Exploring the MOND Paradigm

Olivier Tiret (SISSA)

Contents

- A brief history on the dark matter
- The formulation of MOND
- Some observationnal issues
- N-body simulations and modified gravity
- Perspectives

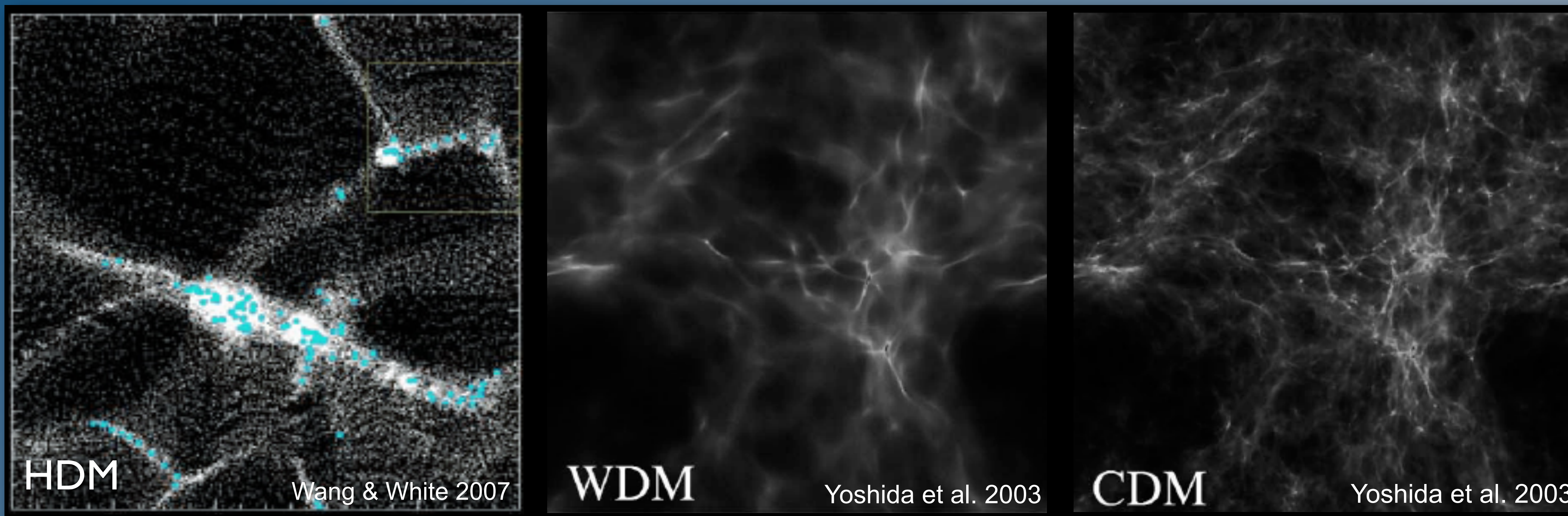
The dark matter

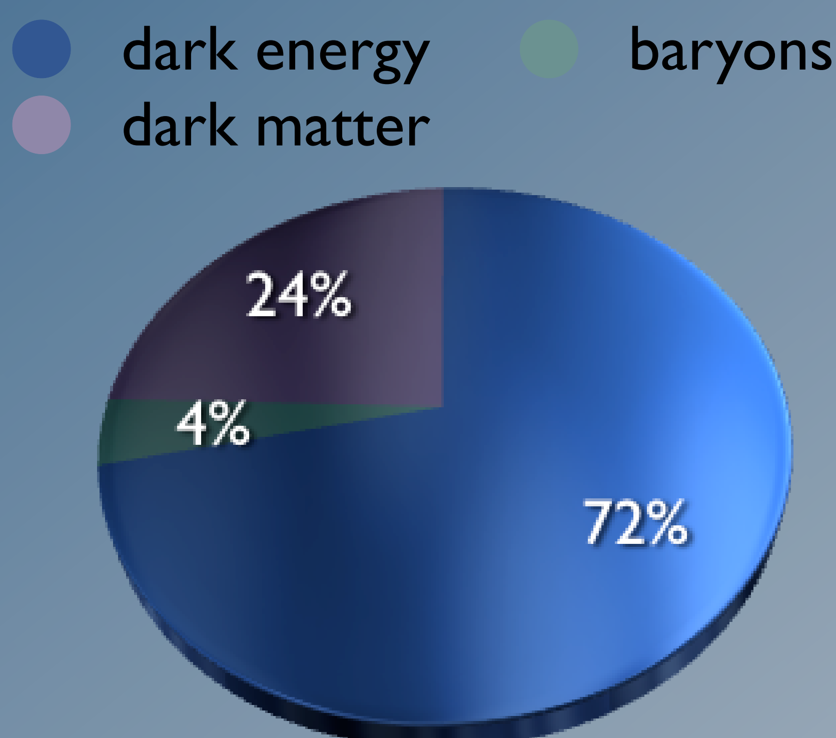
- Velocities of individual galaxies in the Coma Berenice Cluster (Zwicky,1933).
- Also, observations at galactic scale show that galaxies should contain more mass that what it is observed.
- 1970s, generally accepted that galaxies contain "dark matter", and is related to the missing mass in galaxy cluster (Rubin).

The dark matter

- Velocities of individual galaxies in the Coma Berenice Cluster (Zwicky,1933).
- Also, observations at galactic scale show that galaxies should contain more mass that what it is observed.
- 1970s, generally accepted that galaxies contain "dark matter", and is related to the missing mass in galaxy cluster (Rubin).

The dark matter





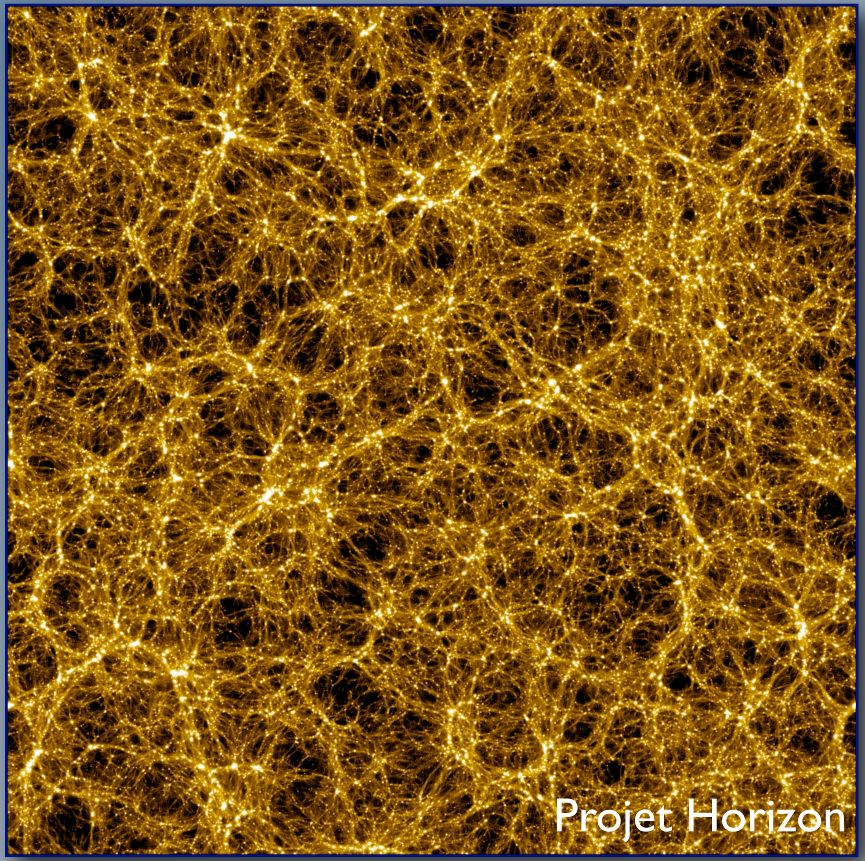
- Several models (HDM, WDM, CDM) for several candidats : neutrinos, WIMPS,…
- Cosmological simulations: ΛCDM

The dark matter

- Sucessfull concerning the large scale structure formation.
- But three problems still persist at galactic scale:
  - Cusp
  - Angular momentum
  - Satellites

Numerical artefact? Feedback?

Mayer et al (2008), Diemand et al (2005)
Strigari et al (2007), Madau et al (2008)

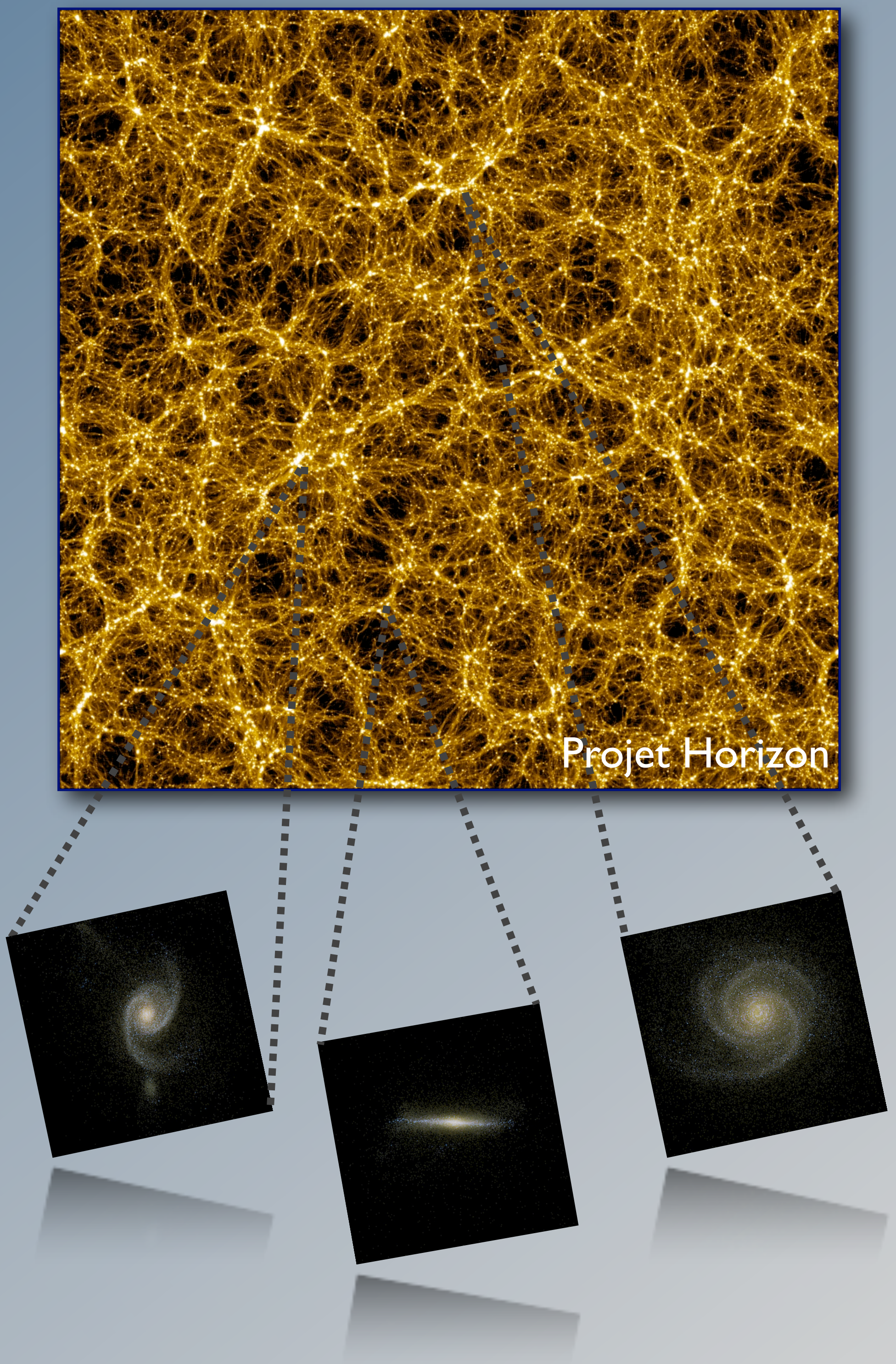


The dark matter

- Sucessfull concerning the large scale structure formation.
- But three problems still persist at galactic scale:
  - Cusp
  - Angular momentum
  - Satellites

Numerical artefact? Feedback?

Mayer et al (2008), Diemand et al (2005)
Strigari et al (2007), Madau et al (2008)

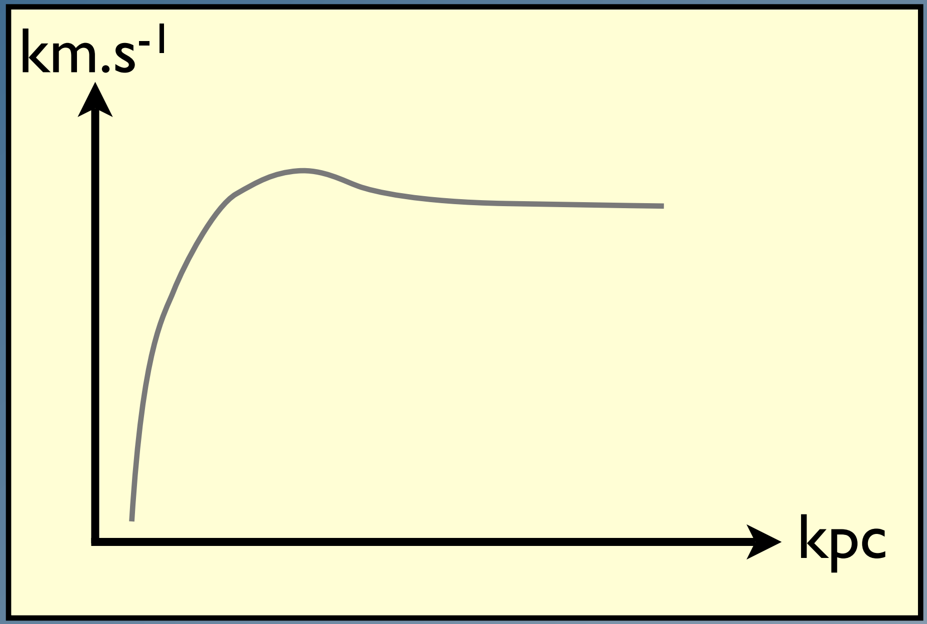


From Newtonian gravity to MOND

- Rotation Curves fit:

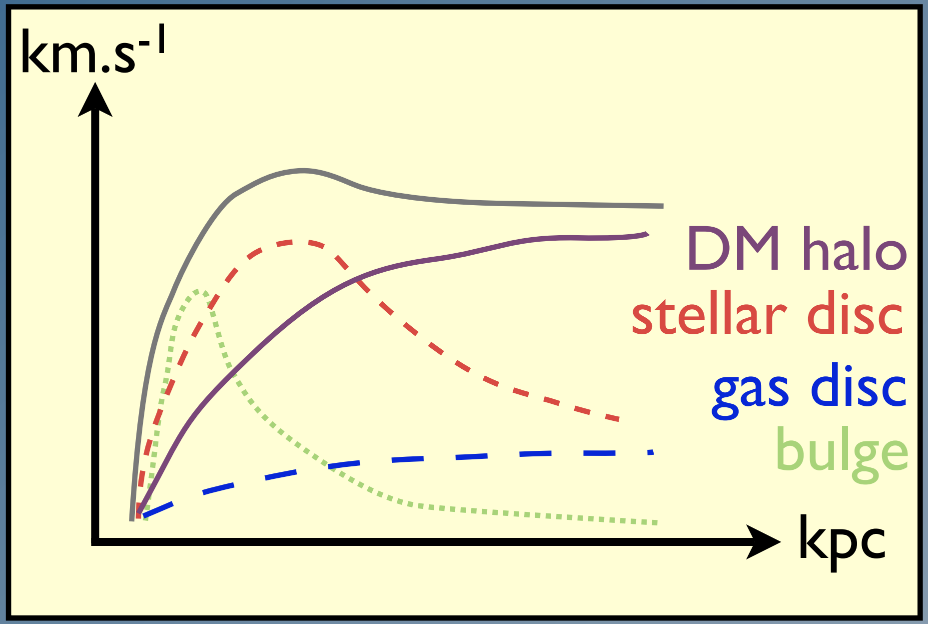
From Newtonian gravity to MOND

- Rotation Curves fit:



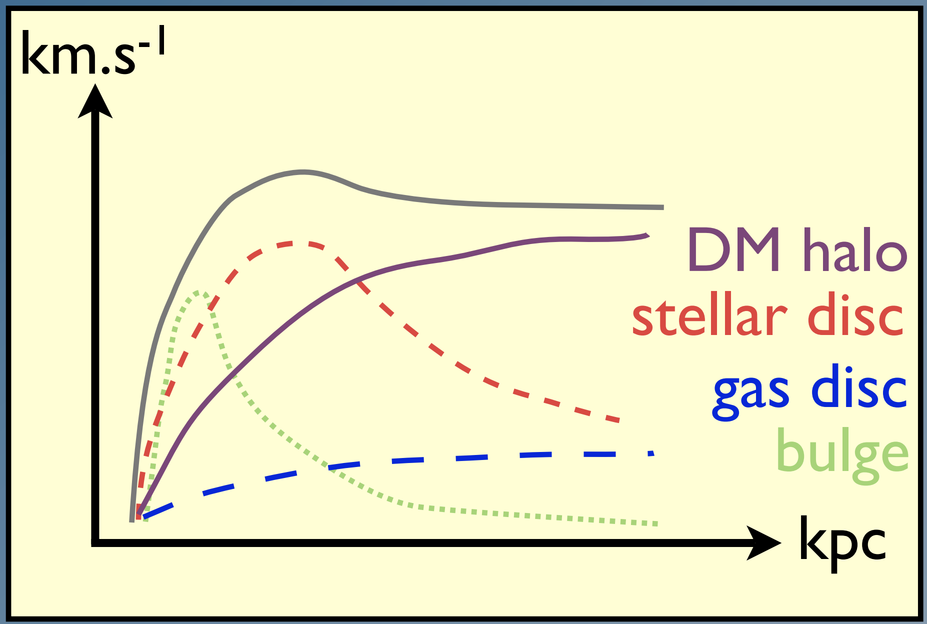
From Newtonian gravity to MOND

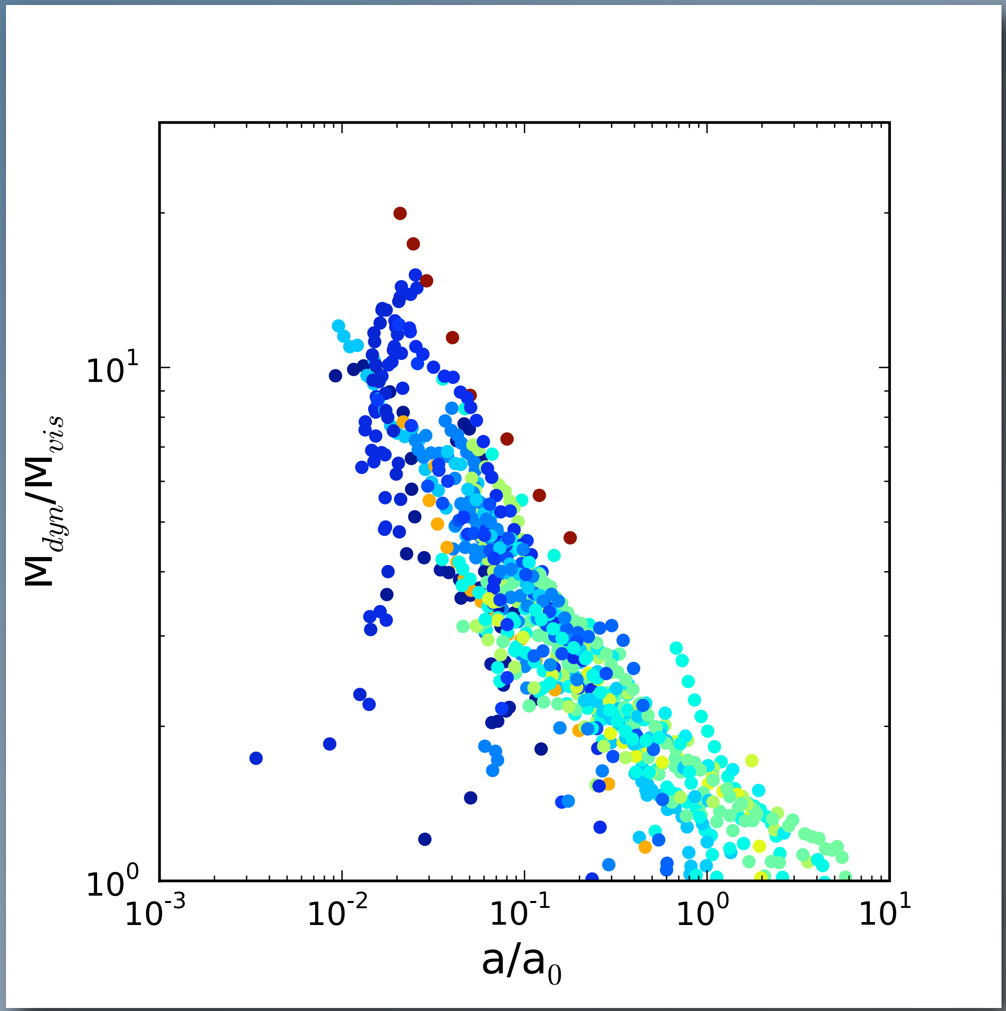
- Rotation Curves fit:



From Newtonian gravity to MOND

- Rotation Curves fit: Mdyn/Mvis

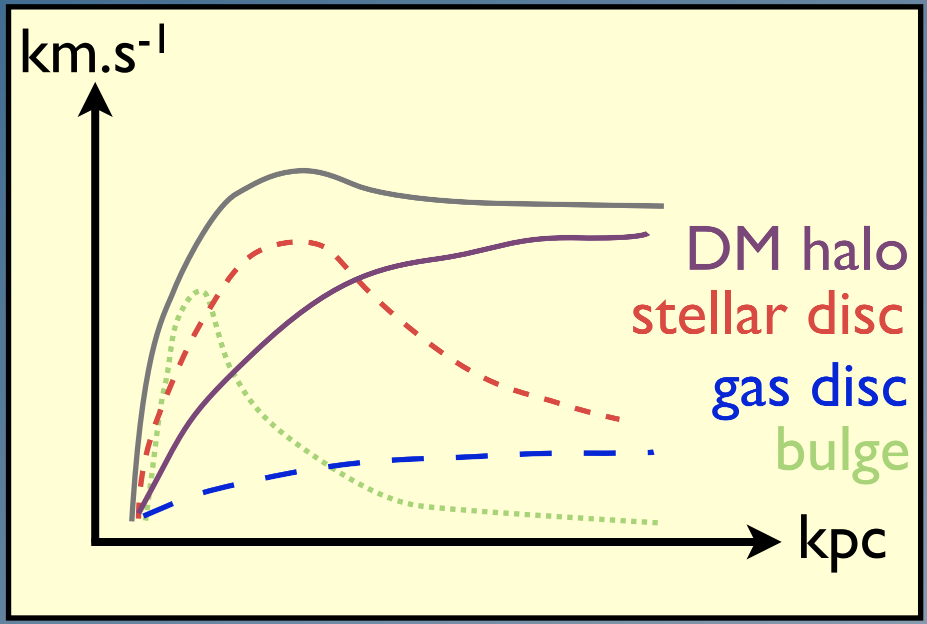


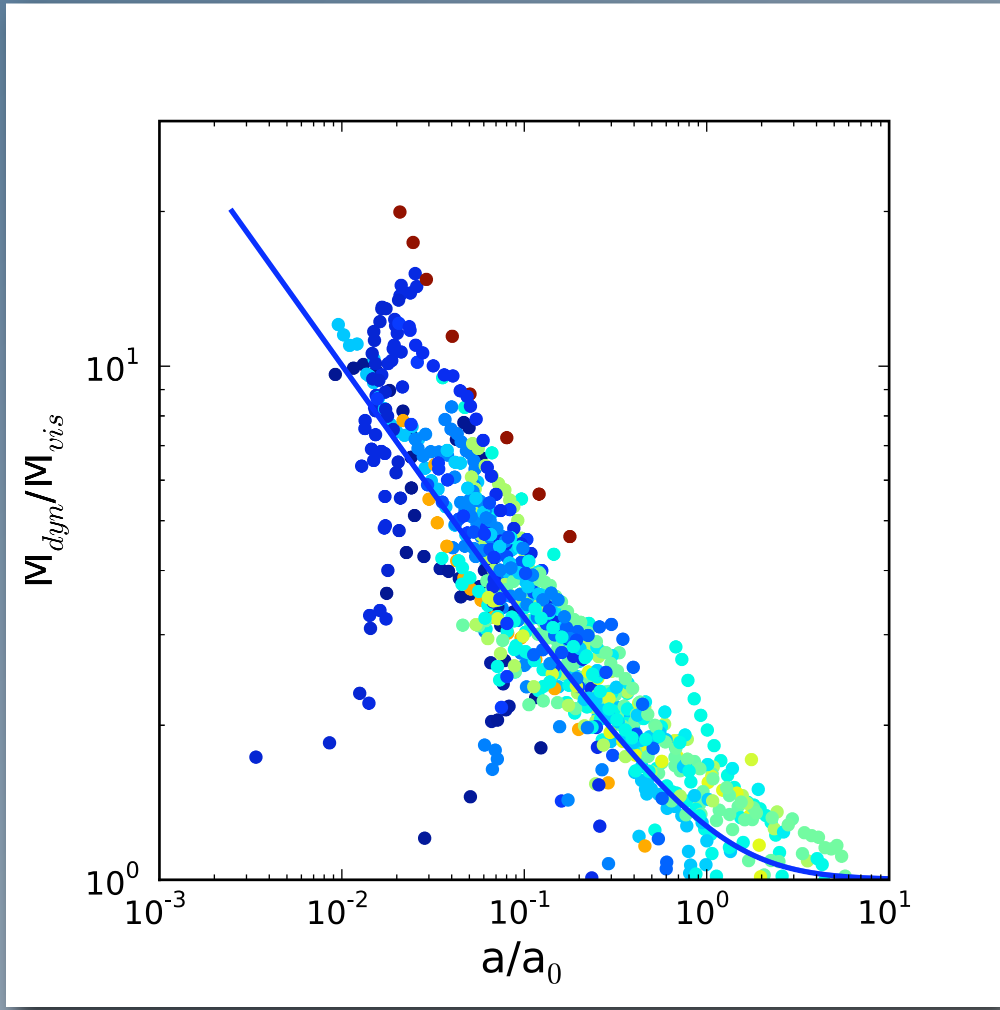


From Newtonian gravity to MOND

- Rotation Curves fit: Mdyn/Mvis

  => Strong correlation with the acceleration



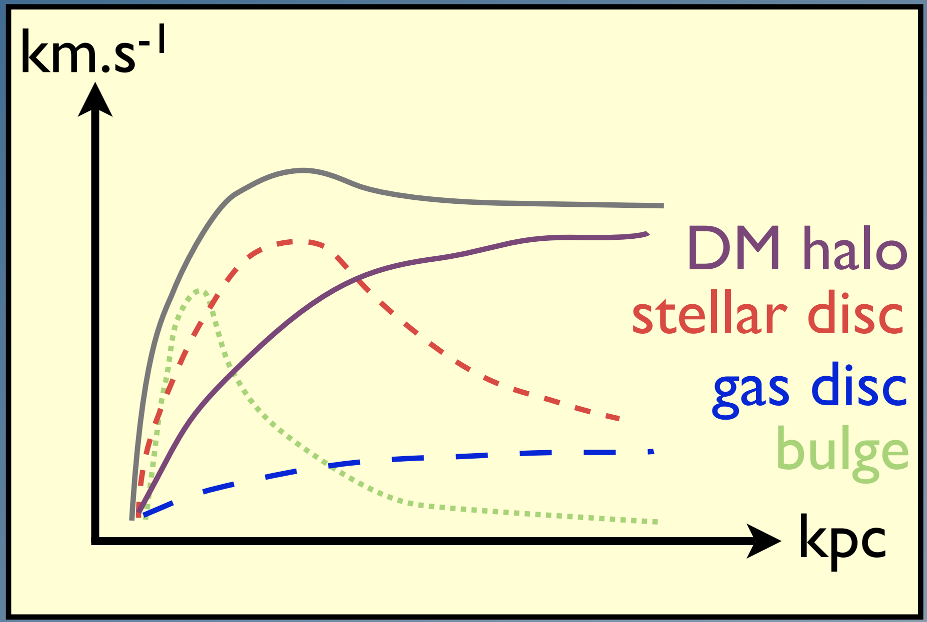


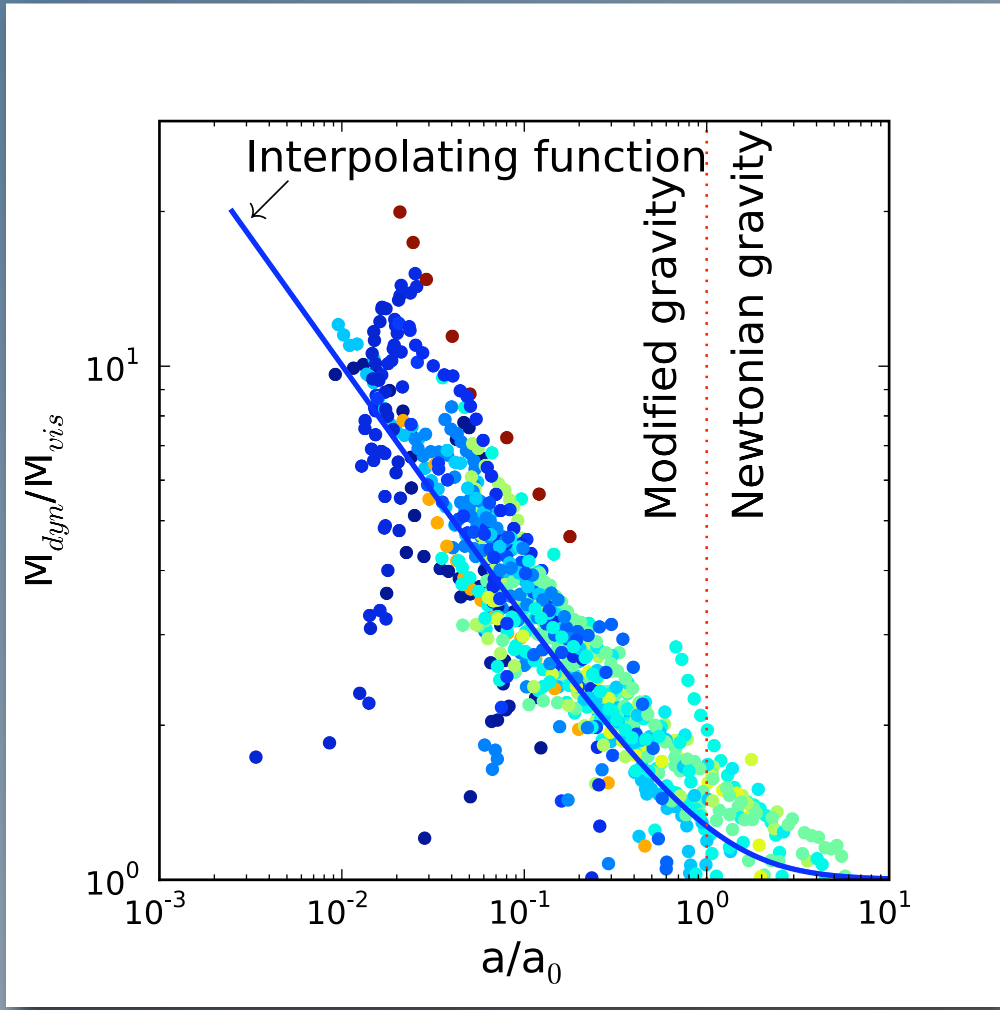
$$\frac{M_{dyn}}{M_{vis}} = f\left(\frac{a_N}{a_0}\right)$$

From Newtonian gravity to MOND

- Rotation Curves fit: Mdyn/Mvis

  => Strong correlation with the acceleration





$$\frac{M_{dyn}}{M_{vis}} = f\left(\frac{a_N}{a_0}\right)$$

$$a_N = a_M \mu(a_M/a_0)$$

Milgrom, 1983: modification of the gravitation law below a critical acceleration $a_0 \sim 1.2 \times 10^{-10}$ m.s^{-2}

MOND

The formulation of MOND: $a_N = a_M \mu(a_M/a_0)$ $x = a_M/a_0$

The μ-function is constrained by the observations

- high accelerations:

$$x \gg 1, \mu(x) \to 1 \qquad a_M = a_N \qquad a_N \propto 1/r^2$$

- low accelerations:

$$x \ll 1, \mu(x) \to x \qquad a_M = \sqrt{a_0 a_N} \qquad a_M \propto 1/r$$

$$\frac{x}{1+x}, \frac{x}{\sqrt{1+x^2}} \cdots$$ simple / standard / ...

MOND

The formulation of MOND: $a_N = a_M \mu(a_M/a_0)$ $\quad x = a_M/a_0$

The μ-function is constrained by the observations

- high accelerations:

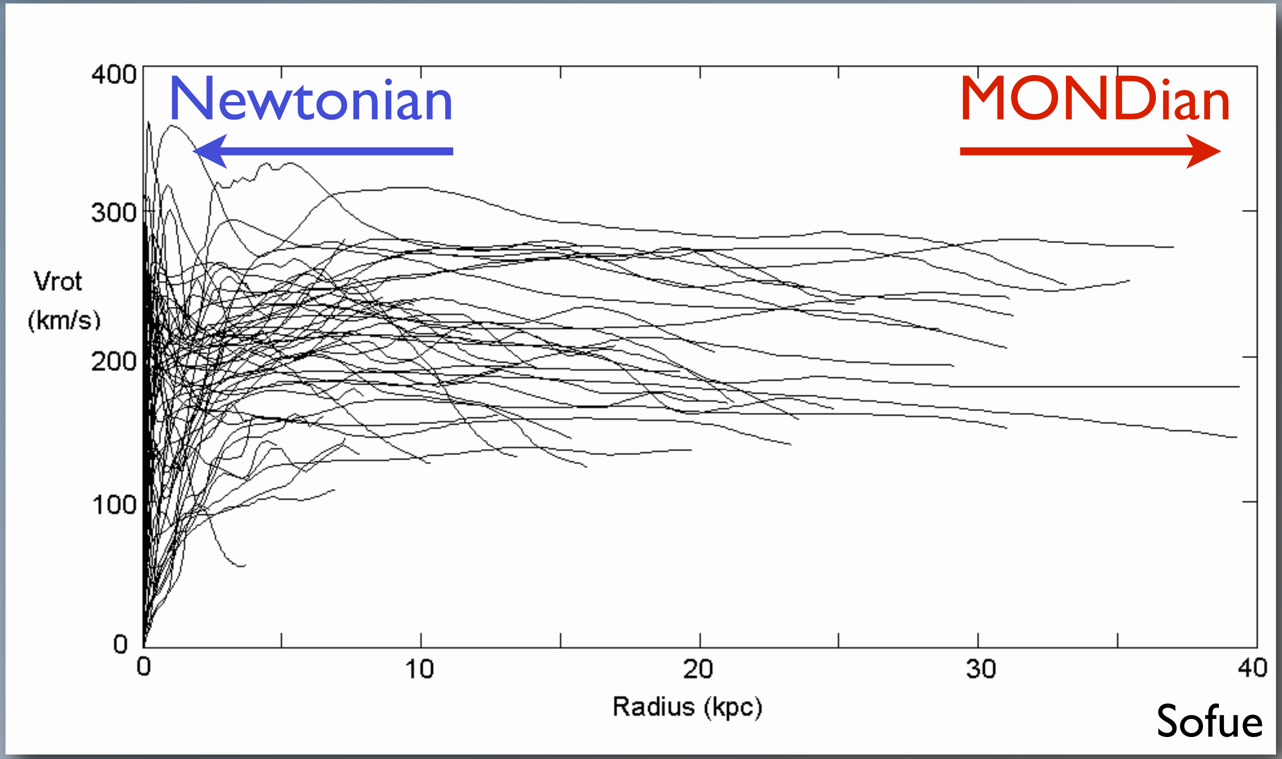
$$x \gg 1, \mu(x) \rightarrow 1 \qquad a_M = a_N \qquad a_N \propto 1/r^2$$

- low accelerations:

$$x \ll 1, \mu(x) \rightarrow x \qquad a_M = \sqrt{a_0 a_N} \qquad a_M \propto 1/r$$

$$\frac{x}{1+x}, \frac{x}{\sqrt{1+x^2}} \cdots$$ simple / standard / ...

$$a_M = v_c^2/r$$
$$v_c^2 \rightarrow cst$$



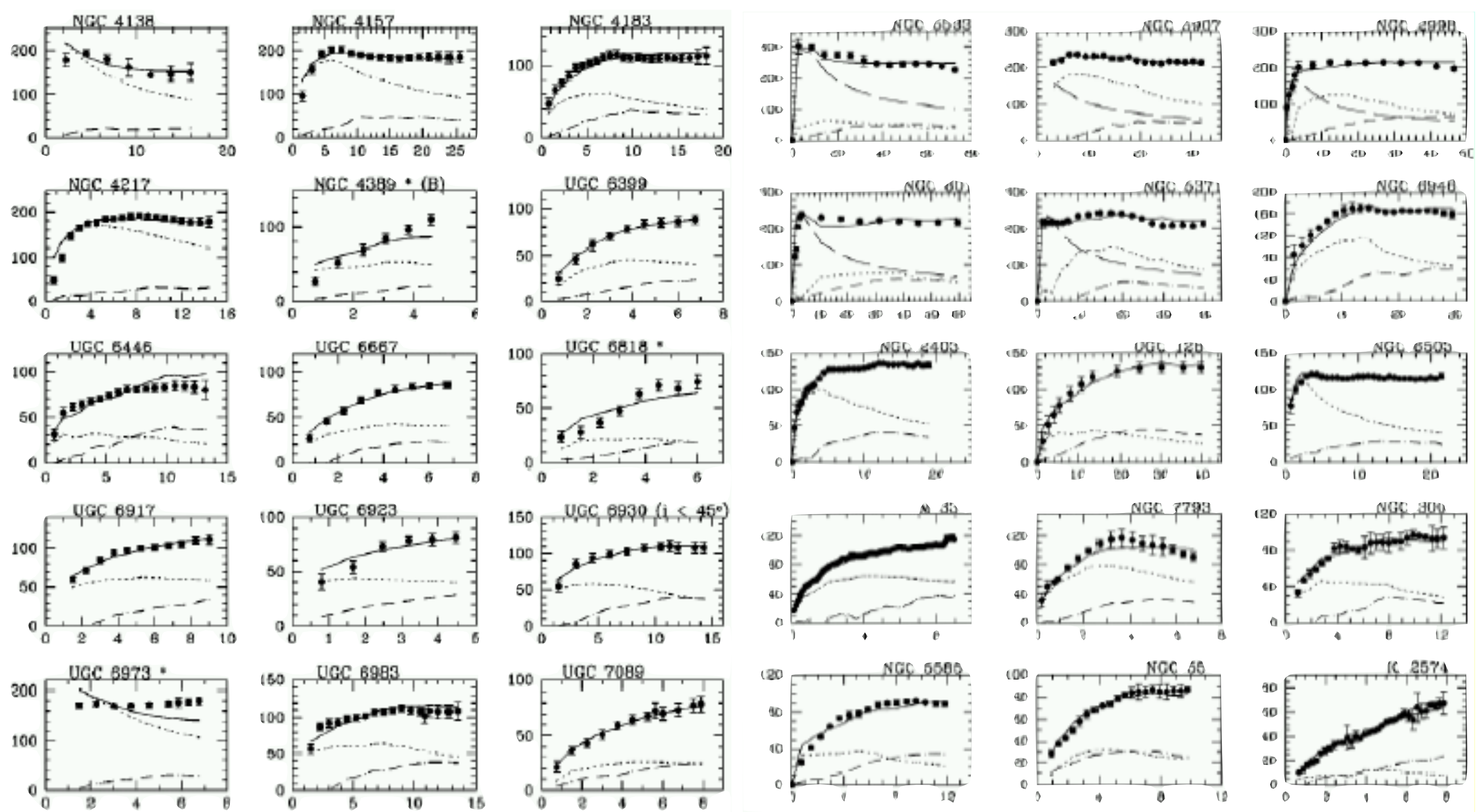
Rotation Curves

<u>MOND:</u>

$\mu(x)$ and a_0 are fixed // *(M/L)* varies

<u>Newtonian gravity:</u>

(M/L), ρ_{DM} , r_{DM} vary



Sanders & McGaugh (2002)

The baryonic Tully-Fisher relation

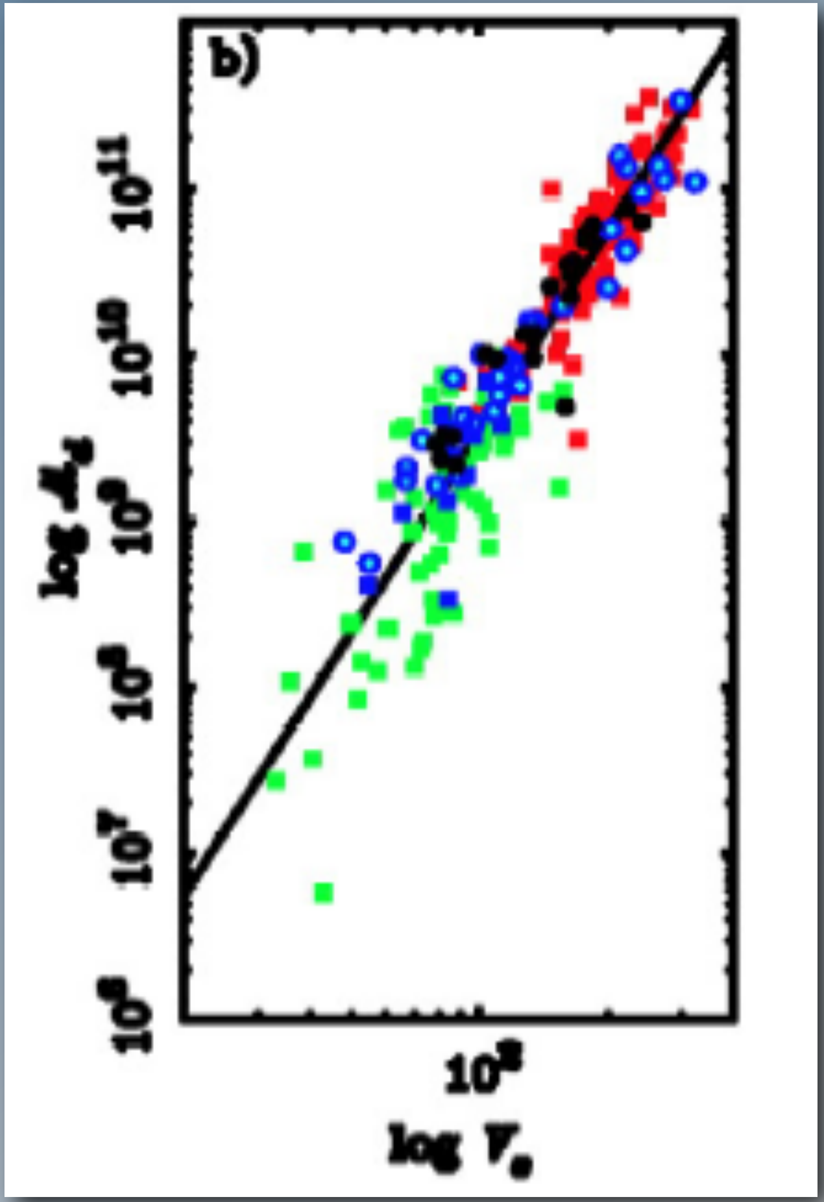
- Tully-Fisher relation: $L \propto v^4$
- Baryonic Tully-Fisher relation: $M \propto v^4$
  - <u>MOND</u>:

    $$v^4 = GMa_0$$

  - <u>Newtonian gravity:</u>

    $$Mv^2 \propto M^2/r$$ Viriel theorem

    $$M \propto r^2$$ Exponential disc



McGaugh et al (2000)

Galaxies Cluster

- The Bullet Cluster
  - MOND

    $M_\nu = 3\ M_X$

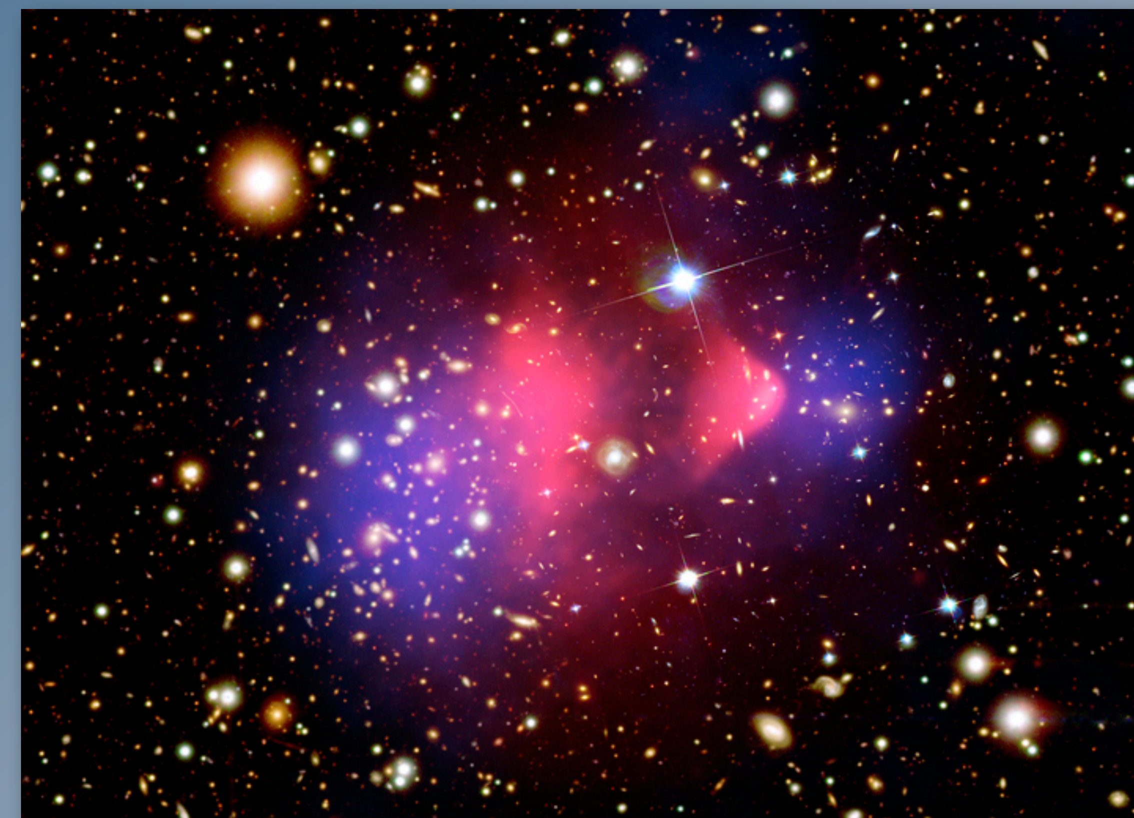
    2eV neutrinos Angus et al. (2007)
  - Newtonian gravity

    dark matter halo

    $M_{DM} = 6M_X$
- Velocity of the chock

  $\Delta v = 4700\ km.s^{-1}$
  - in agreement with MOND
  - too high for Newtonian gravity

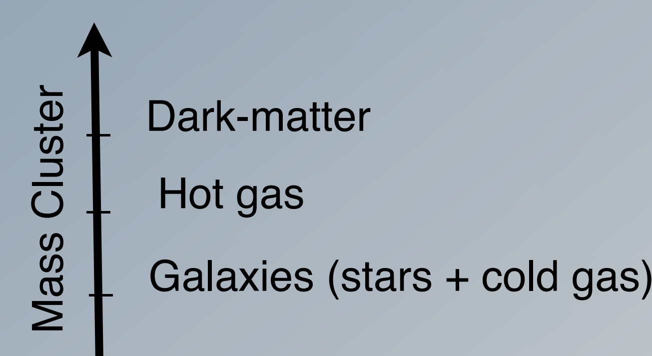
Springer & Farrar (2007), Milosavljevic et al (2007), Mastropietro & Burkert (2008), Angus & McGaugh (2008)



red: gas

blue: gravitationnal lens

Clowe et al (2007)

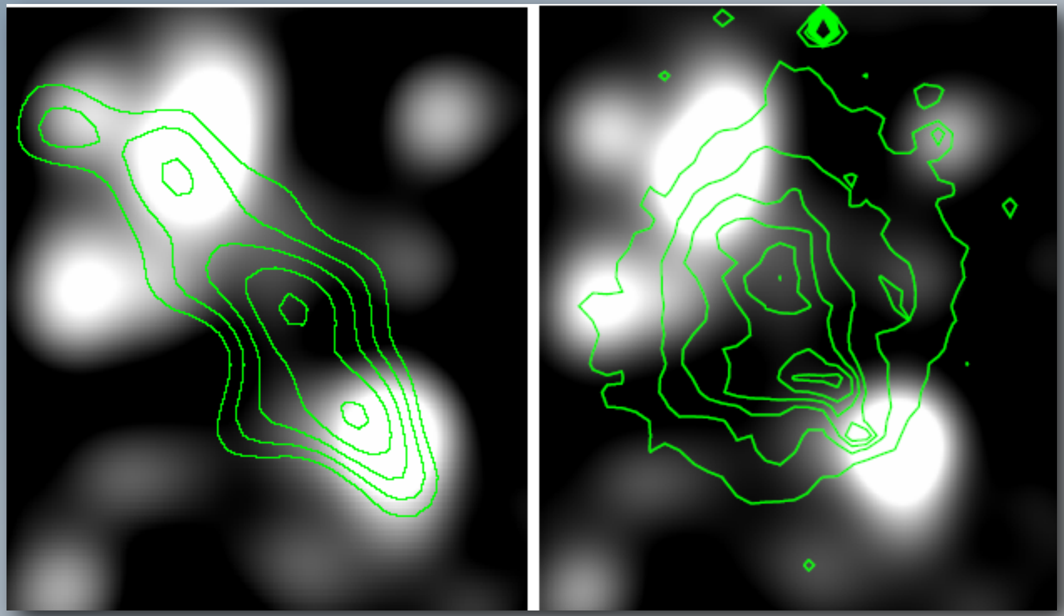


Galaxies Cluster

red: gas   blue: gravitationnal lens

- ABEL 520
  - MOND: lens = gas X
  - Newtonian gravity

  DM ≠ massive galaxies



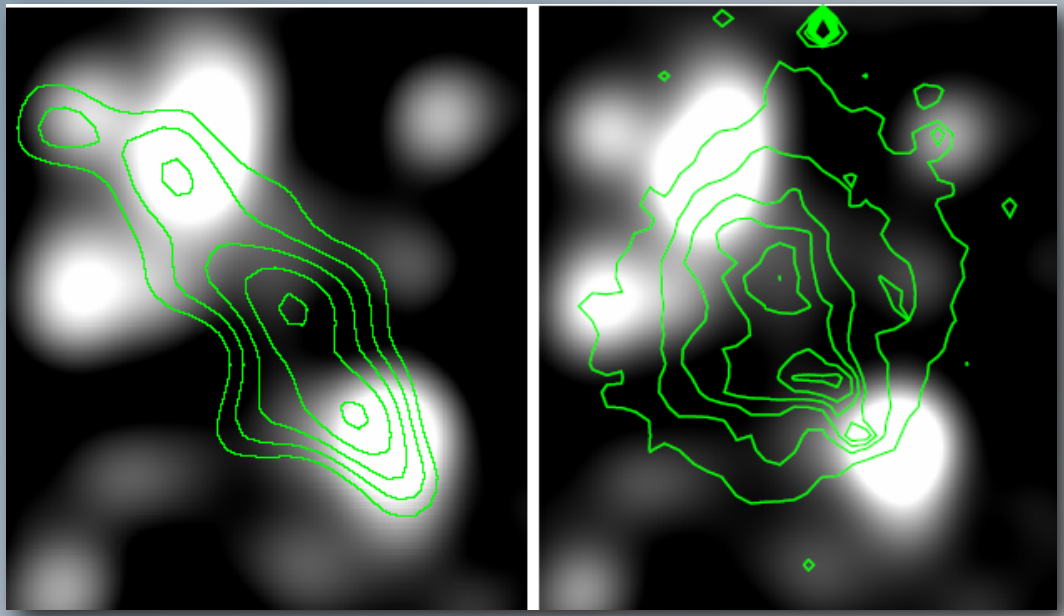
Mahdavi et al (2007)

Galaxies Cluster

X galaxies   red: gas   blue: gravitationnal lens

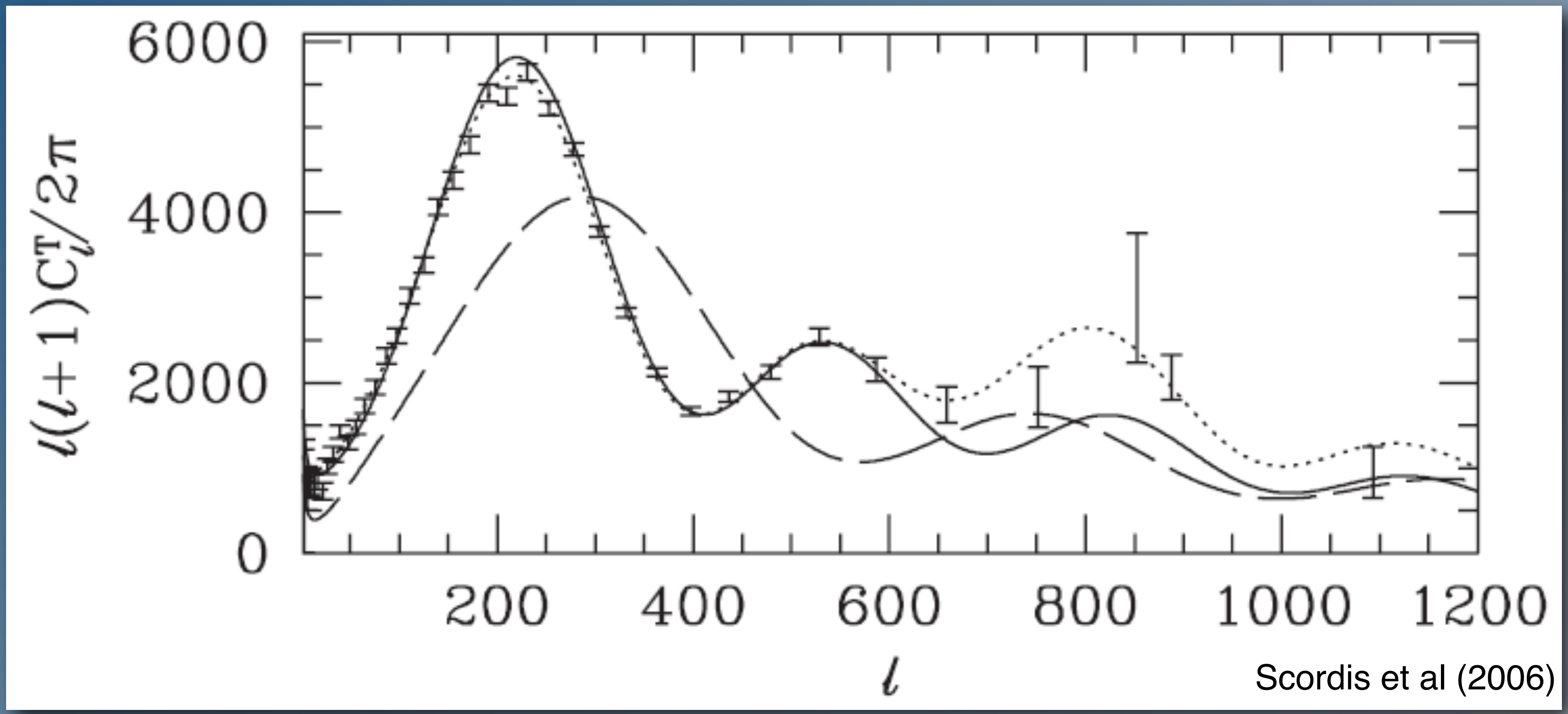
- ABEL 520
  - MOND: lens = gas X
  - Newtonian gravity

  DM ≠ massive galaxies



Mahdavi et al (2007)

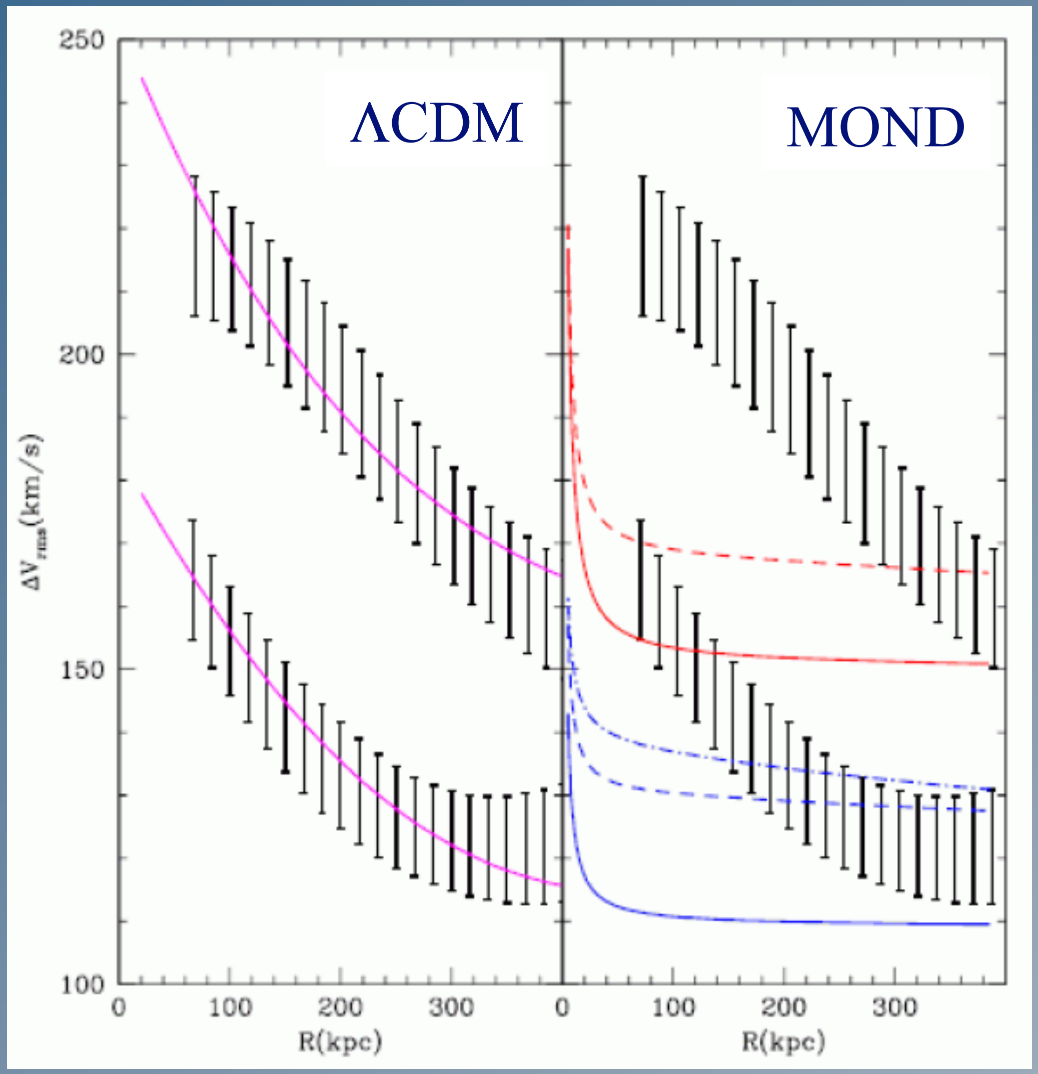
Cosmic Microwave Background



- dashed line (MOND): Ω_b=0.05, Ω_Λ=0.95
- solid line (MOND): Ω_b=0.05, Ω_Λ=0.78, Ω_ν=0.17
- dotted line (ΛCDM): Ω_b=0.05, Ω_Λ=0.72, Ω_{DM}=0.23

Elliptical galaxies

- Test the gravitational field around ellipticals with the movement of galaxy satellites (SDSS).



Klypin & Prada (2007)

Jeans equation:

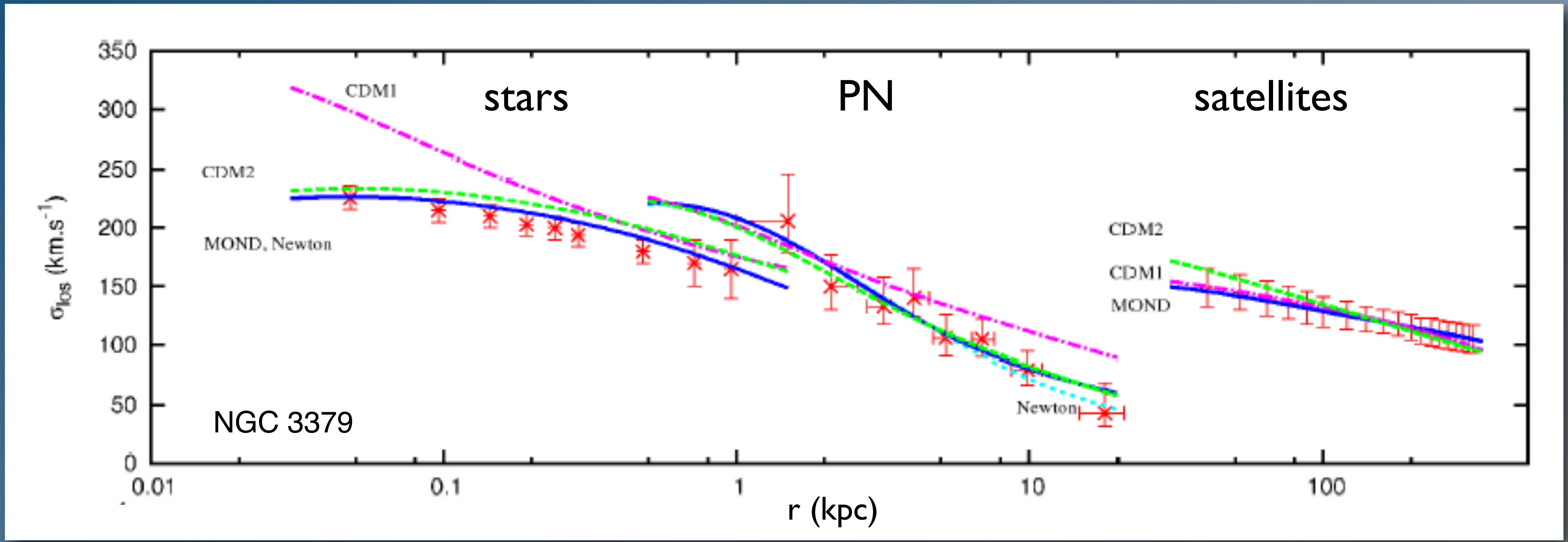
$$\frac{d\sigma^2}{dr} + \sigma^2 \frac{2(\beta + \alpha)}{r} = -g(r)$$

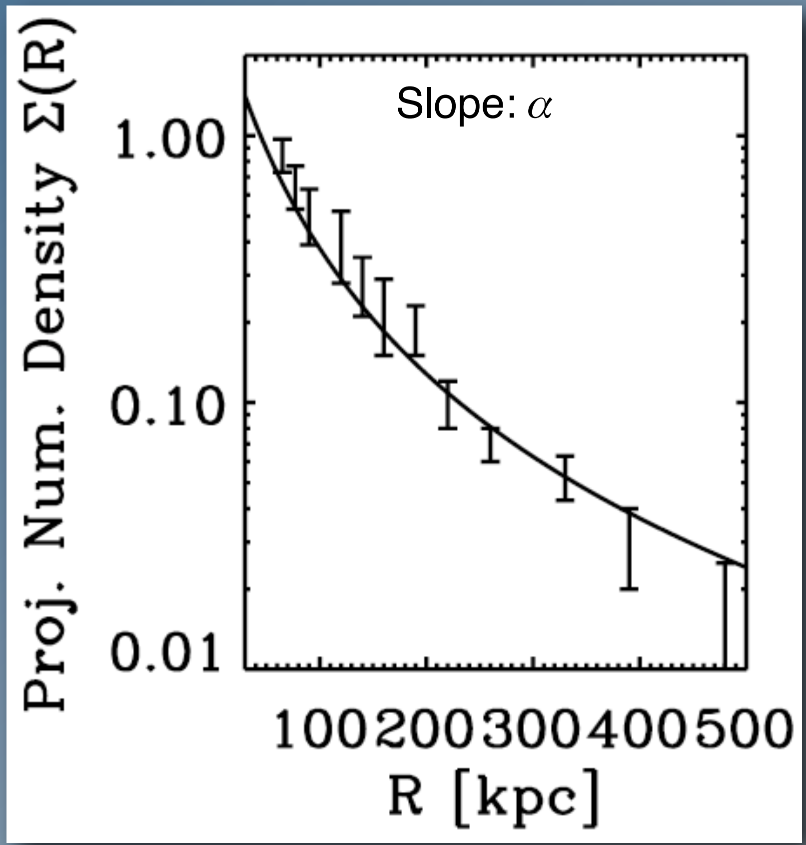
$$\alpha = d\ln\rho / d\ln r$$

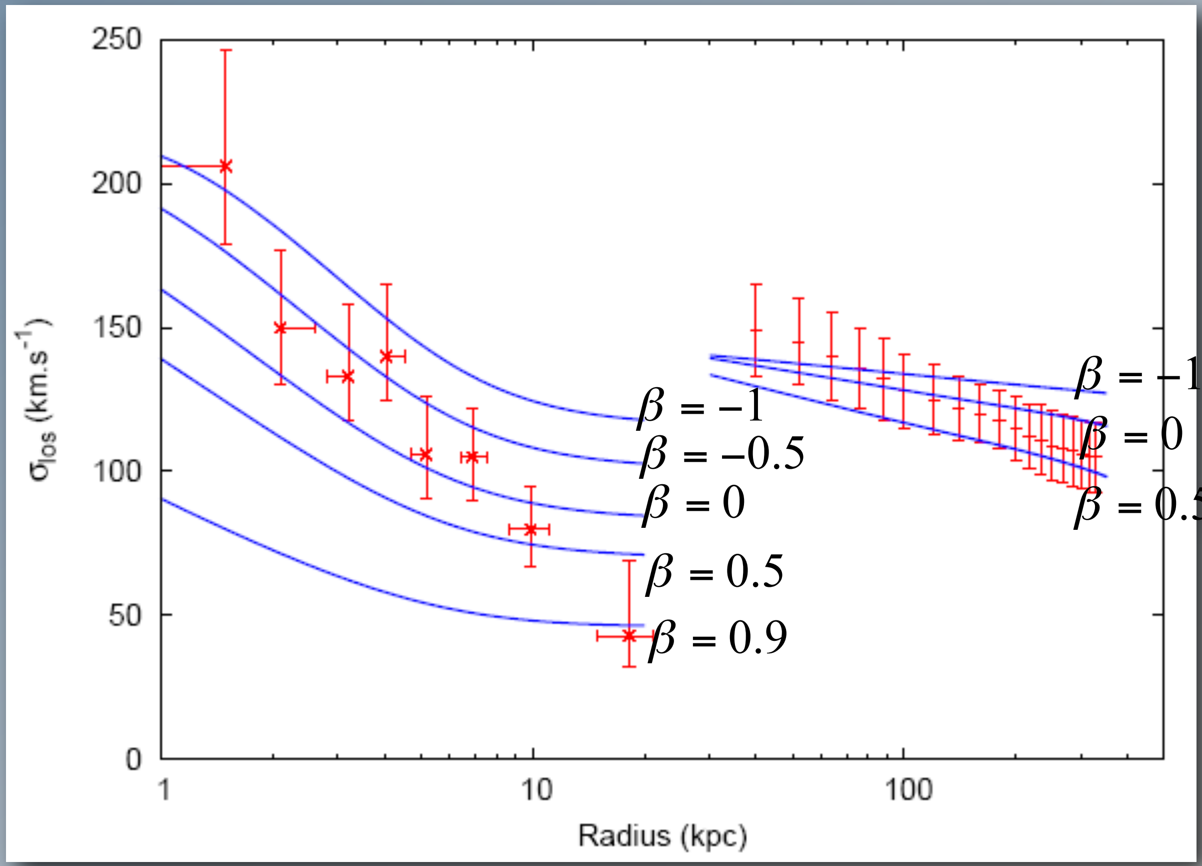
$$\beta = 1 - (\sigma_\theta^2 + \sigma_\phi^2)/2\sigma$$

$$\sigma \longrightarrow cst$$

Elliptical galaxies







Tiret et al (2007)

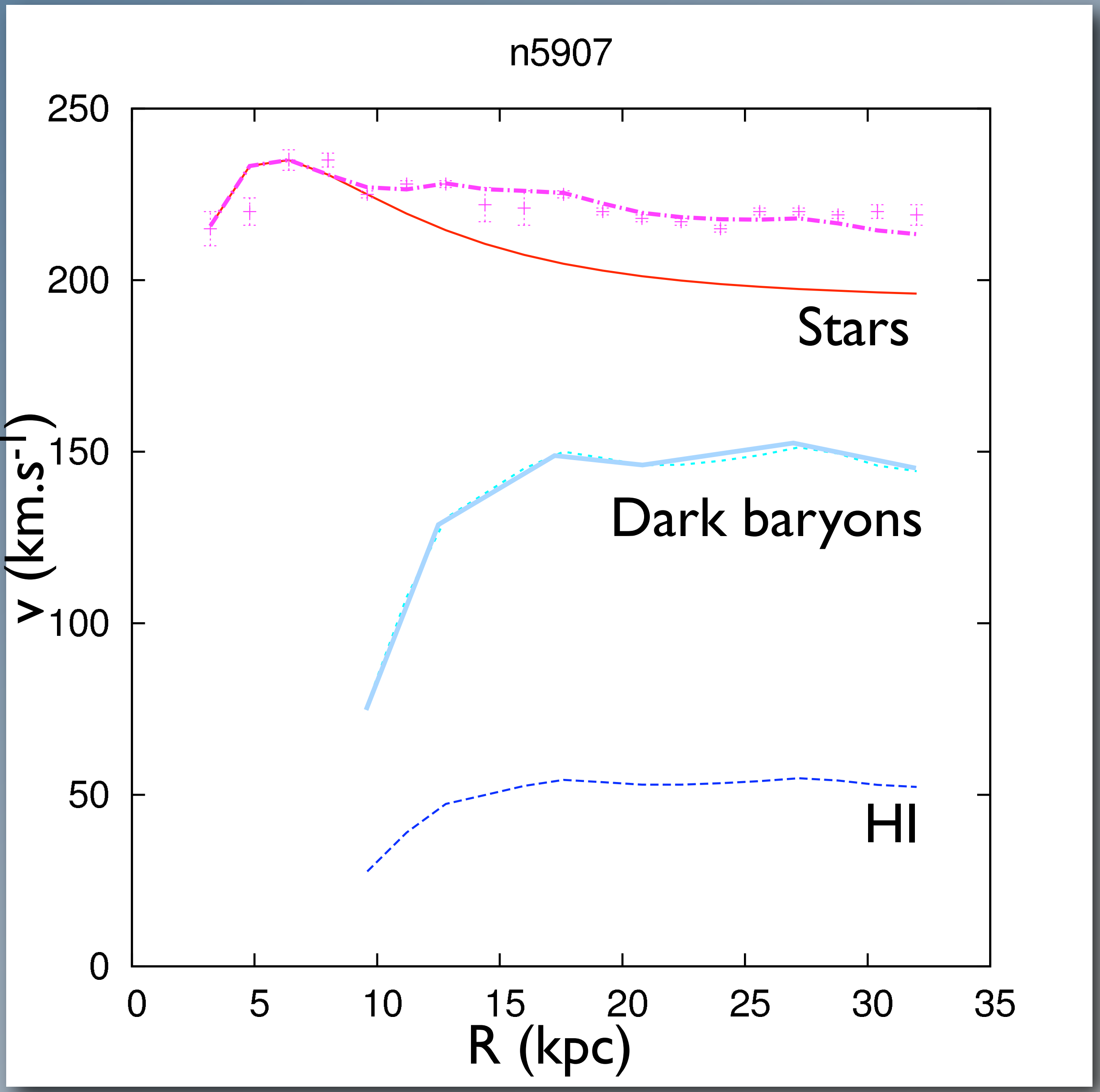
The dark baryons

- Cosmic baryon budget:
  - 6%, stars + gas in the galaxies
  - 30%, Lyman α forest
  - 5-10%, Warm-Hot medium
  - ➡ 50% at least are missing...
    Fukugita et al (1998), Nicastro et al (2005), Danforth et al (2006)
- Some of them are present at galactic scale like the molecule H_2.
  Pfenniger & Combes (1994)
- Compatibility with the MOND phenomenology:
  - which fraction M_{dark}/M_{HI}?
  - value of the critical acceleration a_0?

Tiret & Combes, accepted in A&A

The dark baryons

- Cosmic baryon budget:
  - 6%, stars + gas in the galaxies
  - 30%, Lyman α forest
  - 5-10%, Warm-Hot medium
  - ➡ 50% at least are missing...
    Fukugita et al (1998), Nicastro et al (2005), Danforth et al (2006)
- Some of them are present at galactic scale like the molecule H_2.
  Pfenniger & Combes (1994)
- Compatibility with the MOND phenomenology:
  - which fraction M_{dark}/M_{HI}?
  - value of the critical acceleration a_0?



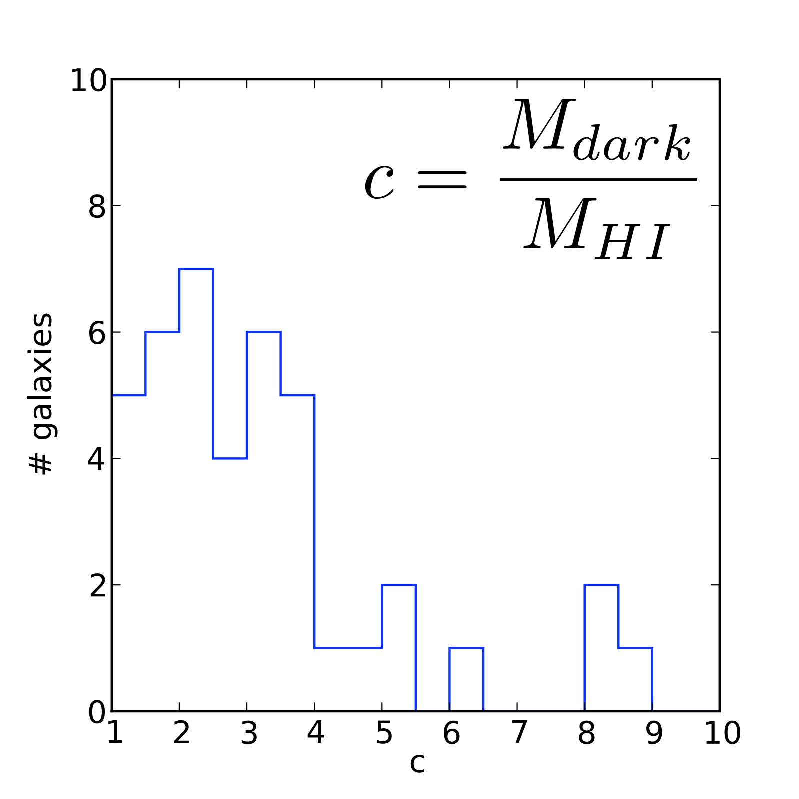
Tiret & Combes, accepted in A&A

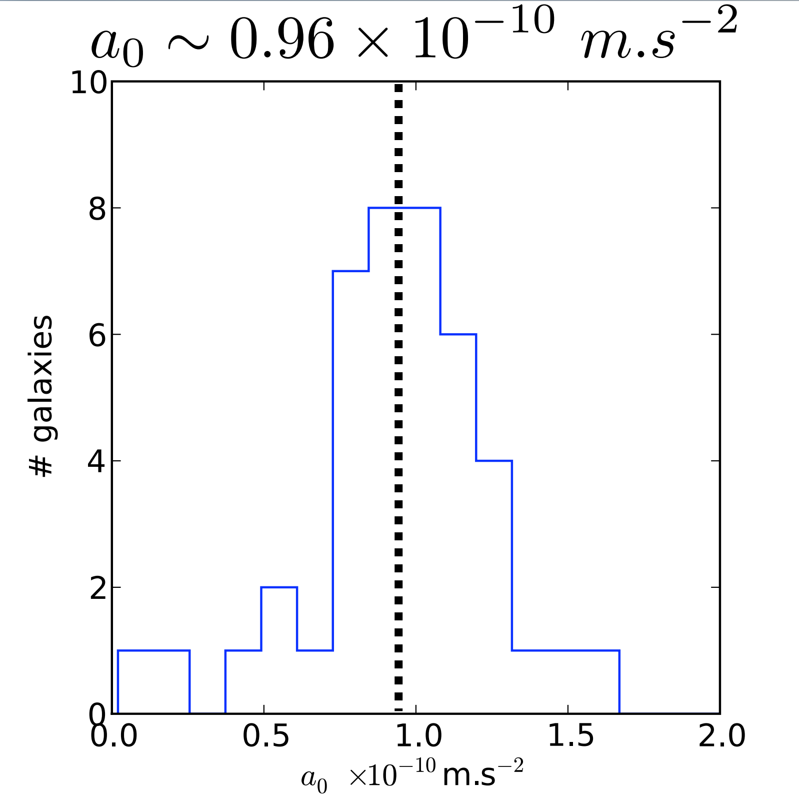
The dark baryons

- Cosmic baryon budget:
  - 6%, stars + gas in the galaxies
  - 30%, Lyman α forest
  - 5-10%, Warm-Hot medium
  - ➡ 50% at least are missing...

    Fukugita et al (1998), Nicastro et al (2005), Danforth et al (2006)
- Some of them are present at galactic scale like the molecule H_2.

  Pfenniger & Combes (1994)
- Compatibility with the MOND phenomenology:
  - which fraction M_{dark}/M_{HI}?
  - value of the critical acceleration a_0?



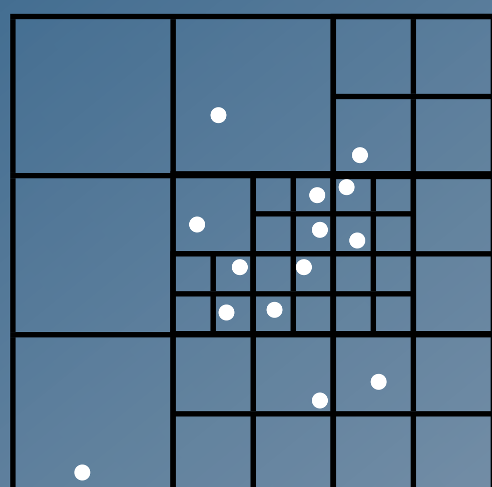


Tiret & Combes, accepted in A&A

Numerical simulations

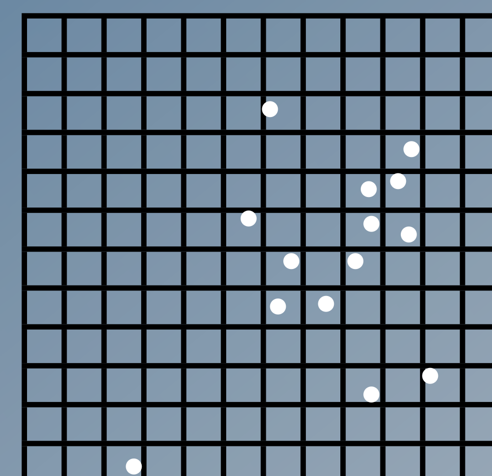
- N-body: needs to compute the gravitationnal force for each particle
- Newtonian codes:

Tree-Code



$$F_i \sim \sum_{near} \frac{G m_i m_j}{r_{ij}^2} + \sum_{far} multipole$$

PM (Particle-Mesh)



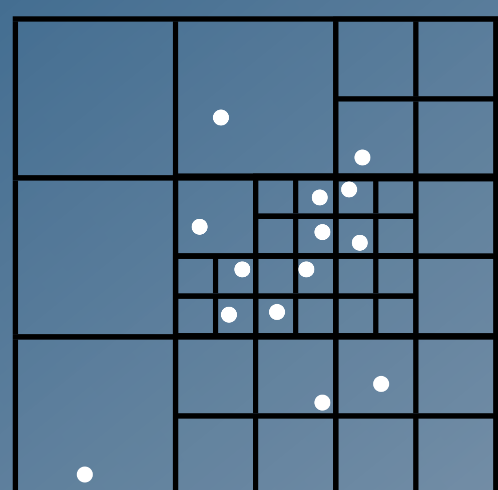
$$\Delta\phi = 4\pi G\rho$$

Fast Fourier Transform (FFT)

Numerical simulations

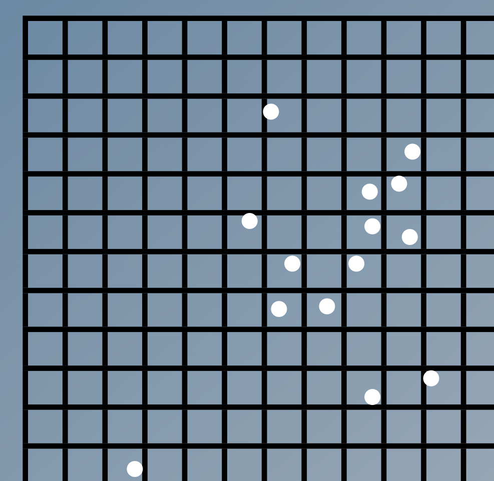
- N-body: needs to compute the gravitationnal force for each particle
- Newtonian codes:

Tree-Code



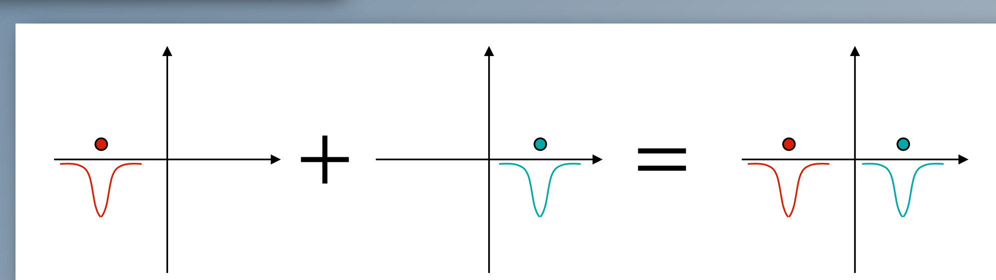
$$F_i \sim \sum_{near} \frac{G m_i m_j}{r_{ij}^2} + \sum_{far} multipole$$

PM (Particle-Mesh)



$$\Delta\phi = 4\pi G\rho$$

Fast Fourier Transform (FFT)

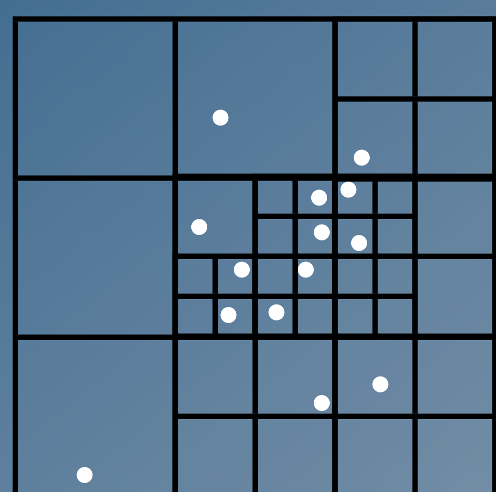


Based on the linearity of the newtonian gravity

Numerical simulations

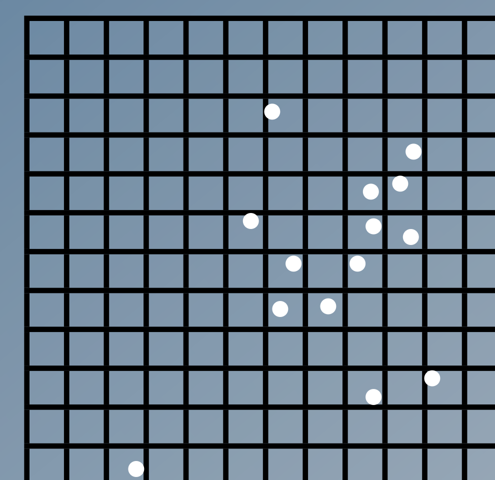
- N-body: needs to compute the gravitationnal force for each particle
- Newtonian codes:

<u>*Tree-Code*</u>



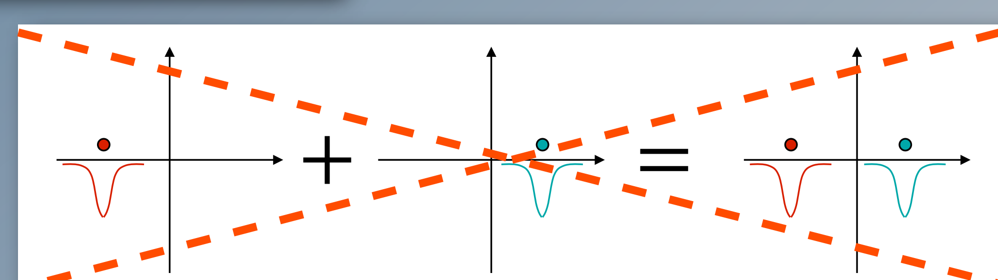
$$F_i \sim \sum_{near} \frac{Gm_i m_j}{r_{ij}^2} + \sum_{far} multipole$$

<u>*PM (Particle-Mesh)*</u>



$$\Delta\phi = 4\pi G\rho$$

Fast Fourier Transform (FFT)



Based on the linearity of the newtonian gravity

- Modified gravity: Non-linear with the mass $a_M = \sqrt{a_0 a_N}$

Modified Poisson Equation

- Lagrangian Formalism:
  - Newtonian gravity:

$$L_N = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1}(\nabla\phi)^2\right\} \longrightarrow \Delta\phi = 4\pi G\rho$$

  - MOND:

$$L_M = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1} a_0^2 \mathcal{F}\left[\frac{(\nabla\phi)^2}{a_0^2}\right]\right\} \rightarrow \nabla\cdot\left\{\mu\left[\frac{|\nabla\phi|}{a_0}\right]\nabla\phi\right\} = 4\pi G\rho$$

AQUAL, Bekenstein & Milgrom (1984)

Modified Poisson Equation

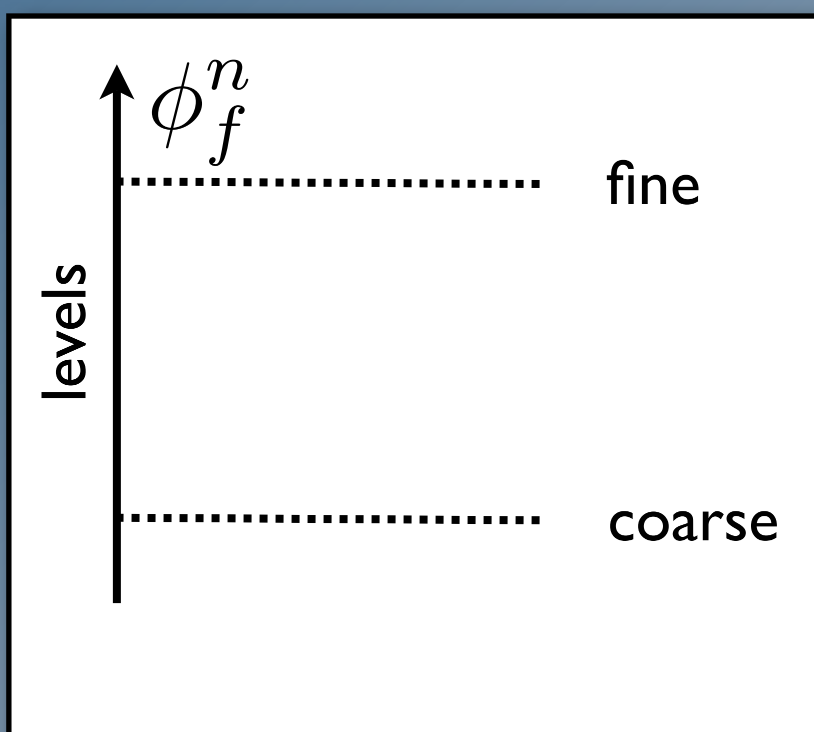
- Lagrangian Formalism:
  - Newtonian gravity:

$$L_N = -\int d^3r \left\{ \rho\phi + (8\pi G)^{-1} (\nabla\phi)^2 \right\} \longrightarrow \Delta\phi = 4\pi G\rho$$

  - MOND:

$$L_M = -\int d^3r \left\{ \rho\phi + (8\pi G)^{-1} a_0^2 \mathcal{F}\left[\frac{(\nabla\phi)^2}{a_0^2}\right] \right\} \rightarrow \nabla\cdot\left\{ \mu\left[\frac{|\nabla\phi|}{a_0}\right] \nabla\phi \right\} = 4\pi G\rho$$

AQUAL, Bekenstein & Milgrom (1984)



Multigrid Technique

$$\mathcal{L}_f \phi_f = \rho_f$$

Modified Poisson Equation

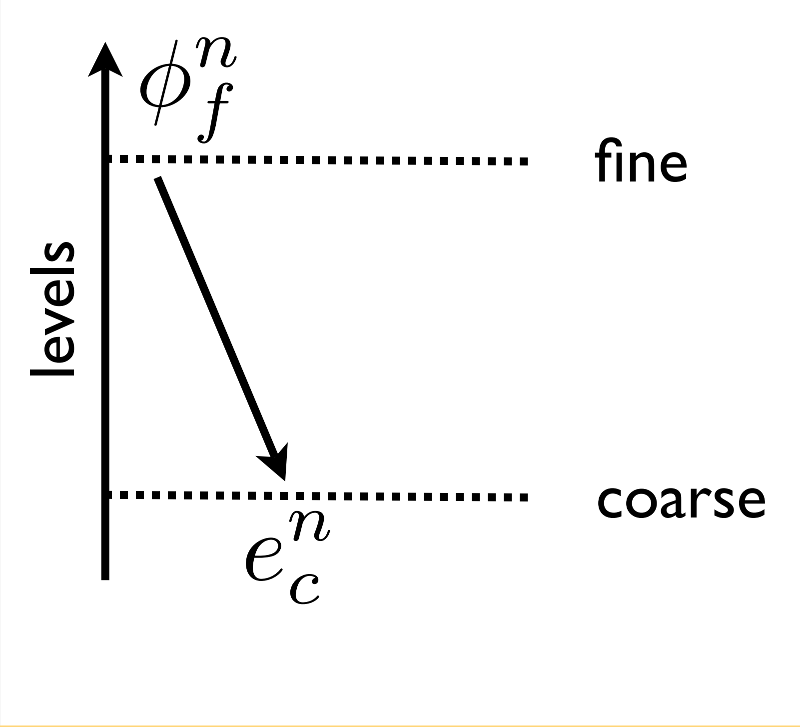
- Lagrangian Formalism:
  - Newtonian gravity:

$$L_N = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1}(\nabla\phi)^2\right\} \longrightarrow \Delta\phi = 4\pi G\rho$$

  - MOND:

$$L_M = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1}a_0^2\mathcal{F}\left[\frac{(\nabla\phi)^2}{a_0^2}\right]\right\} \rightarrow \nabla\cdot\left\{\mu\left[\frac{|\nabla\phi|}{a_0}\right]\nabla\phi\right\} = 4\pi G\rho$$

AQUAL, Bekenstein & Milgrom (1984)



Multigrid Technique

$$\mathcal{L}_f\phi_f = \rho_f$$

$$r_f^n = \rho_f - \mathcal{L}_f\phi_f^n$$

$$\mathcal{L}_c e_c^n = [r_f^n]_c$$

Modified Poisson Equation

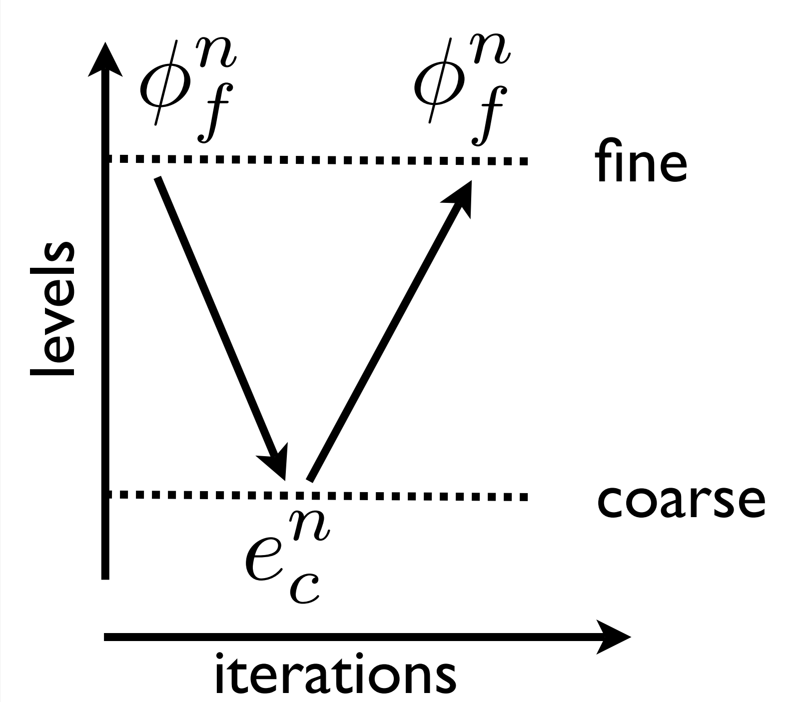
- Lagrangian Formalism:
  - Newtonian gravity:

$$L_N = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1}(\nabla\phi)^2\right\} \longrightarrow \Delta\phi = 4\pi G\rho$$

  - MOND:

$$L_M = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1} a_0^2 \mathcal{F}\left[\frac{(\nabla\phi)^2}{a_0^2}\right]\right\} \rightarrow \nabla\cdot\left\{\mu\left[\frac{|\nabla\phi|}{a_0}\right]\nabla\phi\right\} = 4\pi G\rho$$

AQUAL, Bekenstein & Milgrom (1984)



Multigrid Technique

$$\mathcal{L}_f\phi_f = \rho_f$$

$$r_f^n = \rho_f - \mathcal{L}_f\phi_f^n$$

$$\mathcal{L}_c e_c^n = [r_f^n]_c$$

$$\phi_f^{n+1} = \phi_f^n + [e_c^n]_f$$

Modified Poisson Equation

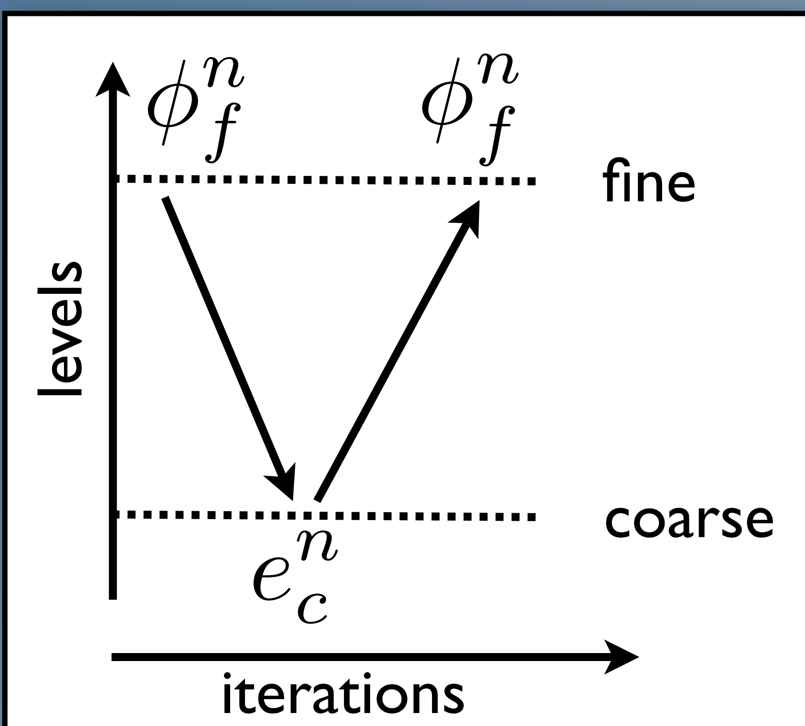
- Lagrangian Formalism:
  - Newtonian gravity:

$$L_N = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1}(\nabla\phi)^2\right\} \longrightarrow \Delta\phi = 4\pi G\rho$$

  - MOND:

$$L_M = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1} a_0^2 \mathcal{F}\left[\frac{(\nabla\phi)^2}{a_0^2}\right]\right\} \rightarrow \nabla\cdot\left\{\mu\left[\frac{|\nabla\phi|}{a_0}\right]\nabla\phi\right\} = 4\pi G\rho$$

AQUAL, Bekenstein & Milgrom (1984)



Multigrid Technique

$$\mathcal{L}_f\phi_f = \rho_f$$

$$r_f^n = \rho_f - \mathcal{L}_f\phi_f^n$$

$$\mathcal{L}_c e_c^n = [r_f^n]_c$$

$$\phi_f^{n+1} = \phi_f^n + [e_c^n]_f$$

$$[r_f]_c = \mathcal{L}_c\phi_c - \mathcal{L}_c[\phi_f^n]_c = \mathcal{L}_c\left(\phi_c - [\phi_f^n]_c\right)$$

Linear multigrid

Modified Poisson Equation

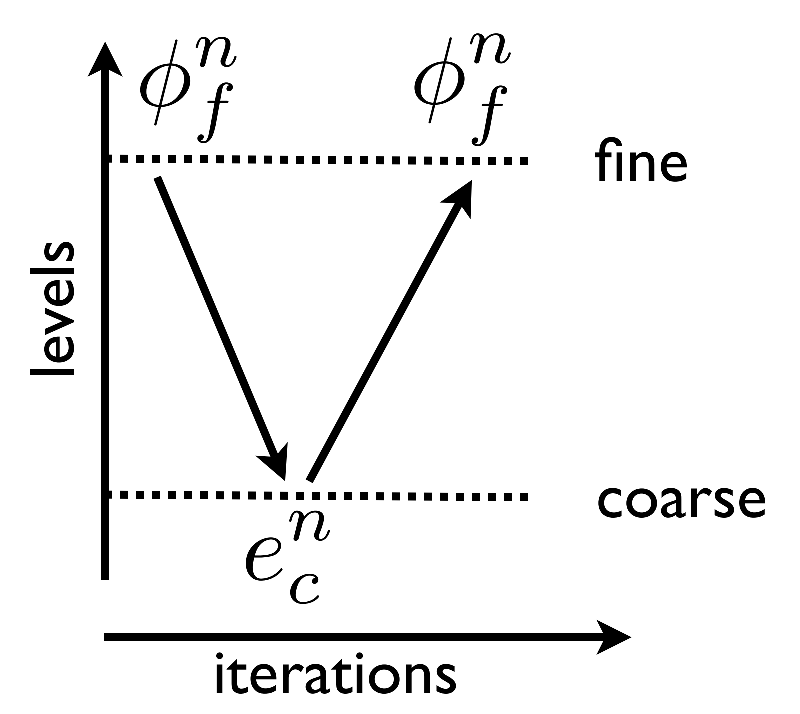
- Lagrangian Formalism:
  - Newtonian gravity:

$$L_N = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1}(\nabla\phi)^2\right\} \longrightarrow \Delta\phi = 4\pi G\rho$$

  - MOND:

$$L_M = -\int d^3r \left\{\rho\phi + (8\pi G)^{-1} a_0^2 \mathcal{F}\left[\frac{(\nabla\phi)^2}{a_0^2}\right]\right\} \rightarrow \nabla\cdot\left\{\mu\left[\frac{|\nabla\phi|}{a_0}\right]\nabla\phi\right\} = 4\pi G\rho$$

AQUAL, Bekenstein & Milgrom (1984)



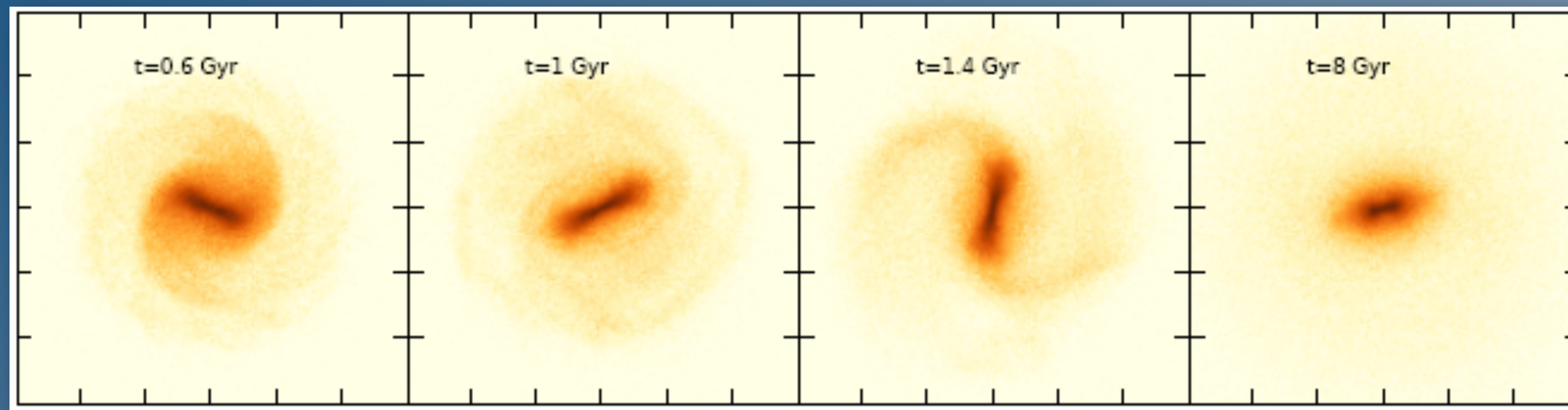
Multigrid Technique

$$\mathcal{L}_f\phi_f = \rho_f$$
$$\tau_c^n = \mathcal{L}_c\phi_c^n - \mathcal{L}_c[\phi_f^n]_c$$
$$\mathcal{L}_c\phi_c^n = \rho_c + \tau_c^n$$
$$e_c^n = \phi_c^n - [\phi_f^n]_c$$
$$\phi_f^{n+1} = \phi_f^n + [e_c^n]_f$$

N-body code

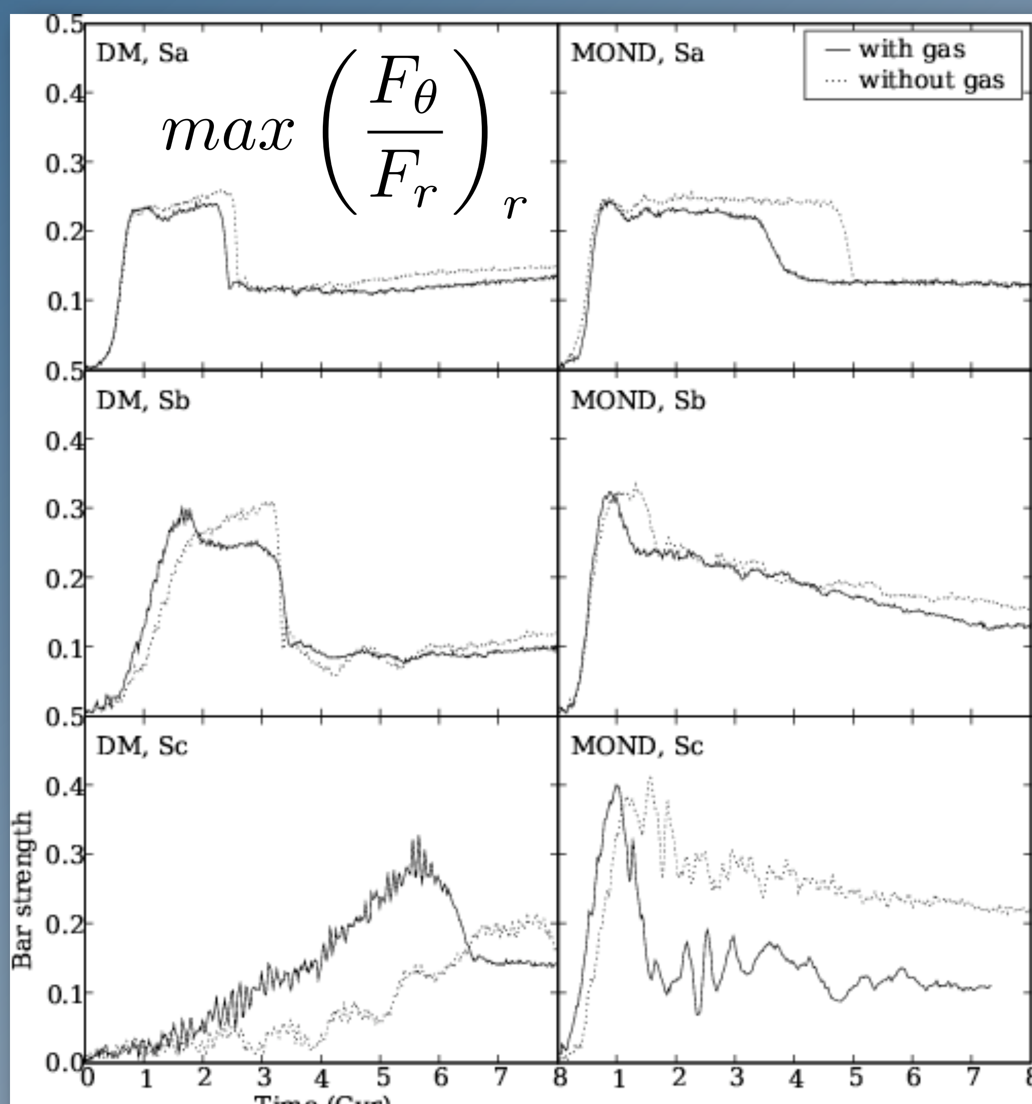
- Potential Solver
  - Gravity: Newtonian / MOND
  - Efficiency: convergence obtained after a few cycles
- Density/Forces/...
  - Cloud In Cell (CIC) interpolation
- Equations of motion
  - leapfrog scheme
- Gas Dynamics
  - sticky-particles

Bar formation

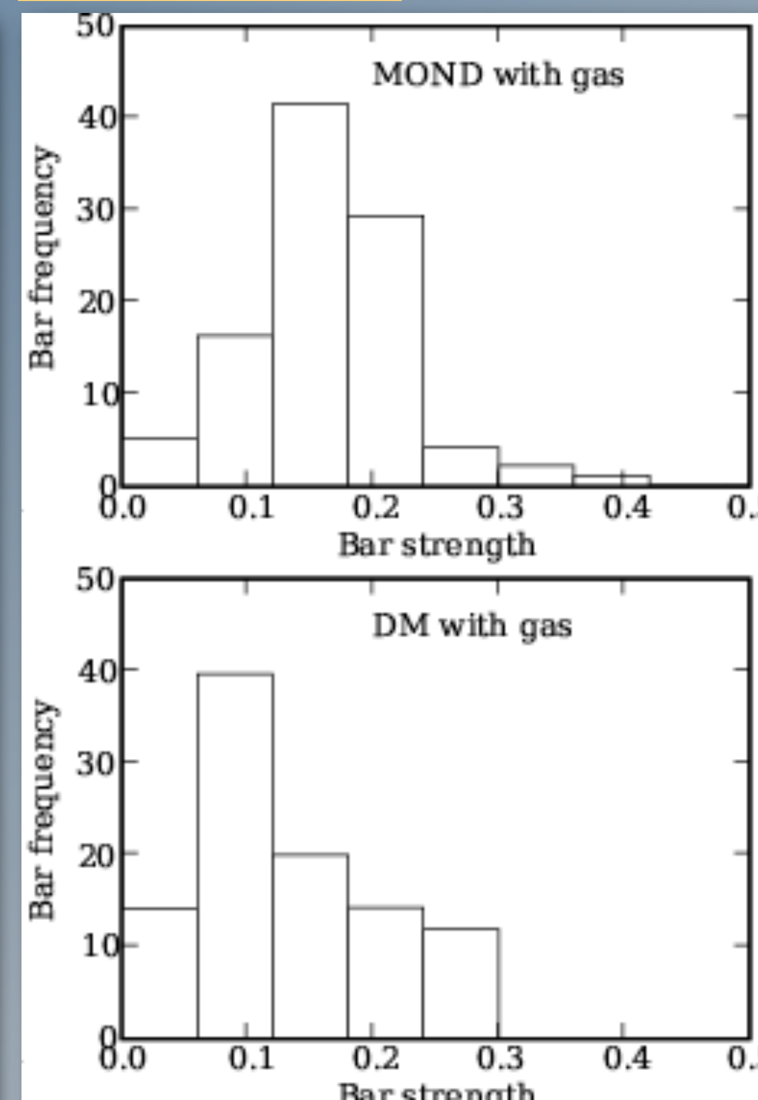


MOND simulations

Bar strengh

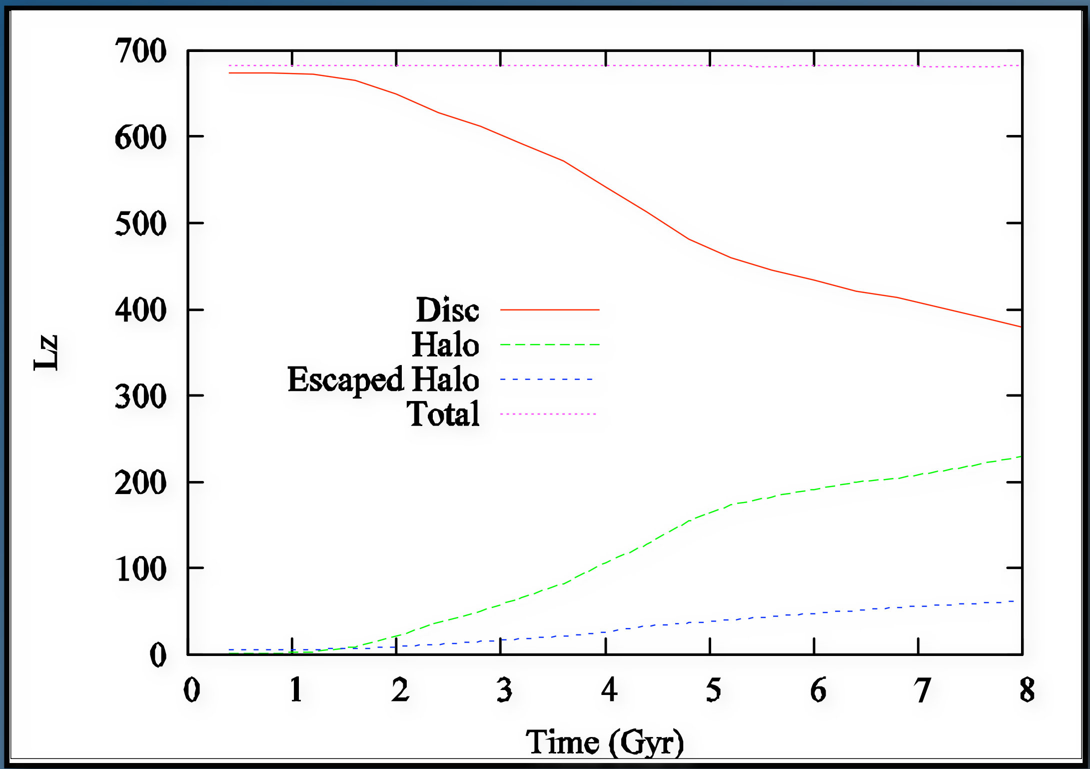


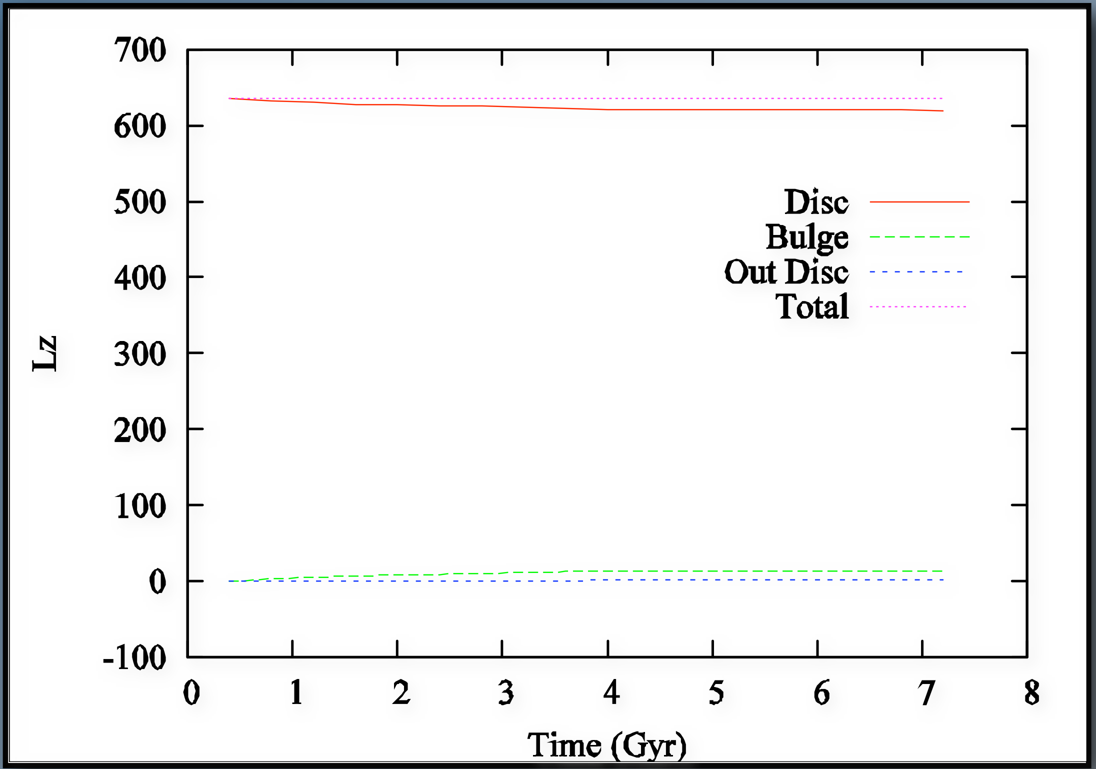
Bar frequency

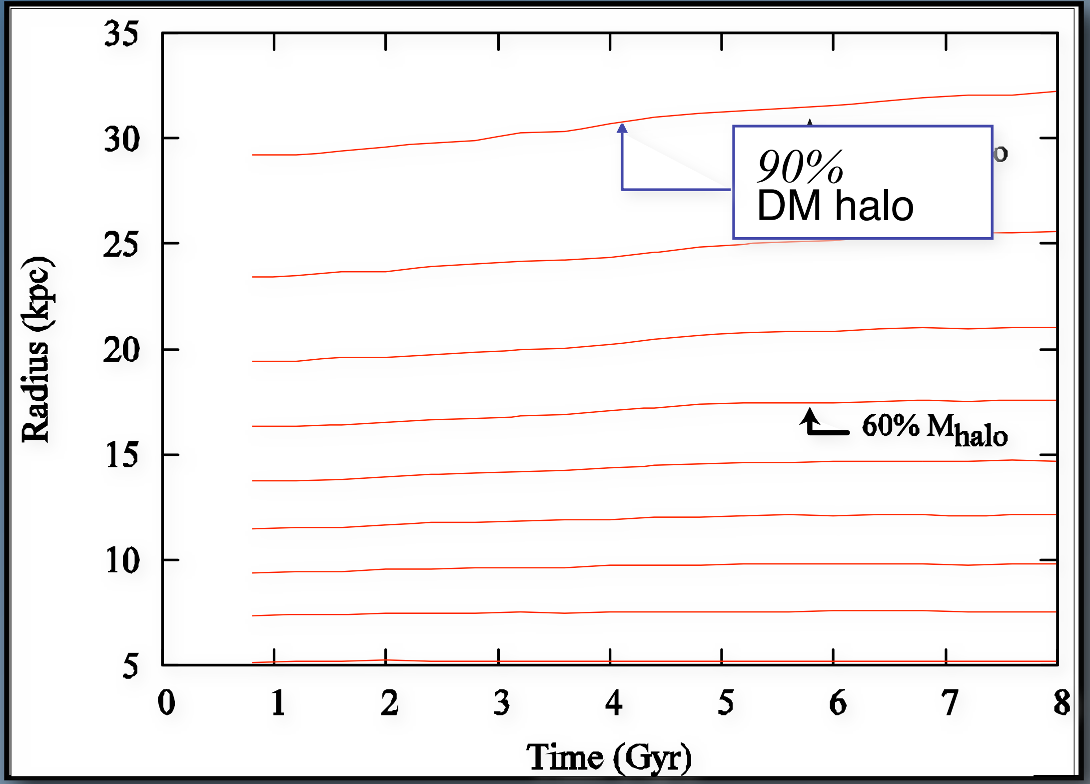


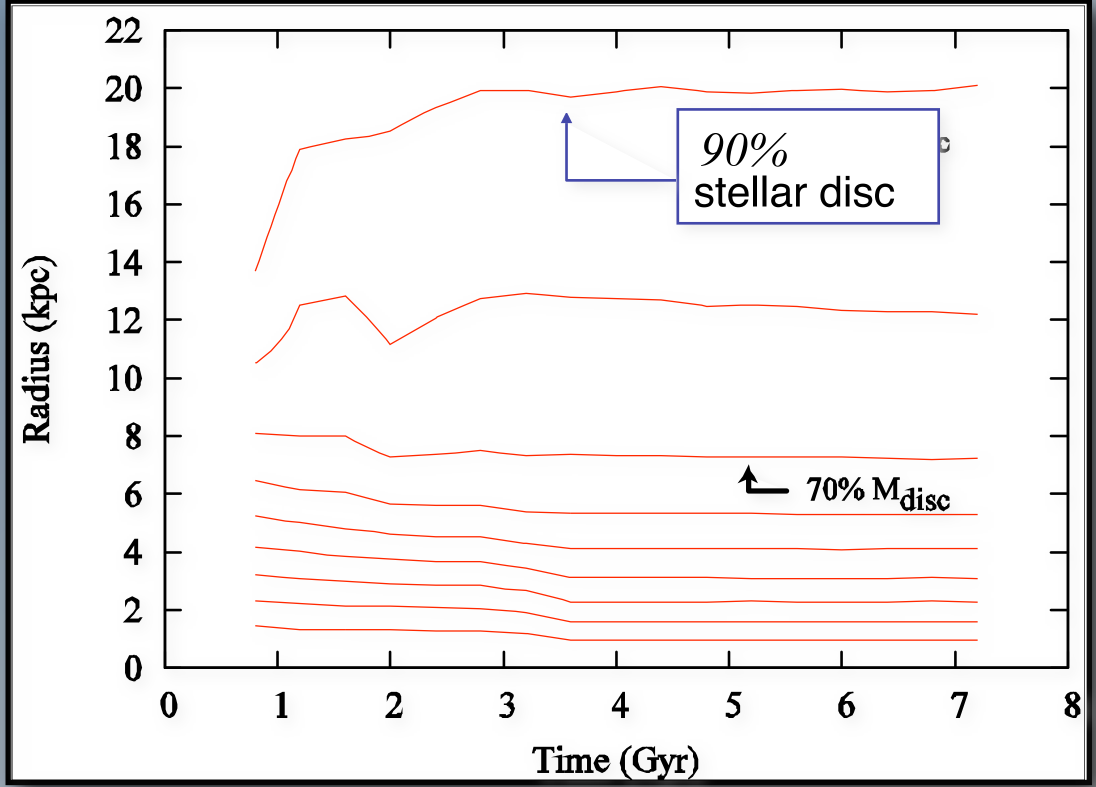
Tiret et Combes (2007)

Angular momentum transferts





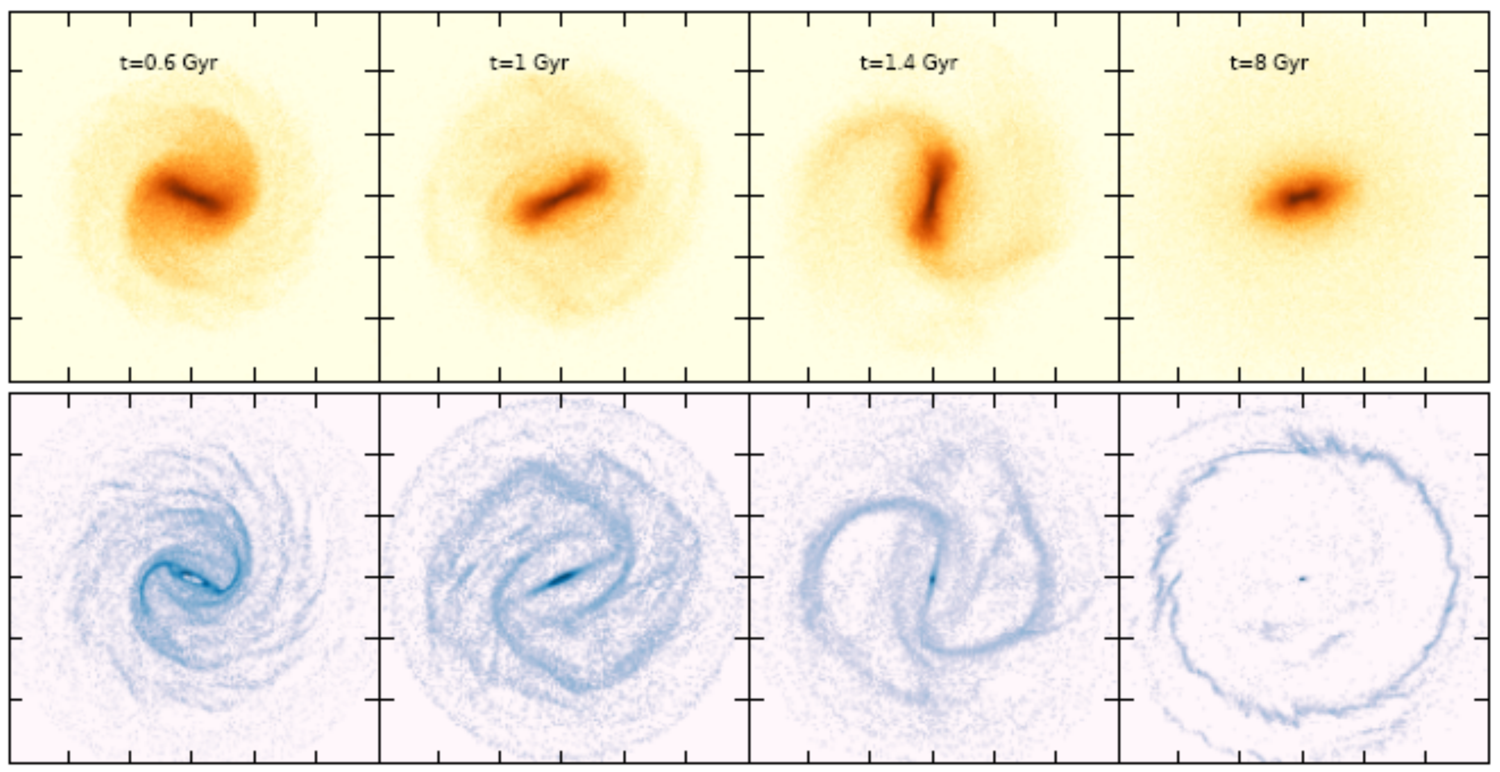


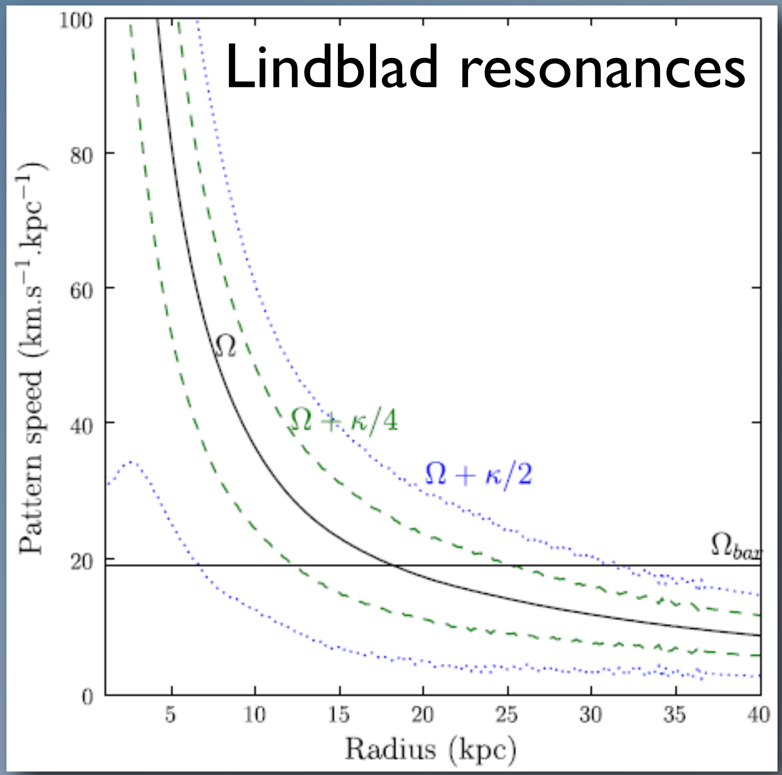


Newtonian gravity

MOND

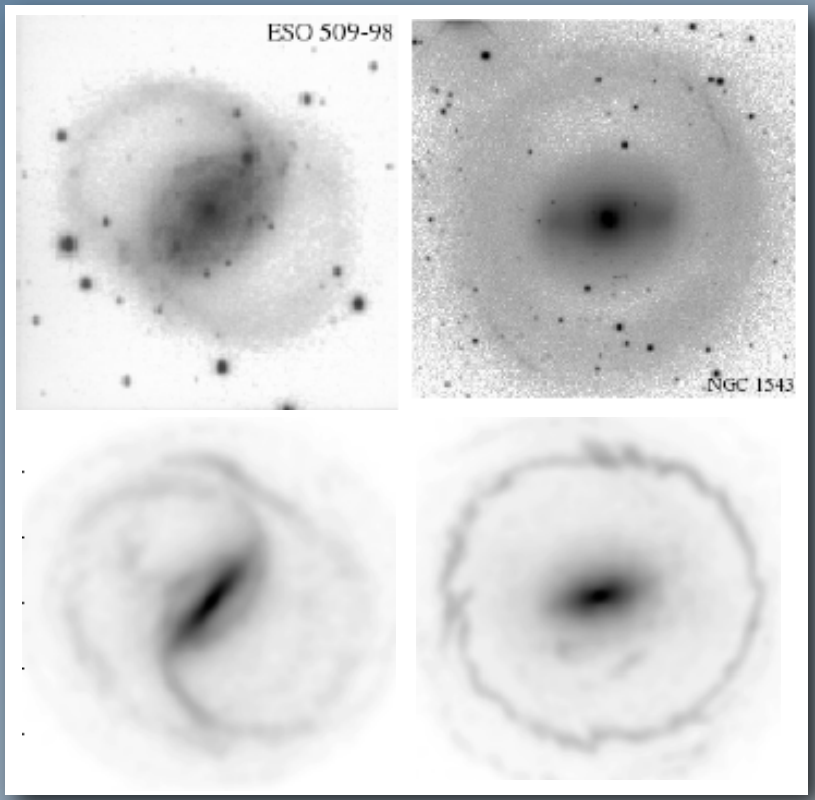
Resonances

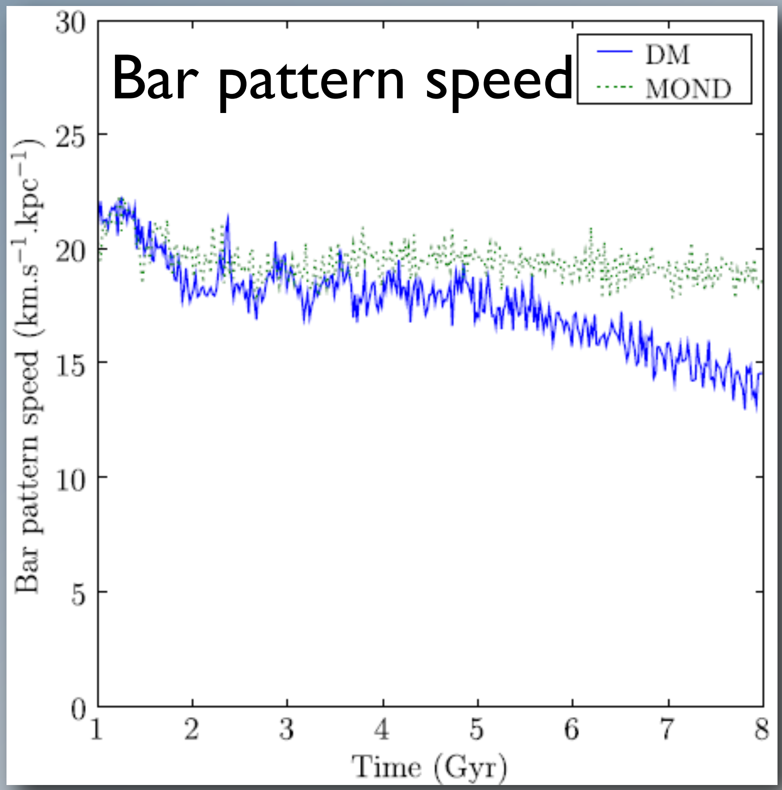




Observations

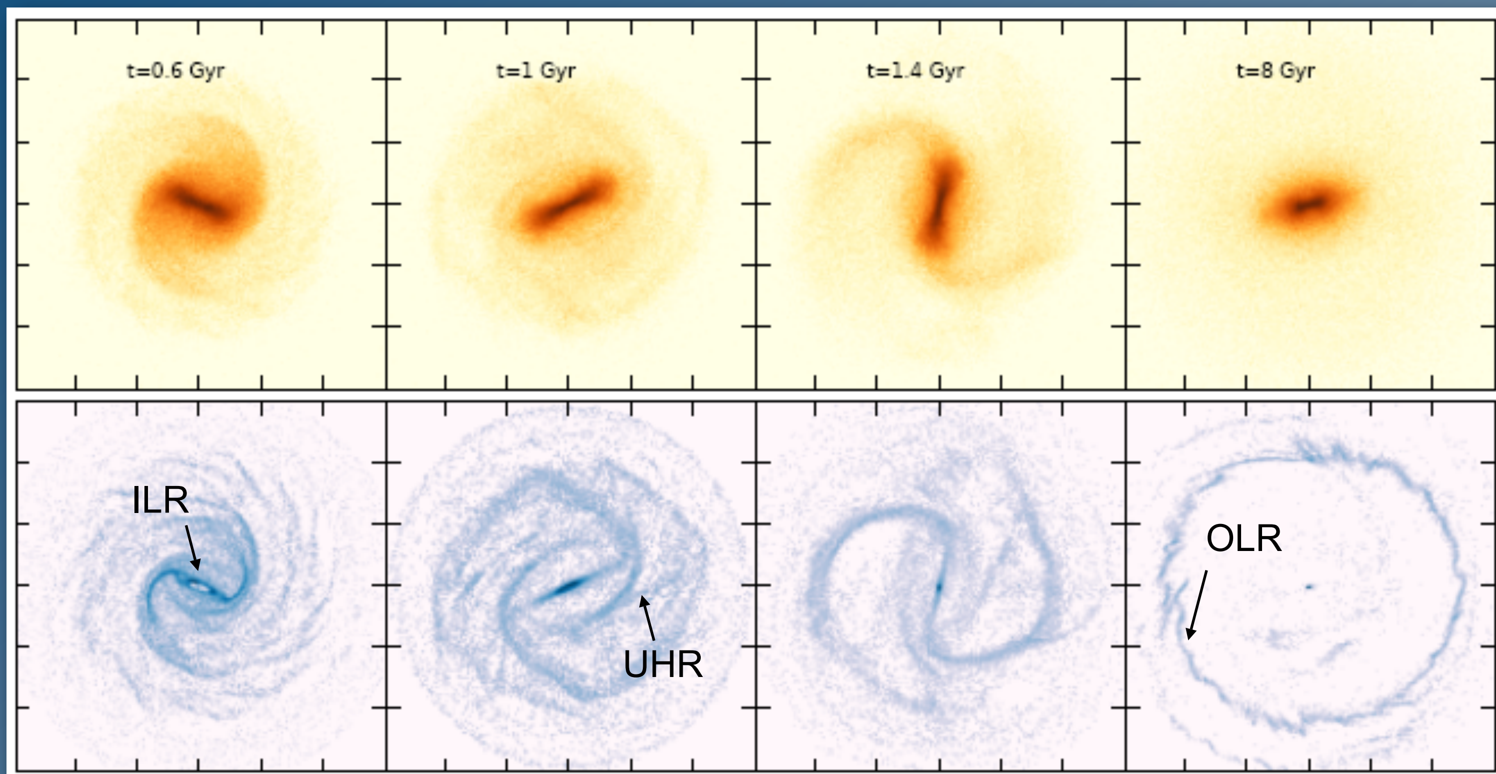
MOND simulations

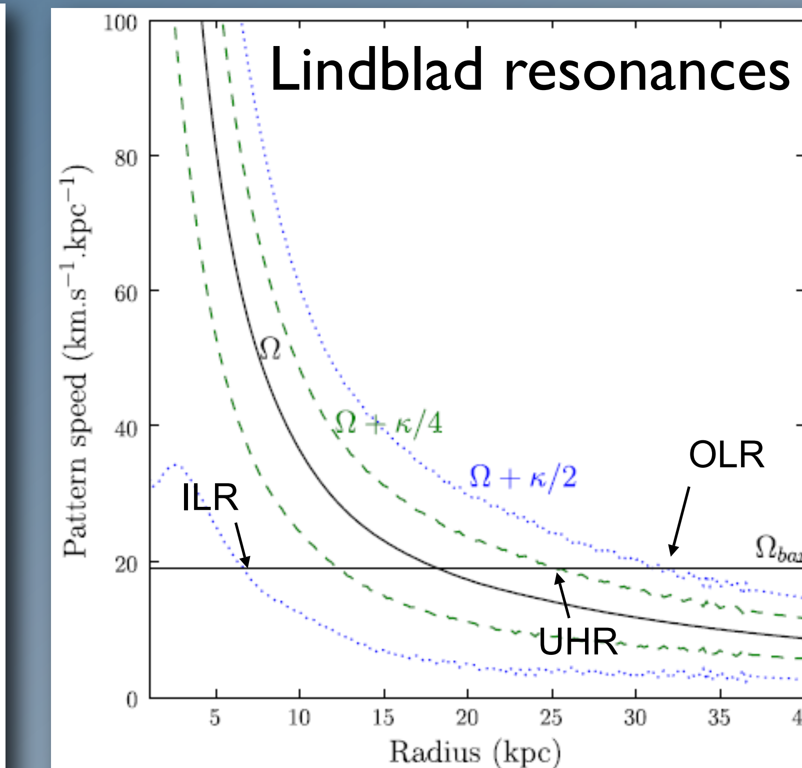




Tiret et Combes (2008)

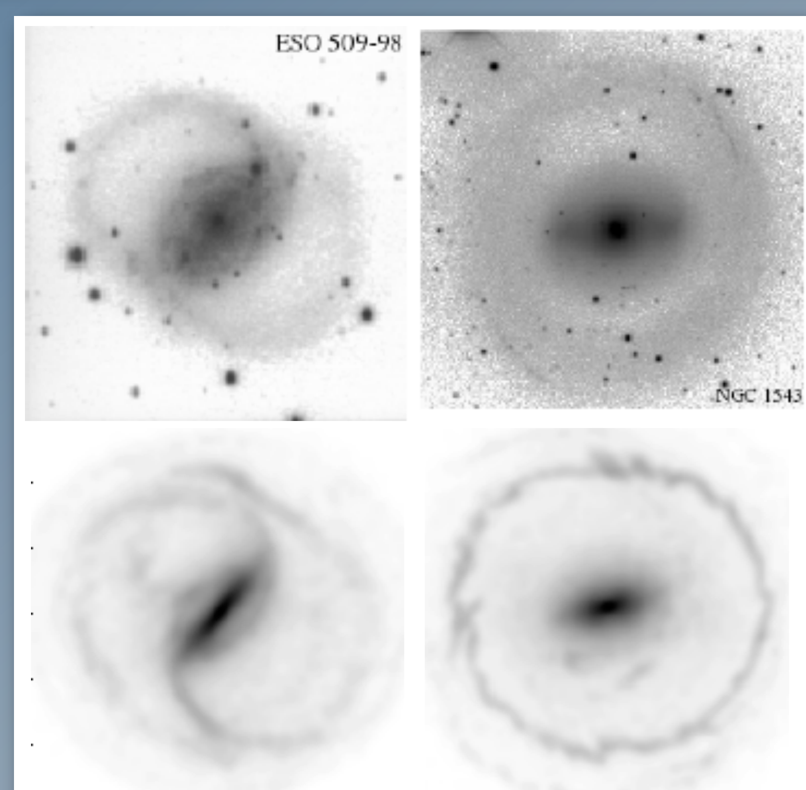
Resonances

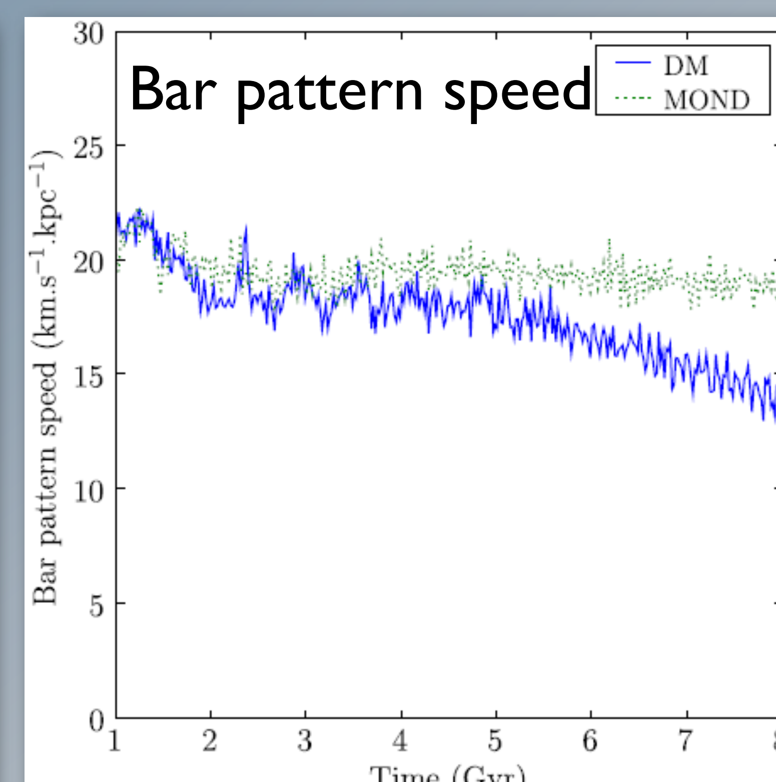




Observations

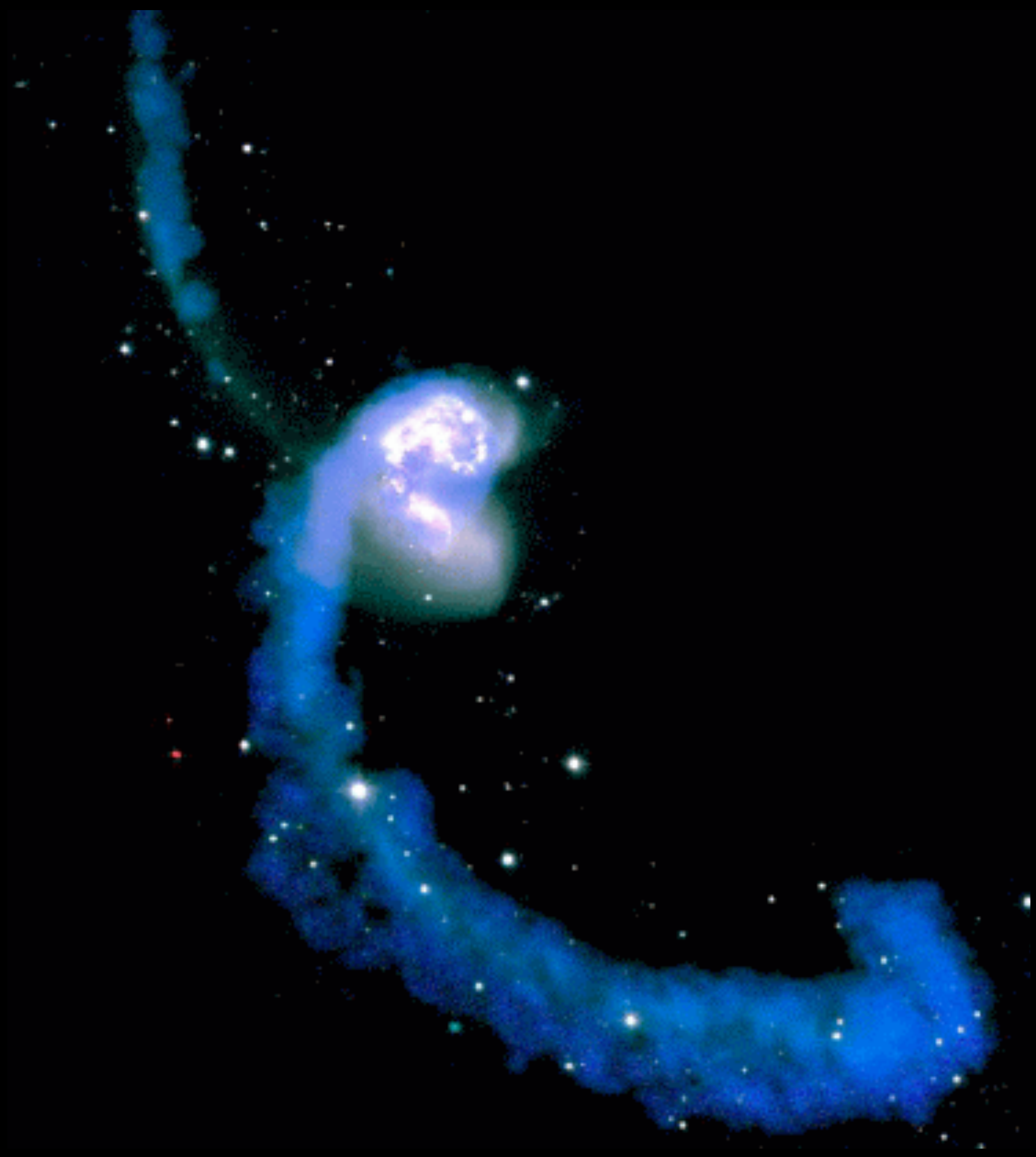
MOND simulations

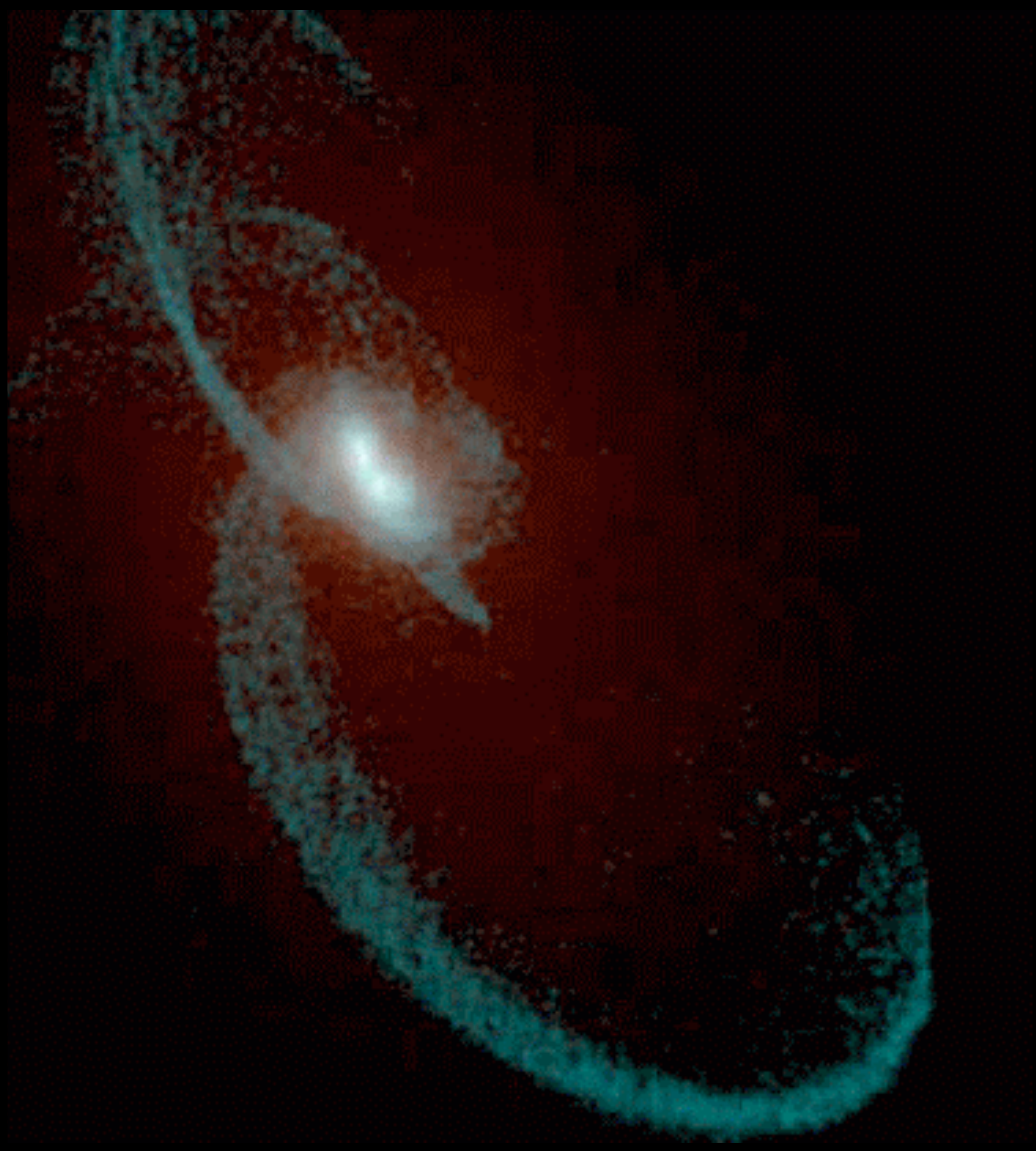




Tiret et Combes (2008)

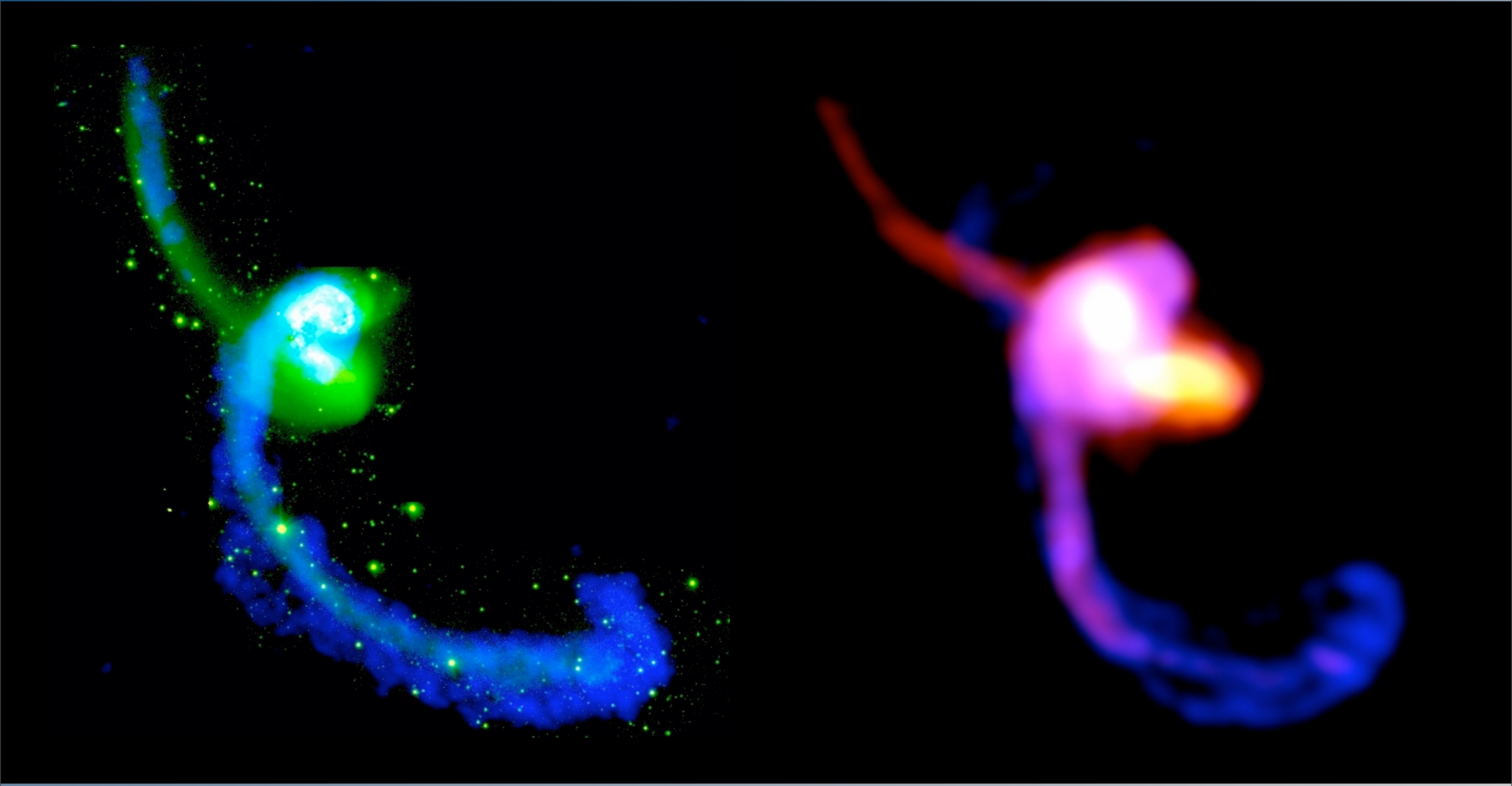
The Antennae galaxies, DM





Barnes (1998)

The Antennae galaxies, MOND

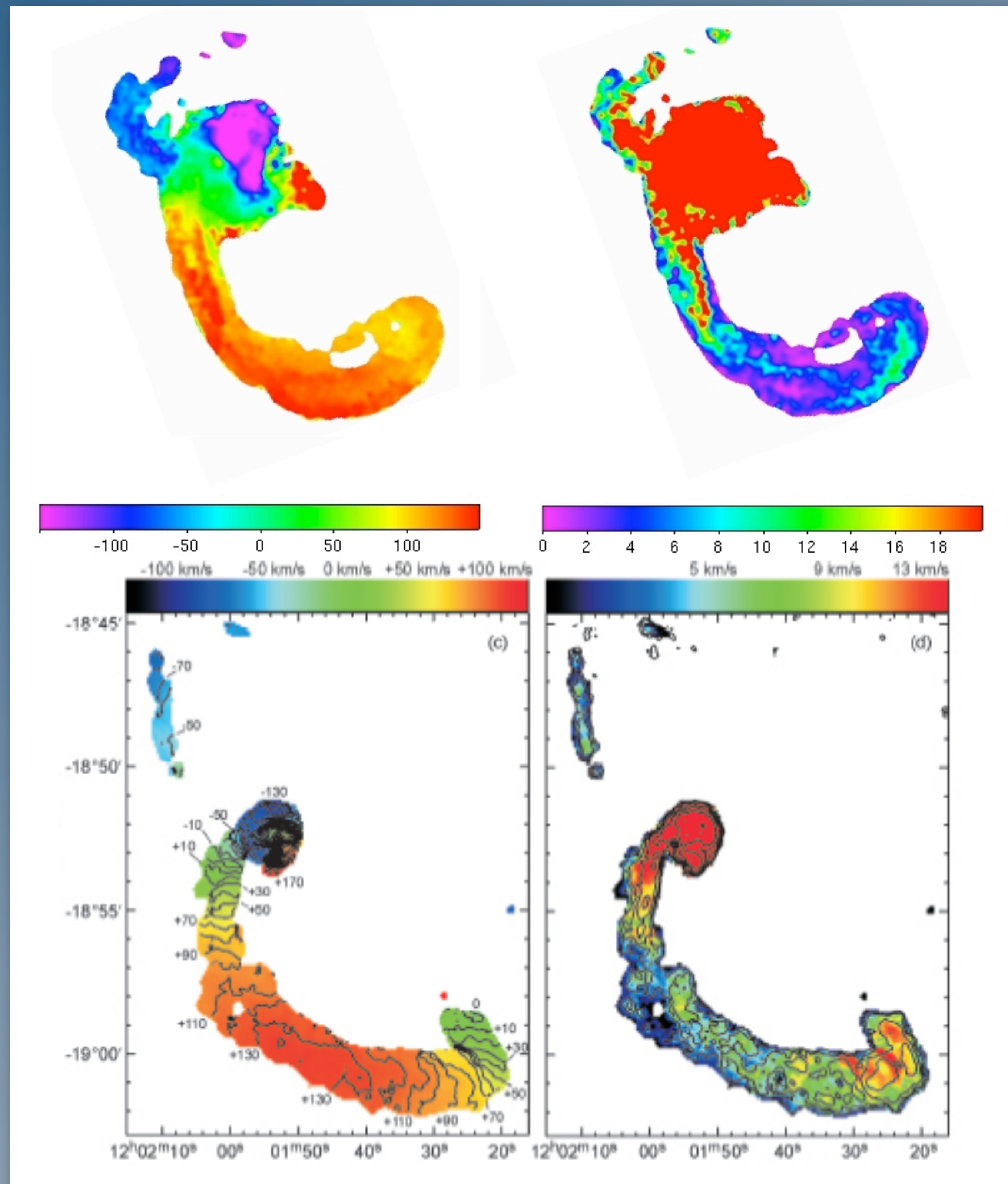


Tiret & Combes, in prep

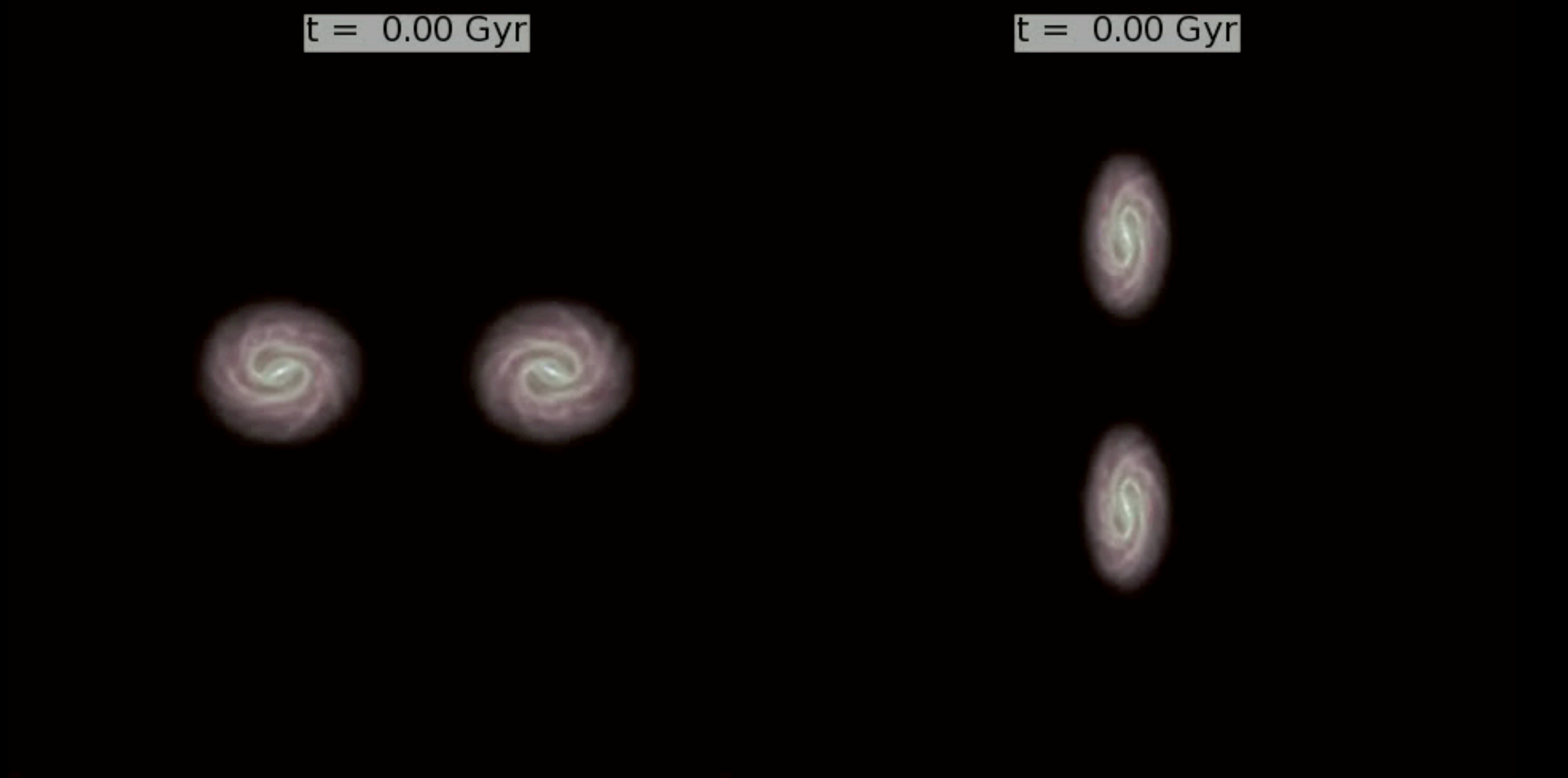
The Antennae galaxies, MOND

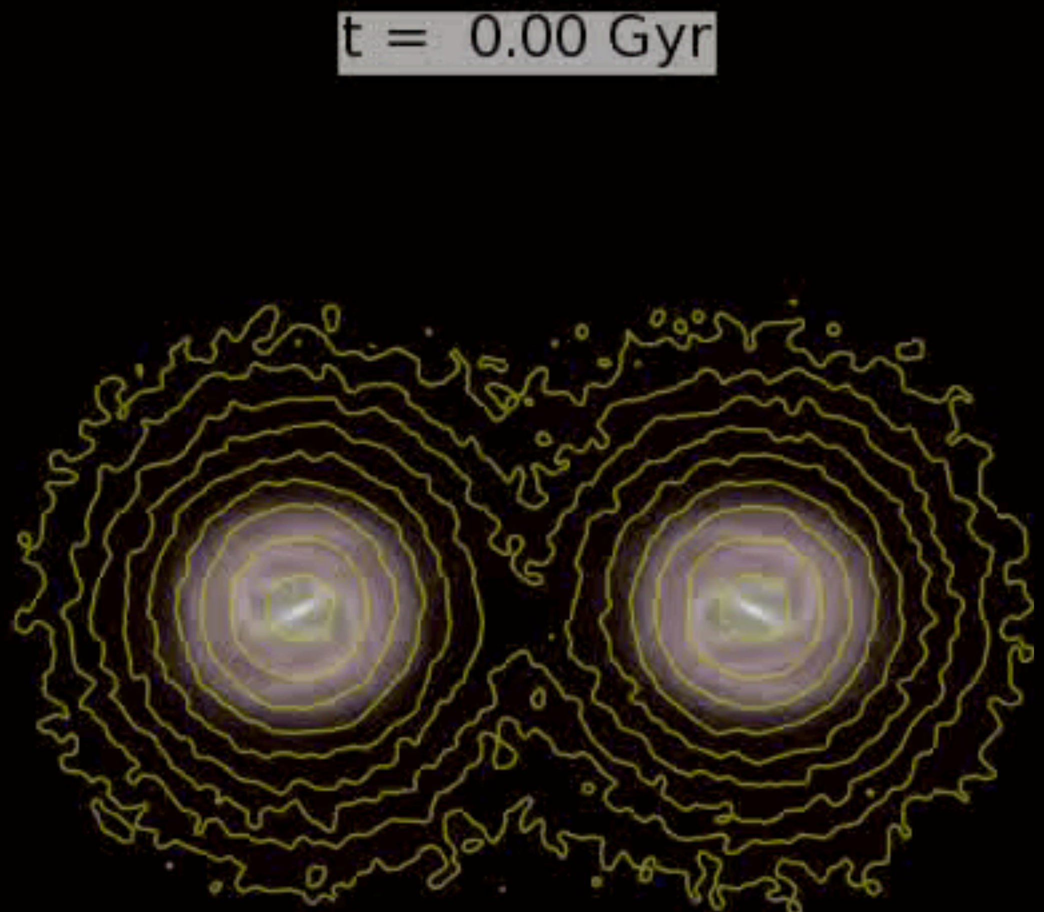
MOND simulations

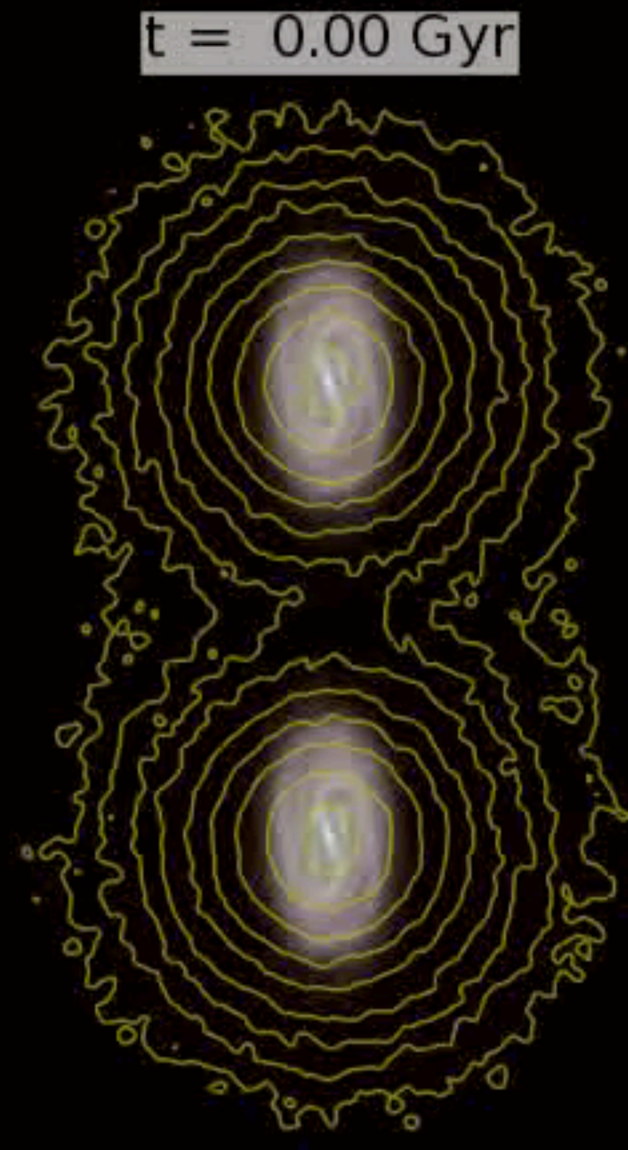
Observations



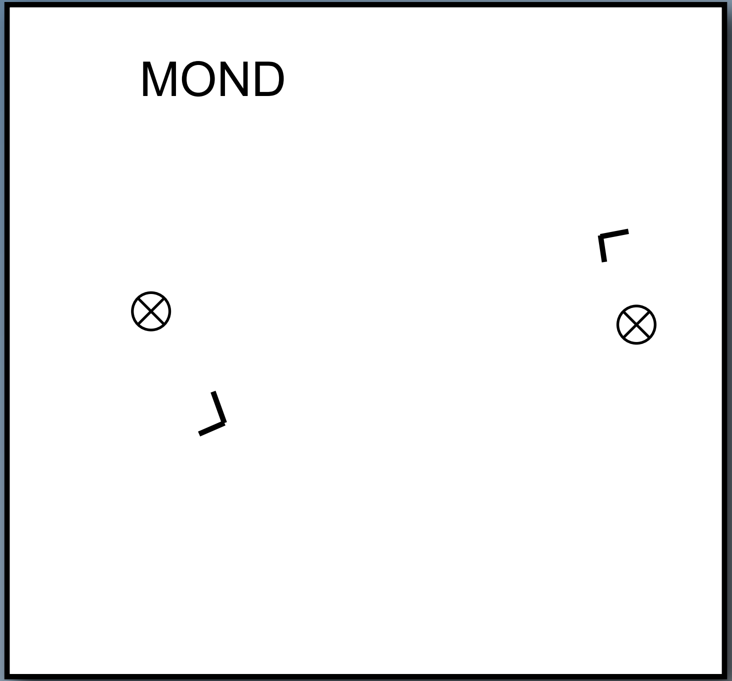
v_{los} σ_{los}

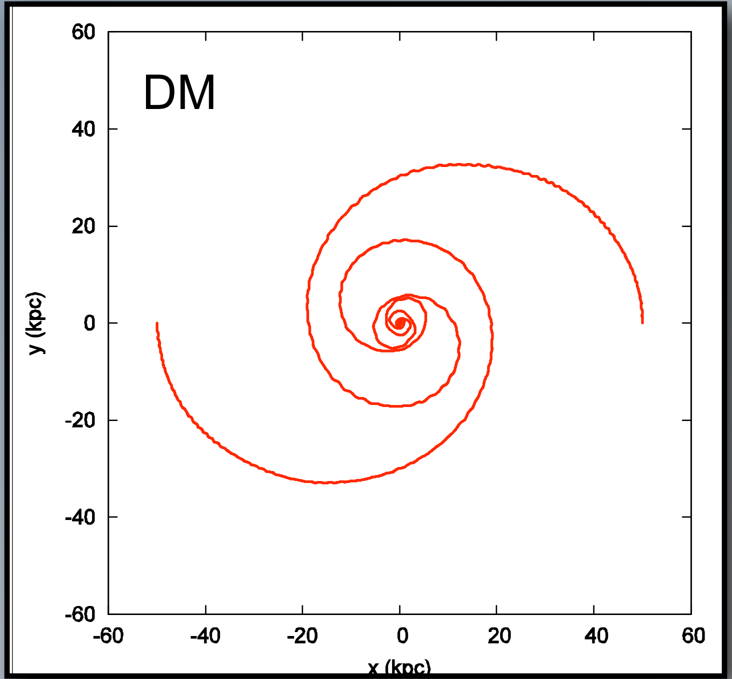

t = 0.00 Gyr
t = 0.00 Gyr


t = 0.00 Gyr

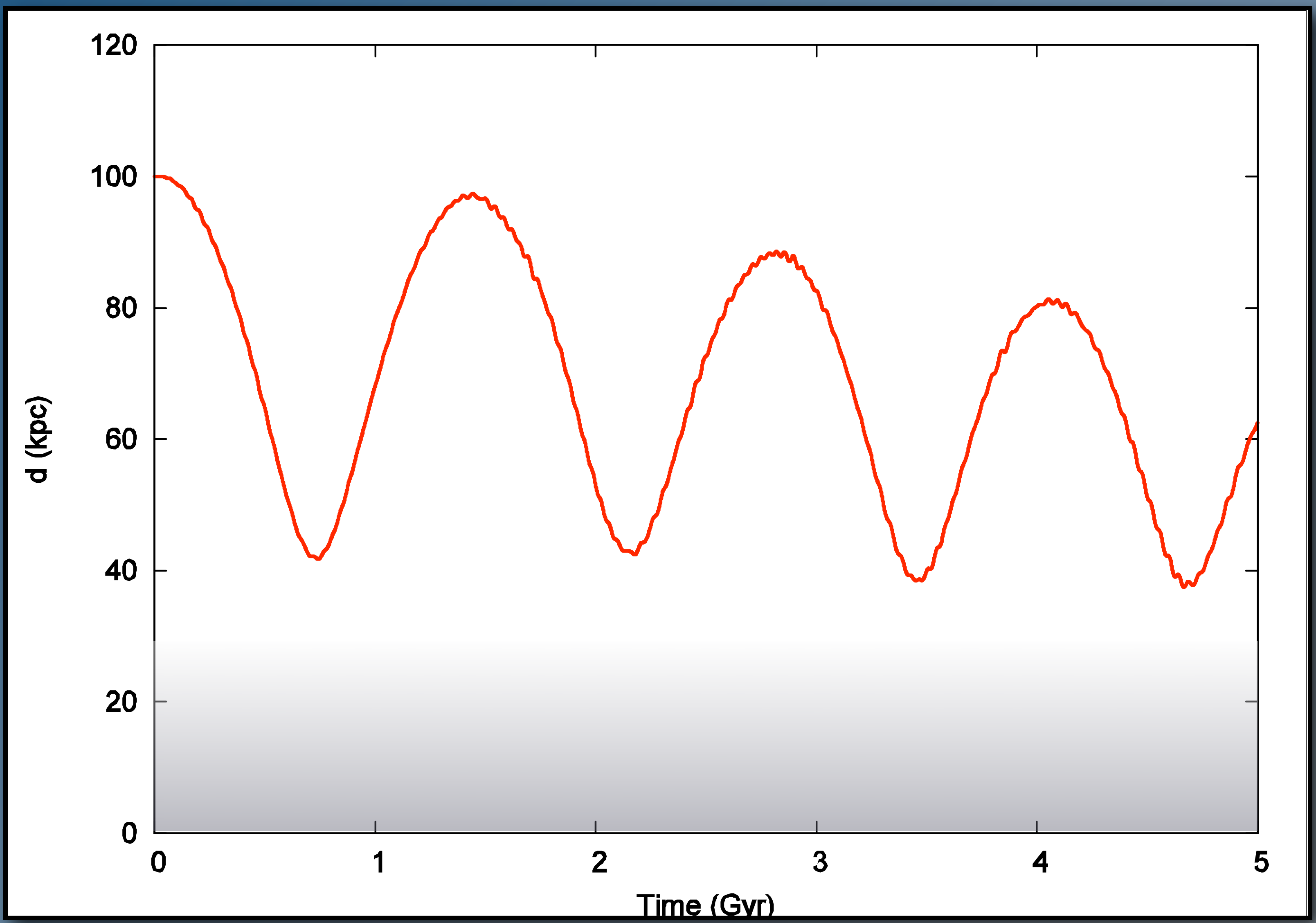

t = 0.00 Gyr

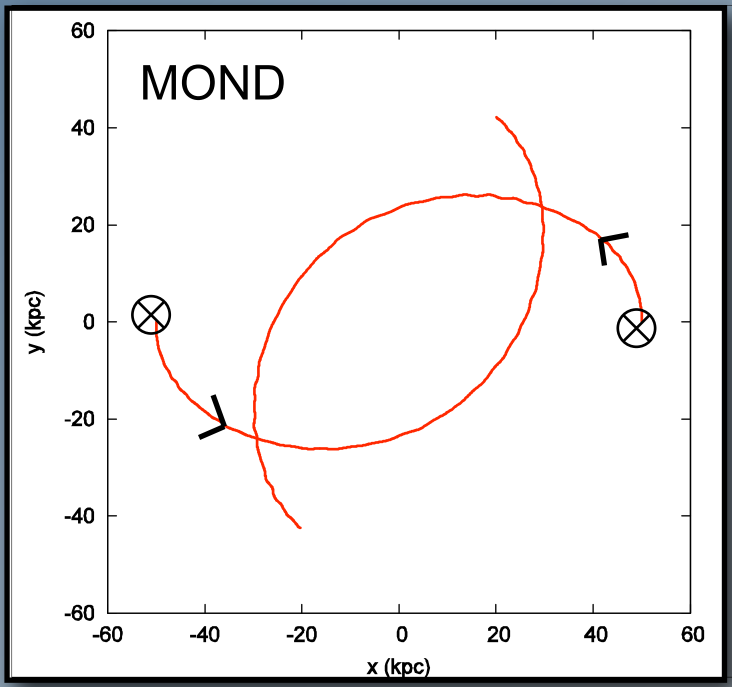
Dynamical Friction

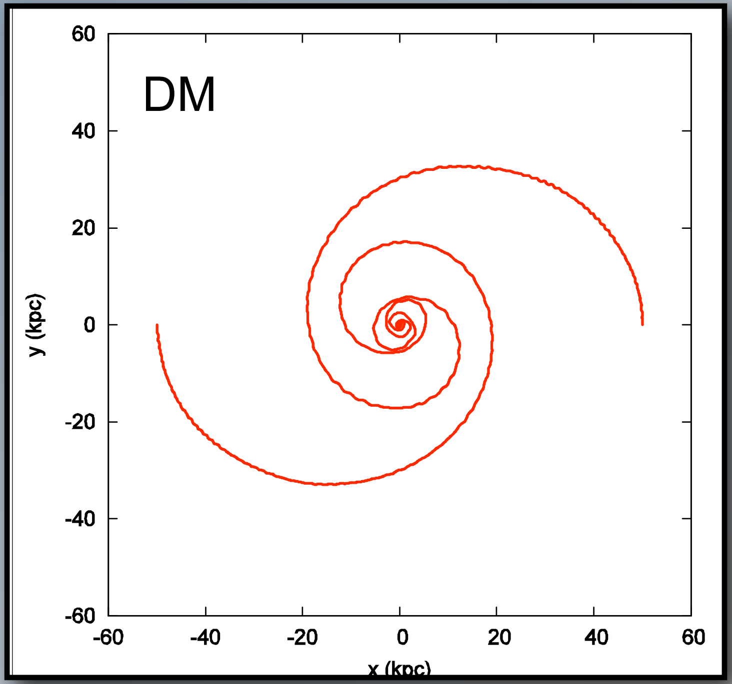




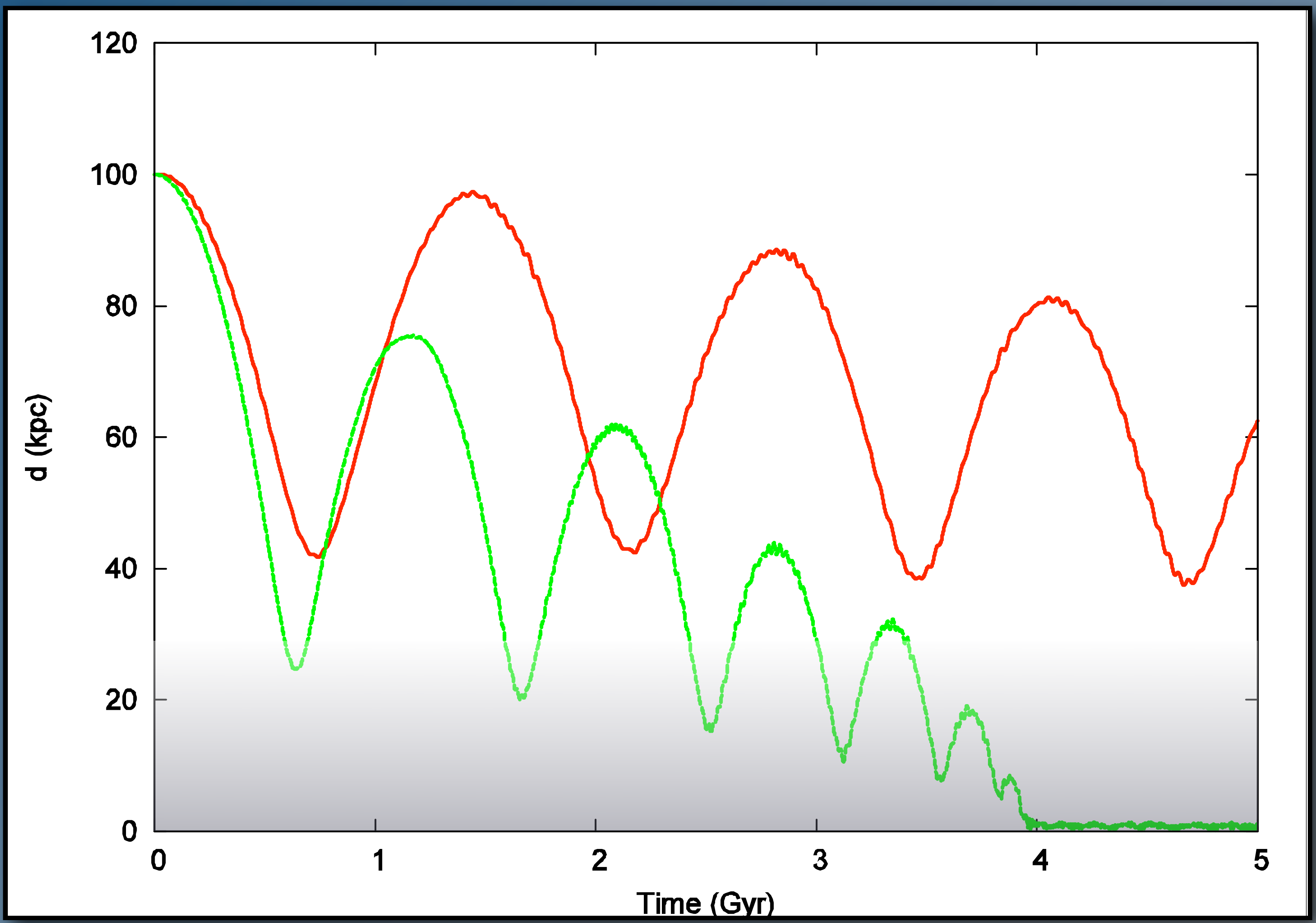
Dynamical Friction

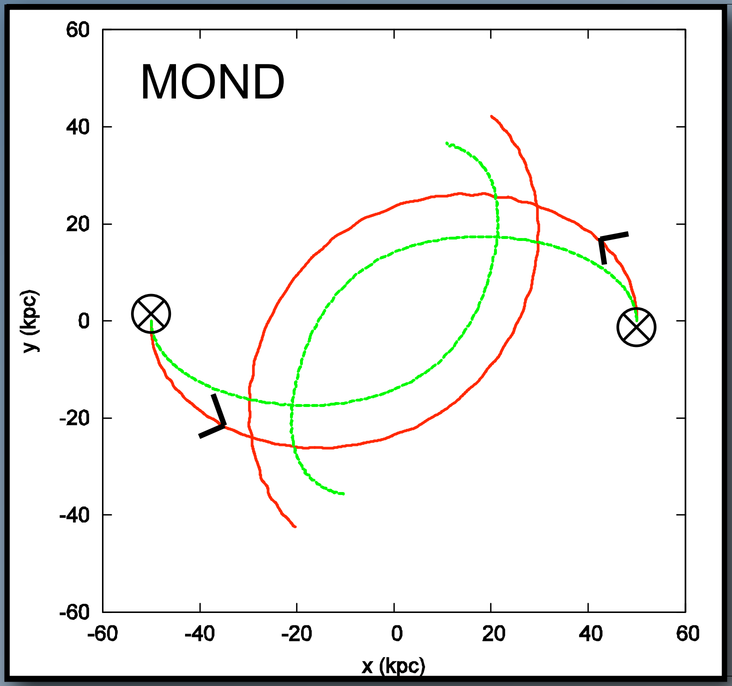


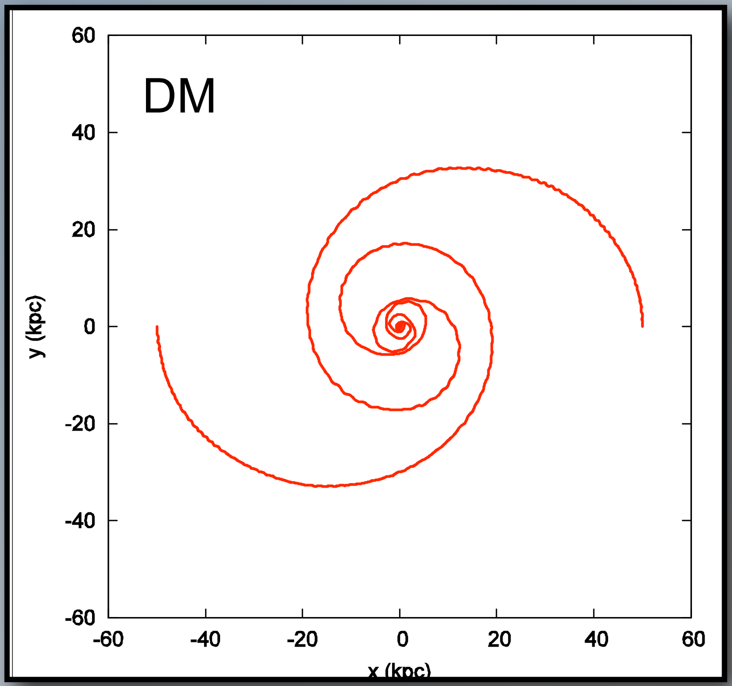




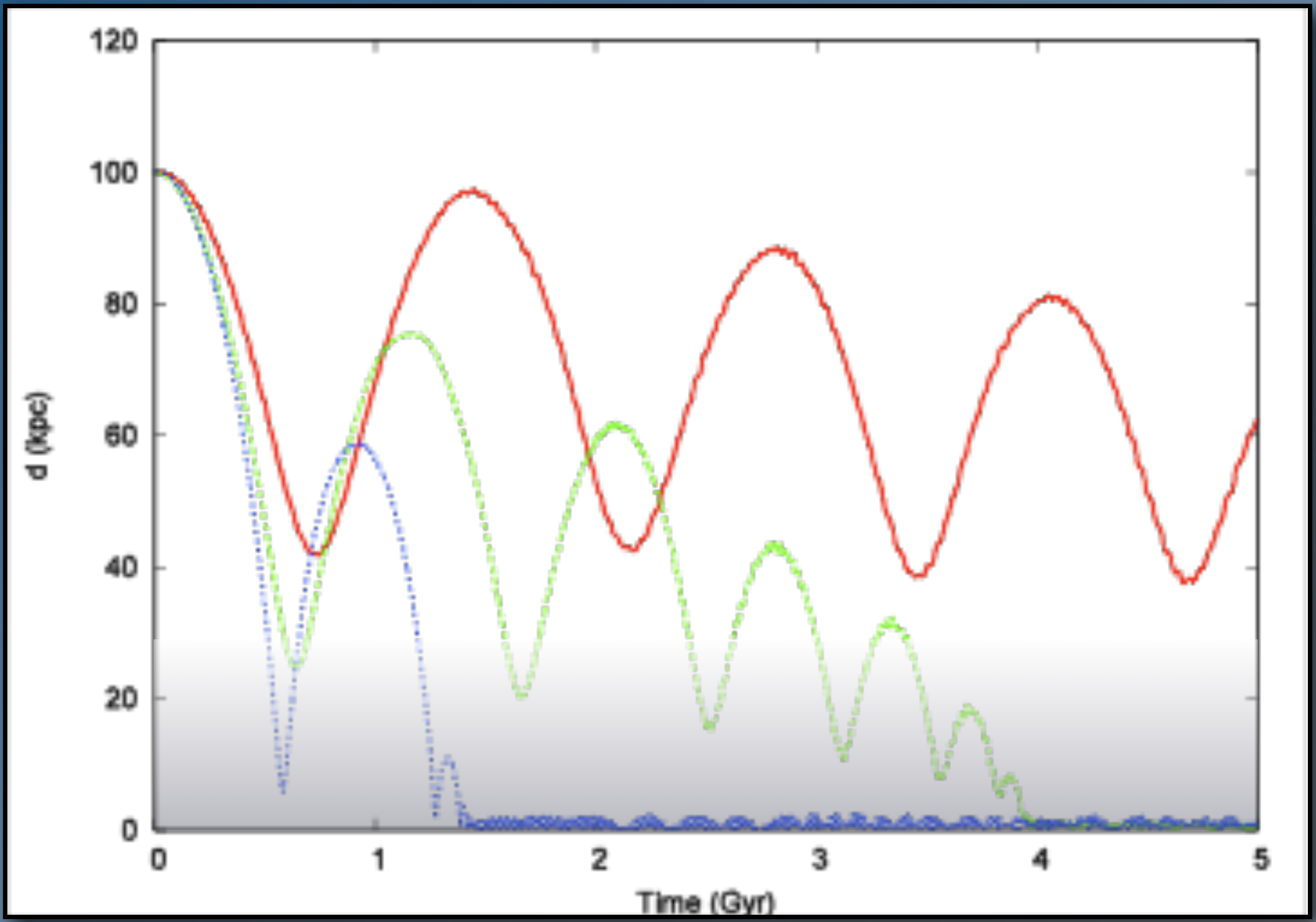
Dynamical Friction

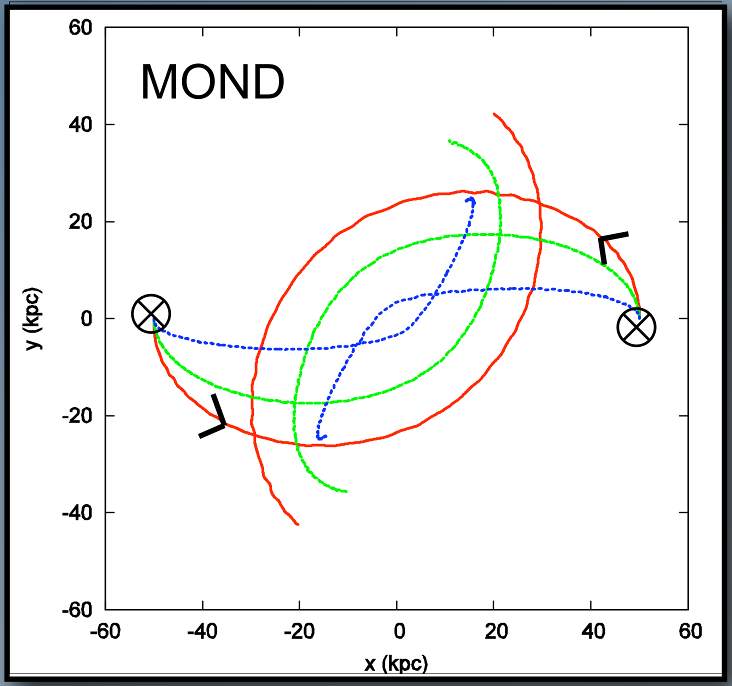


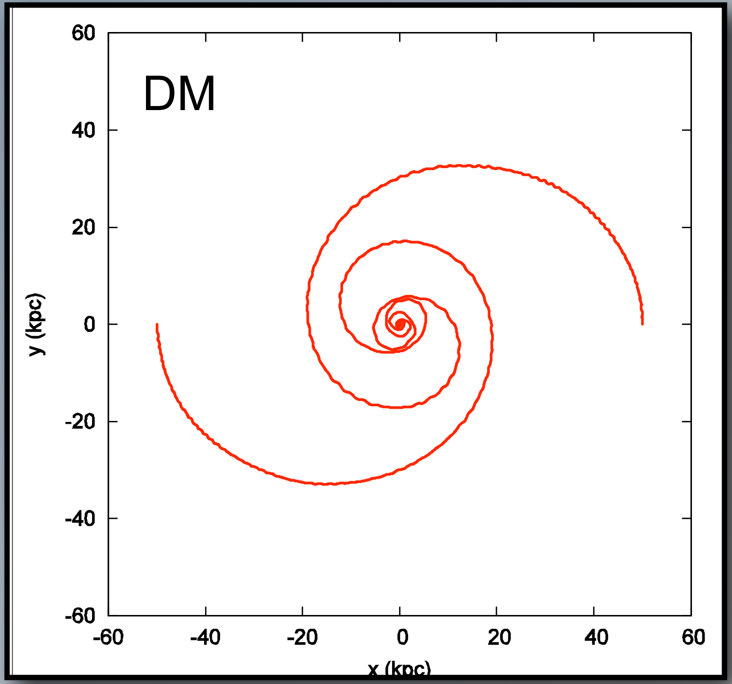




Dynamical Friction







Conclusion

Conclusion

- Numerical code to test MOND
- Newtonian / MONDian gravity: the problem is still degenerated
- It becomes discriminating for interacting galaxies (dynamical frcition)

Conclusion

- Numerical code to test MOND
- Newtonian / MONDian gravity: the problem is still degenerated
- It becomes discriminating for interacting galaxies (dynamical frcition)

Galaxy Evolution

Conclusion

- Numerical code to test MOND
- Newtonian / MONDian gravity: the problem is still degenerated
- It becomes discriminating for interacting galaxies (dynamical frcition)

Galaxy Evolution

Structure Formation

- Cosmology (Llianares et al 2008)
- Boundary conditions / Gas physics / ...
- A "clean" relativistic theory is needed
- Several alternative theories (relativistic) are developed now (Blanchet & Le Tiec 2008, Fuzfa & Alimi 2007, Ferreira et al 2008, ...)