

# Neutrino mass and neutrino dark matter

Do non-relativistic neutrinos  
constitute the dark matter ?

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# Outline

Introduction: What is dark matter DM

A modeling in virial equilibrium

Comparison to a galaxy cluster

Mass, properties, name of DM particle

Nucleosynthesis

About virial equilibrium

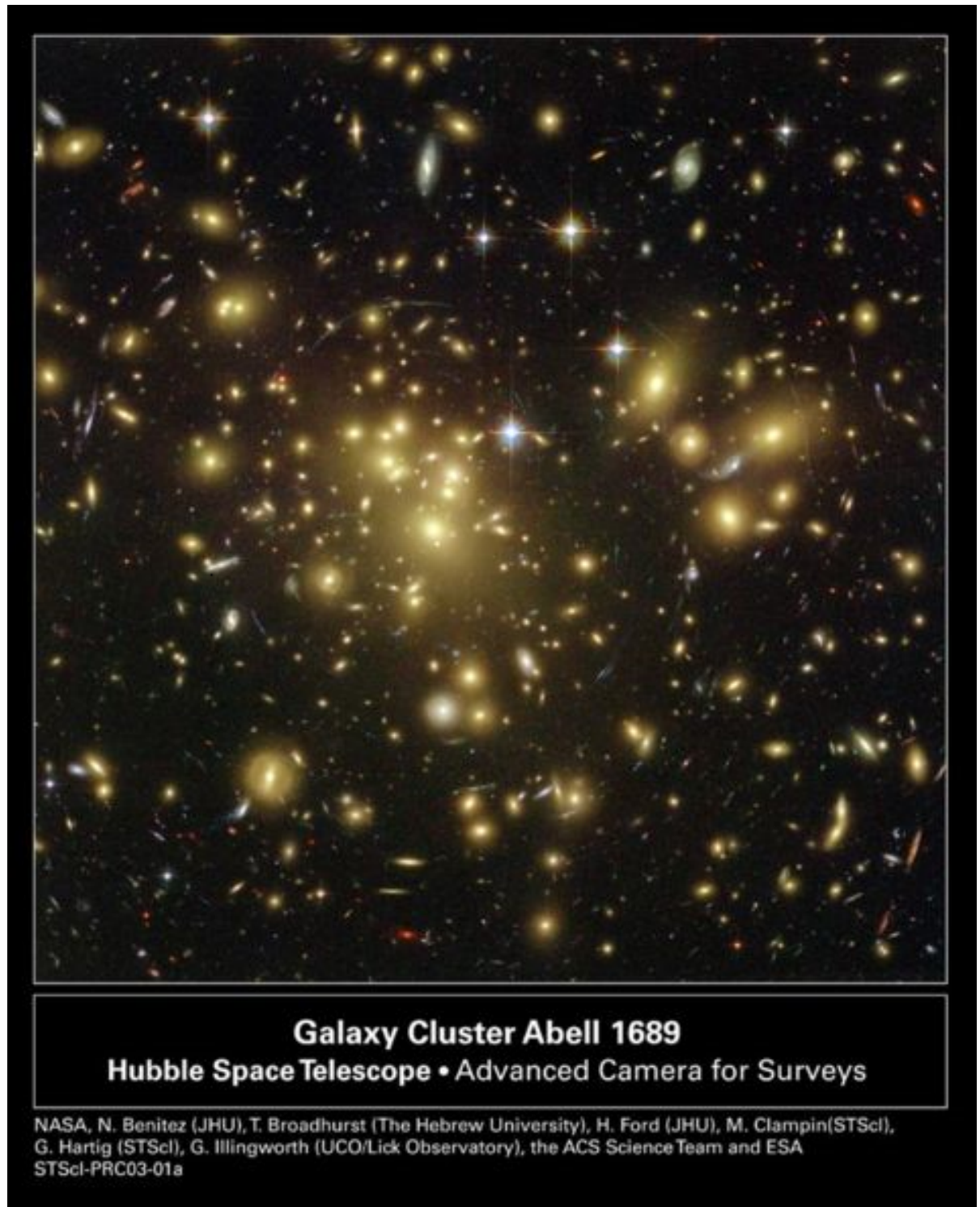
Dark matter condensation on cluster; reionization

Conclusion

# The **two** types of dark matter

- Galactic dark matter:  
MACHOs: Massive Astrophysical Compact Halo Objects: H-He planets
- Cluster dark matter  
WIMPs Weakly Interacting Massive Particles: this talk
- The dark matter of galaxy clusters is non-baryonic.
- Detected from rotation curves, by lensing
- About 20% of total mass of the Universe.
- Not MACHOs or WIMPs but MACHOs and WIMPs !!

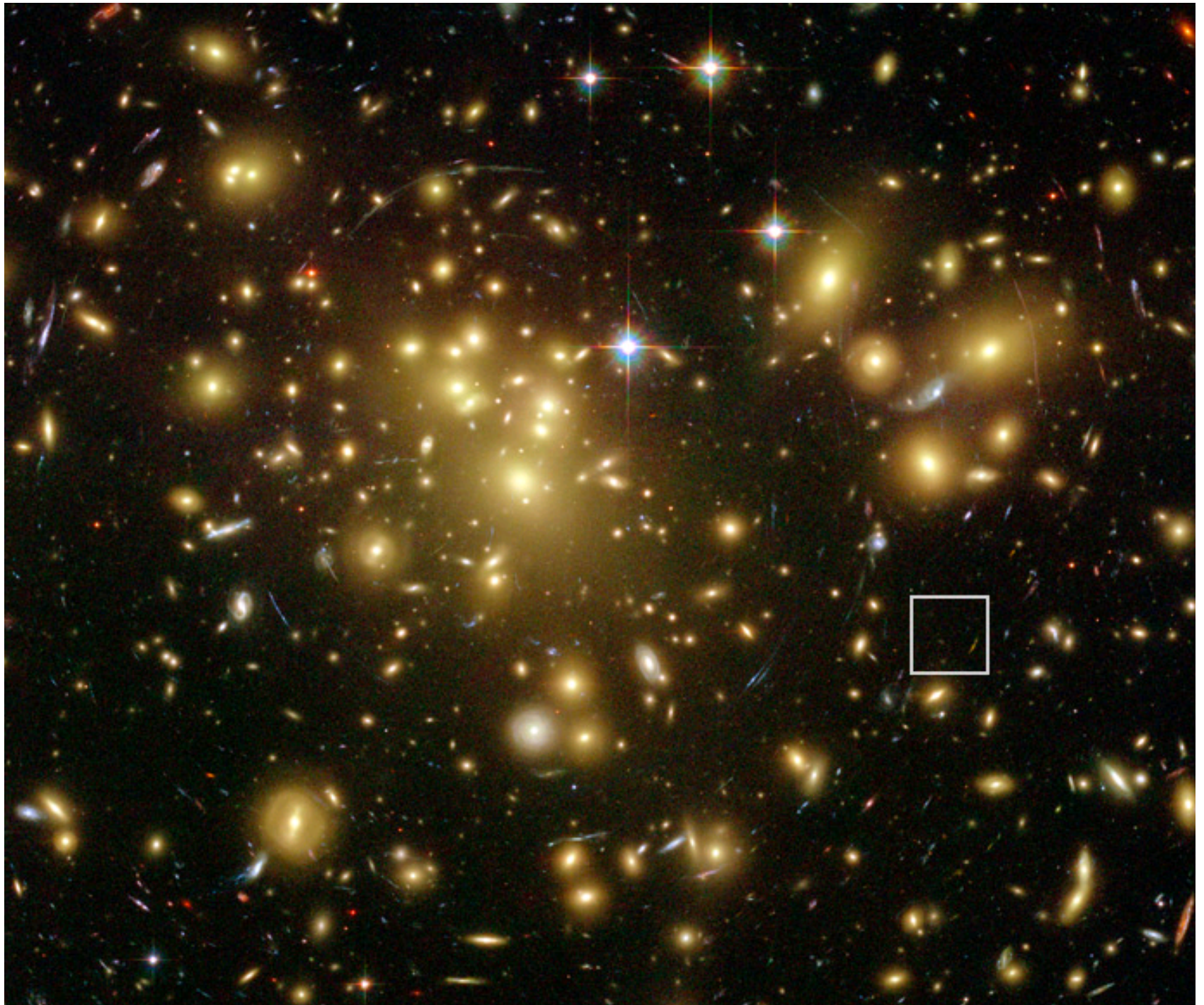
- Abell 1689  
galaxy cluster
- Nearby  $z = 0.184$
- Total mass  
 $1 - 3 \times 10^{15} M_{\odot}$
- Luminous mass  
 $4.4 \times 10^{11} M_{\odot}$
- Baryon poor
- Einstein ring



Incomplete  
Einstein ring  
at  $100/h$  kpc

Abell 1689  
Center

Dark matter  
acts as  
crystal ball



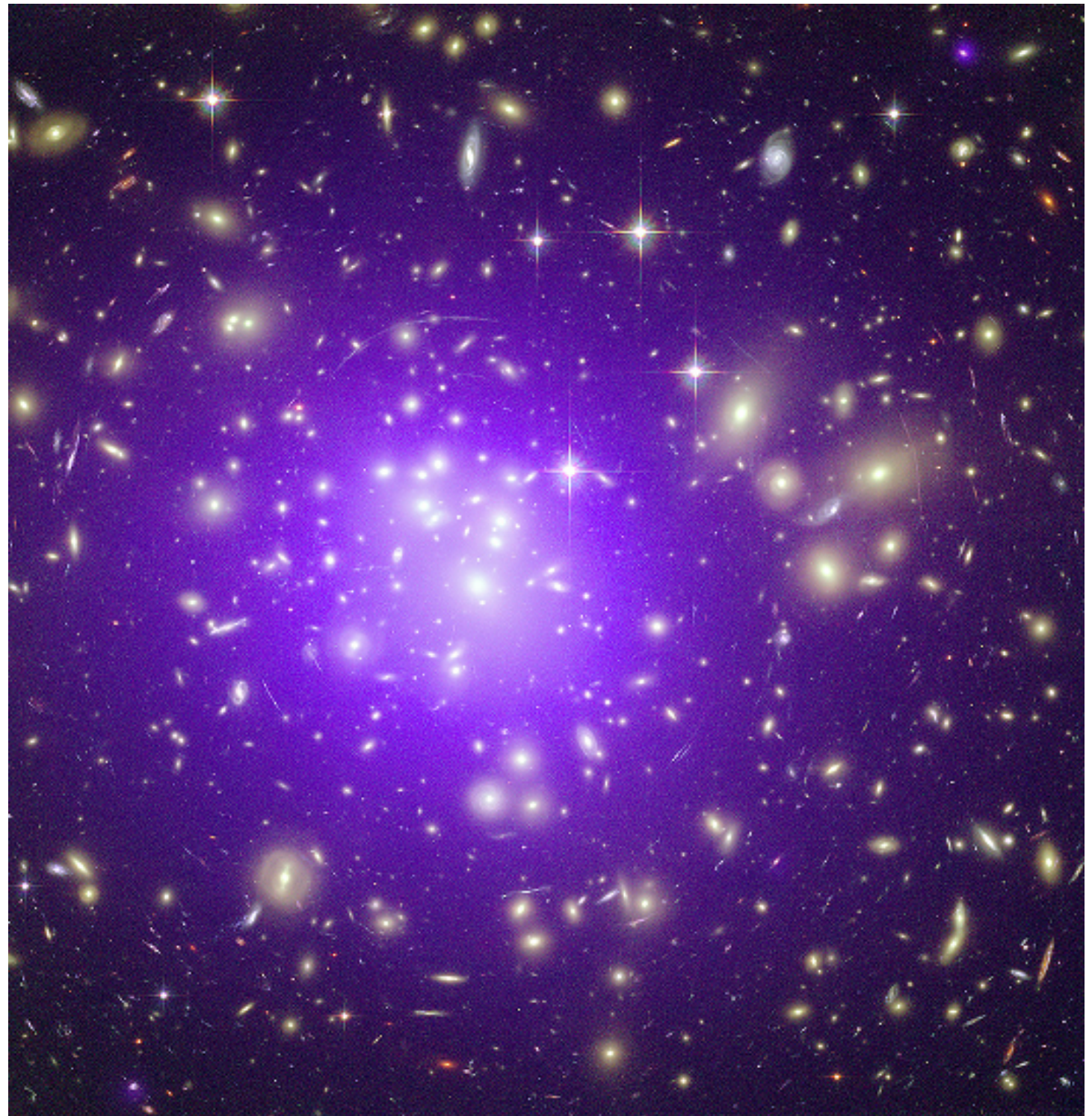
Abell 1689

X-ray  
emitting gas

$T=10 \text{ keV} =$   
 $1.16 \cdot 10^8 \text{ K}$

Three  
components:

Dark matter  
Galaxies  
Gas





# Theory

- Assume that DM comes from fermions in their common gravitational potential  $U(r)$
- Mass  $m$ , degeneracy  $g$ ;  $g = 2(2s+1)$  #families
- Mass density for equilibrium at  $T$

$$\rho_D = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{\bar{g}m}{\exp[(p^2/2m + mU(r) - \mu)/k_B T] + 1}$$

- Gravitational potential  $U(r) = G \int d^3 r' \rho(r') \left( \frac{1}{r'} - \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right)$   
 $U(0)=0$

- Poisson eqn (spherical symmetry)  $U'' + \frac{2}{r}U' = 4\pi G\rho$

- Together they give closed problem for  $U(r)$



# Dark matter (x), Galaxies (G) and gas (g)

- Hydrostatic equilibrium  $\frac{p'_i(r)}{\rho_i(r)} = -\frac{GM(r)}{r^2} \quad i = x, G, g$   
M(r) total mass inside r
- Ideal gas laws  $p_i = \frac{\rho_i}{\bar{m}_i} k_B T_i \quad i = G, g$
- Result  $\rho_i = A_i e^{-\bar{m}_i U / k_B T_i}$
- Virial equilibrium:  
equal velocity dispersions  $(\sigma_v^G)^2 \equiv \frac{T_G}{\bar{m}_G} = \frac{T_x}{m_x} = \frac{T_\alpha}{m_\alpha} = (\sigma_v^g)^2$

# Dimensionless shape

radius, potential

$$x = r/R_*, \phi = U/k_B T,$$

thermal length, scale

$$\lambda_T = \left( \frac{2\pi\hbar^2}{mk_B T} \right)^{1/2}, \quad R_* = \left( \frac{\lambda_T^3 k_B T}{4\pi g G m^2} \right)^{1/2}$$

Dark density

$$\rho_D(r) = -\frac{\bar{g}m}{\lambda_T^3} \text{Li}_{3/2} \left( -e^{\alpha - \phi(x)} \right)$$

polylogarithm  $\text{Li}_\gamma(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\gamma}$

Poisson eqn

$$\phi'' + \frac{2}{x}\phi' = -\text{Li}_{3/2} \left( -e^{\alpha - \phi} \right) + e^{\alpha_G - \beta_G \phi} + e^{\alpha_g - \beta_g \phi}$$

dark matter + Galaxies + gas

Virial equilibrium:

Galaxies

$$\beta_G = 1$$

gas (ionized H, He, 30% solar metallicity)

$$\beta_g = 0.153$$

# Observed quantity in strong and weak lensing

Integrated mass  
along line-of-sight

$$\Sigma(r_{\perp}) = \int_{-\infty}^{\infty} dz \rho \left( \sqrt{r_{\perp}^2 + z^2} \right)$$

Average in (0, r)

$$\bar{\Sigma}(r) = \frac{1}{\pi r^2} \int_0^r dr_{\perp} 2\pi r_{\perp} \Sigma(r_{\perp})$$

Average in (r, r\_m)

$$\bar{\Sigma}(r \rightarrow r_m) = [r_m^2 \bar{\Sigma}(r_m) - r^2 \bar{\Sigma}(r)] / (r_m^2 - r^2)$$

Contrast function

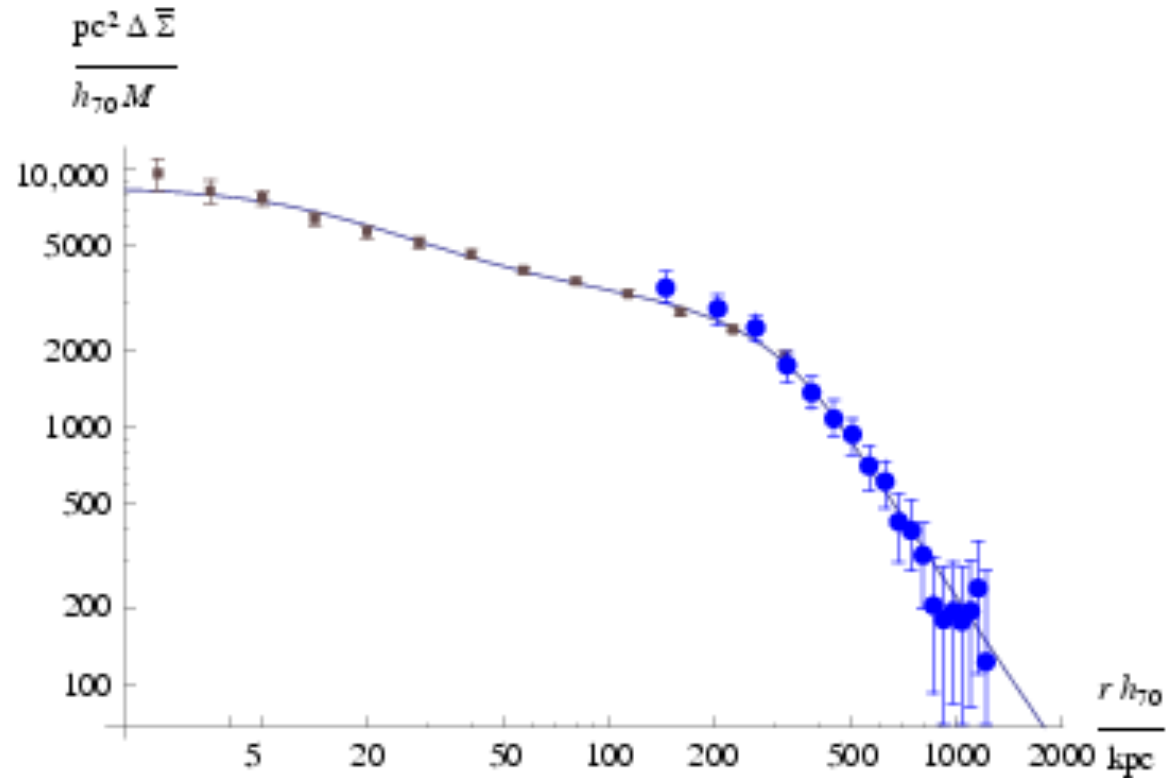
$$\Delta \bar{\Sigma}(r) \equiv \bar{\Sigma}(r) - \bar{\Sigma}(r \rightarrow r_m) = \frac{\bar{\Sigma}(r) - \bar{\Sigma}(r_m)}{1 - r^2/r_m^2}$$

From the model

$$\bar{\Sigma}(r) = A \Phi \left( \frac{r}{R_*} \right), \quad \Phi(x) = \int_0^{\infty} ds \phi'(x \cosh s)$$

# Fit to A1689 lensing data of Tyson and Fischer ApJ 1995

## Limousin et al, ApJ 2007



$$\begin{aligned}
 A &= 59.4 \pm 9.6 h_{70} M_{\odot} \text{pc}^{-2}, & \alpha &= 38.4 \pm 3.1, & \alpha_g &= 2.36 \\
 R_{*} &= 297 \pm 10 h_{70}^{-1} \text{kpc}, & \alpha_G &= 8.26 \pm 0.32, & \bar{\beta}_G &= 1 & \bar{\beta}_g &= 0.153
 \end{aligned}$$

# The mass of the dark matter particle

$$m = \frac{1}{2^{1/8} \bar{g}^{1/4}} \frac{\hbar^{3/4}}{A^{1/8} G^{3/8} R_{\star}^{5/8}} \quad \bar{g} = \text{number of } \textit{available} \text{ modes in cluster}$$

reduced Hubble parameter  $h_{70} = \frac{H_0}{70 \text{ km/s Mpc}}$

$$m = h_{70}^{1/2} \left( \frac{12}{\bar{g}} \right)^{1/4} 1.455 \pm 0.030 \text{ eV} \quad \text{small error, 2.0\%}$$

Previous estimates and searches: keV, MeV, GeV, TeV: excluded

Cosmic density of  $g$  *occupied* modes that once were thermal  $n_F = g \frac{3}{4} \frac{4}{11} \frac{\zeta(3)}{\pi^2} \left( \frac{k_B T_\gamma}{\hbar c} \right)^3 = g 55.977 \text{ cc}^{-1}$

Cosmic matter fraction  $\Omega_x = \frac{n_F m}{\rho_c} = \frac{g}{12} \left( \frac{12}{\bar{g}} \right)^{1/4} h_{70}^{-3/2} 0.1893 \pm 0.0039$

# What is the dark matter particle?

The dark matter fraction  $\Omega_x = \frac{n_F m}{\rho_c} = \frac{g}{12} \left( \frac{12}{\bar{g}} \right)^{1/4} h_{70}^{-3/2} 0.1893 \pm 0.0039$

Early decouplers have small occupation g: Not axions, gravitinos, neutralinos, X-inos

Match possible to WMAP5  $\Omega_D = 0.214 \pm 0.027$  for  $\bar{g} = 12$

(anti) **neutrinos**, left+right-handed, 3 families  $2*2*3=12$  mass = 1.455 eV

Neutrino oscillations cause small mass differences, 0.001 eV or less

Thermal length visible to the eye  $\lambda_{T\nu} = \frac{\hbar\sqrt{2}}{m_\nu\sqrt{GAR_*}} = 0.20 \text{ mm}$

Temperature is low  $T_\nu^A = \pi G m_\nu A R_* / k_B = 0.0447 \text{ K}$

Typical speed is non-relativistic,  $v = 490 \text{ km/s}$

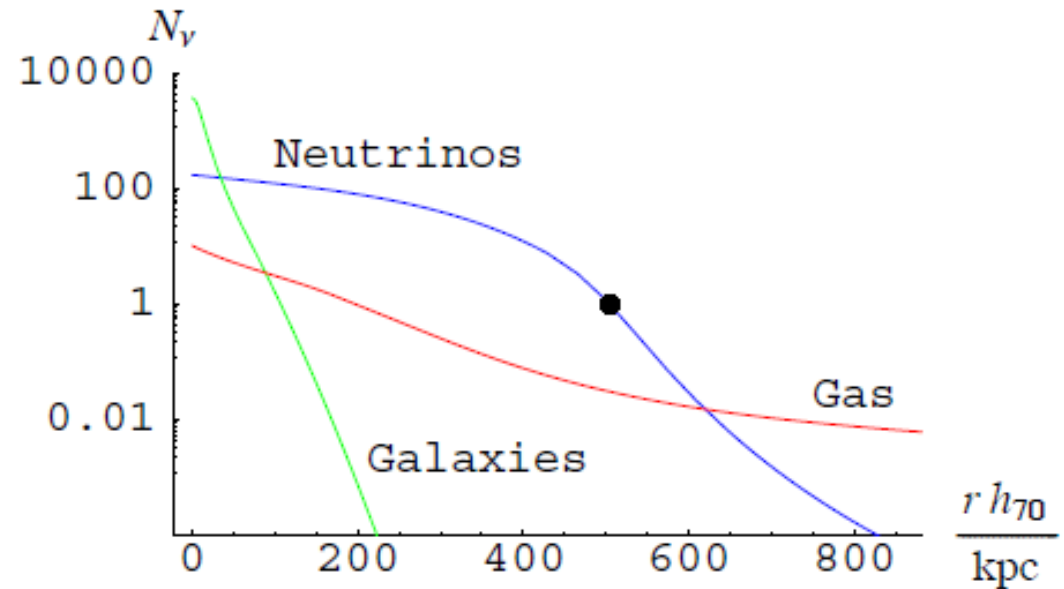
Local density can be enormous: in Abell center: one billion in a few cc

# The biggest quantum structure in the Universe

normalized density:

# neutrinos

per cubic thermal wavelength  
per degree of freedom



Galaxies, gas:

Baryons are poor tracers  
of dark matter *density*,  
even though they do  
trace the *enclosed mass*

$N > 1$ : quantum degenerate nu's

$N = 1$ : quantum-to-classical crossover

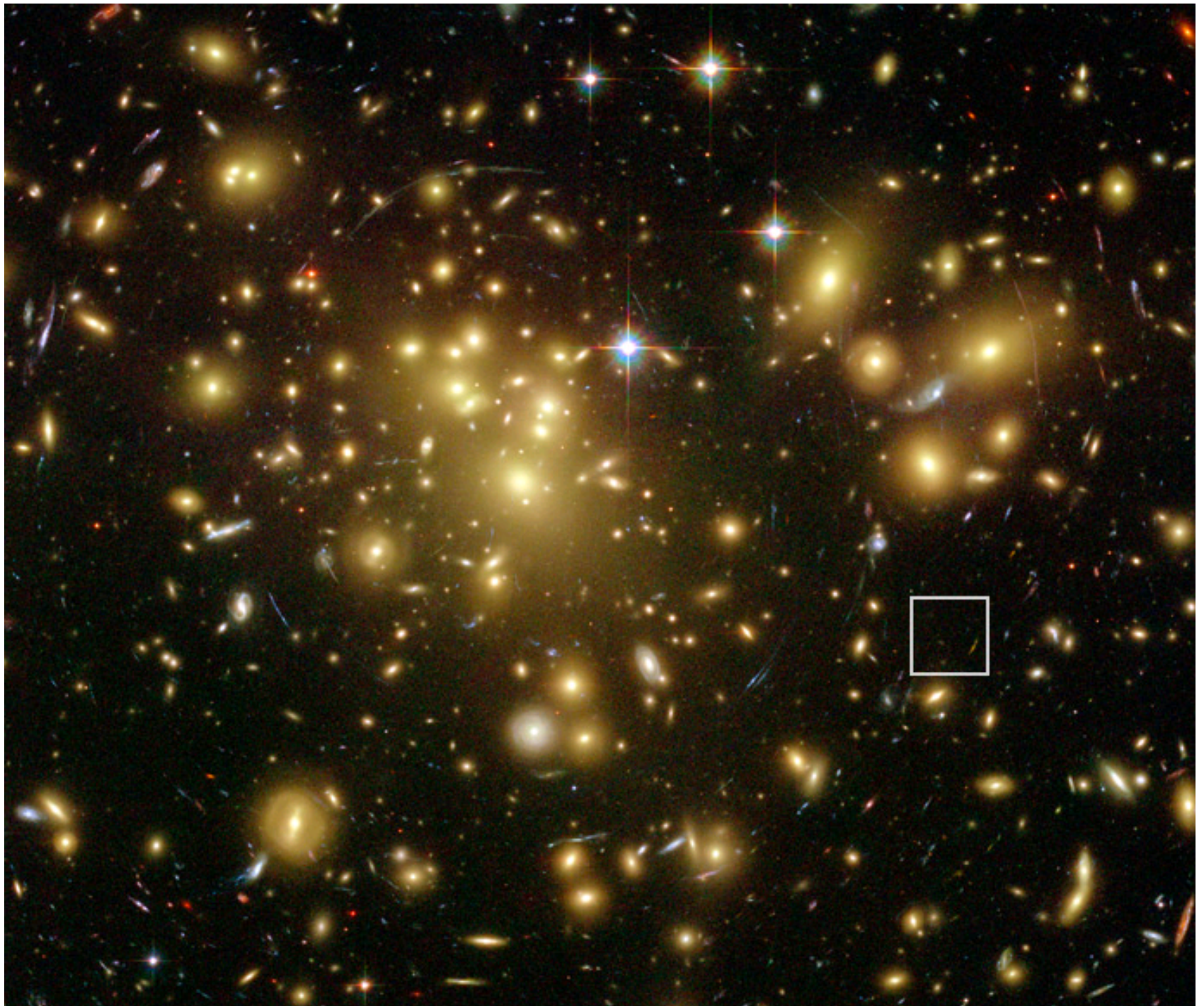
at  $r = 505 \text{ kpc} = 1.6 \text{ million light year}$

$d = 2r = 3.2 \text{ million light year}$

That is pretty big ...

Incomplete  
Einstein ring  
at  $100/h$  kpc

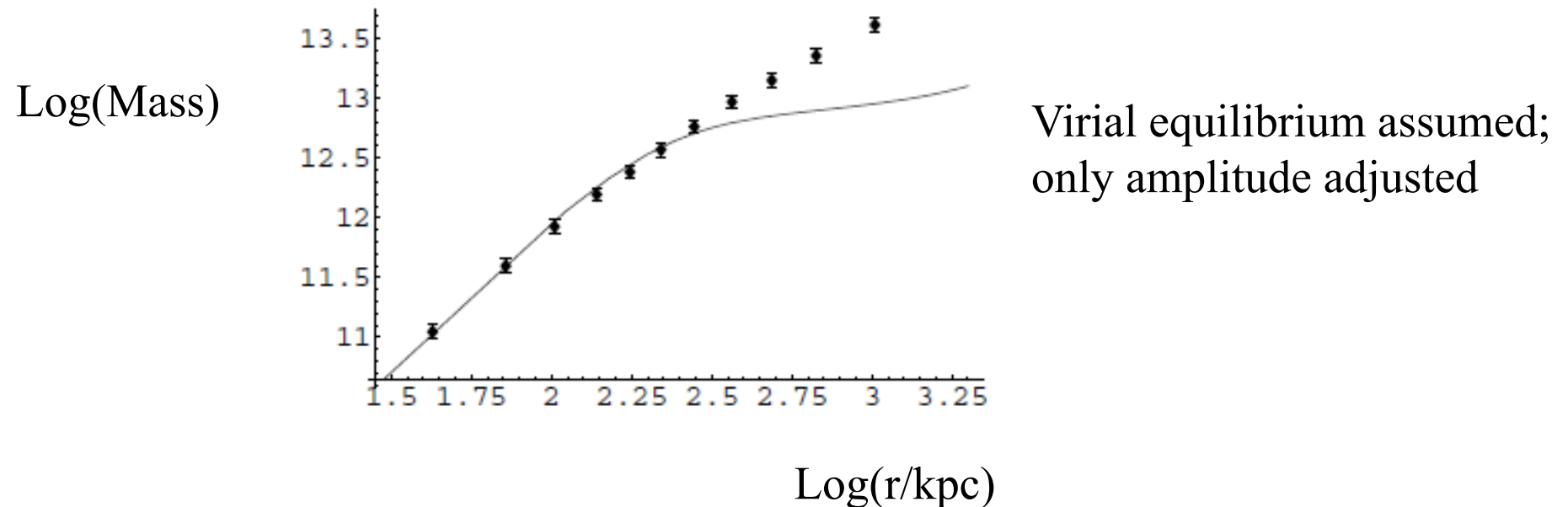
Abell 1689  
Center





# Temperature of gas: $10 \text{ keV} = 10^8 \text{ K}$

Virial T of alpha-particles  $T_\alpha = \frac{m_\alpha}{m_\nu} T_\nu = 9.9 \pm 1.0 \text{ eV}$  overlaps with it.



Cluster radiates in X-rays like a star in light.  
Radiated energy supplied by contraction, as for stars.  
Radiation helps to maintain virial equilibrium.

# Dirac and Majorana masses (3\*3 matrices)

- Dirac mass 1.45 eV, ideally  $m_{\nu_1} = Y_e m_e = 2^{3/4} G_F^{1/2} m_e^2 = 1.4998 \text{ eV}$   
iff  $h=0.744$ : good Hubble value
- Majorana: neutrino + neutrino annihilation  
Lepton number not conserved
- Majorana mass  $> 0.0001 \text{ eV}$  creates right-handed (sterile) neutrinos in period when  $200 \text{ MeV} < T < 1 \text{ MeV}$
- Majorana mass causes neutrinoless double beta-decay.  
Experimental upperbound:  $< 0.2\text{-}0.7 \text{ eV}$

# Sterile neutrinos and nucleosynthesis

- Sterile neutrinos cause enhanced expansion

$$H' = SH, \quad \rho' = S^2 \rho \quad S = \left( \frac{16 + 7g(4/11)^{4/3}}{16 + 42(4/11)^{4/3}} \right)^{1/2} = 1.1854$$

- Faster expansion: too few neutron decays, too much He-4
- If neutrino asymmetry: more neutrinos than anti-neutrinos

$$L \equiv \frac{n_\nu - n_{\bar{\nu}}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)} \left( \xi + \frac{\xi^3}{\pi^2} \right) \quad \xi = \mu / kT$$

- Thus more reactions for neutron decay
- These effects can compensate each other  $n + \nu_e \rightarrow p + e$
- So nucleosynthesis can accommodate right-handed neutrinos

# Effect nu-asymmetry on data Steigman IJMP E, 2005

$$S = \left( \frac{16 + 7g(4/11)^{4/3}}{16 + 42(4/11)^{4/3}} \right)^{1/2} = 1.1854$$

$$\eta_{\text{He}} = \eta_{10} + 100(S - 1) - \frac{575}{4}\xi$$

$$\eta_{\text{D}} = \eta_{10} - 6(S - 1) + \frac{5}{4}\xi$$

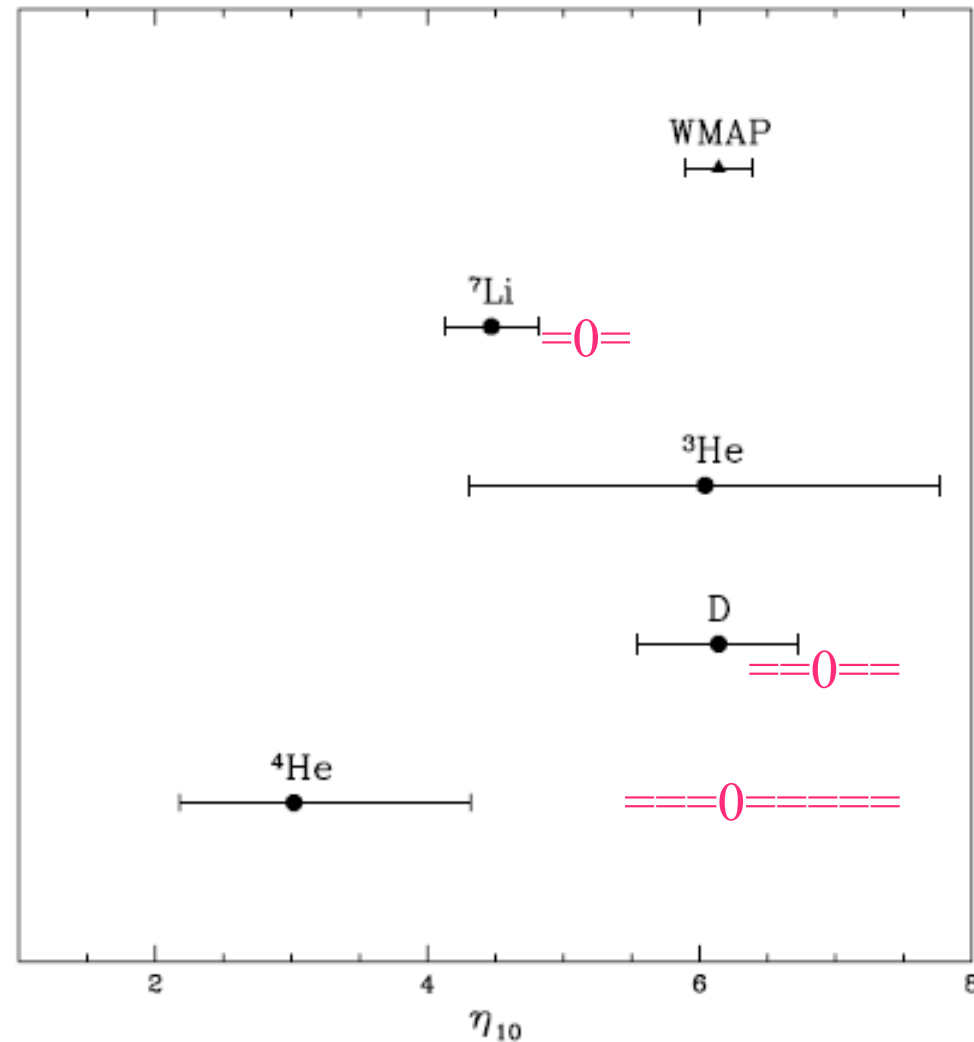
$$\eta_{\text{Li}} = \eta_{10} - 3(S - 1) - \frac{7}{4}\xi$$

$$\Delta N = 3$$

$$S = 1.184$$

$$\xi = 0.152 \quad \text{Adjusting He-4 to WMAP}$$

$$L = 0.106$$



## Why virial equilibrium?

- Lynden Bell: violent relaxation, faster than collisional
- Relaxation in time-dependent potential: every object (individual particle, galaxy) exchanges energy with the whole cluster
- Iff phase space density becomes uniform, then Fermi-Dirac distribution
- X-ray radiation helps to maintain the virial state, as in the sun

# CDM or HDM?

- Cold Dark Matter: heavy particles already clumped at decoupling  $z=1100$
- But light neutrinos are free streaming until trapped by galaxy cluster

$$\partial_t f_\nu + \frac{\mathbf{p} \cdot \partial_{\mathbf{r}} f_\nu}{m_\nu} - \left( \mathbf{p}H + \hat{\mathbf{r}} \frac{Gm_\nu M(r, t)}{r^2} \right) \cdot \partial_{\mathbf{p}} f_\nu = 0$$

- Free streaming  $f_\nu = \frac{1}{e^{\beta \mathbf{p} \cdot \mathbf{v}} + 1}$
- Crossover when Newton force matches Hubble force:  $z = 6 - 7$
- Then cosmic voids loose neutrinos and become empty
- This heats the intracluster gas up to 10 keV, so it *reionizes*, without heavy stars
- Hot Dark Matter paradigm agrees with gravito-hydrodynamics

# Gravitational hydrodynamics

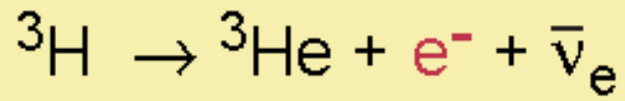
- Structure formation starts in plasma: proto-voids & galaxy clusters
- Explains CMB T-fluctuations of micro-Kelvins
- At decoupling: galaxies; proto-globular clusters 40,000 solar mass
- Some PGCs developed into globular star clusters
- Other PGCs act as ideal gas “particles” around galaxies.
- Explains isothermal model for [galactic dark matter](#): rotation curves, Tully Fisher relation
- (Dwarf) galactic DM [does NOT exclude neutrinos](#).
- Final fragmentation in H-He planets of earth weight.
- Thousands of frozen planets observed in microlensing  
thousands of heated planets in planetary nebulae

# Summary

- Observed DM of Abell 1689 cluster explained by thermal fermions of eV mass. keV, MeV, GeV, TeV, PeV excluded. Thermal bosons (axions) excluded.
- Fitting with global dark matter fraction: 6 active + 6 sterile (anti-)neutrinos  
Mass about 1.45 eV, depends on h. Ideal case:  $m=1.4998$  eV,  $h=0.744$
- Gas temperature  $10^8$  K = alpha-particle temp., gas profile matches automatically
- Baryons are poor tracers of dark matter *density*.
- Free flow into potential well occurs at  $z = 6 - 7$ . Causes reionization,  
without heavy stars
- Neutrino Hot Dark Matter challenges Cold Dark Matter (CDM).
- Dark matter particle was not (and should not) be observed in searches  
ADMX, ANAIS, ArDM, ATIC, BPRS, CAST, CDMS, CLEAN, CRESST, CUORE, CYGNUS, DAMA, DEEP, DRIFT,  
EDELWEISS, ELEGANTS, EURECA, GENIUS, GERDA, GEDEON, GLAST, HDMS, IGEX, KIMS, LEP, LHC, LIBRA,  
LUX, NAIAD, ORPHEUS, PAMELA, PICASSO, ROSEBUD, SIGN, SIMPLE, UKDM, XENON, XMASS, ZEPLIN.  
DAMIC, FERMI, ICECUBE, VERITAS.
- Neutrino mass tested in Katrin search between 0.2 eV – 2 eV (2012).



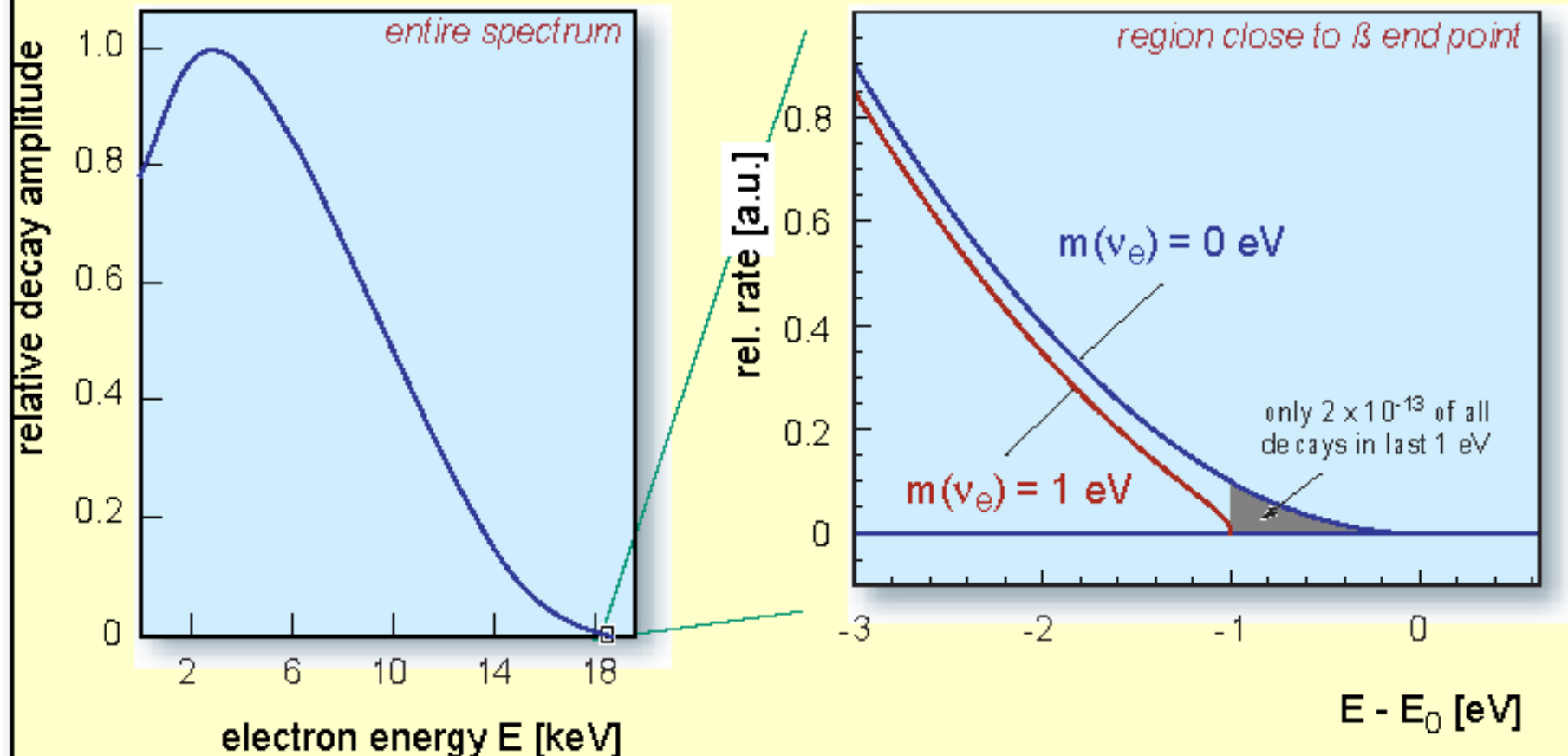
# tritium $\beta$ -decay and the neutrino rest mass



superallowed

half life :  $t_{1/2} = 12.32 \text{ a}$

$\beta$  end point energy :  $E_0 = 18.57 \text{ keV}$



# Karlsruhe TRITium Neutrino Experiments

