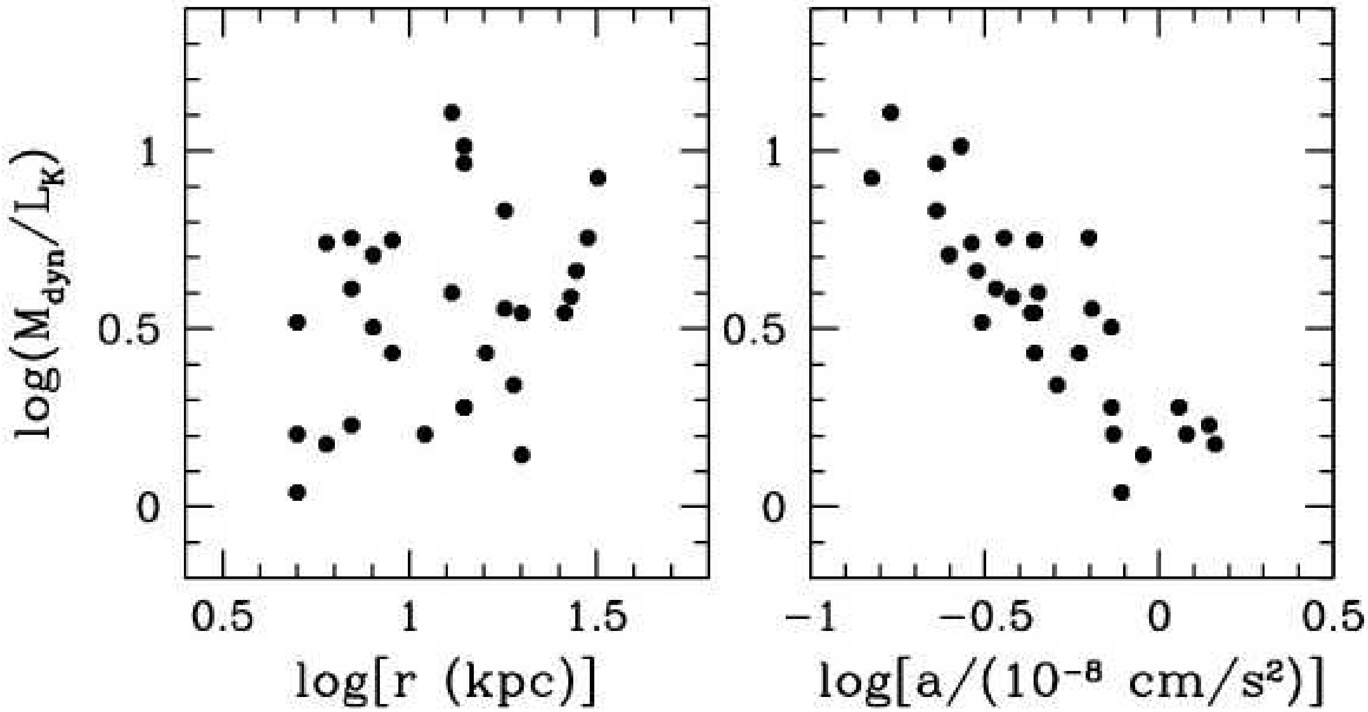


Figure 1: The global Newtonian mass-to-K'-band-luminosity ratio of Ursa Major spirals at the last measured point of the rotation curve plotted first against the radial extent of the rotation curve (left) and then against the centripetal acceleration at that point (right).

MOdified Newtonian Dynamics (MOND)

There is an explanation which does not require fine tuning. It is MOND.

- Theory developed by Milgrom in 1983. It focuses on the acceleration: if it goes below a universal acceleration a_0 then there is a departure from Newton's law which may be interpreted as a mass discrepancy.
- It is based on an ad hoc assumption made to recover the Tully-Fisher law

$$\mathbf{a} = \mathbf{a}_N \rightarrow \mu(a/a_0)\mathbf{a} = \mathbf{a_N}$$

where $\mu(x) = 1$ for $x \gg 1$ and $\mu(x) = x$ for $x \ll 1$.
It "predicts" flat rotational curves and TF rather easily

$$(\frac{V^2}{r})^2 \frac{1}{a_0} = G\frac{M}{r^2} \Rightarrow V^4 = (Ga_0)M$$

which holds whenever the peripheral acceleration goes well below a_0.

- Quite remarkably $a_0 = 1.2 \times 10^{-8} cm/s^2 \simeq H_0 c/7$. Is there a connection between the dynamics of galaxy and cosmology? This question is open and will get even more intriguing when we shall consider the Pioneer anomaly.

Should we take $a_0 \simeq H_0 c$ seriously?

Milgrom warns us from giving too much importance to the approximate relation $a_0 \simeq H_0 c$ or

$$a_0 \simeq c^2/R_H \,,$$

with R_H the Hubble length.
He reminds us of the relation between the acceleration g at the earth surface, the radius of the earth R_e and the escape velocity from earth c_e

$$g = c_e^2/(2R_e)$$

It has the same shape, but it is an effective relation, none of the quantities involved is fundamental. People living at the surface of the earth and with no knowledge of the exterior space would give too much importance to such a relation.

MOND: Lagrangian formulation

- There is a dual almost equivalent description of MOND. Instead of modifying the inertia one can equivalently modify gravity $\mathbf{a} = f(a_N/a_0)\mathbf{a_N}$ they are not equivalent if other types of interactions are considered.
- There is a nice Lagrangian formulation (Bekenstein-Milgrom)

$$S = S_\phi + S_k + S_{in} = -(8\pi G)^{-1} a_0^2 \int F[(\nabla\phi)^2/a_0^2] \mathrm{d}^3 r \mathrm{d}t + \frac{1}{2}\sum_i m_i \int v_i^2 \mathrm{d}t - \int \rho\phi \, \mathrm{d}^3 r$$

which gives instead of the Poisson equation (here $\mu(x) = \mathrm{d}F(y)/\mathrm{d}y|_{y=x^2}$, $F(y) = y^{3/2}$ in the MOND regime and $F(y) = y$ in the Newtonian regime)

$$\nabla \cdot [\mu(|\nabla\phi|/a_0)\nabla\phi] = 4\pi G\rho$$

here $\mathbf{a} = -\nabla\phi$ and $\mathbf{a}_N = -\mu(|\nabla\phi|/a_0)\nabla\phi$, so that $\mu(a/a_0)\mathbf{a} = \mathbf{a}_N$. It has the shape of a continuity equation for a irrotational and compressible fluid (the Poisson equation is obtained in the incompressible limit).
- Being a Lagrangian formulation it admits conservation laws depending on the symmetries.
- Note that at the Lagrangian level it is a modified gravity theory but at the Newton law level it is a modified inertia theory.

MOND: conformal invariance

The replacement of the space metric $\delta_{ij} \to \alpha(x)\delta_{ij}$ sends

$$\mathrm{d}r^3 \to \alpha^{3/2}\mathrm{d}r^3$$

$$(\nabla\phi)^2 \to \alpha^{-1}(\nabla\phi)^2$$

$$\rho \to \alpha^{-3/2}\rho$$

thus the action

$$\int \{-(8\pi G)^{-1}a_0^2 F[(\nabla\phi)^2/a_0^2] - \rho\phi\}\mathrm{d}^3r\mathrm{d}t$$

is conformal invariant in the deep MOND regime where $F(y) = y^{3/2}$. It is still unclear if this conformal invariance has any fundamental meaning. It means that if the acceleration is very small the dynamics is independent of the length scale.

Another prediction of MOND

There is critical value of the surface density

$$\Sigma_m \simeq a_0/G$$

above this value the internal acceleration is in the Newtonian regime (the stars are close together and there are strong ($> a_0$) accelerations between them). Thus in HSB galaxies there should be low mass discrepancies within the optical disk, while for LSB there must be high mass discrepancies. This is the case, HSB galaxies have a Keplerian profile, the rotation curve decline (Keplerian implies $v(r) \sim 1/\sqrt{r}$) and reaches the asymptotic limit, while LSB galaxies have an increasing profile.

Another figure from Sanders (2002)

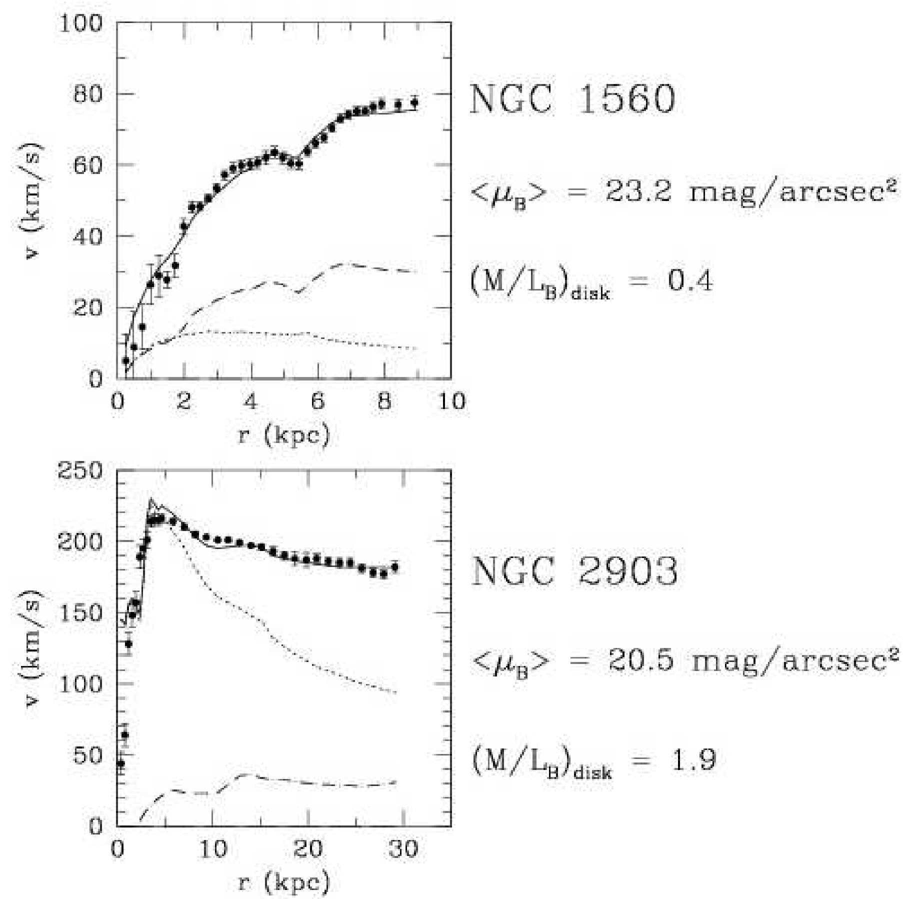


Figure 3: The points show the observed 21 cm line rotation curves of a low surface brightness galaxy, NGC 1650 (Broeils 1992) and a high surface brightness galaxy, NGC 2903 (Begeman 1987). The dotted and dashed lines are the Newtonian rotation curves of the visible and gaseous components of the disk and the solid line is the MOND rotation curve with $a_o = 1.2\times10^{-8}$ cm/s^2– the value derived from the rotation curves of 10 nearby galaxies (Begeman

Fitting rotation curves with MOND

The main success of MOND is its ability to fit even the smallest details of galaxy rotation curves with only the L/M ratio as free parameter.

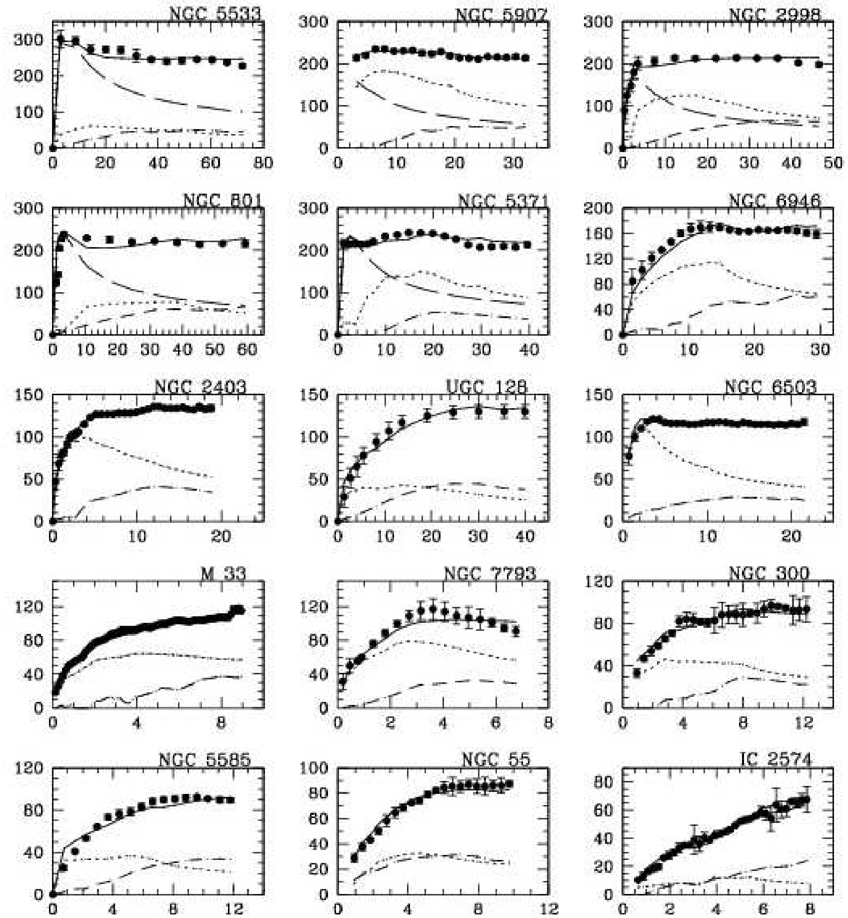


Figure from Sanders (1996) and Block & McGaugh (1998).

Difficulties with MOND

- It is basically a Newtonian spacetime theory, there are relativistic versions but they are quite complicated and not entirely satisfactory. This is naturally so, because a body in free fall in the general relativistic paradigm has *no acceleration* thus all the acceleration entering in MOND are with respect to a Newtonian background.
- It is uncertain as to whether a relativistic formulation can account for gravitational lensing without introducing dark matter.
- Difficult fit of a few rotational curves e.g. NGC 2841.
- MOND predicts incorrect mass discrepancies in galaxy clusters. Some dark matter seems needed.

The Pioneer anomaly

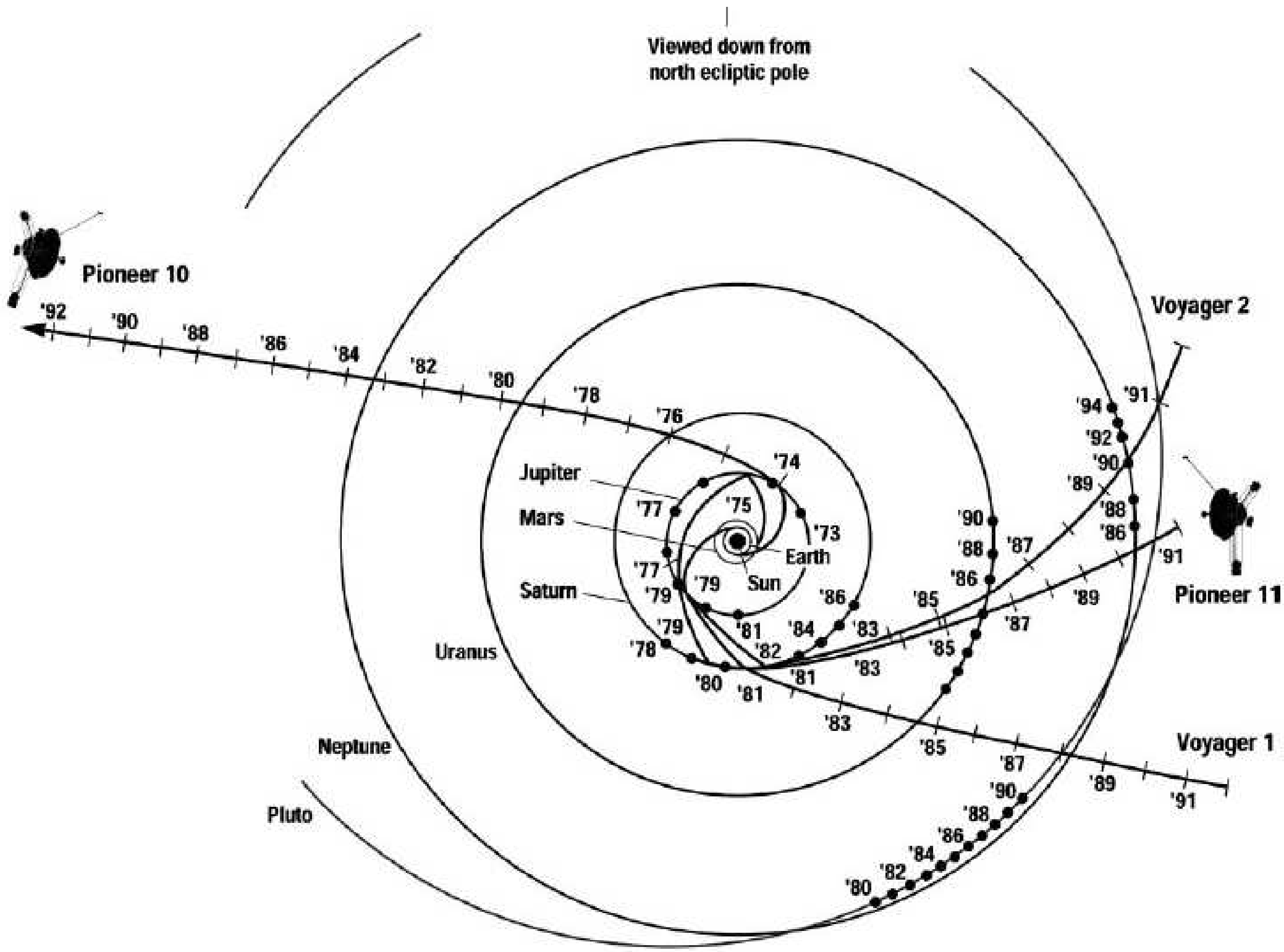
- Pioneer 10, launched in 1972 and Pioneer 11 launched in 1973 after the encounter, respectively, with Jupiter and Saturn, followed hyperbolic orbits near the plane of the ecliptic to the opposite sides of the solar system.
- Since they were spin stabilized no control from earth was required (due to earth controlled manoeuvre the Voyagers have a more complicated telemetry).
- Beginning in 1980 at about 20 AU from the Sun the solar radiation pressure decreased below $5 \times 10^{-8} cm/s^2$ and a steady (from 20 AU to 70 AU) unexpected acceleration of about

  $$a_P = 8.5 \times 10^{-8} cm/s^2$$

  towards the Sun began to be observed. It was considered a curiosity up to 1998 where a first study in Phys. Rev. Lett. brought it to the attention of the scientific community. A detailed report appeared in 2002 in Phys. Rev. D.
- In 1990 at about 30 AU from the Sun the Pioneer 11 radio system failed so since then no further Doppler data is available. For Pioneer 10, the data was collected up to 1998 and the last signal was received on January 2003.
- Similar anomalous accelerations were detected for Ulysses and Galileo spacecraft but the results are not conclusive.

The trajectories

Figure from Anderson et al. (2002)

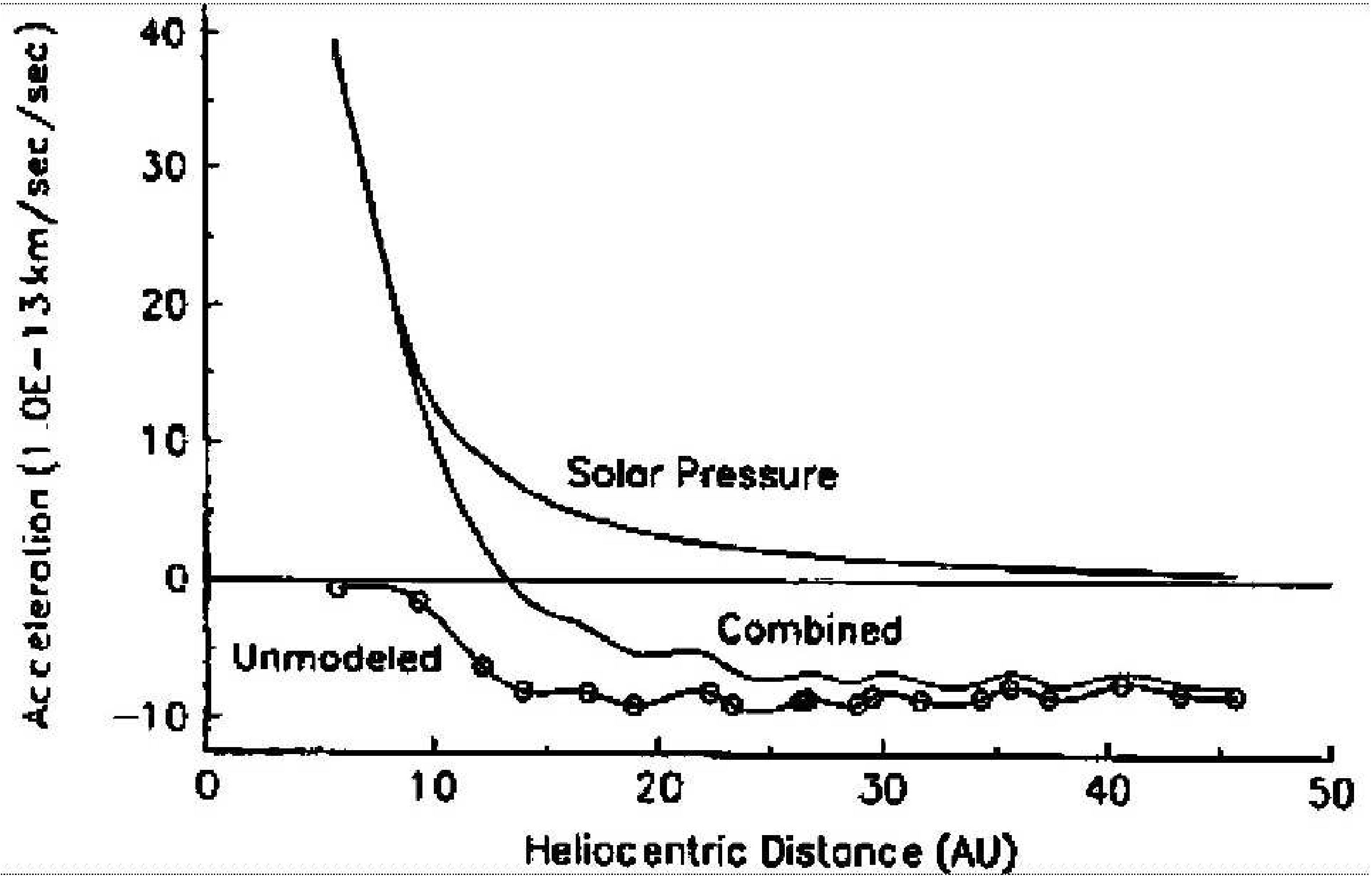


The data and its interpretation

- A stable signal at about 2.2GHz produced by the *DSN timing system* is sent to the spacecraft, here it is multiplied by 240/221 by a transponder and sent back to earth. The signal is modulated so as to give at the same time not only Doppler data, but also ranging data.
- The velocity is inferred from the Doppler data. Of course there is redshift, and since the redshift decreases the spacecraft is actually slowing down its velocity. The problem is that it decreases more that expected, i.e. there is an unexpected *blue drift*.
- The ranging data gives directly the distance of the spacecraft. There are less measures but they must be compatible with the integrated Doppler data.
- The data so obtained is compared with that expected from the post-Newtonian equations of motion.

The anomalous acceleration

Figure from Anderson et al. (2002)



So far no explanation for the anomaly.

Just a list of checked possibilities

- Heat reflecting off the spacecraft
- Kuiper's belt gravity
- Frequency stability of clocks
- Solar radiation pressure and wind
- Propulsive explusion of gas
- Dark matter
- Electromagnetic Lorentz forces
- Drag caused by dust or unseen material

Main difficulty

- The anomaly is too large to have gone undetected in planetary orbits. The Viking mission provided radio-ranging measurements to an accuracy of about 12 m.

Thus *if the anomaly is a real acceleration then the equivalence principle does not hold*.

The ubiquitous acceleration $H_0 c$

There is a numerical coincidence between $a_P/c = (2.8 \pm 0.4) \times 10^{-18}\mathrm{s}^{-1}$ and the Hubble constant,

$$H_0 = (72 \pm 8)\mathrm{km}/(\mathrm{s\ Mpc}) = (2.3 \pm 0.3) \times 10^{-18}\mathrm{s}^{-1}.$$

but several authors confirm that according to general relativity the expansion of the universe as no affect, at least not at this order, in the dynamics inside the solar system.
Recall that the Tully-Fisher relation can be written

$$V_\infty^4 = (Ga_0)M$$

and $a_0 \simeq K_1^2 a_p$ with $K_1^{-2} \simeq 7$.

- We have no idea why $H_0 c$ enters the dynamics of galaxies and the solar system, however, *can we at least show that a_0 and a_P should be related?*

A study of a minimal modification of Newton's law

Consider a minimal modification of Newton's law

$$V(r) = -G\frac{m}{r}(1 + f(\beta r)),$$

where $\beta \in [L]^{-1}$ is the new dimensional parameter, and $f(x)$ a function, with x dimensionless parameter. Add two conditions

(i) Test particles in the field of an heavy body of mass m have at small distances an acceleration $\mathbf{a} = a(r)\hat{\mathbf{e}}_r$ with $a(r) = -G\frac{m}{r^2} - a_P$, where a_P is a universal constant that does not depend on m, and r is the distance between the test particle and the mass m.

(ii) $\lim_{r \to +\infty} ra(r) = -v_\infty^2 = cnst.$

Condition (i) leads to the Pioneer anomaly, condition (i) to galactic flat rotational curves.

Constraints on the function f imposed by (i) and (ii)

$$V(r) = -G\frac{m}{r}(1 + f(\beta r)),$$

First, the Newtonian limit implies $f(0) = 0$. Note that since V is defined only up to a constant, the function $f(r)$ is defined only up to linear terms in r. Note also that there is a rescaling freedom in the definition of f and β, indeed let $\lambda \in \mathbb{R} - \{0\}$ and redefine

$$\begin{aligned} \bar{f}(x) &= f(\lambda x), \\ \bar{\beta} &= \beta/\lambda, \end{aligned}$$

then $\bar{f}(\bar{\beta}r) = f(\beta r)$. Under the assumption $|f''(0)| \neq 0$, we use this freedom to fix $|f''(0)| = 2$ and $\beta > 0$.

Condition (i)

The acceleration field $\mathbf{a} = a(r)\hat{\mathbf{e}}_r = -\nabla V$ is given by (the derivatives are with respect to x)

$$a(r) = -G\frac{m}{r^2}(1+f) + G\frac{m\beta}{r}f'.$$

Let us consider the condition (i). Taylor expanding $f(x)$ and $f'(x)$ at $x=0$ we obtain the acceleration field at small distances

$$a(r) = -\frac{Gm}{r^2} + \frac{Gm}{2}\beta^2 f''(0) + Gm\beta^2 O(\beta r). \tag{1}$$

The condition (i) is satisfied iff $f''(0) < 0$, which due to our normalization implies $f''(0) = -2$, and

$$\beta^2 = a_P/Gm$$

where a_P is a universal constant independent of m. Consider the spacecraft Pioneer in the solar system. The last term on the right-hand side of Eq. (1), is much smaller than the second one since $a_P/(Gm_\odot/d^2) < 10^{-3}$, where $d < 87AU$ is the Pioneer distance from the Sun and $m_\odot$ is the mass of the Sun.

Condition (ii)

Recall

$$a(r) = -G\frac{m}{r^2}(1+f) + G\frac{m\beta}{r} f'.$$

The condition (ii) implies ($x = \beta r$)

$$\lim_{r\to+\infty} \frac{r a(r)}{G m \beta} = \lim_{x\to+\infty} (f' - f/x) = -\frac{v_\infty^2}{G m \beta}.$$

Note that x is a dimensionless parameter, it follows that as $r \to \infty$, $f(\beta r) \to f_\infty(\beta r)$ a function that solves the differential equation

$$f'_\infty - f_\infty/x = -K_1, \qquad K_1 \in \mathbb{R}^+,$$

and $v_\infty^2 = K_1 G m \beta$. Using the relation between β and a_P we obtain the Tully-Fisher relation

$$v_\infty^4 = (K_1^2 a_P G) m,$$

which expresses the proportionality between the mass (and hence the luminosity) of the spiral galaxy and the fourth power of the asymptotic rotational velocity.

Dimensional argument

By the dimensional argument K_1 must be of the order of unity and indeed we find $K_1^{-2} \simeq 7$. In conclusion

- *Given a minimal modification of the Newtonian potential, if the Pioneer anomaly is assumed for test particles in the field of an heavier body (condition (i)) and the galactic flat rotational curves are assumed (condition (ii)) then the Tully-Fisher relation follows with the right order of magnitude for the coefficient of proportionality.*

Thus if the Pioneer anomaly is real, the Pioneer anomalous acceleration and the coefficient of the Tully-Fisher relation have to be related in the found experimental way.

The relation with MOND

The modified potential dynamics is related to the MOND theory Let us introduce the Newtonian acceleration $g_N = Gm/r^2$, the MOND characteristic acceleration $a_0 = K_1^2 a_P$ and the function $z(y)$, (with $y = 1/(K_1 x)^2 = g_N/a_0$)

$$z(y) = y[1 + f(\frac{1}{K_1\sqrt{y}})] - \frac{\sqrt{y}}{K_1} f'(\frac{1}{K_1\sqrt{y}}),$$

then

$$a(r) = -G\frac{m}{r^2}(1+f) + G\frac{m\beta}{r}f'.$$

can be rewritten

$$a/a_0 = -z(g_N/a_0).$$

If $z(y)$ has an inverse $I(z) = z\mu(z)$, $I(z(y)) = y$, it reduces to a MOND theory where

$$\mu(z) \sim 1 - \frac{1}{K_1^2 z}, \quad \text{as} \quad z \to +\infty.$$

Since the differences between MOND and our derived dynamics are only minimal these calculations prove that a MOND theory subject to the above constraint follows from (i) and (ii).

Is there any trace of the Pioneer anomaly in galactic rotational curves?

We have seen that the Pioneer anomaly leads to the right TF relation even with the right value of the proportionality constat. Everything seems fine but are the rotational curves compatible with the additional constraint

$$\mu(z) \sim 1 - \frac{1}{K_1^2 z}, \quad \text{as} \quad z \to +\infty.$$

Recent studies of the best μ give some information.
The simple choice (Famaey and Binney)

$$\tilde{\mu}(z) = z/(1+z) \Rightarrow anomalous\ acceleration$$

is particularly successful in fitting the Milky Way and the galaxy NGC3198, in particular, it proved superior than the traditional choice

$$\breve{\mu} = z/\sqrt{1+z^2} \Rightarrow no\ anomalous\ acceleration.$$

But the function $\tilde{\mu}$ gives good results only up to values $z \lesssim 5$, while for the Pioneers we are in the range $z \sim 10^3$. At that range Famaey and Binney argue that a transition should have already taken place to the function $\breve{\mu}$.
Thus it seems that the Pioneer anomalous acceleration does not show up in the dynamics of galaxies.

Conclusions

- The imposition of the Pioneer anomaly and the flat rotational curves leads naturally to Tully-Fisher with the right coefficient. This fact strongly suggests a connection between the two most interesting departures from Newtonian gravity.
- The implied gravitational theory is MOND subject to an additional constraint on μ.
- However, a more detailed analysis of the rotational curves proves that this constraint is not satisfied. The Pioneer anomaly seems not present in the rotational curves of Galaxies.

The problem of a possible connection between the Pioneer anomaly and the flat rotational curves of galaxies is open.