

# Measuring the dynamical properties of self-gravitating systems in their outer regions through the caustic technique

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intro

CT

mass

members

remarks



Abell 1689 observed by the Hubble Space Telescope

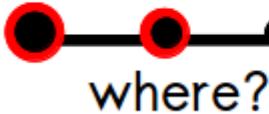
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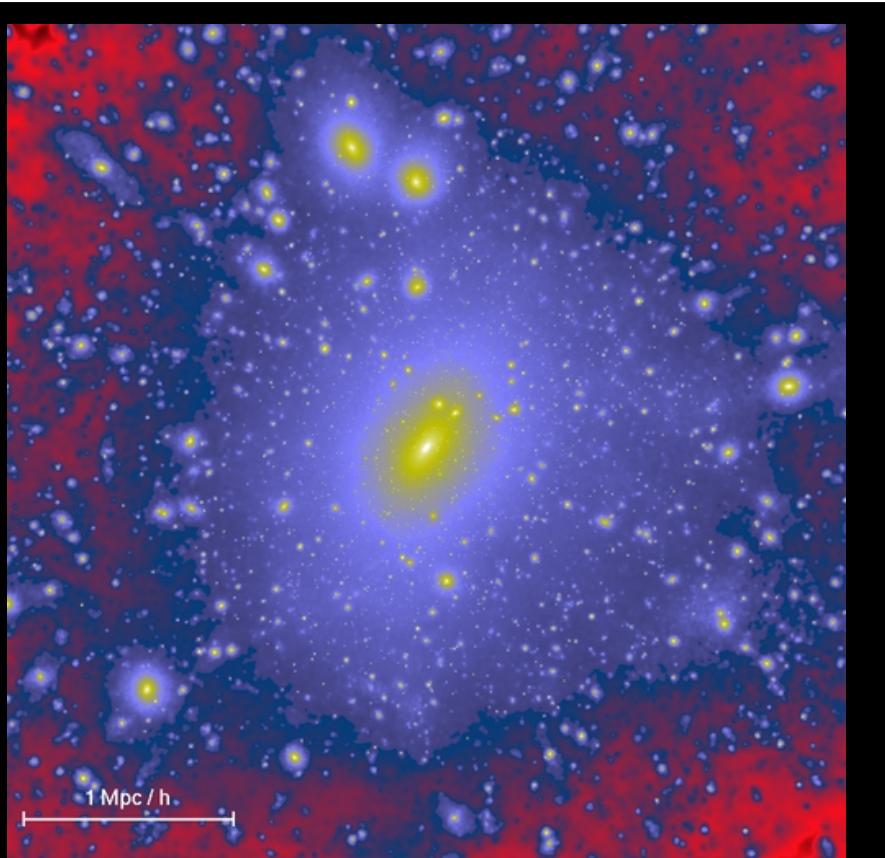
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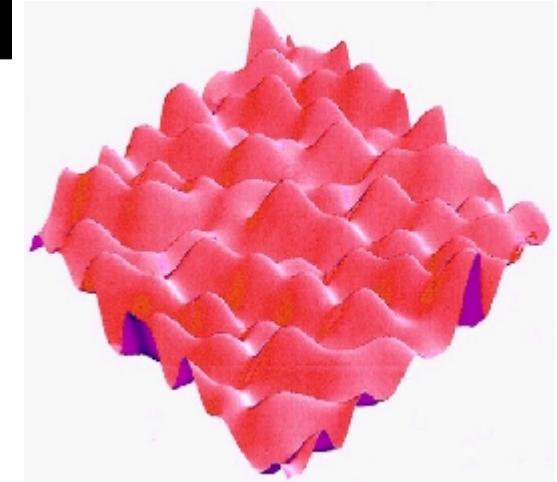
remarks



## Formation of large-scale structure in the Universe



Millennium Simulation, Volker Springel, 2005



Early Universe  
(small perturbations)

ripples evolve independently

they interact with  
others in non-linear ways

the small over-density fluctuations attract additional  
mass as the Universe expands

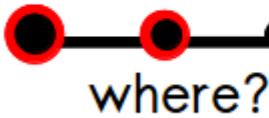
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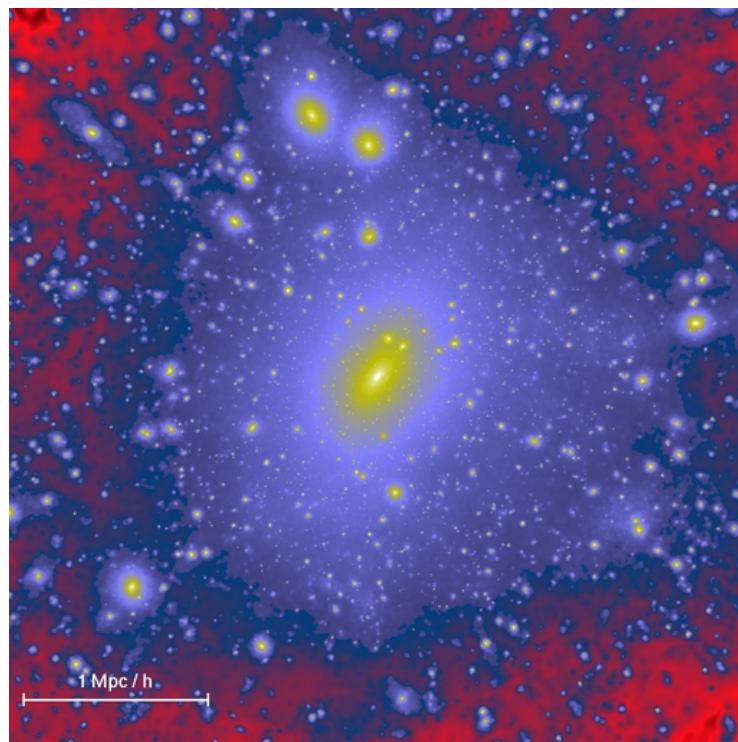


## Formation of large-scale structure in the Universe

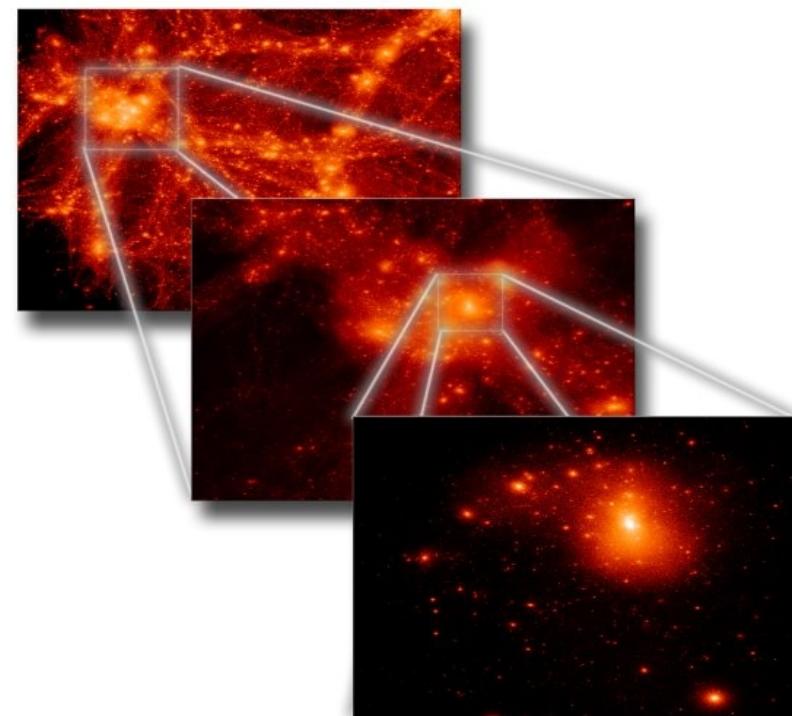
Gravitational instability produces high peaks of the density field



merger of small clumps at the intersection of a filamentary large-scale structure



Millennium Simulation, Volker Springel, 2005



Simulation by Gauss Centre for Supercomputing  
Gottlöber, Khalatyan, Klypin, 2008



**GALAXY CLUSTER**

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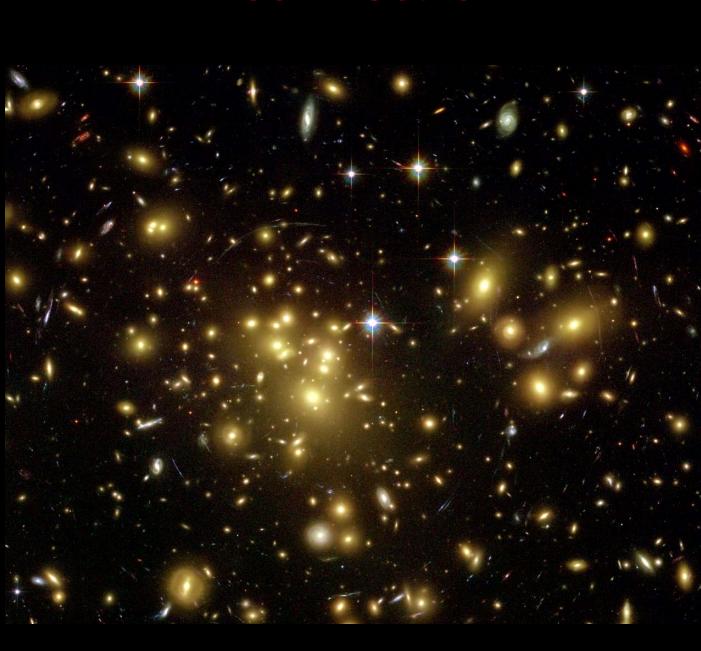
what?

**cosmological structure**  
large-scale galaxy-environment matter distribution  
galaxy formation

## parameters

largest gravitationally bound systems in the Universe

galaxy-environment connection



a few Mpc  
 $\sim 10^{14-15} M_{\odot}$

hundreds of galaxies



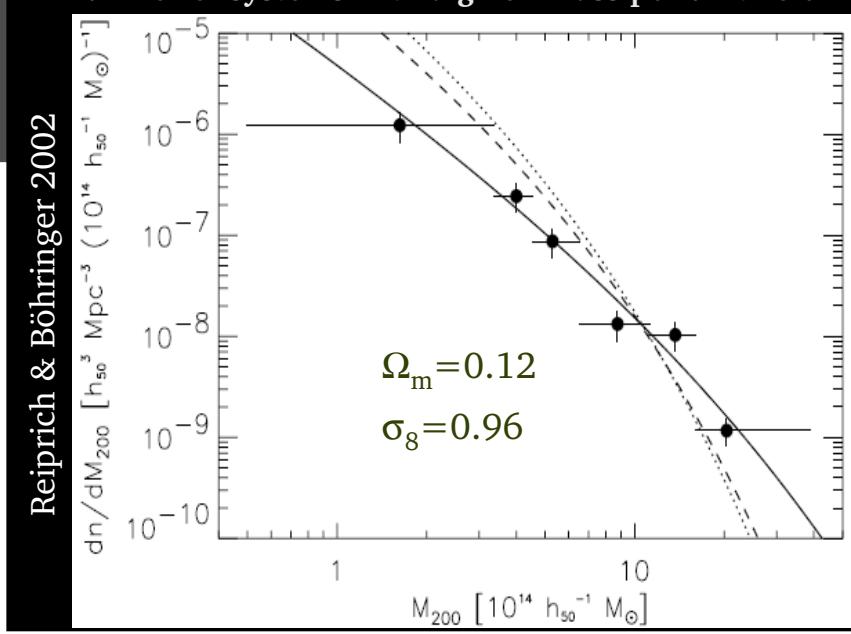
~80% - DM  
~20% - hot diffuse plasma  
- stars, dust, cold gas

High-mass tail of mass function

number density

measurement of cosmological parameters

number of systems with a given mass per unit volume



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how?

## mass distribution

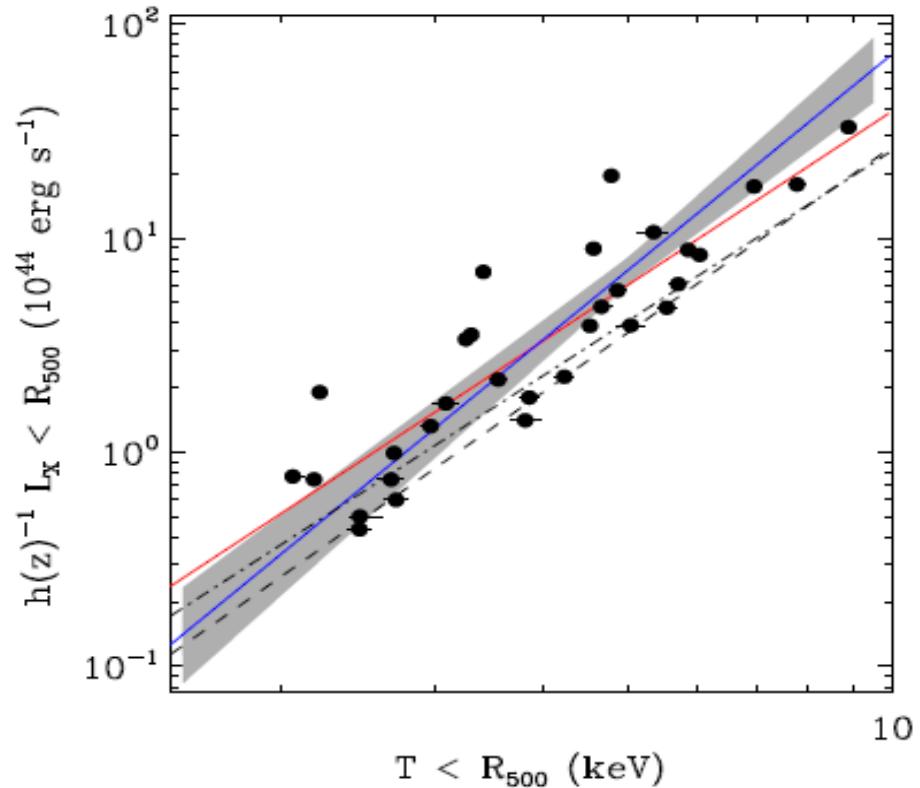
small scales → assumption of dynamical equilibrium

Intermediate scales  
(1-10 Mpc/h)

large scales → small over-densities → linear theory

can we assume dynamical equilibrium?

from scaling relations... ~yes



Pratt et al. 2009

however there are...

spatially inhomogeneous thermal and non thermal emission

kinematic and morphological segregation of galaxies

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## how to measure the mass of galaxy clusters ?

Virial theorem

Jeans Equation

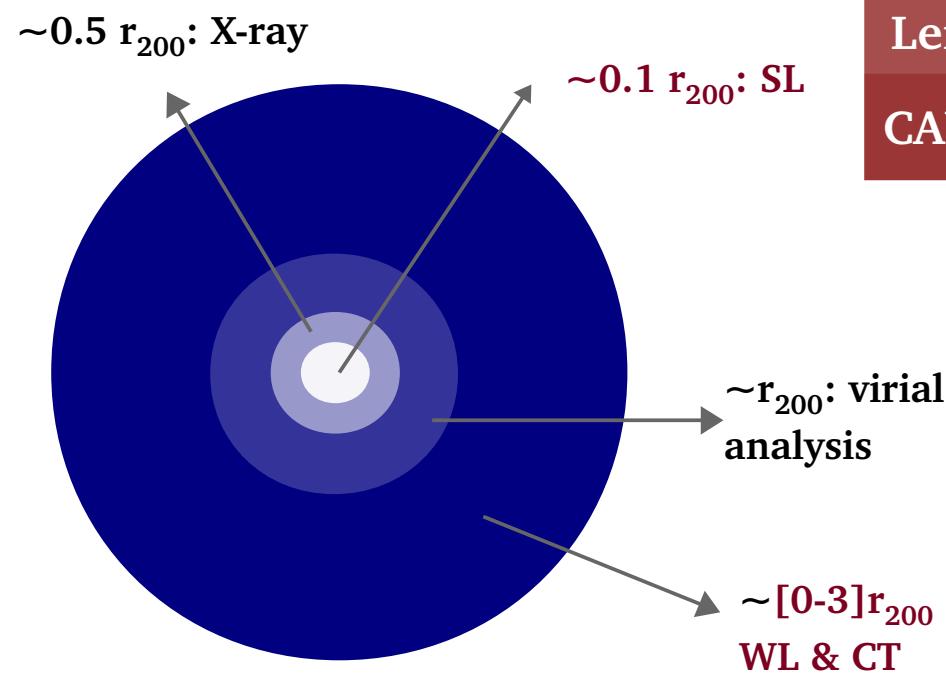
Scaling relations

X-ray temperature

Strong and Weak  
Lensing

CAUSTIC TECHNIQUE

No assumption of  
dynamical equilibrium



$r_{200}$ : radius enclosing a matter density 200 times the critical density of the Universe  $\sim 277.5 h^2 M_\odot / \text{Mpc}^3$

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total mass within a radius

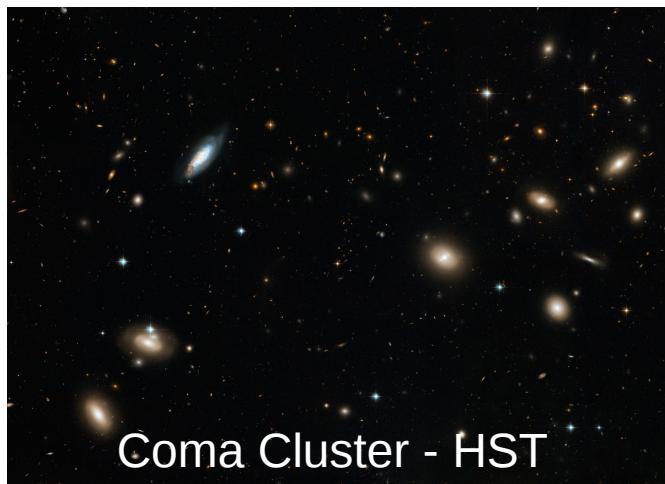
mass profile

from l.o.s. velocities and position

virial theorem

$$M = \frac{3\sigma^2 R}{G}$$
$$R = \frac{\pi N(N-1)}{2} \left( \sum_i \sum_{i>j} \frac{1}{r_{ij}} \right)^{-1}$$

scaling relations



Coma Cluster - HST

Jeans equation

$$M(< r) = \frac{\langle v_r^2 \rangle r}{G} \left[ \frac{d \ln \rho_m}{d \ln r} + \frac{d \ln \langle v_r^2 \rangle}{d \ln r} + 2\beta(r) \right]$$

$$\beta(r) = 1 - \frac{\langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle}{2\langle v_r^2 \rangle}$$

mass-anisotropy degeneracy  
assumption of relation between the  
galaxy density profile and the  
mass density profile

dynamical equilibrium + sphericity

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how?

total mass within a radius

mass profile

from X-ray

X-ray temperature

isothermal ICM → important  
departure from assumption in  
some clusters

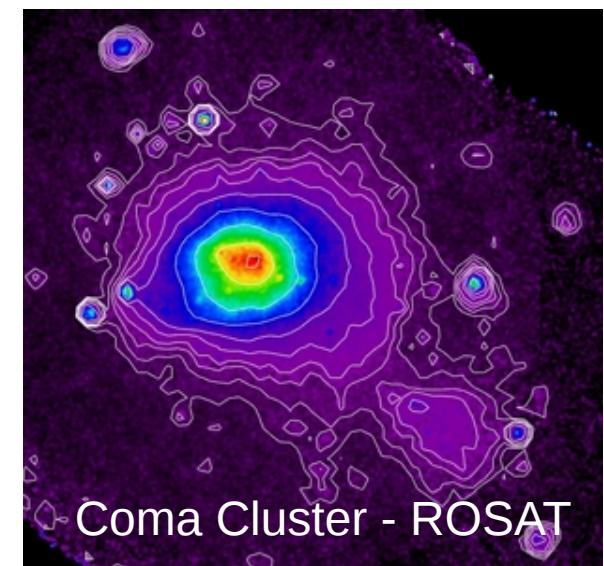
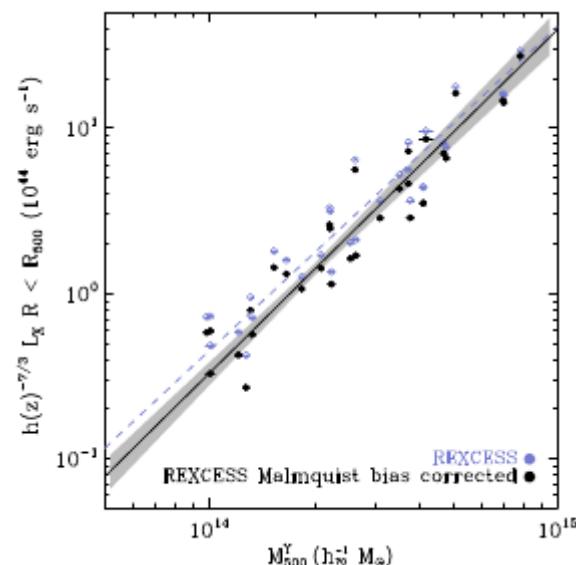
X-ray spectrum

$$M(< r) = \frac{kTr}{G\mu m_p} \left[ \frac{d \ln \rho_{gas}}{d \ln r} + \frac{d \ln T}{d \ln r} \right]$$

scaling relations

the complex thermal  
structure of the ICM can bias  
the estimate.

Pratt et al. 2009



dynamical equilibrium + sphericity

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how?

total mass within a radius

mass profile

from lensing signal

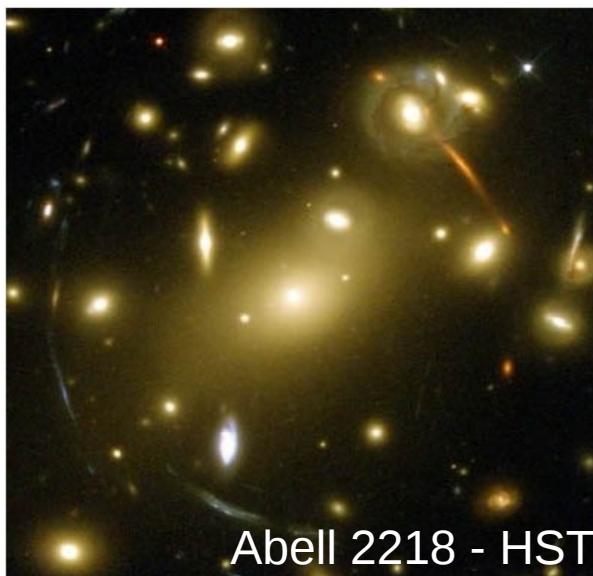
Strong lensing  
multiple images, arcs

$$M(< r) = \frac{rc^2}{4G}\alpha$$

Weak lensing

tangential distortion of the shape of  
galaxies

disadvantage: the signal intensity depends on the distances between observer,  
lens and source



Abell 2218 - HST

from l.o.s. velocities and position

Caustic technique

in hierarchical models of structure  
formation, the velocity field surrounding the  
cluster is not perfectly radial, as expected in  
the spherical infall model



it is possible to extract the escape velocity  
of galaxies from the redshift diagram

dynamical equilibrium + sphericity

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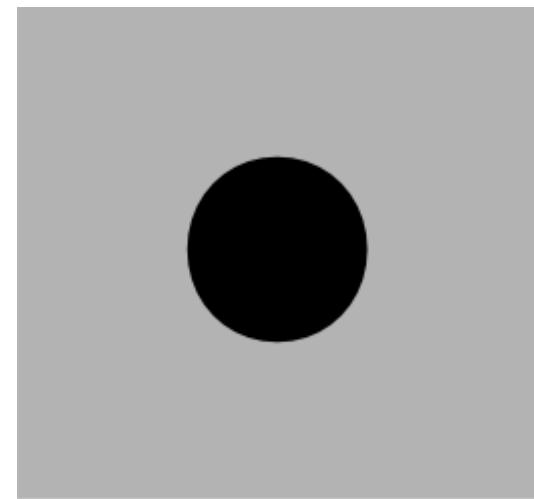
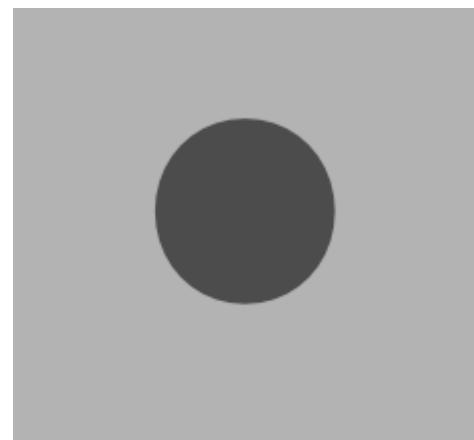
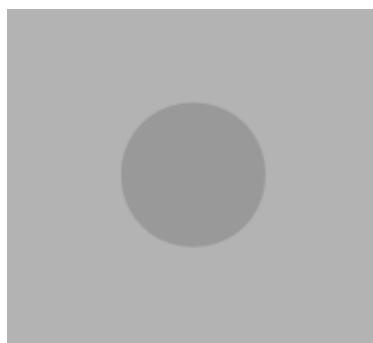
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theory



evolves like a Friedmann model (expanding medium)

for any small density perturbation there will be a competition between its self-gravity (which is attempting to increase the density) and the general expansion of the universe (which decreases the density)

structures will be formed if, at some time, the spherical region ceases to expand with the background universe and begins to collapse

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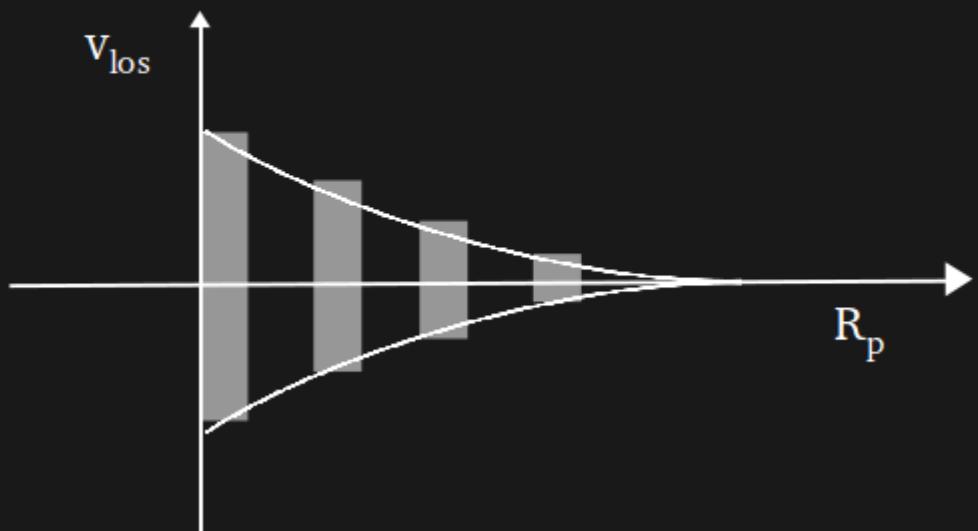
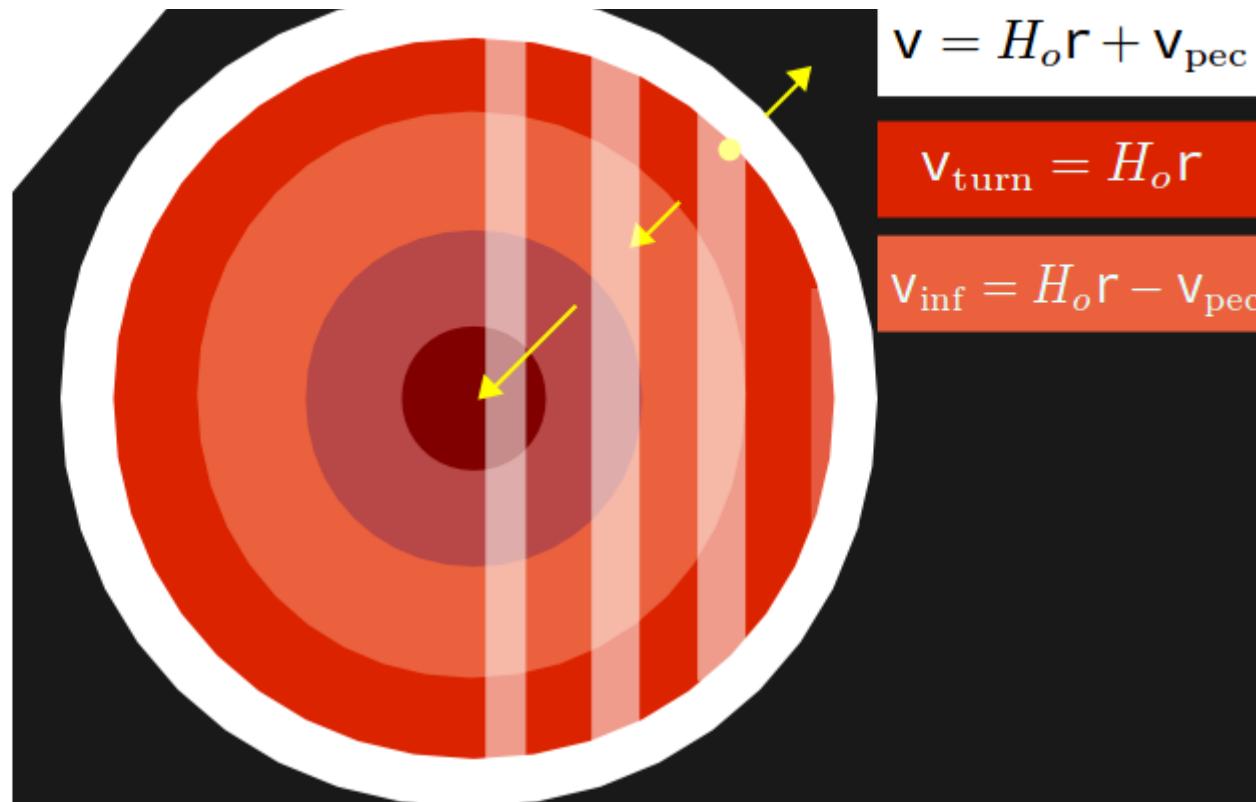
theory

When observed in redshift space, the infall pattern around a rich cluster appears as a “trumpet” whose amplitude  $\mathcal{A}(\theta)$  decreases with  $\theta$  (Kaiser, 1987)

from spherical infall model

(Regös & Geller, 1989)

$$\mathcal{A}(\theta) \sim \Omega_0^{0.6} r f(\delta) \sqrt{-\frac{d \ln f(\delta)}{d \ln r}}$$



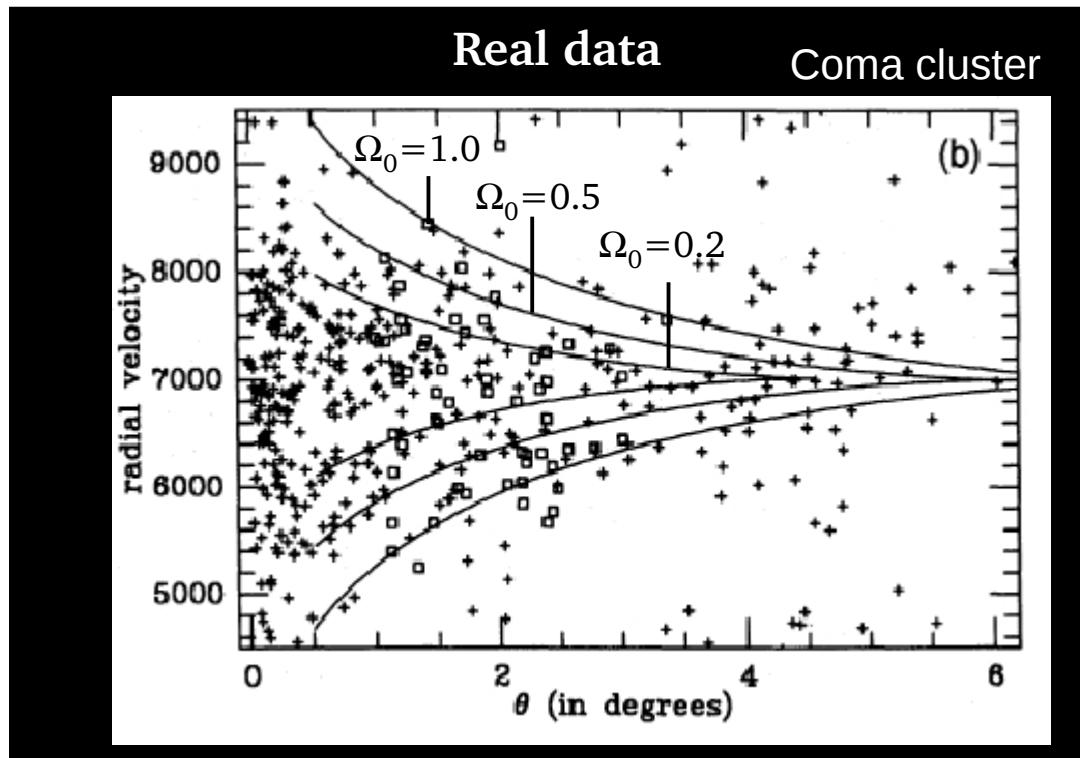
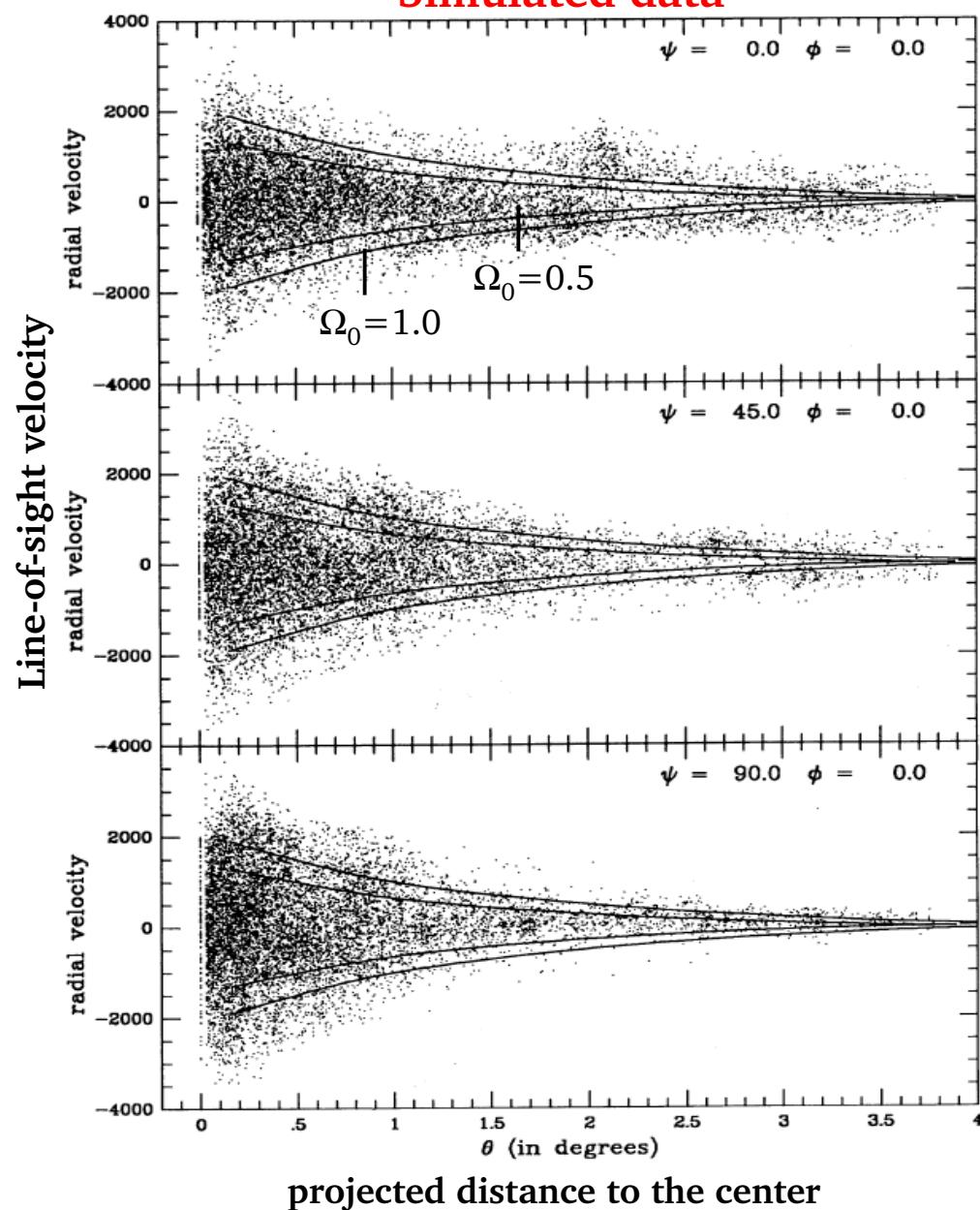
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van Haarlem et al. 1993

but clusters accrete mass anisotropically  
 → the velocity field can have a substantial  
 non-radial random component

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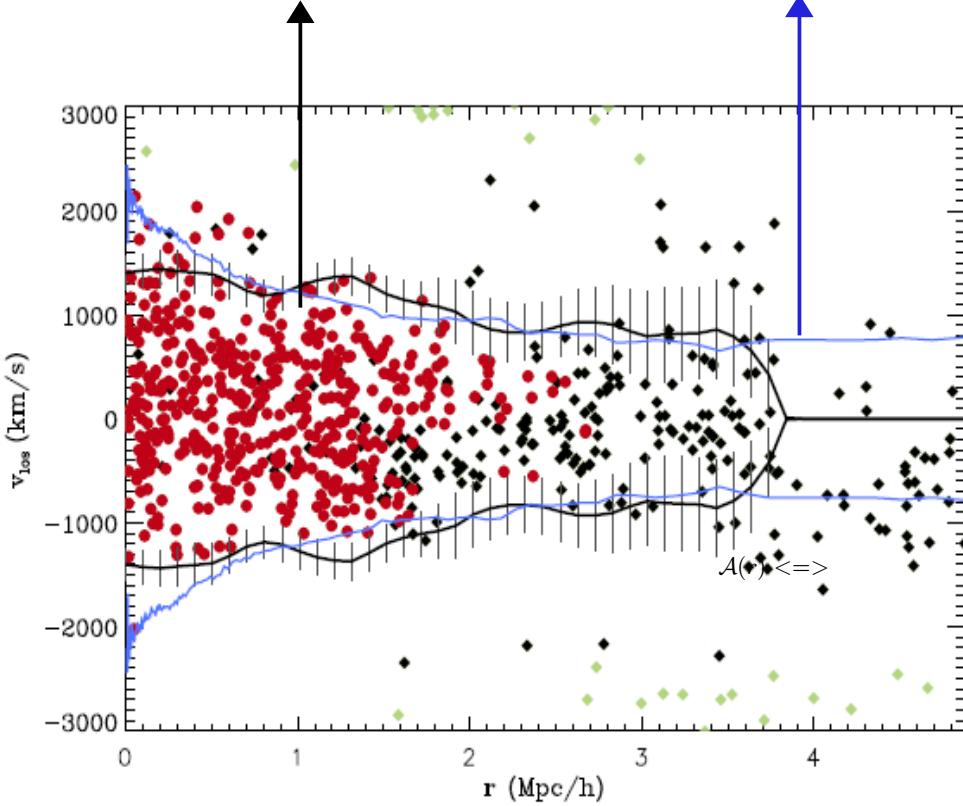
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$$\mathcal{A}_{\text{infall model}} < \mathcal{A}_{\text{non-radial}}$$

The random components increase the caustic amplitude when compared to the spherical model

but clusters accrete mass anisotropically  
 → the velocity field can have a substantial non-radial random component

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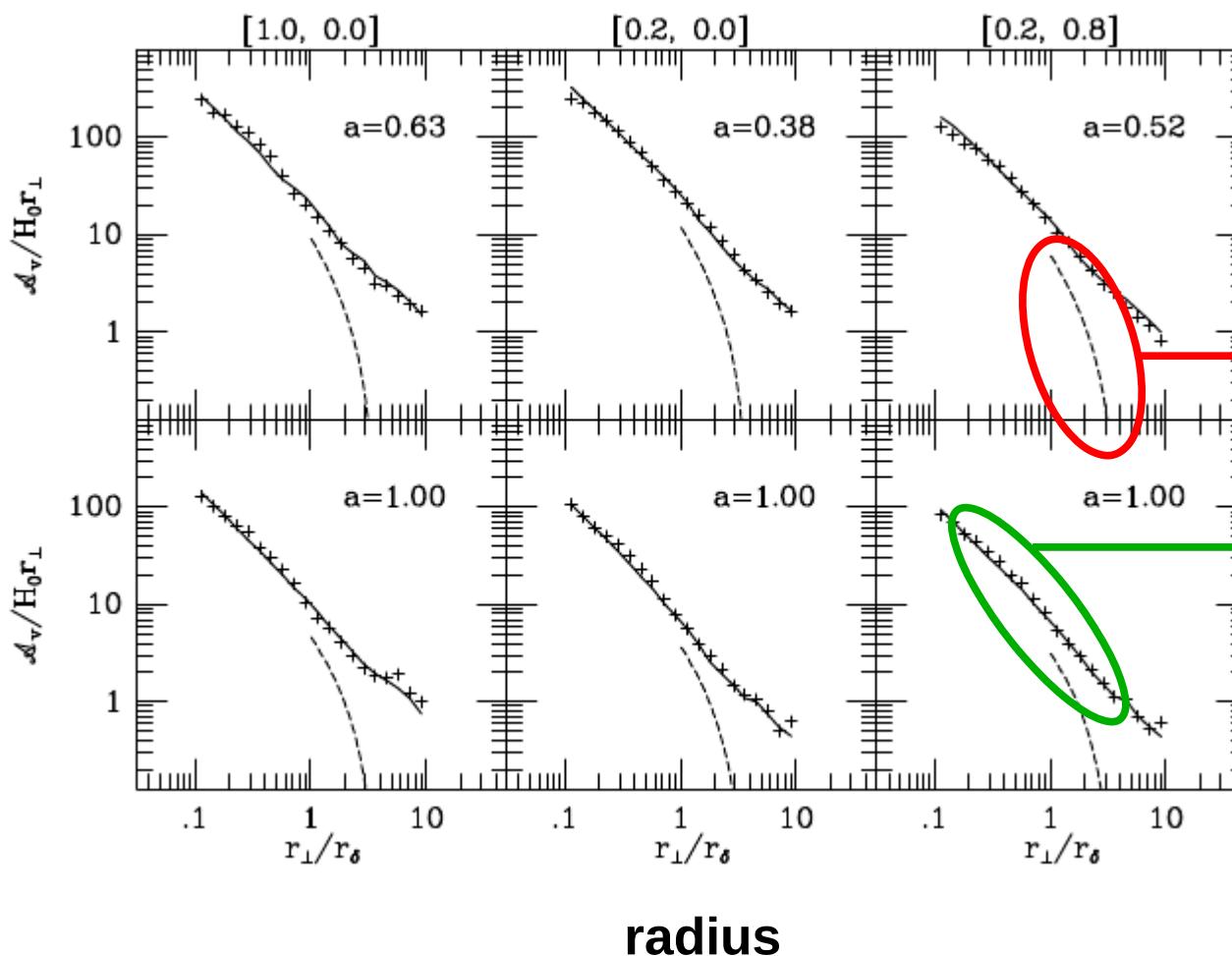
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theory

 $\mathcal{A}(r) \Leftrightarrow \text{escape velocity}$ 
flat with  $\rho_c$ 

open

flat with low  $\rho$ 

out of equilibrium

spherical infall  
model

caustic amplitude

in equilibrium

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**Interpretation:**  $\mathcal{A}(\theta)$  is the average over a volume  $d^3\mathbf{r}$  of the square of the l.o.s. component of the escape velocity

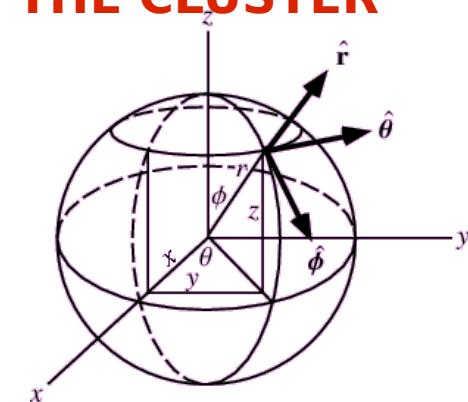
$$\mathcal{A}^2(r) = \langle v_{esc,los}^2 \rangle$$

$$\langle v_{esc,los}^2 \rangle = -2\phi(r)g^{-1}(\beta)$$

$$\beta(r) = 1 - \frac{\langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle}{2\langle v_r^2 \rangle}$$

**HOLDS INDEPENDENTLY OF THE DYNAMICAL STATE OF THE CLUSTER**

$$\mathcal{A}_{\text{infall model}} < \mathcal{A}_{\text{non-radial}}$$



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## Mass estimate

$$\mathcal{A}^2(r) = \langle v_{esc,los}^2 \rangle$$

$$\langle v_{esc,los}^2 \rangle = -2\phi(r)g^{-1}(\beta)$$

mass of an infinitesimal shell

$$G \, dm = -2\phi(r)\mathcal{F}(r) \, dr = \mathcal{A}^2(r)g(\beta)\mathcal{F}(r) \, dr$$

where  $\mathcal{F}(r) = \frac{-2\pi G \rho(r)r^2}{\phi(r)}$  and

$\mathcal{F}_\beta(r) = \mathcal{F}(r)g(\beta)$  is a slowly changing function of  $r$

theoretical framework of the  
**CAUSTIC TECHNIQUE**

$$GM(< r) = \mathcal{F}_\beta \int_0^r \mathcal{A}^2(r) \, dr$$

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theory

Developed in the '90s  
(Diaferio & Geller, 1997;  
Diaferio, 1999)

Problem: it requires  
hundreds of galaxy  
redshifts

nowadays the  
required data are  
easily collectable

## CAUSTIC TECHNIQUE

Can be applied for

- MASS/POTENTIAL ESTIMATES
- IDENTIFICATION OF MEMBERS
- [ IDENTIFICATION OF SUBSTRUCTURES ]

to simulated and real data

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1

arrange the galaxies in a binary tree according to a hierarchical method

2

select two thresholds to cut the tree: the largest group obtained from the upper-level threshold identifies the cluster candidate members

3

build the redshift diagram of all the galaxies in the field; locate the caustics, and identify the final cluster members

4

the caustic amplitude determines the escape velocity and mass profiles

four steps

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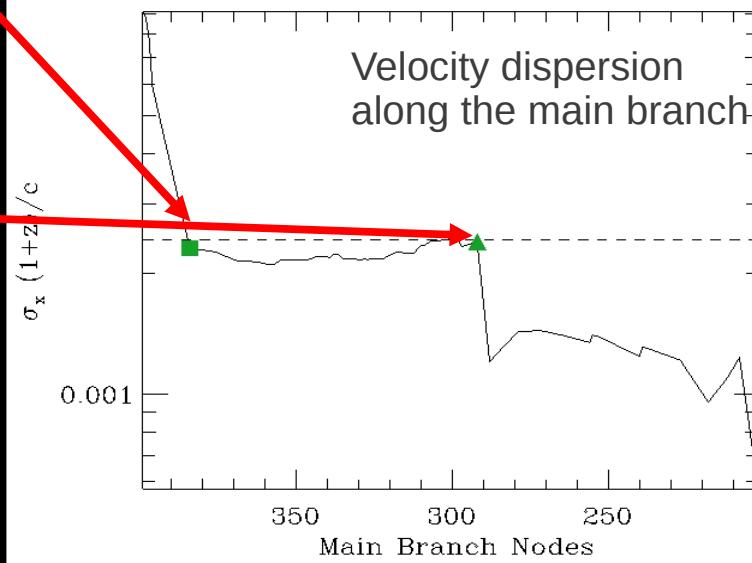
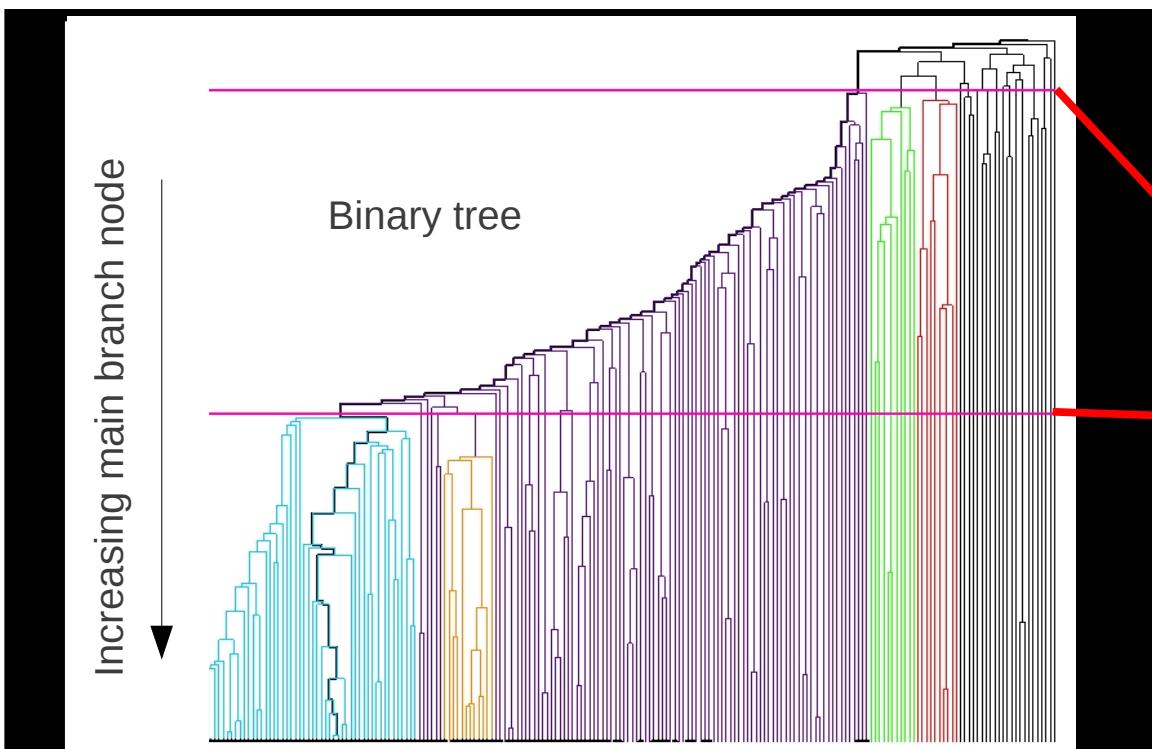
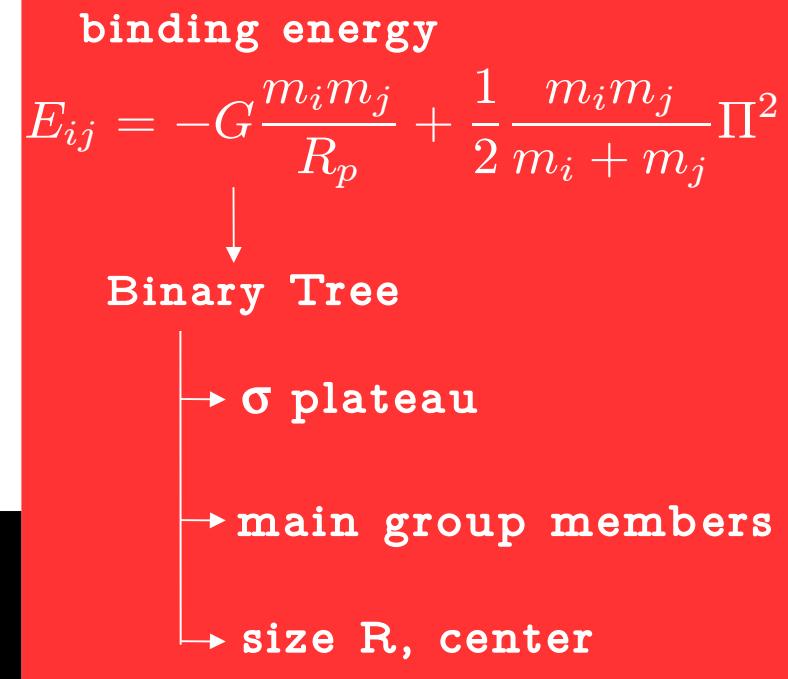
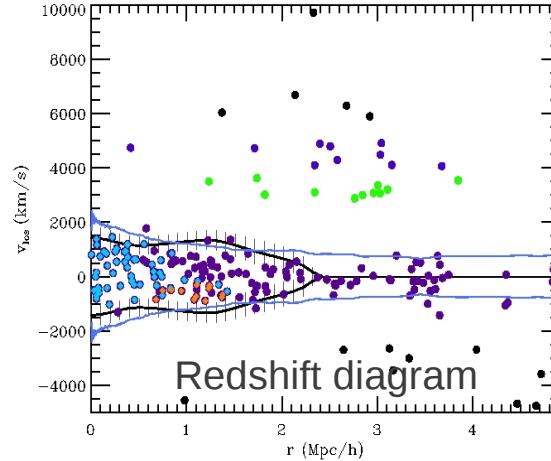
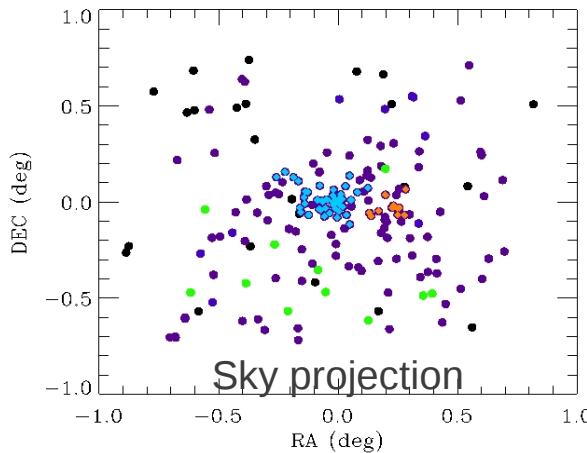
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1

# Binary tree & $\sigma$ -plateau



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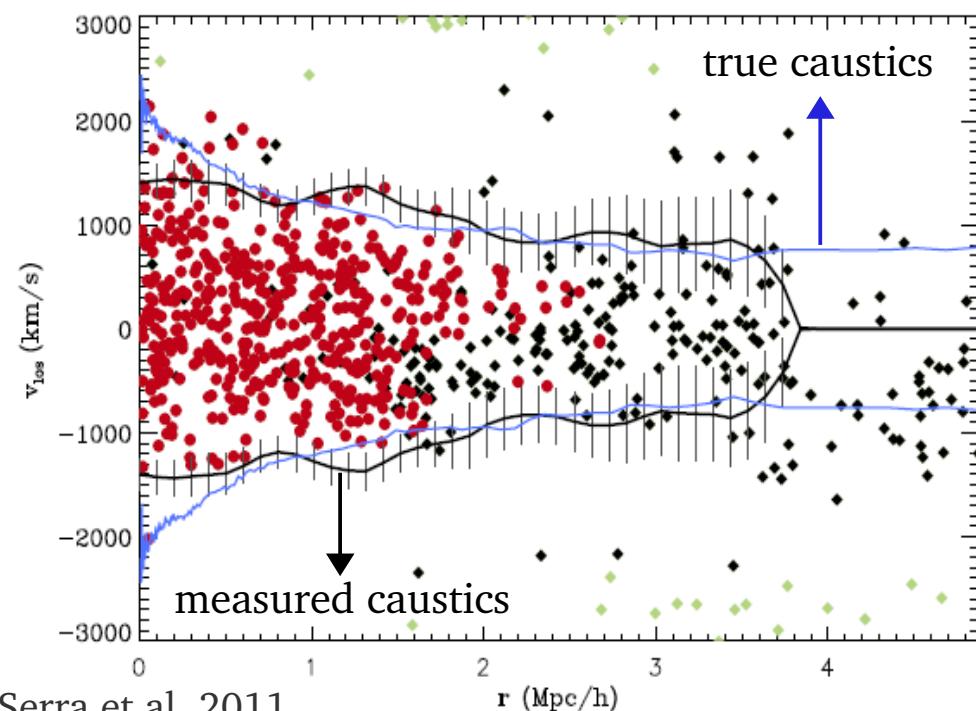
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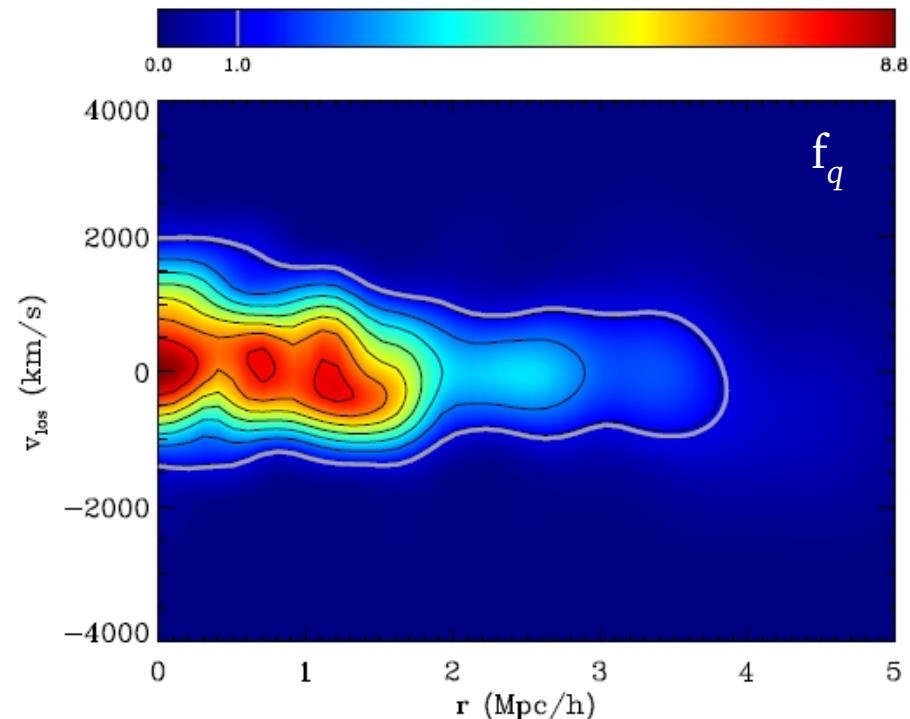
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2

## Redshift diagram



Serra et al. 2011



3

## Caustic location

we choose the parameter  $\kappa$  that  
determines the correct caustic location  
as the root of the equation

$$S(\kappa) \equiv \langle v_{\text{esc}}^2 \rangle_{\kappa, R} - 4 \langle v^2 \rangle = 0$$

distribution of  $N$  galaxies

$$f_q(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^N \frac{1}{h_i^2} K \left( \frac{\mathbf{x} - \mathbf{x}_i}{h_i} \right)$$

$$\mathbf{x} = (r, v)$$

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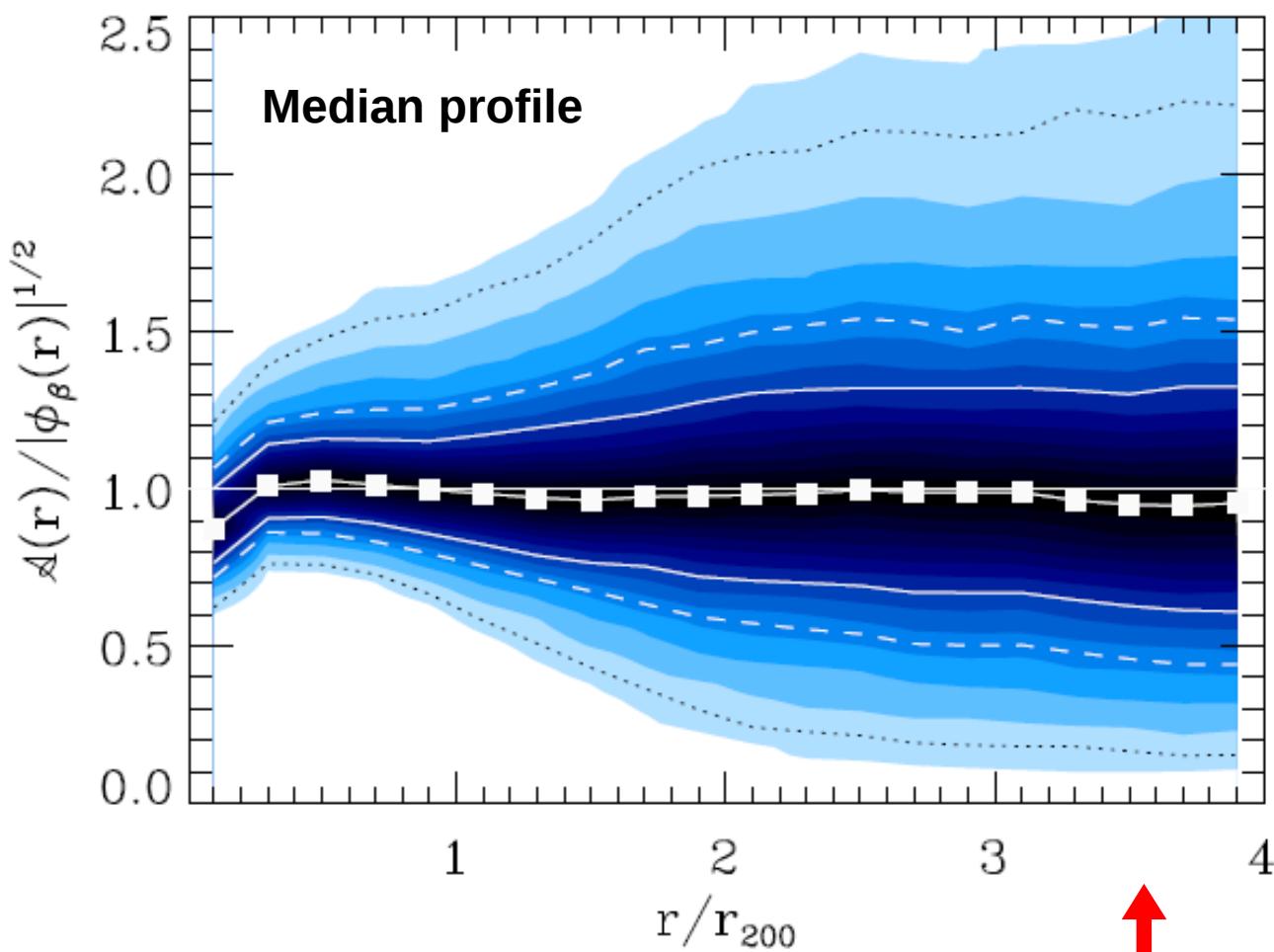
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app

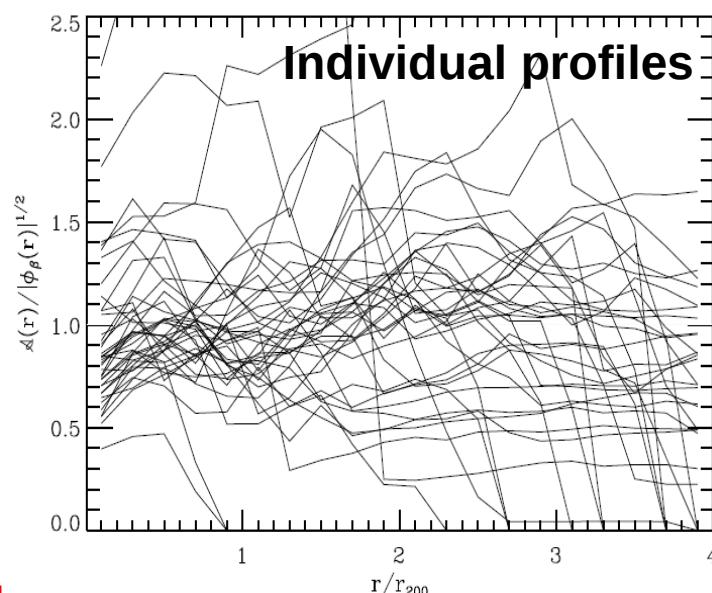
3

## Gravitational potential profiles



Escape velocity: better than 10% up to  $r \sim r_{200}$

3000 simulated clusters with  
 $M_{200} \geq 10^{14} h^{-1} M_\odot$   
from simulation by Borgani et al. 2004



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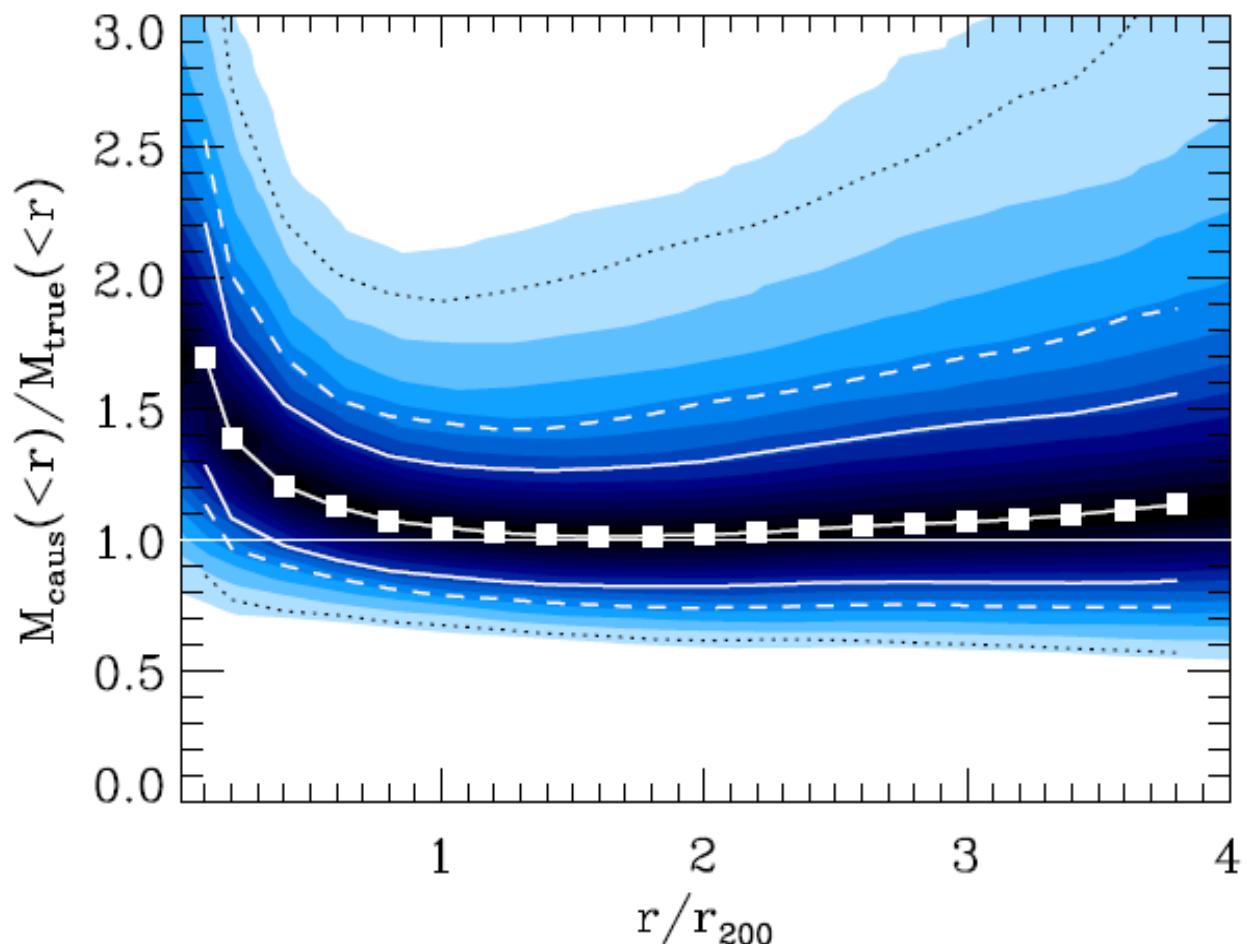
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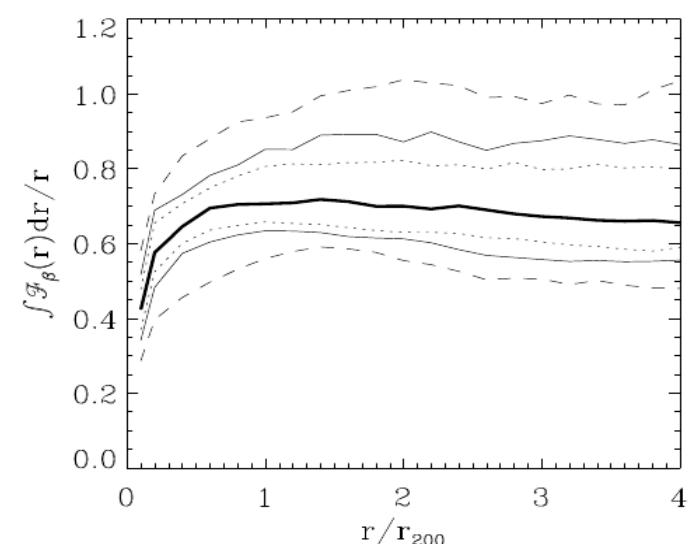
4

## Mass profiles



- (0.6-4)  $r_{200} \rightarrow$  better than 15%
- $r < 0.6 r_{200} \rightarrow$  overestimation of the mass up to 70%

3000 simulated clusters with  
 $M_{200} \geq 10^{14} h^{-1} M_\odot$



$$GM(< r) = \mathcal{F}_\beta \int_0^r \mathcal{A}^2(r) dr$$

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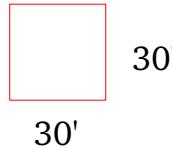
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# Stacked cluster

$$|v_{\text{los}}^x - v_{\text{clus los}}^{\text{clus}}| = 2000 \text{ km/s}$$



30'

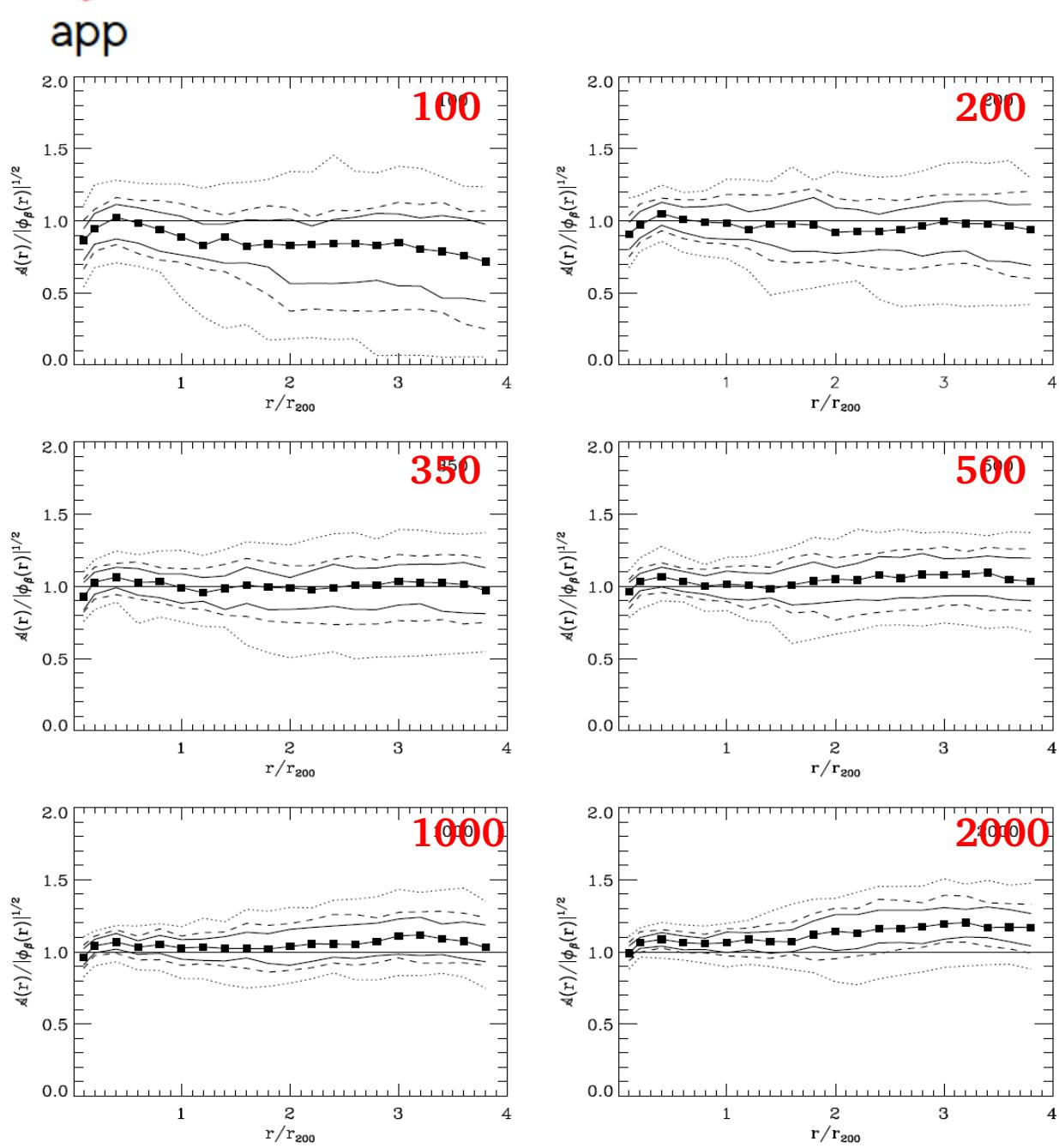
30'

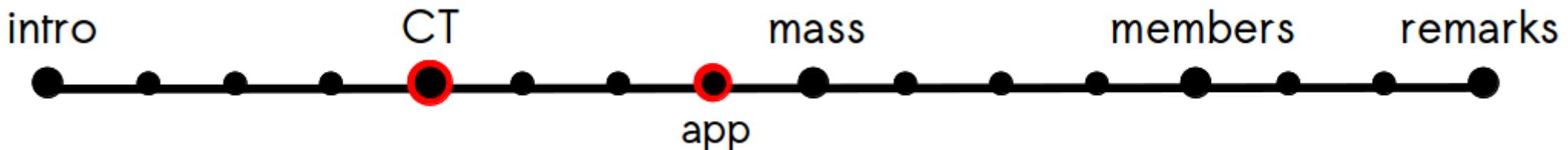
$$z^{\text{clus}} = 0.1$$

$$30' \rightarrow 2.46 \text{ Mpc/h}$$

spread decreases with  
increasing number of galaxies

measured caustic amplitude/true caustic amplitude





## Applications to real systems

Coma (Geller et al. 1999) → NFW profile fits cluster density profile

## Cluster and Infall Region Nearby Survey (CAIRNS) (Rines et al. 2003)

CIRS (Rines & Diaferio, 2006): 72 X-ray selected clusters with galaxy redshifts extracted from DR4-SDSS. Largest sample of clusters have been measured out to  $\sim 3r_{200}$  → virial mass function → cosmological parameters consistent with WMAP (Rines et al. 2007, 2008)

Groups of galaxies: 16 groups – NFW profile confirmed (Rines & Diaferio, 2008)

## 43 stacked clusters from 2dF (Biviano & Girardi, 2003)

Unrelaxed systems: Shapley superclusters (Reisenegger et al. 2000, Davidzon et al. in prep), Fornax cluster (Drinkwater et al. 2001), A2199 (Rines et al. 2002)

Coma and CL0024 to measure  $w_{\text{DM}}$  (Serra & Dominguez, 2011)

Individual systems (Mahdavi et al. 2005; Lemze et al. 2009; Lu et al. 2010)

HeCS (Hectospec Cluster Survey; Rines et al. 2012): clusters in the redshift range  $0.1 < z < 0.3$ ; more than 20,000 new redshifts; 17 clusters with WL mass profiles

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lensing

requires

wide-field redshift  
survey

wide-field  
photometric  
survey

sufficiently dense  
survey

redshift where  
signal is  
sufficiently strong

yields

3D mass profile  
affected by  
projection

projected mass  
profile along the  
line of sight

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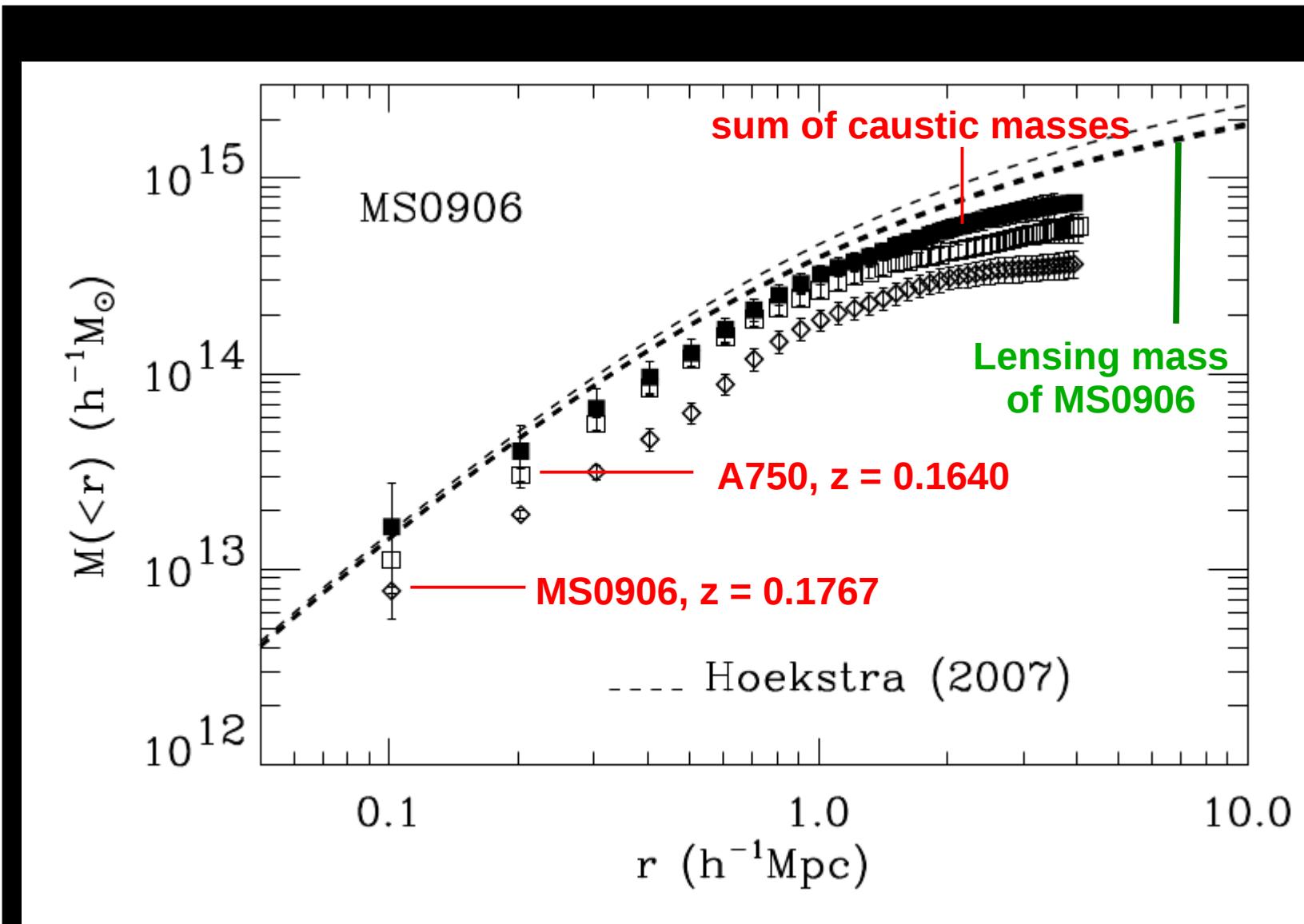
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ind. case



Geller et al. submitted

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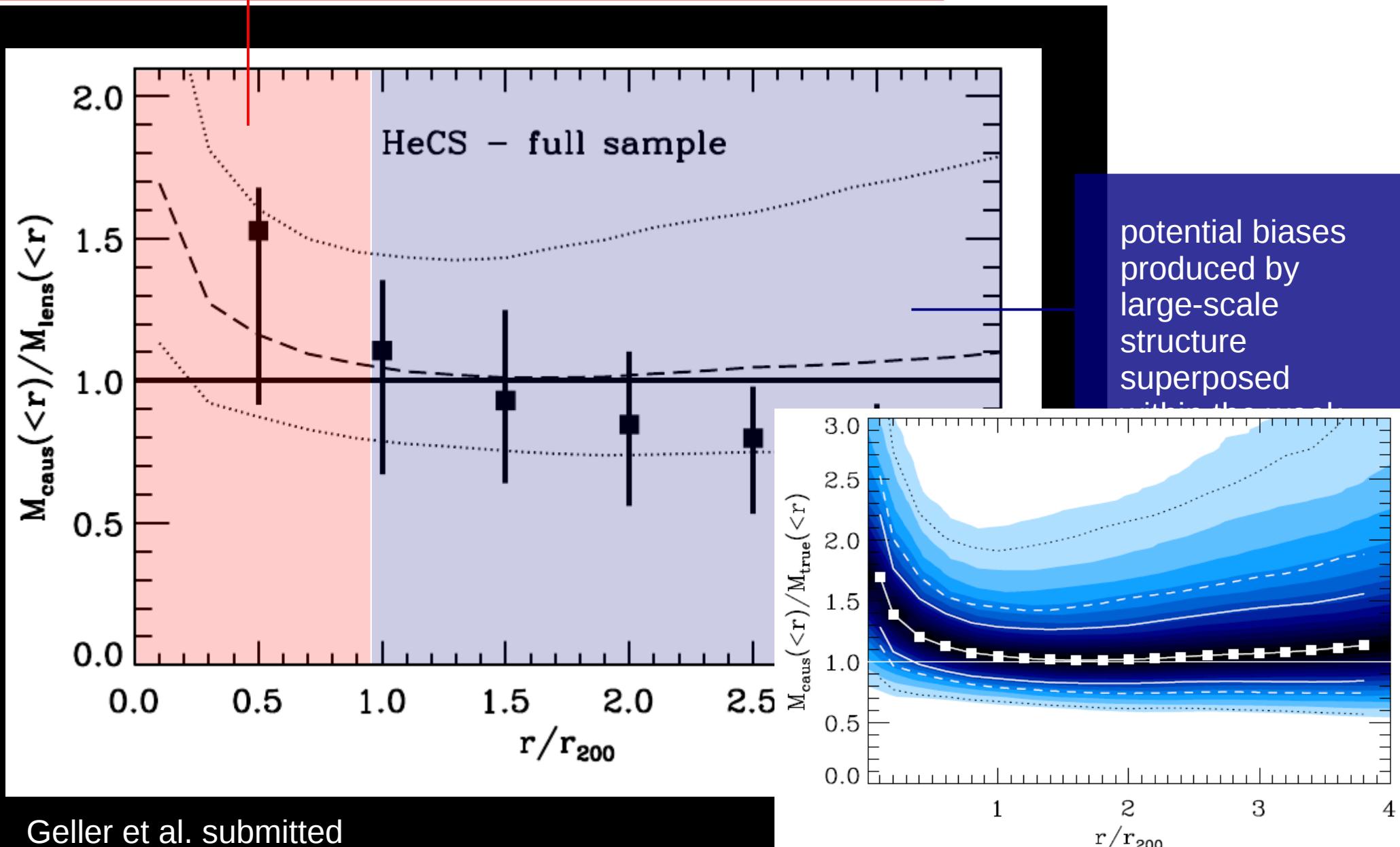
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HeCS

the caustic technique overestimates the mass in the inner regions



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# Membership

To study the dependence of properties on the environment we need to know whether a galaxy is member of a cluster

line-of-sight velocity

$3\sigma$  clipping (Yahil & Vidal 1977): velocity distribution close to gaussian

gap method (Zabludoff et al. 1990;  
Beers et al. 1990)

adaptive kernel method (Pisani 1993)  
probability density underlying the data

gaussian plus a constant function: fit to the velocity distribution (McKay et al. 2002)

+ projected radius

velocity of a galaxy depends on the gravitational potential profile of the cluster

escape velocity profiles → caustic technique

+ dependence of the VD on the projected distance to the center (Prada et al. 2003)

fit to  $\sigma_{\text{los}}$  according to the Jeans formalism ( $\beta = 0$ ) (Lokas et al. 2006)

$v_{\text{max}}(R)$  from the mass distribution  
(den Hartog & Katgert, 1996)

boundary lines  $\pm v_{\text{lim}}(R)$  on the velocity diagram (Wojtak et al. 2007)

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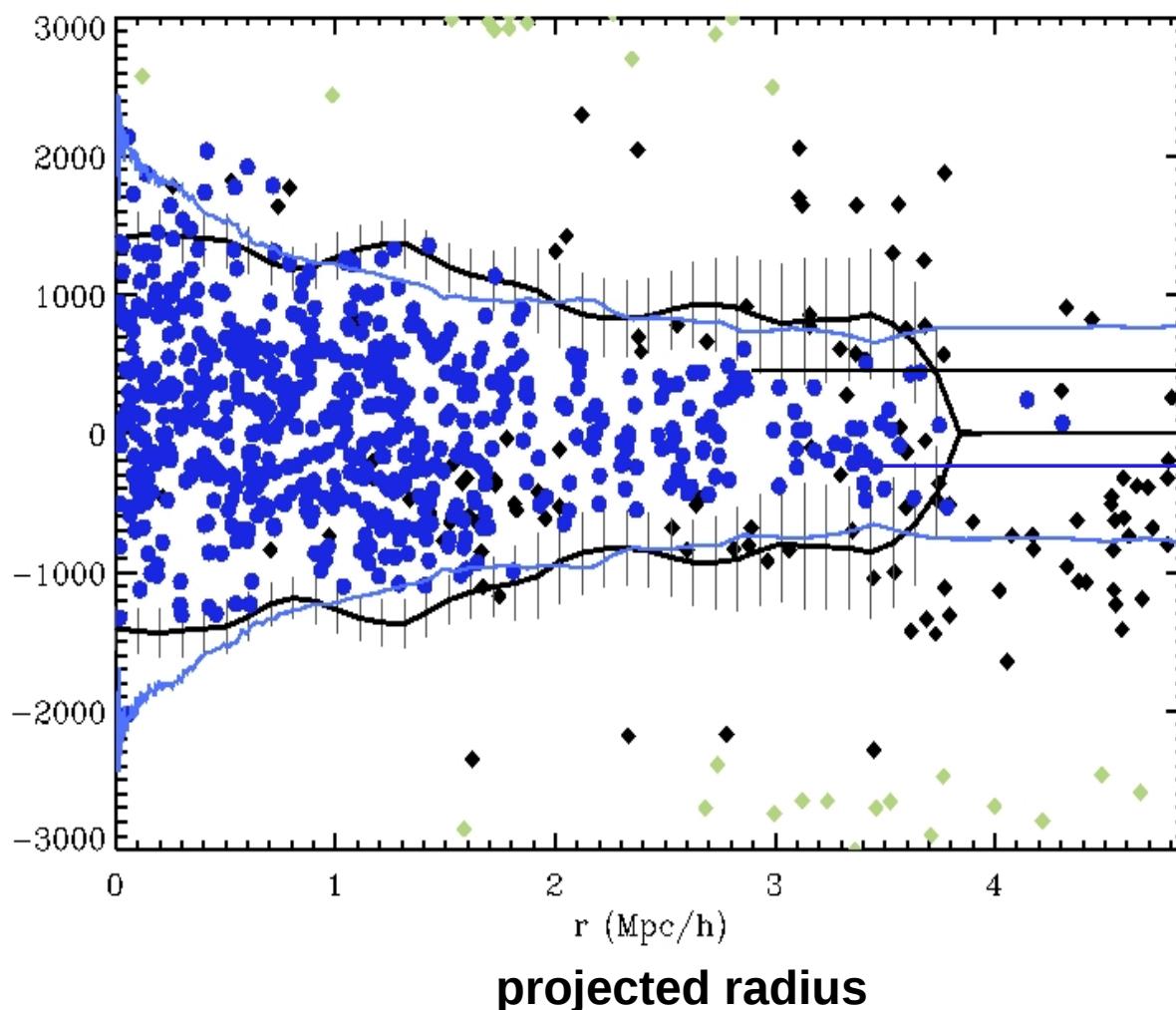
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ID

# Membership

I.o.S. velocity



→ galaxies within the caustics  
→ bound galaxies

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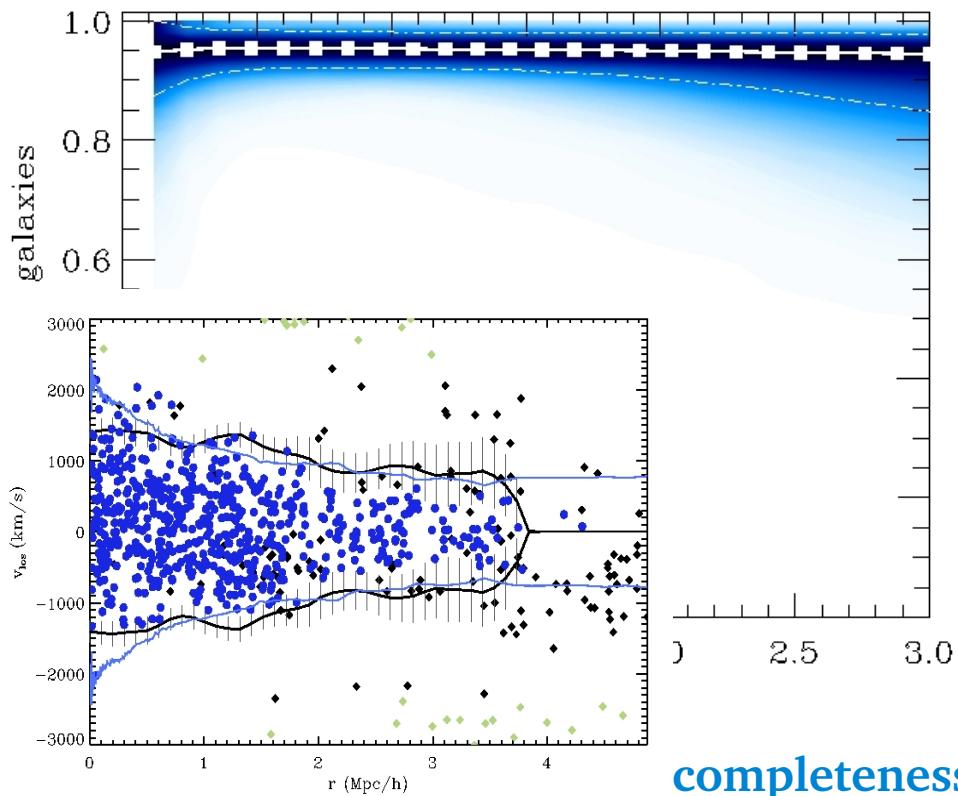
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galaxies within caustics    compared with    bound galaxies



completeness

contamination

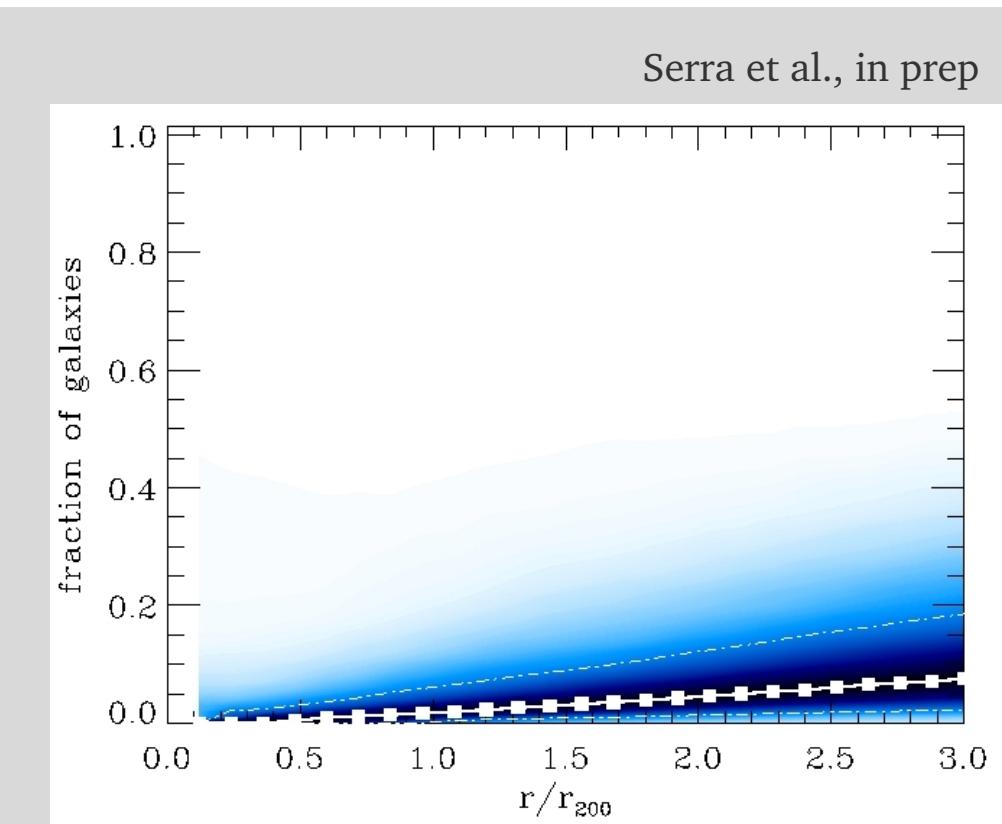
	$-1\sigma$	median	$+1\sigma$
--	------------	--------	------------

	$-1\sigma$	median	$+1\sigma$
--	------------	--------	------------

$r_{200}$	0.921	0.956	0.984
$2r_{200}$	0.908	0.951	0.981
$3r_{200}$	0.875	0.947	0.980

$r_{200}$	0.005	0.020	0.066
$2r_{200}$	0.015	0.047	0.126
$3r_{200}$	0.027	0.080	0.193

Serra et al., in prep



Serra et al., in prep

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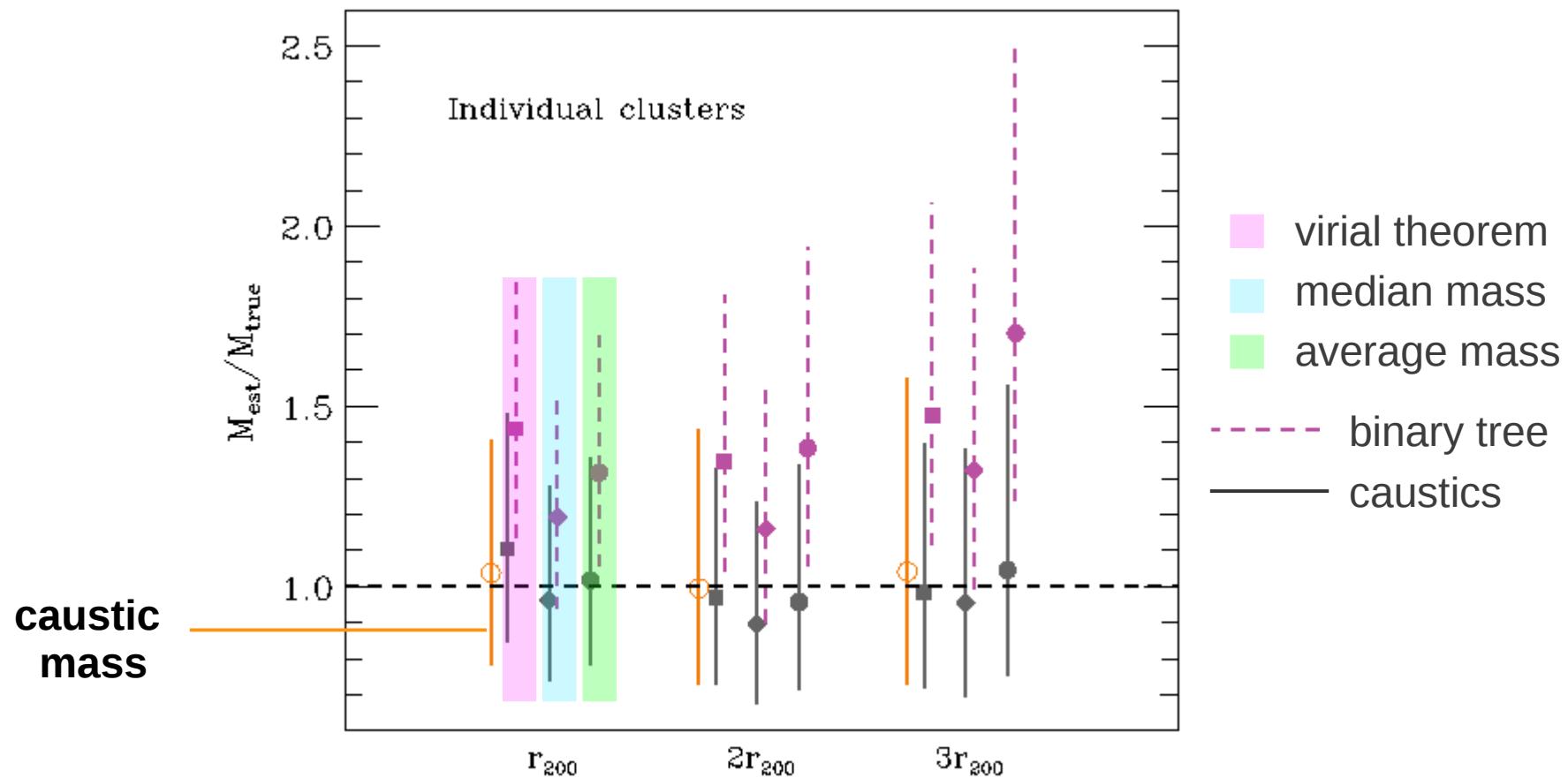
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# Mass estimates



The caustic location performs systematically better in removing interlopers and, on average, the bias in the mass estimate is minimized

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## Remarks

the caustic technique and gravitational lensing are the only two methods available to measure the mass profile of clusters beyond the virial radius without assuming dynamical equilibrium

~200 gxs in a field of 2.46 Mpc/h x 2.46 Mpc/h are enough to have an accurate escape velocity profile

$F_\beta(r)$  is not constant in the inner parts of the cluster → overestimation of the mass

spread due to projection → but the formal errors account for that

$$\delta M_i = \sum_{j=1,i} |2m_j \delta \mathcal{A}(r_j) / \mathcal{A}(r_j)|$$

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CT

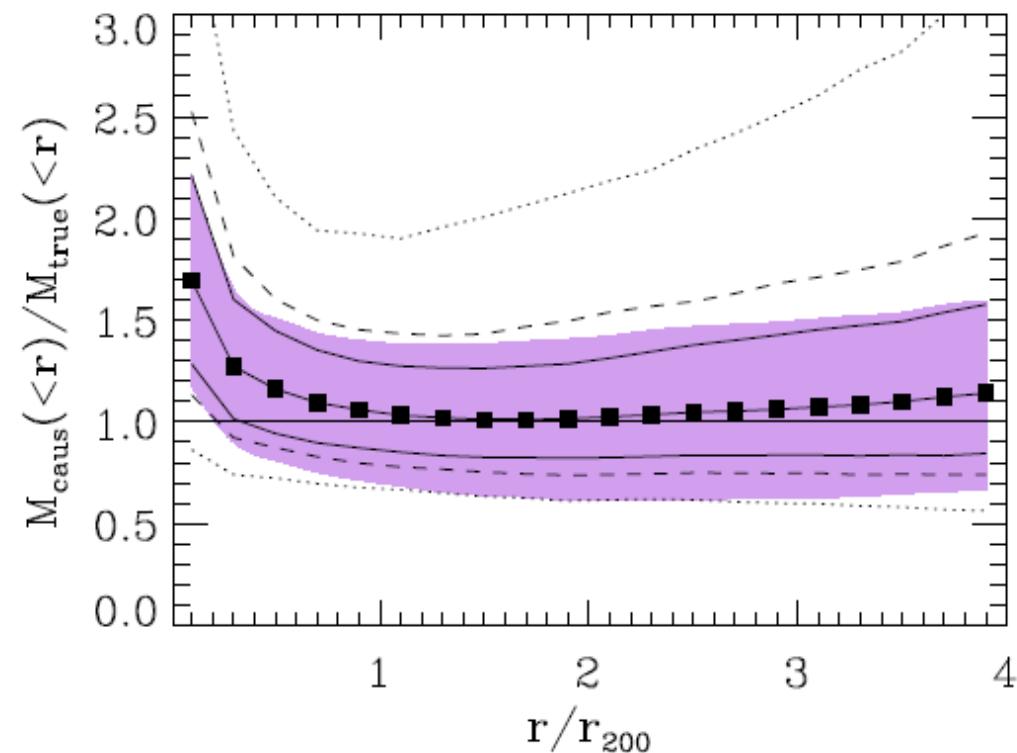
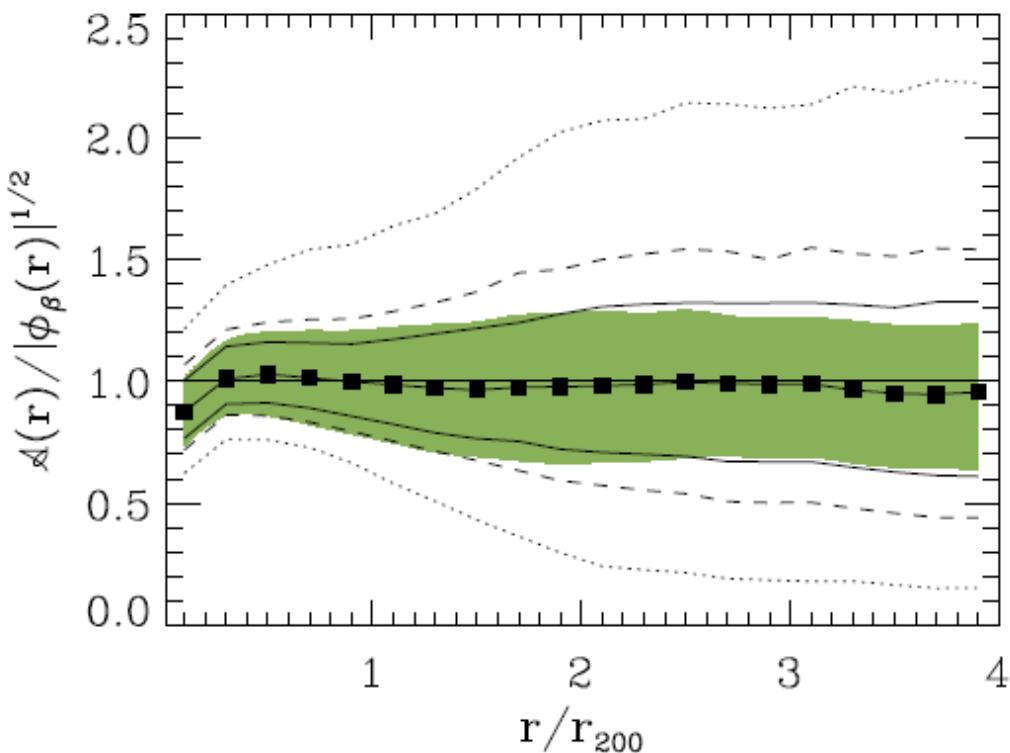
mass

members

remarks

# Remarks

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$$\delta M_i = \sum_{j=1,i} |2m_j \delta \mathcal{A}(r_j) / \mathcal{A}(r_j)|$$

intro

CT

mass

members

remarks



## Remarks

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the applications of the caustic technique to a large sample of simulated clusters demonstrated that the escape velocity is recovered with  $\sim 25\%$   $1-\sigma$  uncertainty and the mass profile with  $\sim 50\%$   $1-\sigma$  uncertainty

the median ratio of the caustic and weak lensing mass profile in the 19 HeCS clusters is within the 68% confidence limits of the ratio between the true and caustic mass profiles derived from N-body simulations. At radii  $< r_{200}$ , the caustic approach overestimated the mass. Near the virial radius ( $\sim 1.3r_{200}$ ) the profiles agree to  $\sim 30\%$ .

thank you!