

# COSMOLOGICAL TESTS of GENERAL RELATIVITY

OATS-UNI/TS  
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in collaboration with: L. Pogosian, G. Zhao, K. Koyama, A. Hojjati, .....



# Motivations and Outlook

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• WHY?

because we can!

cosmic acceleration

• HOW?

what to look for?

$f(R)$ : modified perturbation  
dynamics  $\longrightarrow$

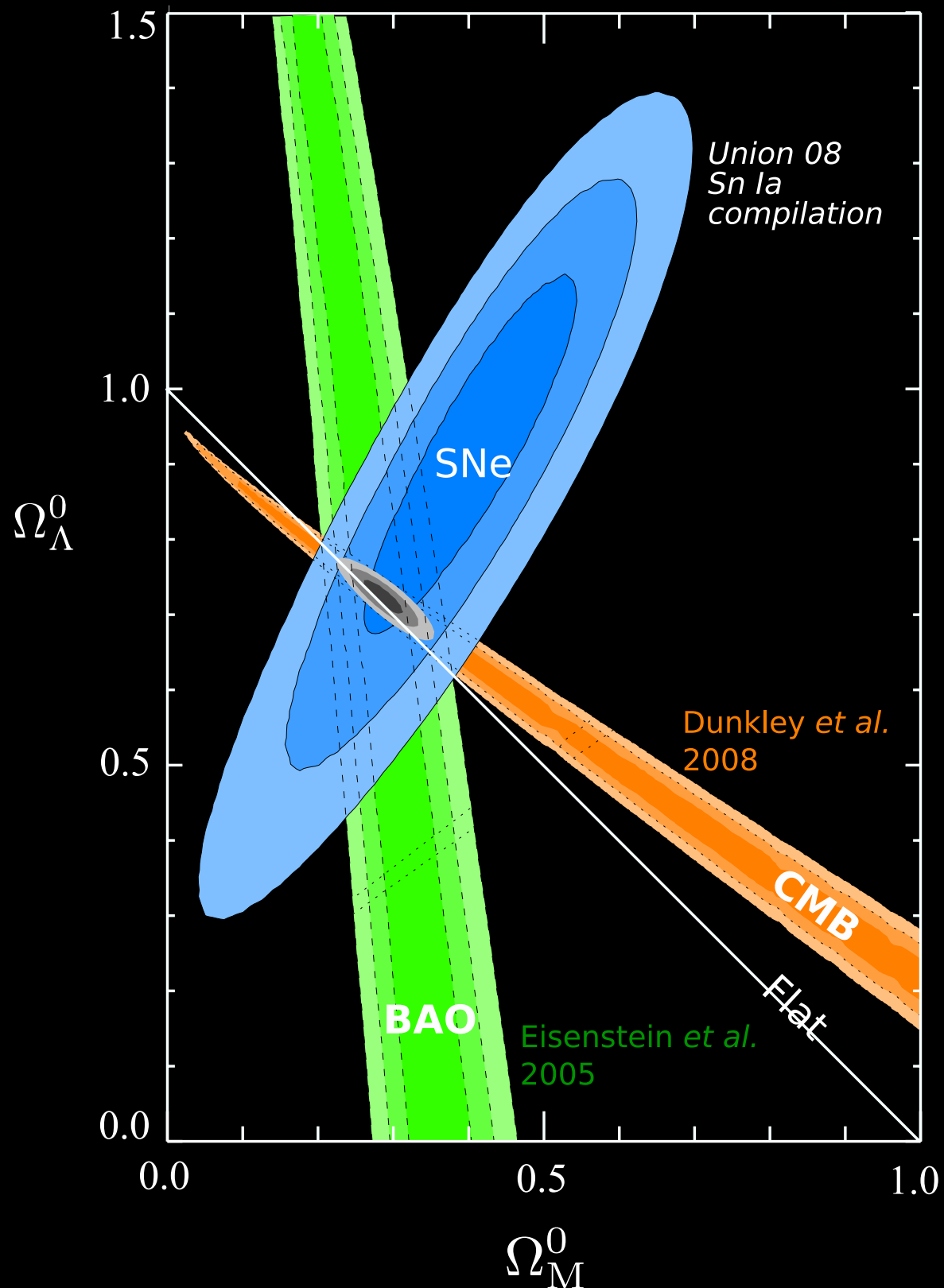
tomographic surveys will map the  
evolution of matter perturbations  
and gravitational potentials from the  
matter dominated epoch until today

how to be model-independent?

**Principal Component Analysis**

# Cosmic Acceleration

Supernova Cosmology Project  
Kowalski, et al., *Ap.J.* (2008)



**SN Ia, CMB, LSS**

+

homogeneity and isotropy on large scales



**COSMIC ACCELERATION**

+

**standard GR**



$$\Omega_m^0 \approx 0.3$$

$$\Omega_X^0 \approx 0.7$$

$$\left( \Omega_i^0 \equiv \frac{\rho_i^0}{\rho_{cr}^0} \right)$$

# ...cosmic acceleration...

- Cosmic Acceleration:  $\Lambda$ ? Modified Gravity? Dark Energy?

from the **DETF** (Albrecht et al. '06)

1. The goal is to determine the very nature of the dark energy that causes the Universe to accelerate and seems to comprise most of the mass-energy of the Universe.

2. Toward this goal, our observational program must

Determine as well as possible whether the accelerating expansion is consistent with being due to a cosmological constant

If the acceleration is not due to a cosmological constant, probe the underlying dynamics by measuring as well as possible the time evolution of the dark energy by determining the function  $w(a)$ .

c. Search for a possible failure of general relativity through comparison of the effect of dark energy on cosmic expansion with the effect of dark energy on the growth of cosmological structures like galaxies or galaxy clusters.

.....

I. We strongly recommend that there be an aggressive program to explore dark energy as fully as possible, since it challenges our understanding of fundamental physical laws and the nature of the cosmos.

II. We recommend that the dark energy program have multiple techniques at every stage, at least one of which is a probe sensitive to the growth of cosmological structure in the form of galaxies and clusters of galaxies.

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by measuring  
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c. Search  
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## US NATIONAL RESEARCH COUNCIL'S DECADEAL SURVEY (2010)

Cosmic Acceleration remains one of the  
main challenges for Modern Cosmology

**LSST** and **WFIRST** ranked top of funding  
priority list, respectively in the category of  
large ground- and space-based experiments

with

dynamics  
governing the

effect of

energy as  
laws and

...more...

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ESA COSMIC VISION 2020

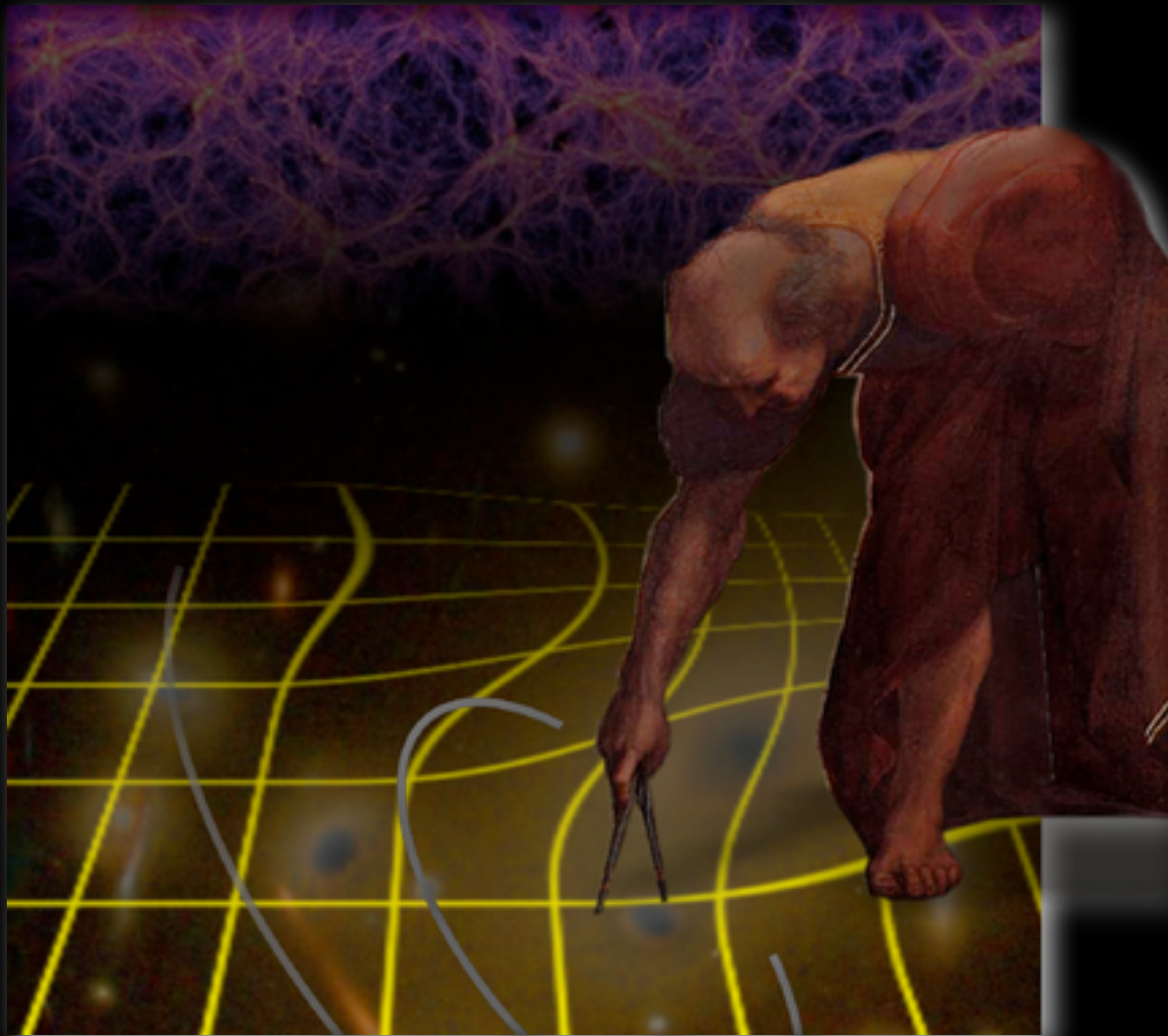
...

What is the Universe formed of?

**EUCLID** selected as one of the two M-class  
missions

...more...

## EUCLID: Mapping the geometry of the universe



1. Together, dark matter and dark energy pose some of the most important questions in fundamental physics today.

....

Euclid is a high-precision survey mission optimised for two independent cosmological probes:

1. **Weak Gravitational Lensing**  
from a high-resolution imaging survey

.....

2. **Baryon Acoustic Oscillations**  
in Galaxy Clustering measured via a massive spectroscopic redshift survey

*final approval received June 2012, to be launched in 2019 ...*

# What can we measure cosmologically?

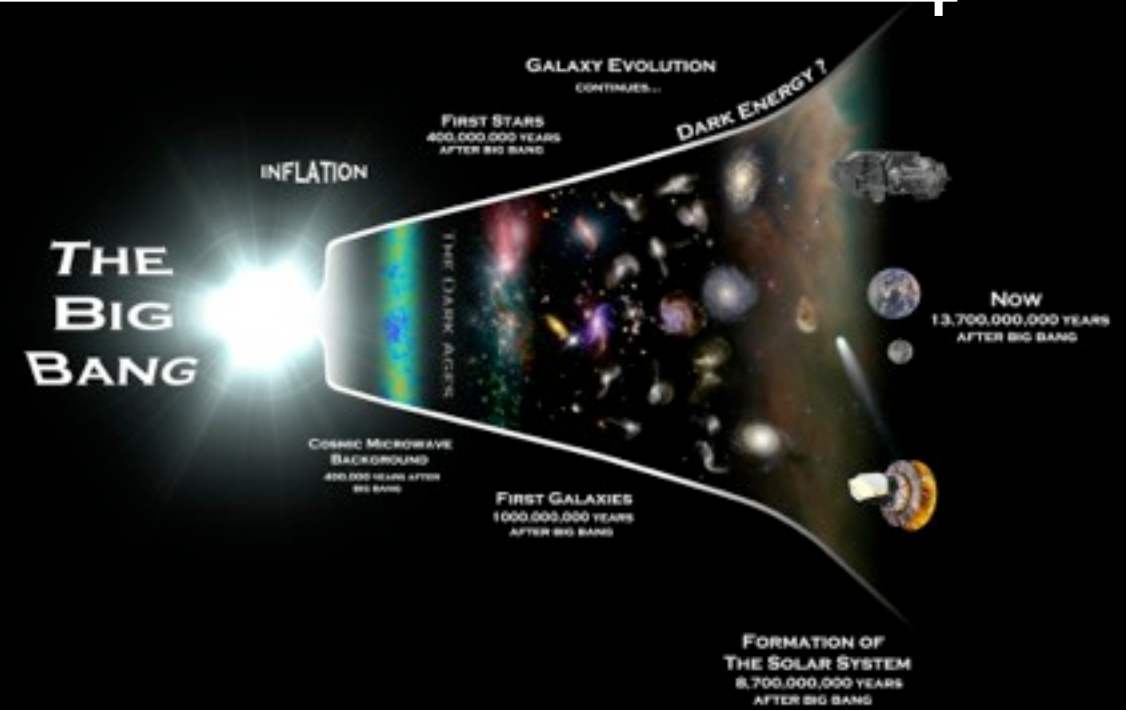
$$ds^2 = -a^2(\tau) \left[ (1 + 2\Psi(\tau, \vec{x}))d\tau^2 - (1 - 2\Phi(\tau, \vec{x}))d\vec{x}^2 \right]$$



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expansion history:  $a(\tau)$

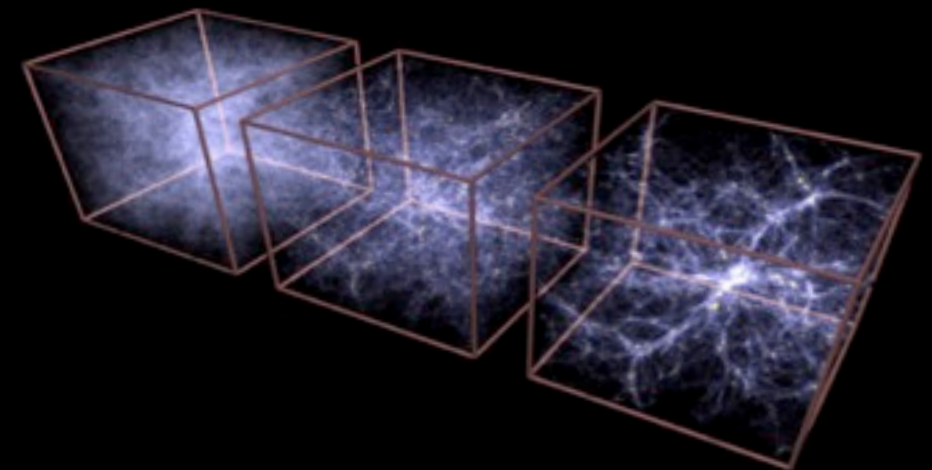
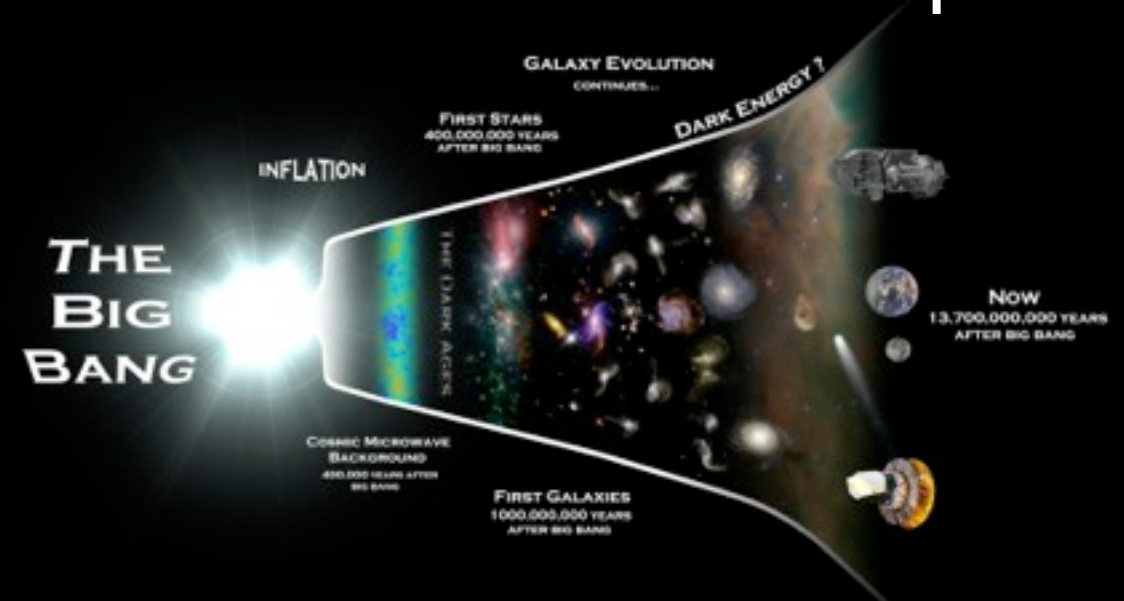


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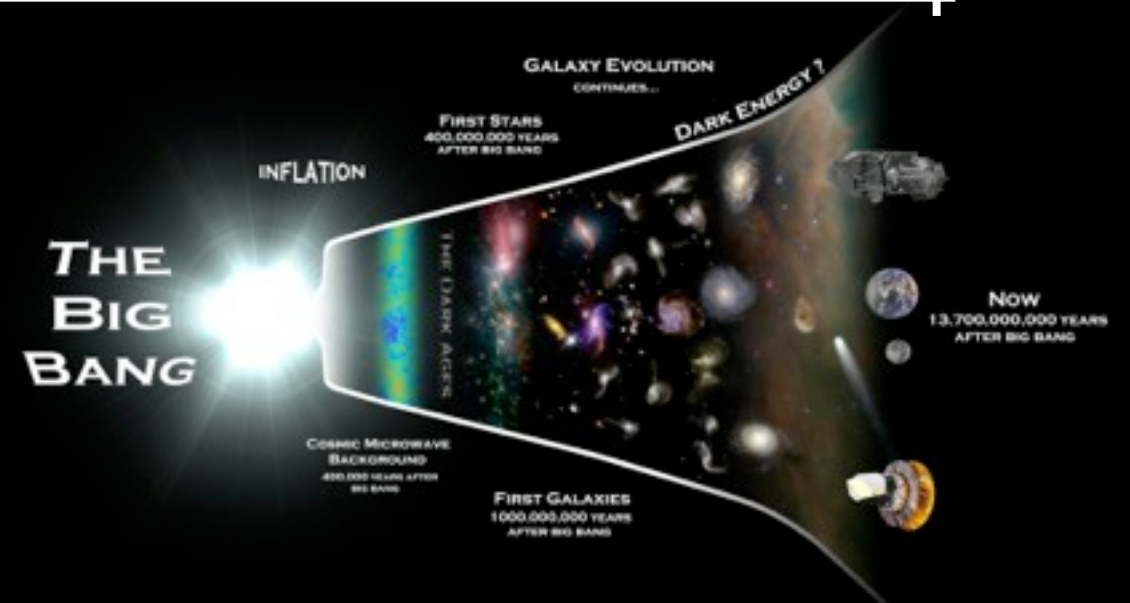
non-relativistic dynamics  
(growth of structure, pec. vel.):  $\Psi$



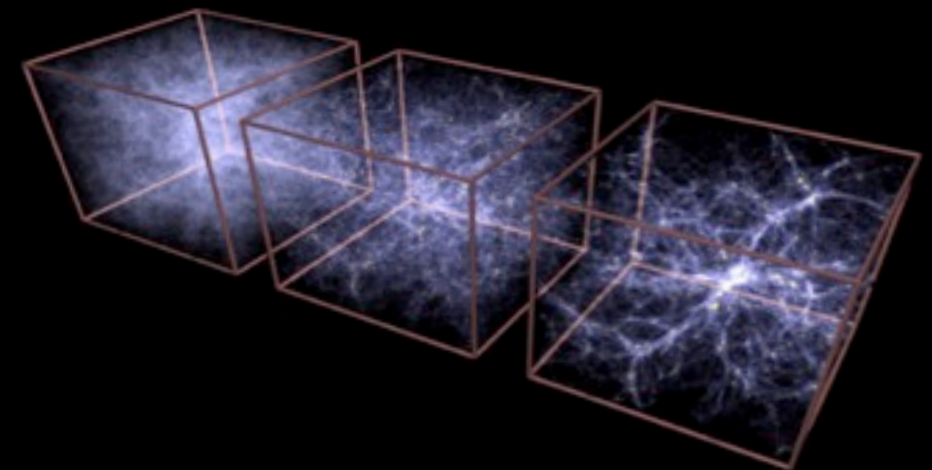
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relativistic dynamics  
(weak lensing, ISW):  $\Phi + \Psi$



# and what can we test?

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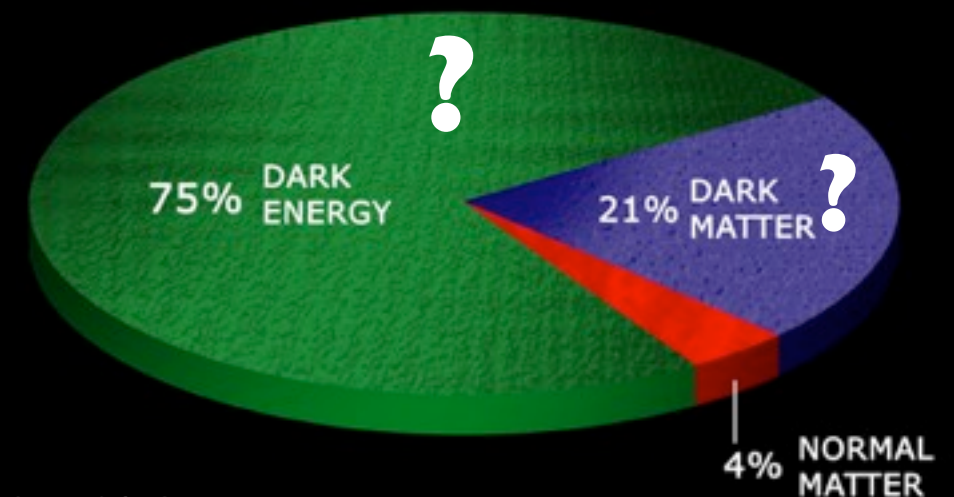
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... if we knew the matter content of the universe we could really test the theory ...



... short of that, we can either assume something on the dark sector or start with a **test of the cosmological concordance model** LCDM ...

... it is based on GR :  $G_{\mu\nu} = \frac{T_{\mu\nu}}{M_P^2}$

$$T_0^0 = -(\rho + \delta\rho)$$

$$T_j^0 = (\rho + p)v_j$$

$$T_j^i = (p + \delta p)\delta_j^i + \frac{1}{2}(\rho + p)(\nabla^i\nabla_j - \frac{1}{3}\delta_j^i\Delta)\pi$$

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LCDM:

$$w_{\text{eff}} = -1 \quad \Phi = \Psi \quad \Psi = -\frac{a^2}{k^2} \frac{\rho\Delta}{2M_P^2}$$



---

What have we learned  
from  
 $f(R)$  gravity?



# f(R) Gravity

$$S = \frac{M_P^2}{2} \int dx^4 \sqrt{-g} [R + f(R)] + \int dx^4 \sqrt{-g} \mathcal{L}_m [\chi_i, g_{\mu\nu}]$$

(S.Carroll, V.Duvvuri, M.Trodden & M.S.Turner, Phys.Rev.D70 043528 (2004),  
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*dynamical !*

# Dynamics of Linear Perturbations...Sub-Horizon

$$\delta''_m + \left(1 + \frac{H'}{H}\right) \delta'_m + \frac{k^2}{a^2 H^2} \Psi = 0$$

$$\frac{\Phi}{\Psi} = \frac{1 + 2 \frac{k^2}{a^2} \frac{f_{RR}}{F}}{1 + 4 \frac{k^2}{a^2} \frac{f_{RR}}{F}}$$

$$F \equiv 1 + f_R$$

$$k^2 \Psi = - \frac{3}{2} \underbrace{\frac{1}{F} \frac{1 + 4 \frac{k^2}{a^2} \frac{f_{RR}}{F}}{1 + 3 \frac{k^2}{a^2} \frac{f_{RR}}{F}}}_{\text{time and scale dependent rescaling of Newton constant}} E_m \delta_m$$

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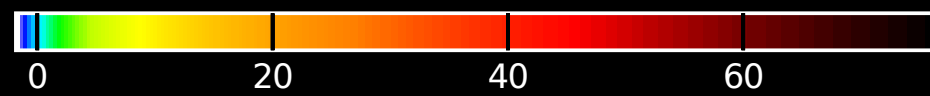
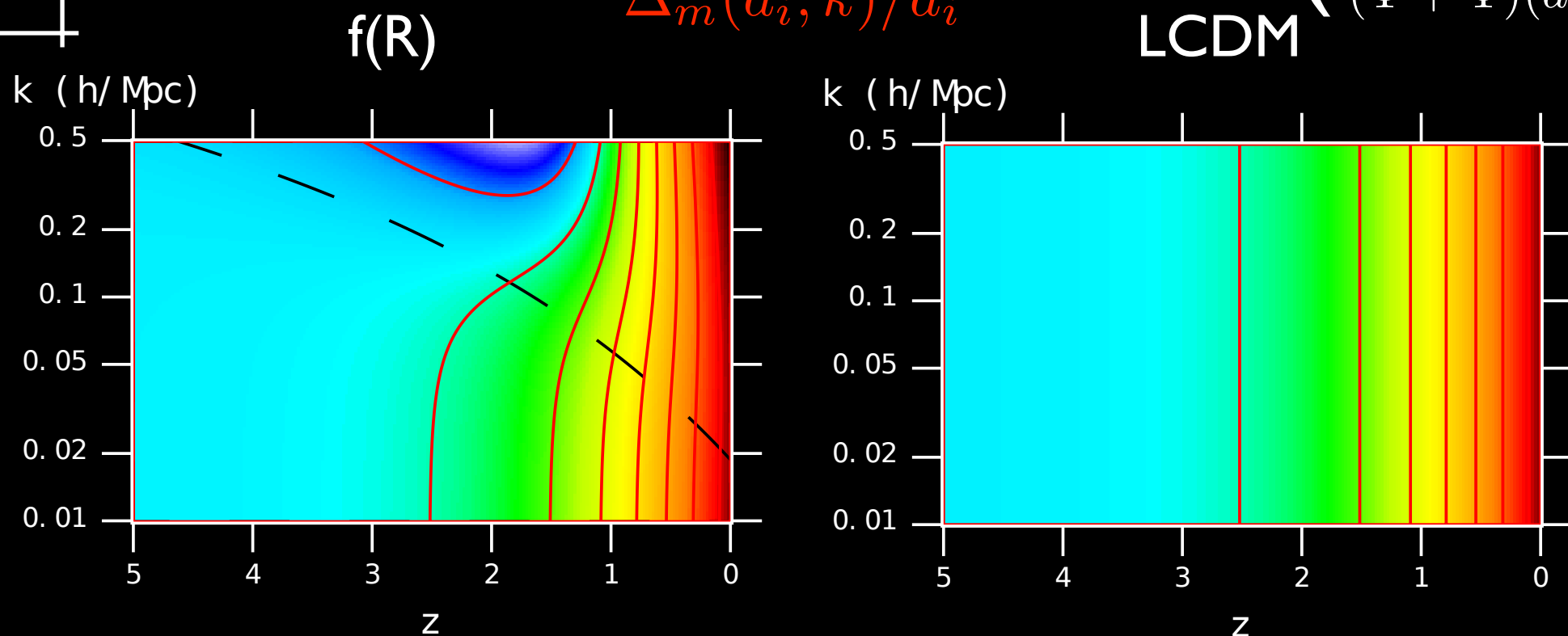
time and scale dependent  
rescaling of Newton constant

$$w_{\text{eff}} = -1$$

$$f_R^0 = -10^{-4}$$

$$\frac{\Delta_m(a, k)/a}{\Delta_m(a_i, k)/a_i}$$

$$\left( \frac{(\Phi + \Psi)(a, k)}{(\Phi + \Psi)(a_i, k)} \right)$$



(Pogosian and S., Phys.Rev.D 77(2008))

# Characteristic signatures

---

Overall we observe a **scale-dependent pattern of growth**

The modifications introduced by  $f(R)$  models are similar to those introduced by more general scalar-tensor theories and models of **coupled DE-DM**

The dynamics of perturbations is richer, and **different observables are described by different functions**, not by a single growth factor!



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$$f(R): \quad w_{\text{eff}} \approx -1 \quad \Phi \neq \Psi \quad \Psi \neq -\frac{a^2}{k^2} \frac{\rho \Delta}{2M_P^2}$$

# ...therefore...

---

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test:

the relation between **matter**  
**and gravitational potential**  $\Psi \leftrightarrow \Delta$

the relation between the **gravitational**  
**potential and the curvature of space**  $\Psi \leftrightarrow \Phi$

...and this will be possible with future **tomographic**  
**surveys**

# On Parametrizing

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...for each known model of gravity, we could derive predictions and compare them with observations...

...or we could determine observationally some “**trigger parameters**” designed to detect a breakdown of the cosmological standard model.

Some examples of these are:

Linder's  $\gamma$ :  $f = \frac{d \ln \delta}{d \ln a} \equiv \Omega_m(a)^\gamma$   
(Astropart.Phys.28 (2007))

Zhang et al.'s  $\epsilon_q$ :  $\langle E_G \rangle \equiv \frac{\nabla^2(\Phi + \Psi)}{3H_0^2 \beta \delta/a}$   
(Phys.Rev.Lett.99 (2007))

Any disagreement between the observed trigger parameter and its LCDM value would indicate some sort of modification of growth

# On Parametrizing

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We could be more ambitious and try to perform a global, model-independent fit to all the data...

...we need a complete and consistent set of equations for calculating predictions for all the observables  
...a possible set up is the following...

# General Dynamics of Linear Perturbations

Scalar perturbations in **Newtonian** gauge

$$ds^2 = -a^2(\tau) (1 + 2\Psi) d\tau^2 + a^2(\tau) (1 - 2\Phi) d\vec{x}^2$$

Energy-momentum conservation eqs.

$$\begin{aligned} \nabla_\mu T^\mu_\nu = 0 \quad & \delta' + \frac{k}{aH} v - 3\Phi' = 0 \\ & v' + v - \frac{k}{aH} \Psi = 0 \end{aligned}$$

Einstein eqs.

$$\text{Poisson:} \quad k^2 \Psi = -\mu(a, k) \frac{a^2}{2M_P^2} \rho \Delta$$

$$\text{anisotropy:} \quad \frac{\Phi}{\Psi} = \gamma(a, k)$$

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gravitational slip

$$\varpi \equiv \frac{\Psi}{\Phi} - 1$$

Caldwell et al., Phys.Rev.D**76**, 023507 (2007)



# General Dynamics of Linear Perturbations

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In LCDM  $\mu=1=\gamma$ , however in other models in general they are functions of time and space.

We expect them to differ from unity in:

- **Scalar-tensor theories (e.g.  $f(R)$ , Chameleon)** (Brax et al., Amendola, L., Song et al., Pogosian et al., Bean et al., Tsujikawa)
- **DGP and higher-dimensional gravity** (Afshordi et al., Lue et al., Song et al., Cardoso et al., Koyama et al., Maartens et al.)
- **LCDM + massive neutrinos** (Lesgourgues et al., Brookfield et al., Hannestad et al., Melchiorri et al., Pettorino et al.)
- **DE which clusters and/or carries anisotropic stress** (Koivisto et al., Bean et al., Mota et al.)

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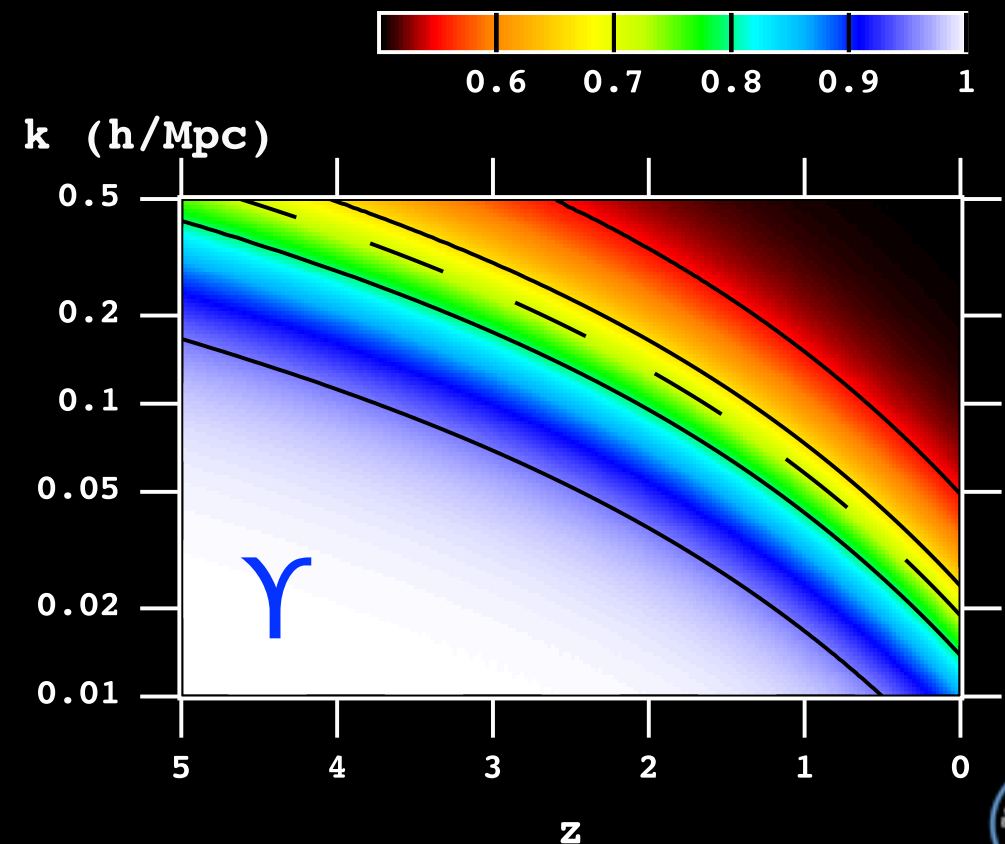
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For instance in **scalar-tensor gravity**:

$$\mu(a, k) = \frac{1 + \left(1 + \frac{1}{2}\alpha'^2\right) \frac{k^2}{a^2 m^2}}{1 + \frac{k^2}{a^2 m^2}}$$

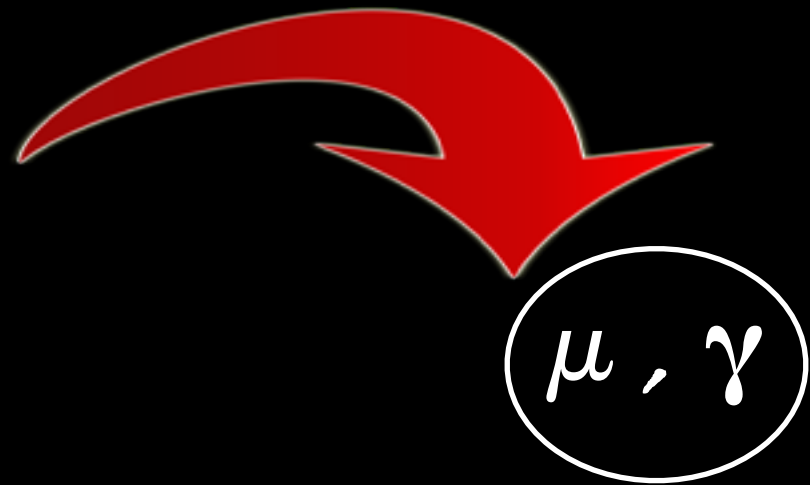
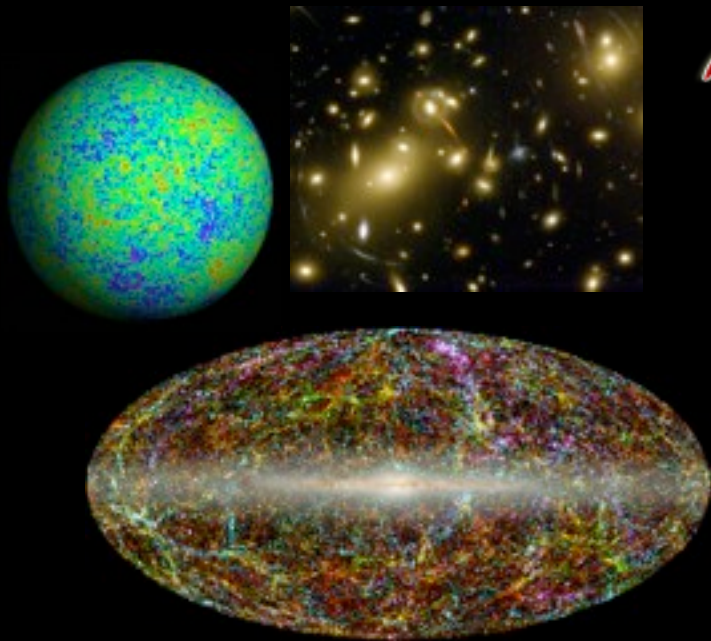
$$\gamma(a, k) = \frac{1 + \left(1 - \frac{1}{2}\alpha'^2\right) \frac{k^2}{a^2 m^2}}{1 + \left(1 + \frac{1}{2}\alpha'^2\right) \frac{k^2}{a^2 m^2}}$$



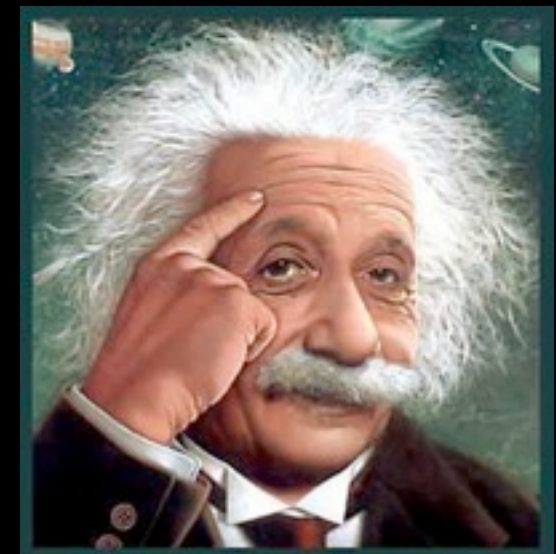
# A final note ...

Let me stress that these functions provide us with a consistent set of equations to perform a fit to any data and test for potential departures from LCDM, however...

DATA



THEORY



# Searching for modified growth patterns

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What is the potential of current and upcoming tomographic surveys to detect departures from GR (LCDM, quintessence) in the growth of structure?



What is the potential of the surveys to constrain the functions  $\mu$  and  $\gamma$  ?

# How to treat the functions themselves?

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For some recent work in this direction see:

Zhao et al., PRL 103 (2009)

Daniel et al., PRD 80 (2009)

Guzik et al., PRD 81 (2010)

Bean, R. et al., PRD 81 (2010)

Pogosian et al., PRD 81 (2010)

Zhao et al., PRD 81 (2010)

Daniel et al., PRD 81 (2010)

Baker et al. arXiv:1209.2117

Hojjati et al., PRD 85 (2012)

Baker et al. PRD 84 (2010)

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We want to stay as much as possible  
model-independent and generic.

Therefore we will treat  $\mu$  and  $\gamma$  as **two unknown functions of time and scale** and determine **how many d.o.f.** of these functions can be (well) constrained by upcoming surveys.

Also, a take-home result will be to determine the “**sweet spots**” in space and time where the experiments are most sensitive to departures from GR. Inversely, this can be used to guide survey-design in order to test specific candidate models.

# Forecasting Constraints

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## PRINCIPAL COMPONENT ANALYSIS

(A.J.S.Hamilton and M.Tegmark, astro-ph/9905192, MNRAS'00  
D.Huterer and G.Starkman, astro-ph/0207517, PRL'03)

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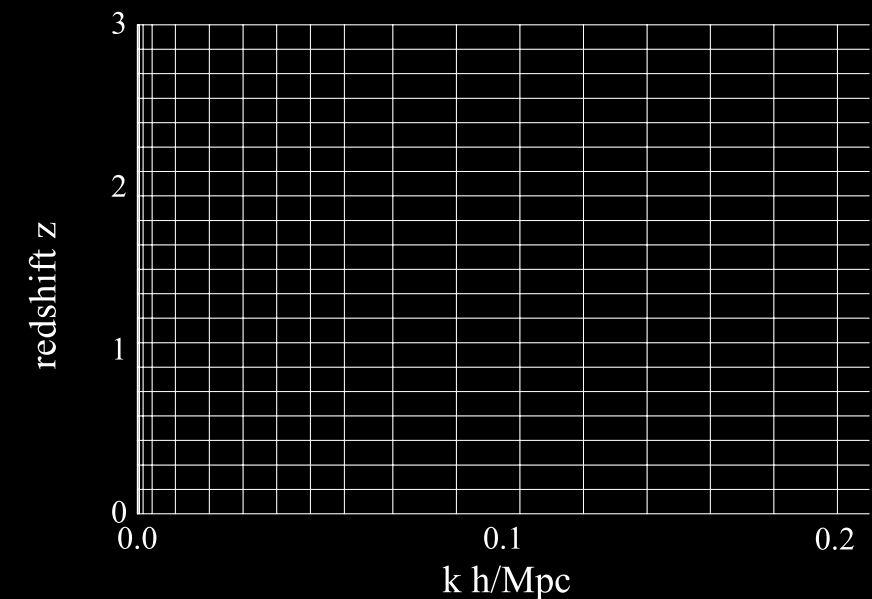
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(A.J.S.Hamilton and M.Tegmark, astro-ph/9905192, MNRAS'00  
D.Huterer and G.Starkman, astro-ph/0207517, PRL'03)

### Procedure:

- discretize  $\mu$  and  $\gamma$  on a  $(k,z)$  grid
- treat their values in each pixel,  $\mu_{ij}$  and  $\gamma_{ij}$ , as free parameters
- discretize  $w$  on the same grid and treat  $w_i$  as free parameters
- vary the 6 “vanilla” parameters and linear bias parameters (one for each bin)
- calculate the Fisher Matrix to forecast the covariance of  $\sim 840$  parameters





# Observables

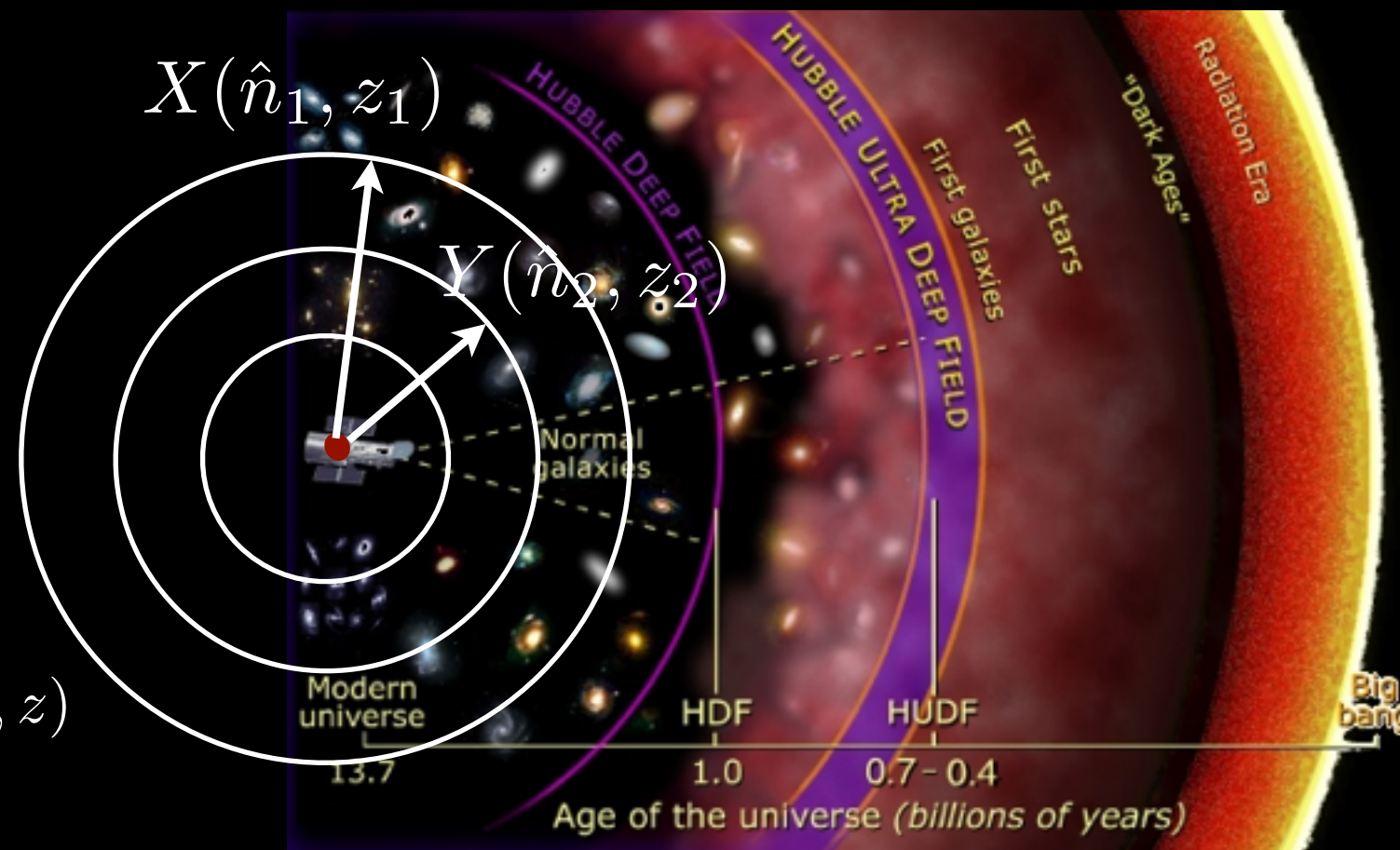
Upcoming and future tomographic surveys will map the evolution of matter perturbations and gravitational potentials from the matter dominated epoch until today.

We wish to combine multiple-redshift information on **Galaxy Count**, **Weak Lensing**, **CMB** and their cross correlations

Therefore the observables are the **ANGULAR POWER SPECTRA:**

$$C_l^{XY} = 4\pi \int \frac{dk}{k} \Delta_{\mathcal{R}}^2 I_l^X(k) I_l^Y(k)$$

$$I_l^X(k) = c_{x\mathcal{R}} \int_0^{z_*} dz W(z) j_l[kr(z)] \tilde{X}(k, z)$$





# Theory & Surveys

THEORY:

$$\left\{ \begin{array}{l} \Delta_m'' + \mathcal{H}\Delta_m' + k^2\Psi = 0 \\ k^2\Psi = -\frac{a^2}{2M_P^2}\mu(a, k)\Delta_m \\ \Phi = \gamma(a, k)\Psi \end{array} \right.$$



theoretical **predictions**  
for the observables

(from Boltzmann  
integrator **MGCAMB**)

(Zhao et al., Phys.Rev.D79 (2008))

# Theory & Surveys

## THEORY:

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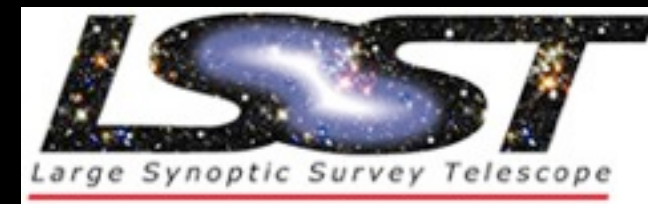
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## SURVEYS:

- SNeIa (**JDEM**) + CMB (**Planck**):  
expansion history
- Weak Lensing (WL) surveys (**DES**,  
**LSST**):  
maps of  $(\Phi + \Psi)$  at different epochs
- Galaxy Number Counts (GC) (**DES**,  
**LSST**):  
maps of  $\Delta$  at different epochs
- Galaxy Number Counts x CMB:  
ISW effect:  $(\Phi + \Psi)'$  at different epochs



# Principal Components of $\mu$

...marginalizing over the other parameters...

- invert it, consider only its  $\mu$  block and **diagonalize** it to find uncorrelated combinations of  $\mu_{ij}$

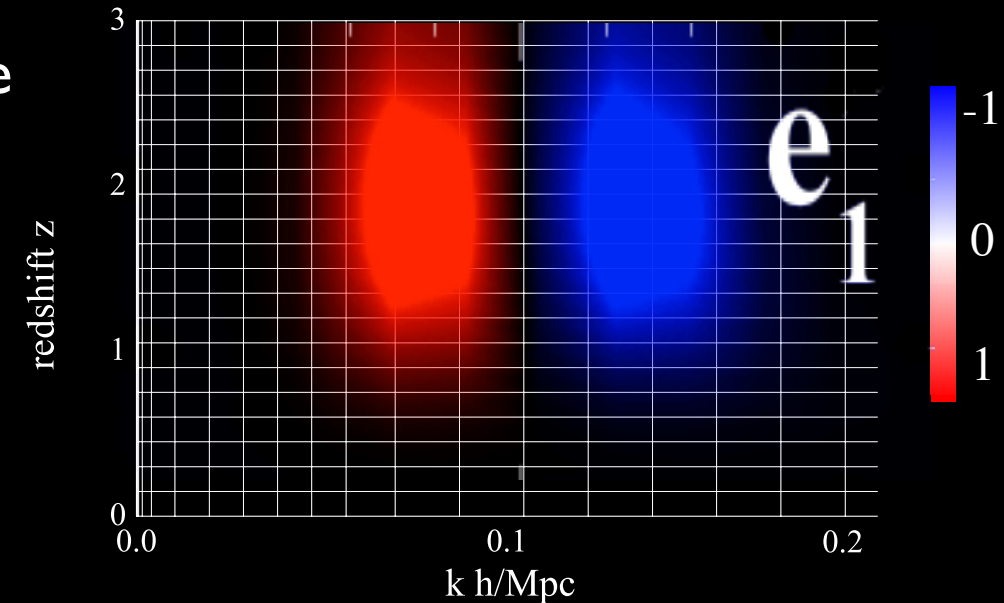
- each eigenmode represents a **surface in the (k,z) space**

- they form an orthonormal basis for the function  $\mu$ :

$$\mu(k, z) - 1 = \sum_m \alpha_m e_m(k, z)$$

- the eigenvalues of the PCs correspond to the **variances** of the expansion coefficients

$$\lambda_m = \sigma^2(\alpha_m)$$



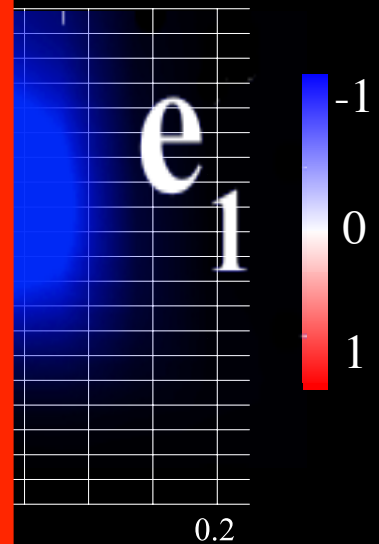
# Principal Components of $\mu$

...marginalizing over the other parameters...

## WHAT ARE PRINCIPAL COMPONENTS ?

They are simply a way of understanding a likelihood surface around its peak, which is a function of model parameters. By finding the eigenvalues and eigen-vectors of the curvature or Fisher matrix, we can find which combinations of parameters are best constrained by a given set of observations.

When there is no natural choice of parameters, we'd rather let **data** tell us which modes of the functions are best measured. Therefore we proceed **pixelizing** our functions and use the PCA approach to determine the **combinations of pixels which are constrained observationally**



$$\lambda_m = \sigma^2(\alpha_m)$$

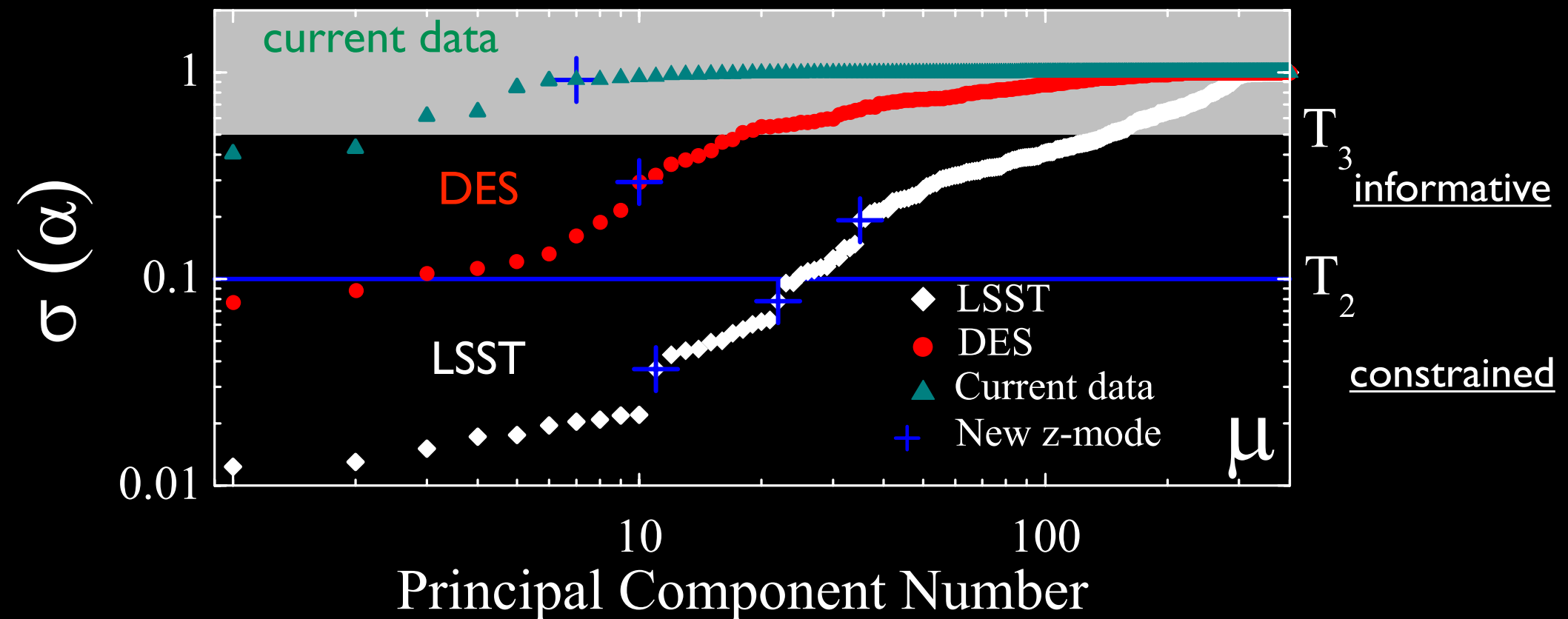
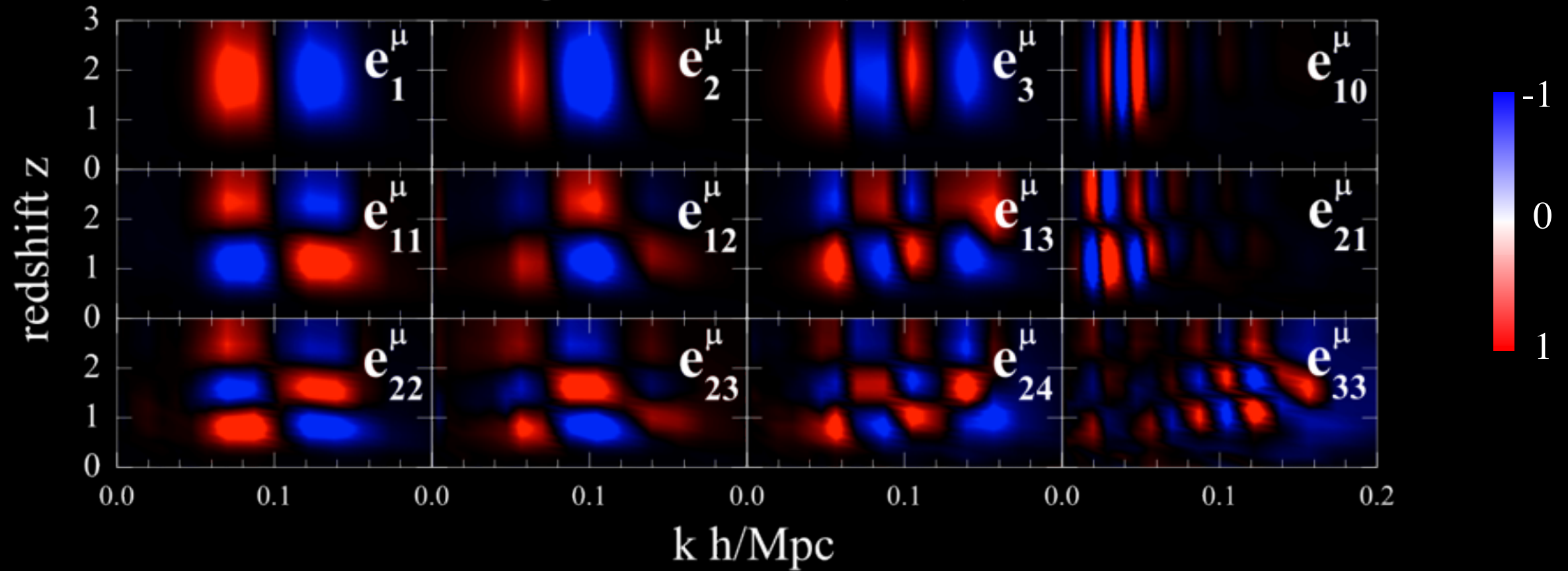
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Series of eigenmodes and the  
uncertainty on the corresponding  
expansion parameters

# Principal Components of $\mu$

...marginalizing over the other parameters...

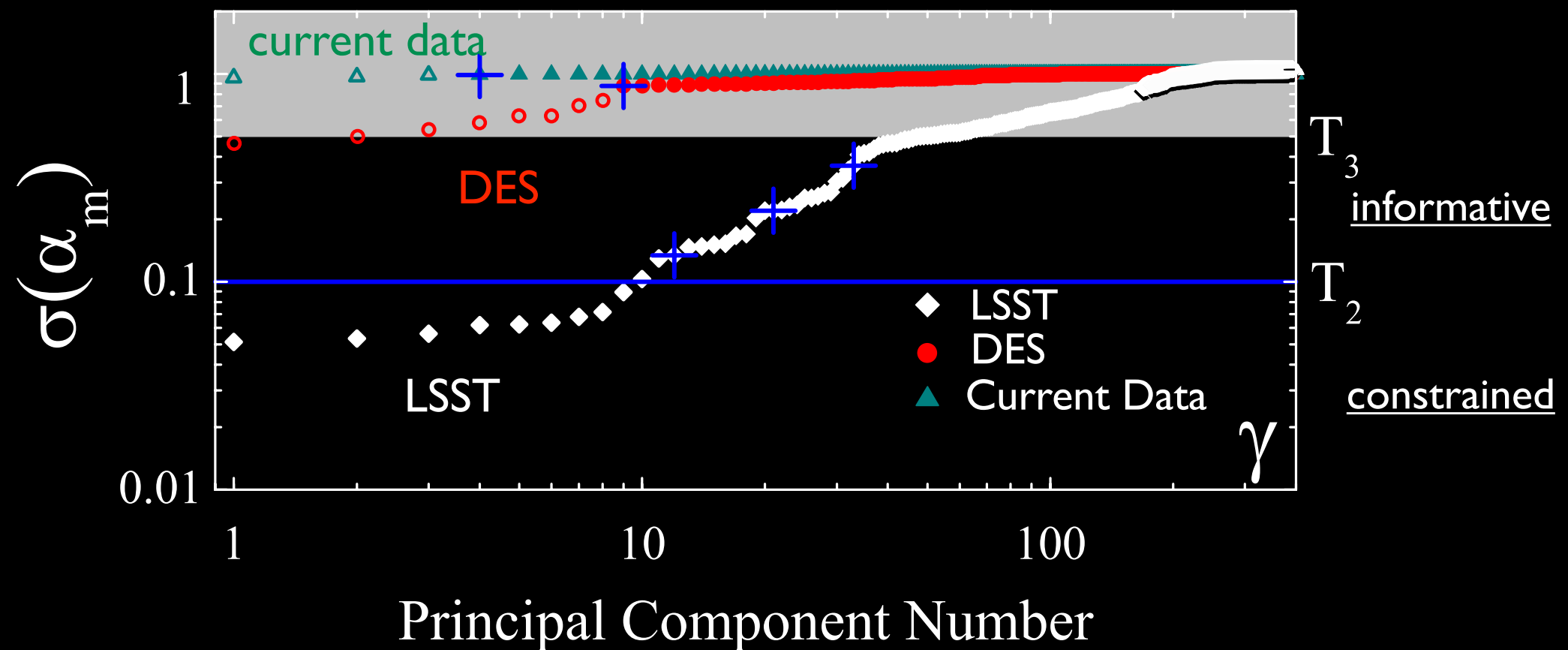
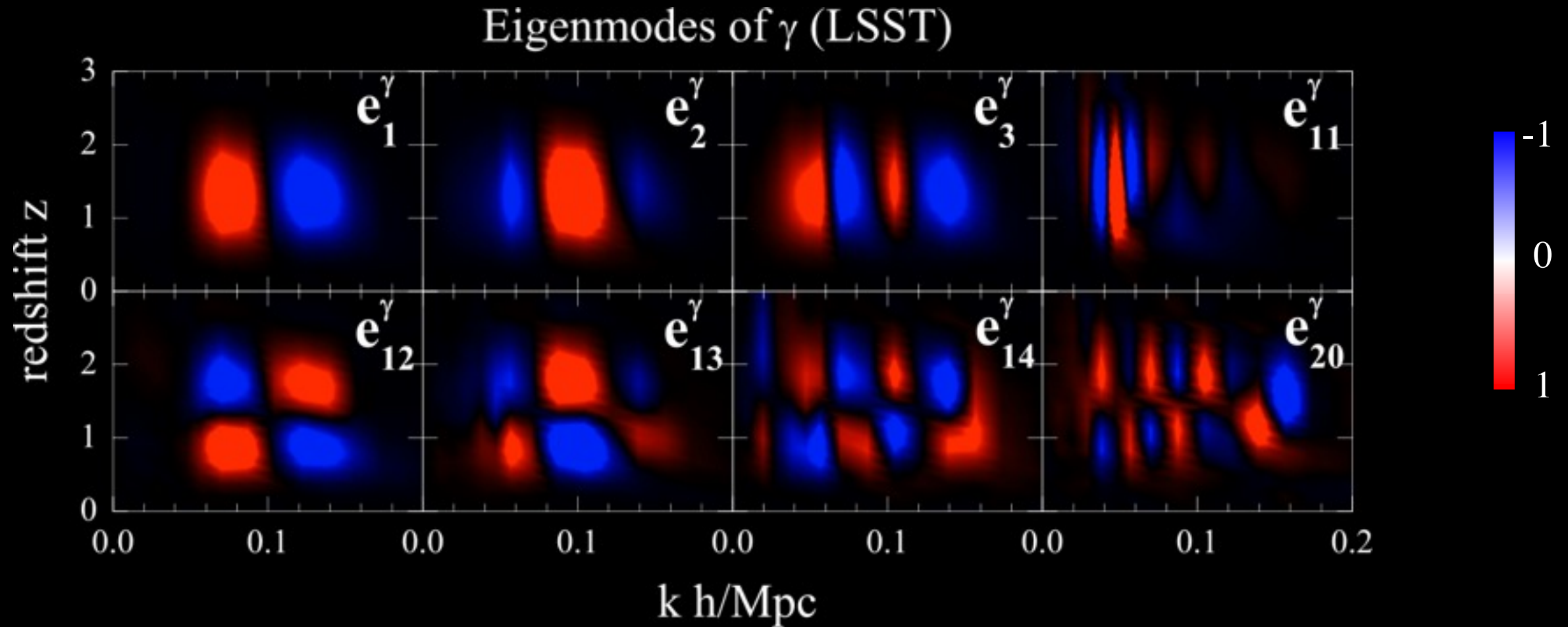
## Eigenmodes of $\mu$ (LSST)





# Principal Components of $\gamma$

...marginalizing over the other parameters...



# Information

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- data is mostly sensitive to **scale-dependent** features  
(  $\rightarrow$  non degenerate with  $w(z)$ !!!)
- $\mu$  eigenmodes go deeper in redshift  
(accumulation effect)
- $\mu$  is better constrained (GC)
- current data are basically blind to  $\Upsilon$
- LSST will have a higher-sensitivity to MG and will be more sensitive to scale-dependent features



# What if we want to constrain ANY departure from LCDM?

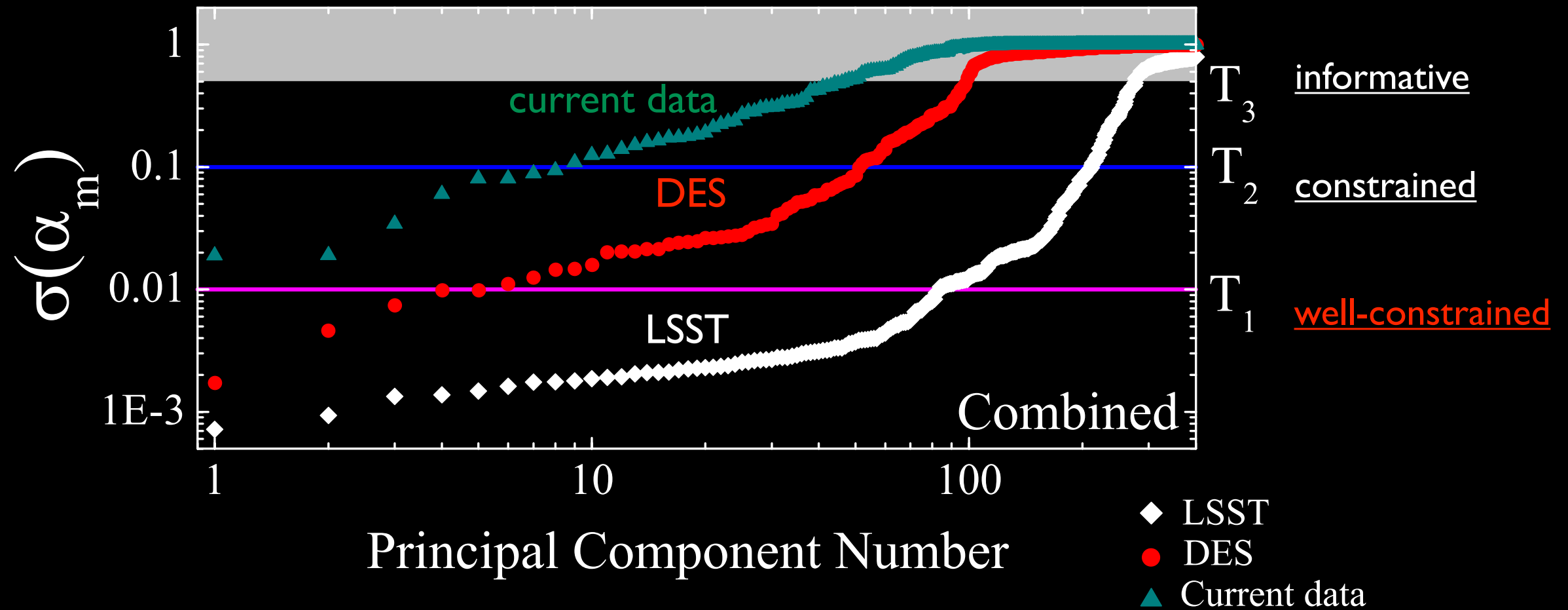
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So far we have determined how well we can separately constrain  $\mu$  and  $\gamma$ .

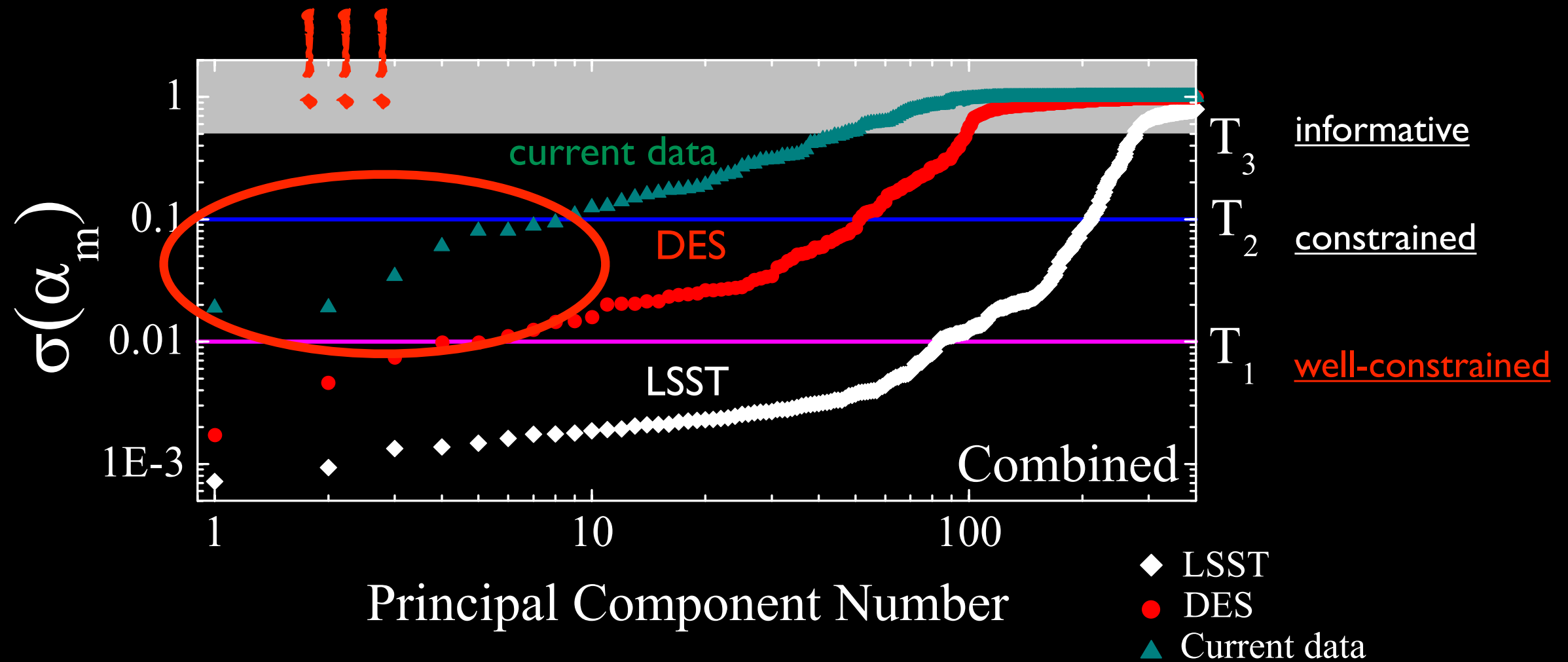
Therefore we have thrown away all the information that cannot distinguish between them.

To determine how well we can constrain **any departure** from LCDM we can keep that info and find the **combined** eigenmodes of  $\mu$  and  $\gamma$ .

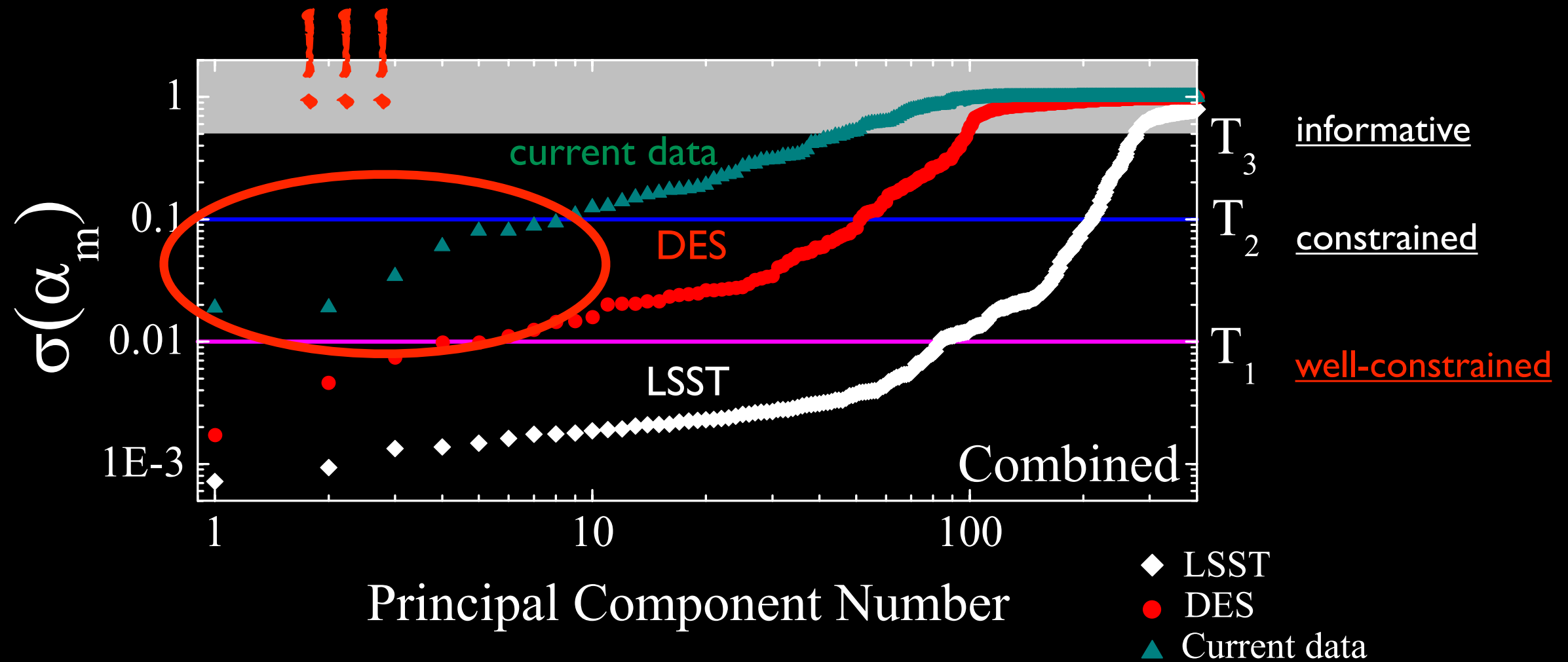
# Combined eigenmodes of $\mu$ and $\gamma$



# Combined eigenmodes of $\mu$ and $\gamma$



# Combined eigenmodes of $\mu$ and $\gamma$



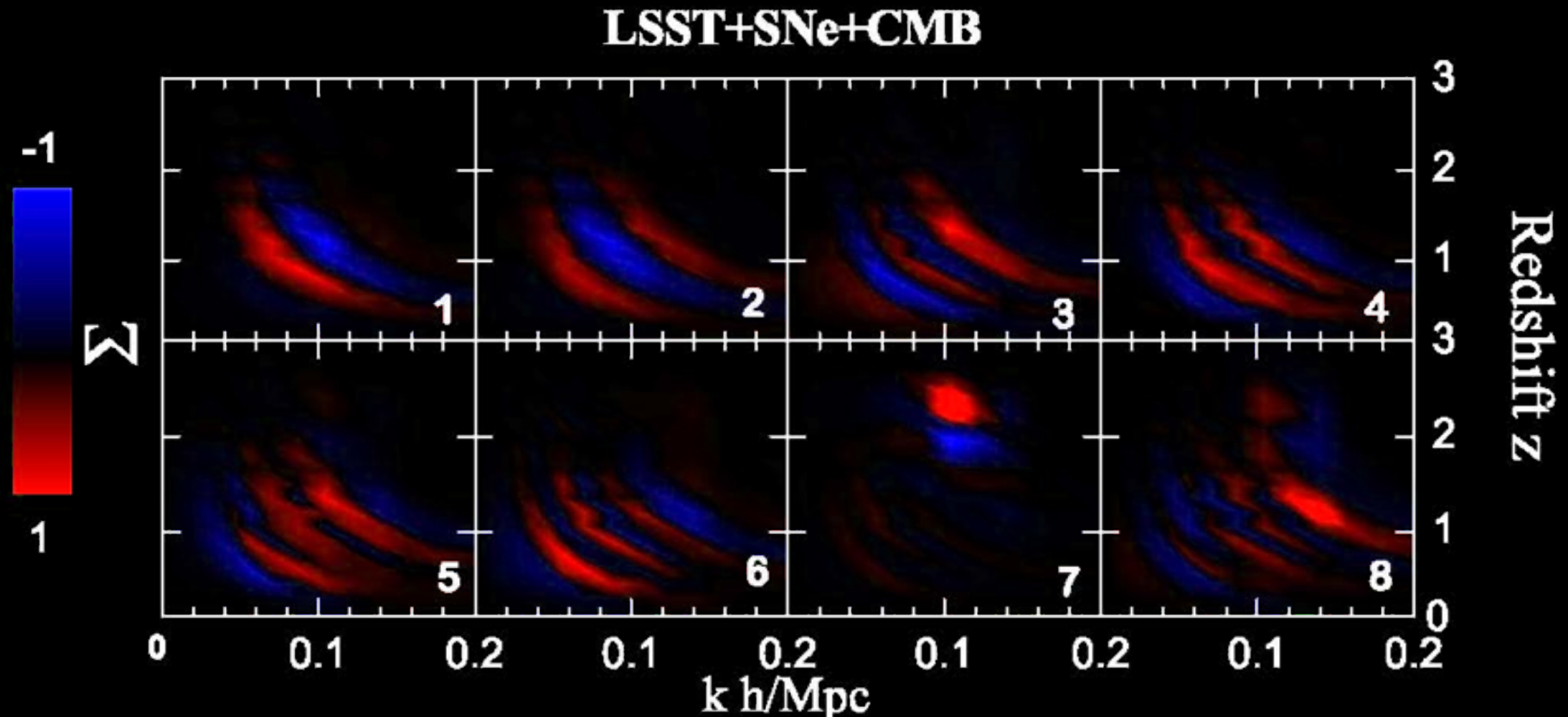
Current data can already put some constraints on the combination of  $\mu$  and  $\gamma$ , and they show consistency with LCDM except for some “systematics” in the WL (CFHTLS) data.

(Zhao et al., Phys.Rev.D81:103510 (2010) [astro-ph/1003.0001])

# Eigenmodes of $\Sigma$

$$k^2 (\Phi + \Psi) = -\Sigma(a, k) \frac{a^2}{2M_P^2} \rho \Delta$$

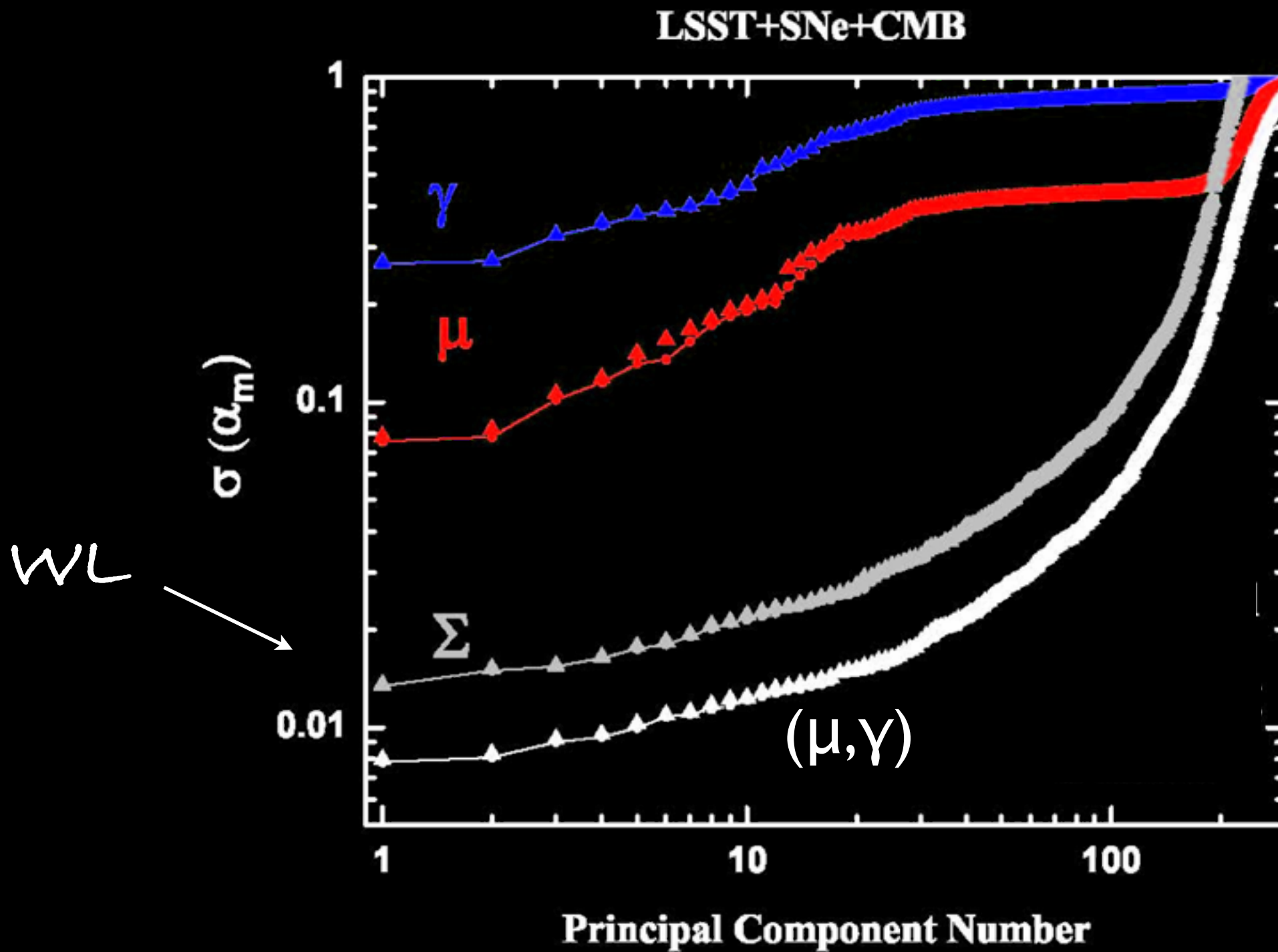
directly related to WL and ISW



$k - z$  degeneracy (from WL kernel)

peaks at low redshift

# Comparing Uncertainties

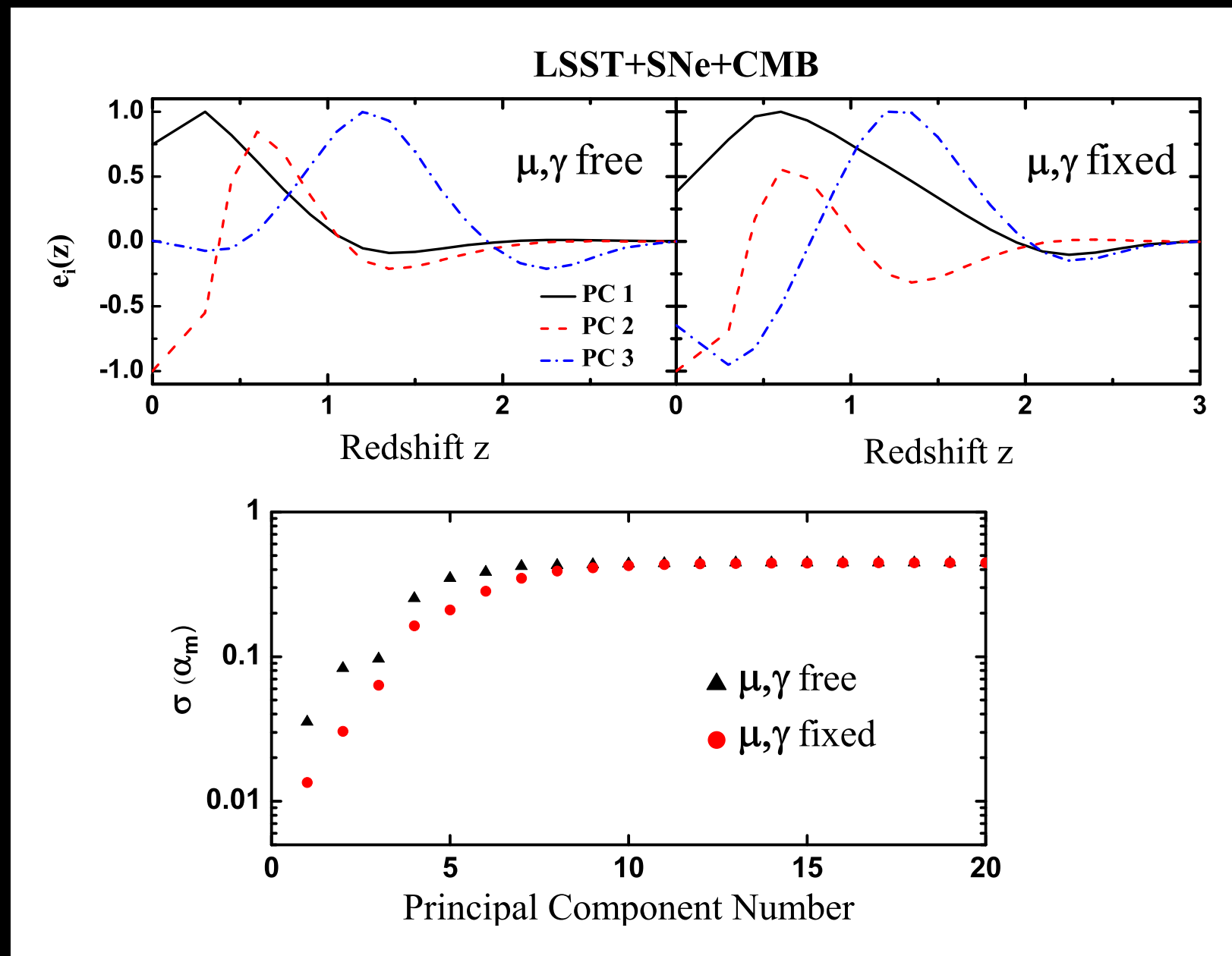


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What happens to the constraints on  
the equation of state  $w(z)$ ?

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# Degeneracy with $w(z)$



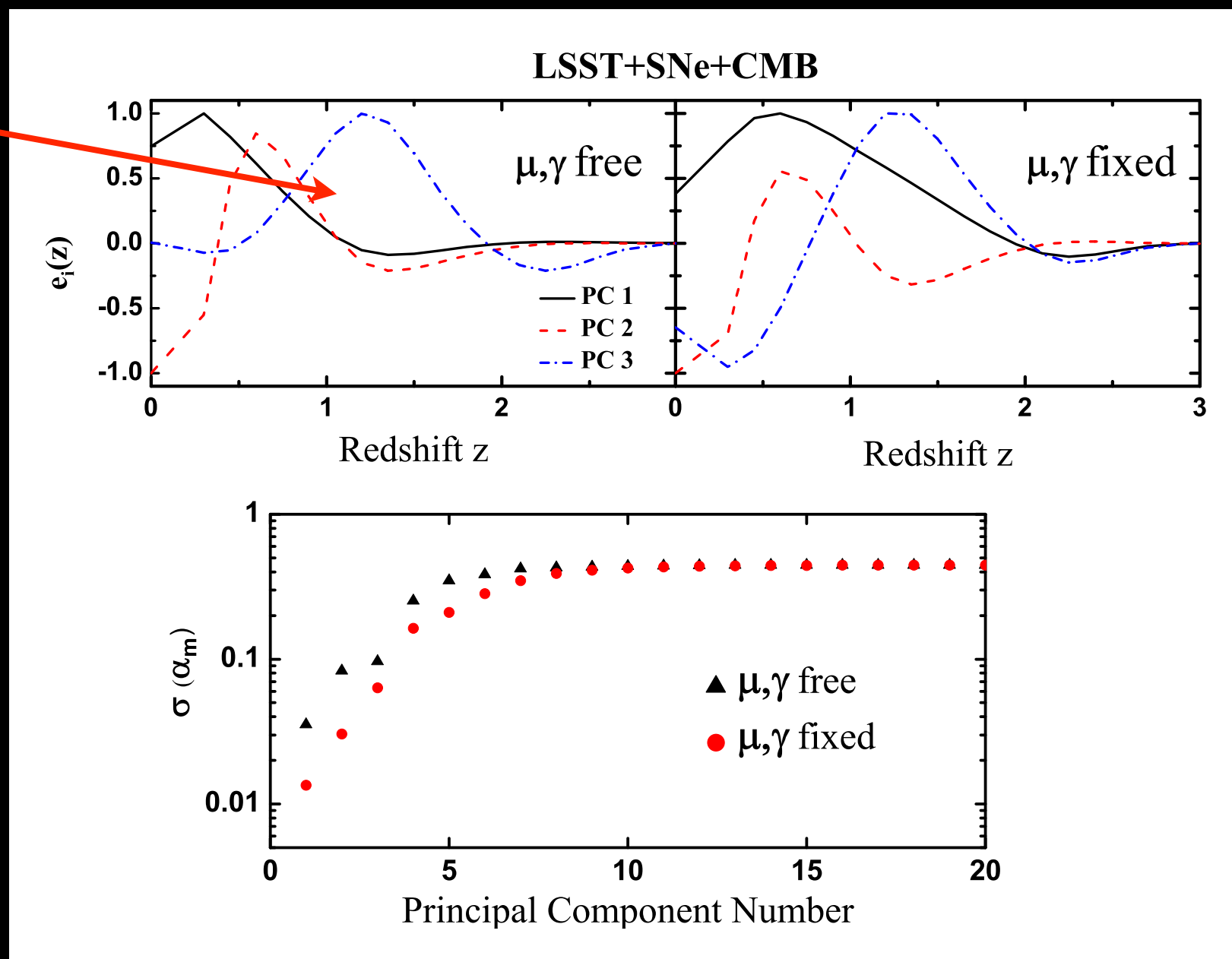
letting the MG parameters vary squeezes the best constrained eigenmodes of  $w(z)$  towards low redshift

overall the effects are not dramatic, and future surveys will measure *both*  $w(z)$  and MG functions



# Degeneracy with $w(z)$

LSS



letting the MG parameters vary squeezes the best constrained eigenmodes of  $w(z)$  towards low redshift

overall the effects are not dramatic, and future surveys will measure *both*  $w(z)$  and MG functions

# Including Systematics

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using the model of Huterer, Takada, Bernstein and Jain,  
MNRAS **366**, 101 (2006)

- photo-z errors:

- I. distortion of the overall distribution

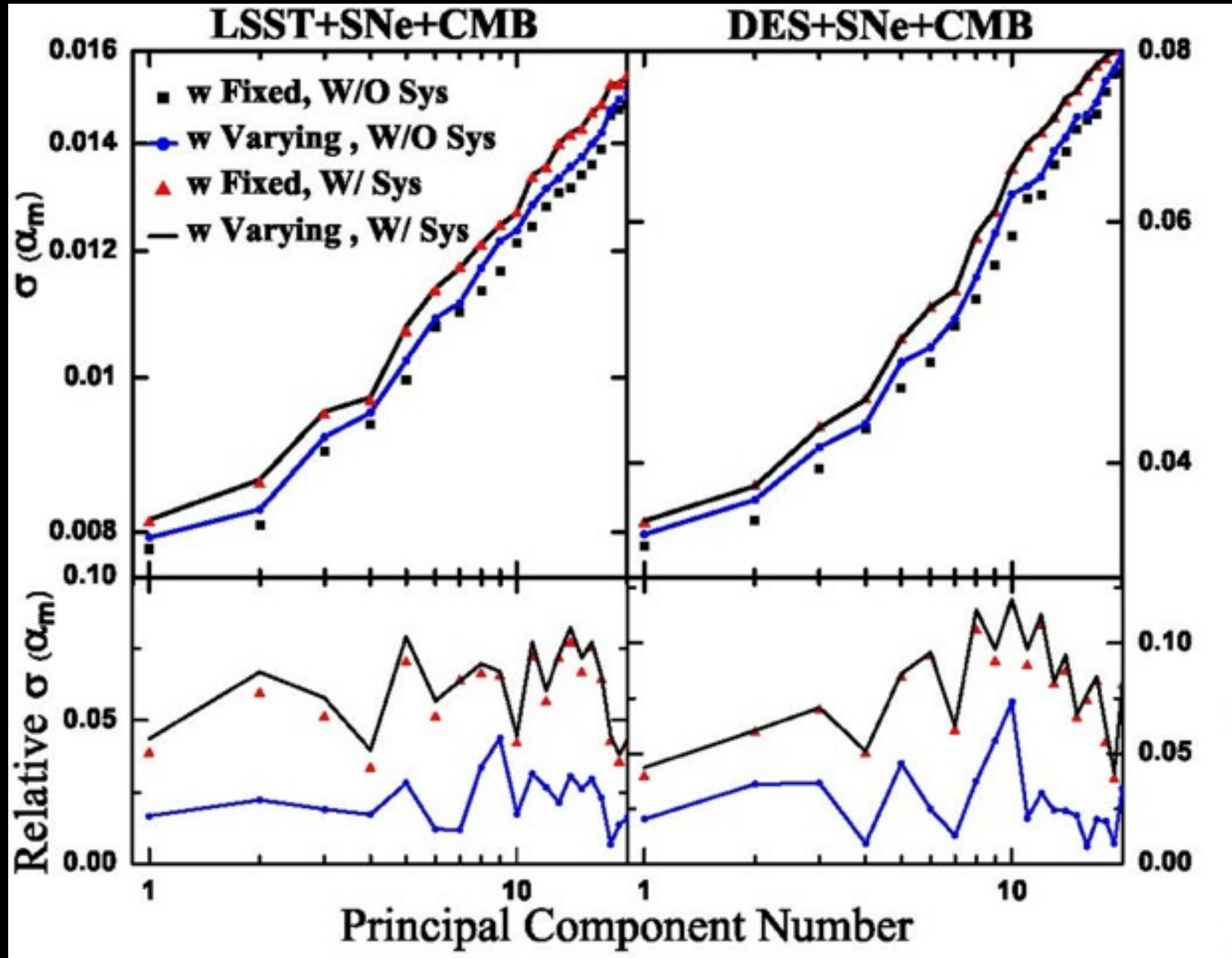
- II. redshift-bin centroids uncertainty

- III. z-scatter

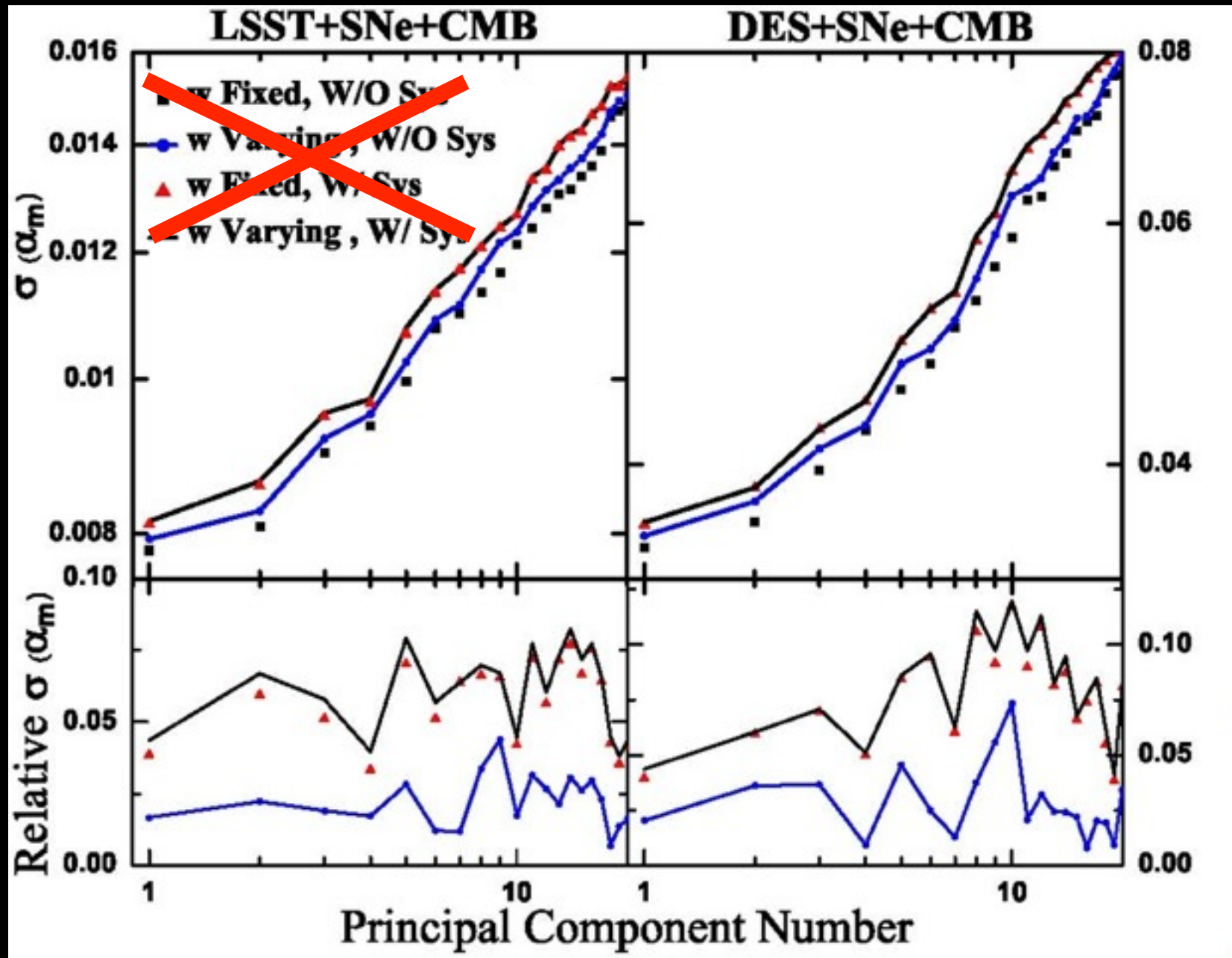
- additive errors (GC & WL)

- multiplicative errors (WL)

# Systematics



# Systematics



most of the scale-dependent info is preserved

a noticeable, but not dramatic, dilution of constraints

errors on best constrained modes are degraded by  $< 10\%$   
for both LSST and DES

# Summary

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Testing GR on cosmological scales is an exciting prospect that will be enabled by upcoming data

We have developed the 2D Principal Component Analysis of cosmological perturbations to study the constraints on modifications of GR that one can expect from future data sets:

This model-independent analysis shows that future surveys will offer a **wealth of information on the relations between mass, gravitational potential and curvature of space**

Data is somewhat more sensitive to **scale-dependent** than time-dependent modifications of growth.  $\Sigma$  is the best constrained

Future surveys will measure **both**  $w(z)$  and modified growth

PCA can also help us determine the **optimal number of parameters** to fit to a set of data. It is also a good way to compress info from surveys

TO DO: (scale-dependent bias), redshift-space distortion measurements to break degeneracies, **can we improve parametrization?**

In conclusion...

It is both a challenging and an exciting time for Cosmology...a wealth of high-precision information will be soon available...we should get ready to test it, keeping an eye on both linear and non-linear scales.

They offer complementary ways of testing gravity.

For the cosmological tests there is still a lot of work that can and should be done, to address systematics, degeneracy among parameters, to include other data such as PV, etc..

For smaller scales, screening mechanisms, of which the Chameleon is an example, might offer interesting tests and several scenarios need to be analyzed and worked out.

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THANK YOU!

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